Q1-1

According to the character of fundamental matrix

$$\tilde{x}_2^T F \tilde{x}_1 = 0$$

Where $\tilde{x}_1$, $\tilde{x}_2$ are the homogeneous coordinate of x_1, x_2

x_1, x_2 are origins

$$\tilde{x}_1 = \tilde{x}_2 = [0, 0, 1]^T$$

$$[0, 0, 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$[0, 0, 1] \begin{bmatrix} F_{13} \\ F_{23} \\ F_{33} \end{bmatrix} = 0$$

$$F_{33} = 0$$

Q 1.2

the translation is parallel to x-axis

the translation matrix $t = [t_x, 0, 0]^T$

the cross product matrix:

$$t_{cross} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

the pure rotation matrix:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the essential matrix:

$$E = t_{cross} R = t_{cross} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

the epipolar lines:

$$l_1^T = \tilde{x}_2^T E = [x_2, y_2, 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = [0 \;\; t_x \;\; -t_x y_2]$$

$$l_2^T = \tilde{x}_1^T E^T = [0 \;\; -t_x \;\; t_x y_1]$$

$$\left.\begin{array}{l} \text{line 1: } t_x y - t_x y_2 = 0 \\ \text{line 2: } t_x y - t_x y_1 = 0 \end{array}\right\} \text{both lines parallel to x-axis}$$

Q 1.3

Let the real world point $p = [x \ y \ z]^T$

At two different time, the corresponding points in the frame of reference camera are:

$$p_1 = R_1 p + t_1 \qquad p_2 = R_2 p + t_2$$

$$\Rightarrow p = R_1^{-1}(p_1 - t_1)$$

$$\Rightarrow p_2 = R_2 R_1^{-1}(p_1 - t_1) + t_2$$

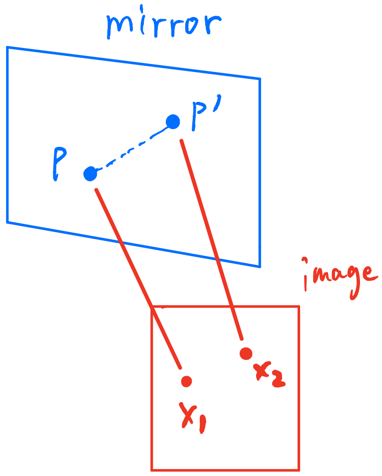
$$= \underbrace{R_2 R_1^{-1}}_{R_{rel} = R_2 R_1^{-1}} p_1 \underbrace{- R_2 R_1^{-1} t_1 + t_2}_{t_{rel} = - R_2 R_1^{-1} t_1 + t_2}$$

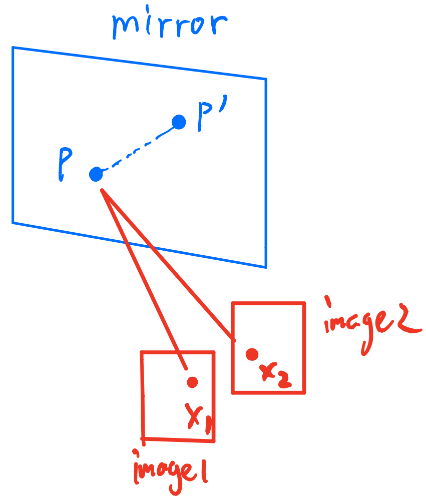
essential matrix: $E = t_{rel} \times R_{rel}$

fundamental matrix: $F = (K^{-1})^T E K^{-1}$

$$= (K^{-1})^T t_{rel} \times R_{rel} K^{-1}$$

Q 1.4





Let the real point is p, the reflection point is p'

Let the corresponding points of p in image 1 and 2 are x_1, x_2

$$\tilde{x}_2^T F \tilde{x}_1 = 0 \xrightarrow{\text{take transpose}} \tilde{x}_1^T F^T \tilde{x}_2 = 0$$

image 2 is a reflection of Image 1

$M_2 = TM_1$, T is the transition matrix of reflection

$$T^T T = I$$

So $\tilde{x}_1 = KM_1\tilde{p}$ $\quad \tilde{x}_2 = KTM_1\tilde{p}$

$$\tilde{x}_2^T F \tilde{x}_1 + \tilde{x}_1^T F^T \tilde{x}_2$$

$$= \tilde{p}^T M_1^T T^T K^T F K M_1 \tilde{p} + \tilde{p}^T M_1^T K^T F^T K T M_1 \tilde{p}$$

$$= \tilde{p}^T M_1^T (T^T K^T F K + K^T F^T K T) M_1 \tilde{p} = 0$$

$$T^T K^T F K + K^T F^T K T = 0$$

$$K^T F K + K^T F^T K = 0$$

$$F + F^T = 0$$

$$F = -F^T$$

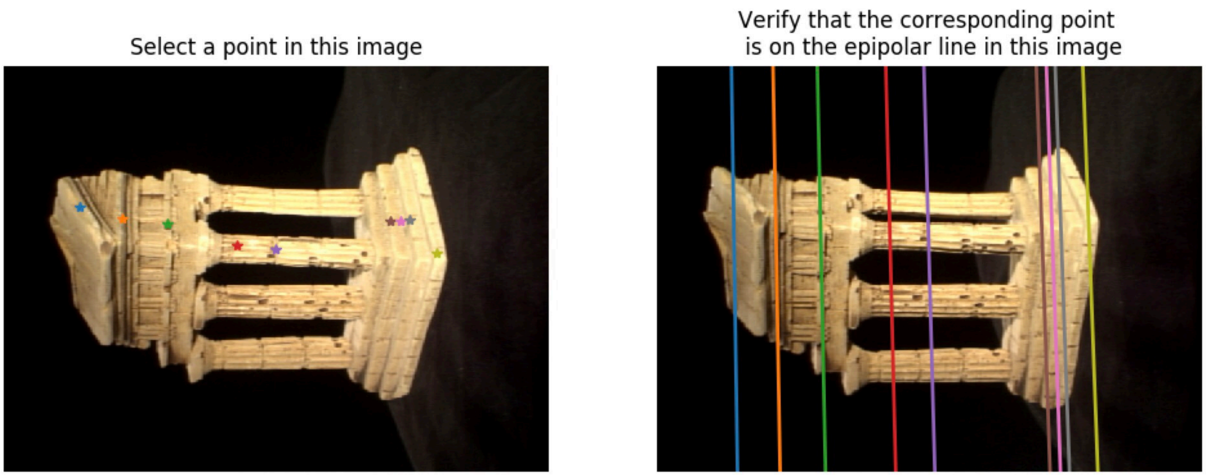
Therefore, F is a skew-symmetric matrix

Q 2.1

Recovered F :

```
[[ 9.78833283e-10 -1.32135929e-07  1.12585666e-03]
 [-5.73843315e-08  2.96800276e-09 -1.17611996e-05]
 [-1.08269003e-03  3.04846703e-05 -4.47032655e-03]]
```

Example Output :

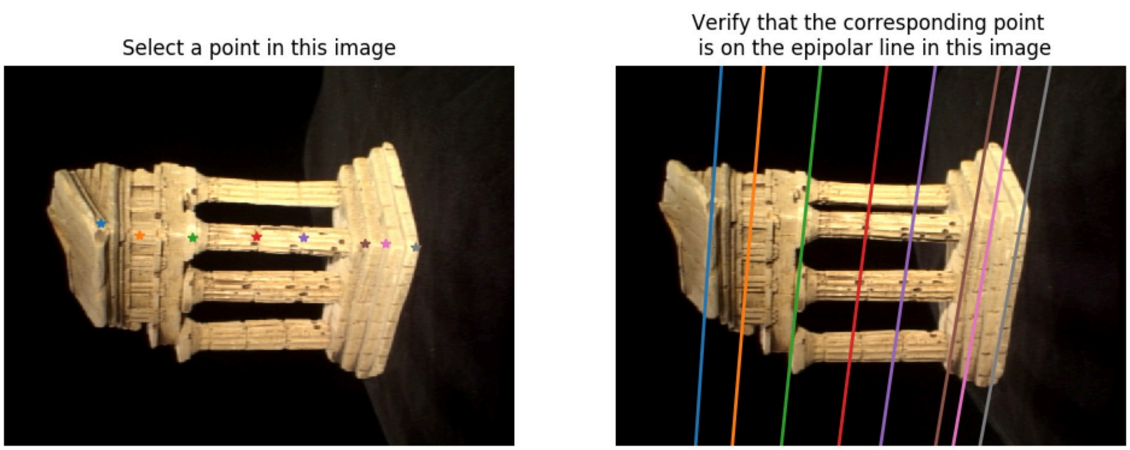


Q 2.2

Recovered F

```
[[-1.86459741e-07  8.99779890e-07 -1.47814380e-03]
 [-4.14517577e-07  3.69048936e-08 -4.64993878e-05]
 [ 1.44724192e-03 -3.43986055e-05  6.50571417e-03]]
```

Example Output



Q3.1

essential matrix E:

```
[[ 2.26268684e-03 -3.06552495e-01  1.66260633e+00]
 [-1.33130407e-01  6.91061098e-03 -4.33003420e-02]
 [-1.66721070e+00 -1.33210351e-02 -6.72186431e-04]]
```

Q 3.2

Let

$$C_1 = \begin{bmatrix} C^1_{11} & \cdots & C^1_{14} \\ \vdots & \ddots & \vdots \\ C^1_{31} & \cdots & C^1_{34} \end{bmatrix}_{3\times 4} \qquad C_2 = \begin{bmatrix} C^2_{11} & \cdots & C^2_{14} \\ \vdots & \ddots & \vdots \\ C^2_{31} & \cdots & C^2_{34} \end{bmatrix}_{3\times 4}$$

$$\tilde{w}_i = [x_i \ y_i \ z_i \ 1]^T \quad \tilde{x}_{i1} = [x_{i1} \ y_{i1} \ 1]^T \quad \tilde{x}_{i2} = [x_{i2} \ y_{i2} \ 1]^T$$

$$C_1 \tilde{w}_i = \lambda_{i1} \tilde{x}_{i1} \quad ①$$

$$C_2 \tilde{w}_i = \lambda_{i2} \tilde{x}_{i2} \quad ②$$

$$① \Rightarrow \lambda_{i1} x_{i1} = C^1_{11} x_i + C^1_{12} y_i + C^1_{13} z_i + C^1_{14}$$

$$\lambda_{i1} y_{i1} = C^1_{21} x_i + C^1_{22} y_i + C^1_{23} z_i + C^1_{24}$$

$$\lambda_{i1} = C^1_{31} x_i + C^1_{32} y_i + C^1_{33} z_i + C^1_{34}$$

$$\Rightarrow (C^1_{11} - C^1_{31} x_{i1}) x_i + (C^1_{12} - C^1_{32} x_{i1}) y_i + (C^1_{13} - C^1_{33} x_{i1}) z_i$$
$$+ (C^1_{14} - C^1_{34} x_{i1}) = 0 \quad ③$$

$$(C^1_{21} - C^1_{32} y_{i1}) x_i + (C^1_{22} - C^1_{32} y_{i1}) y_i + (C^1_{23} - C^1_{33} y_{i1}) z_i$$
$$+ (C^1_{24} - C^1_{34} y_{i1}) = 0 \quad ④$$

Similar to ②

$$(C^2_{11} - C^2_{31} x_{i2}) x_i + (C^2_{12} - C^2_{32} x_{i2}) y_i + (C^2_{13} - C^2_{33} x_{i2}) z_i$$

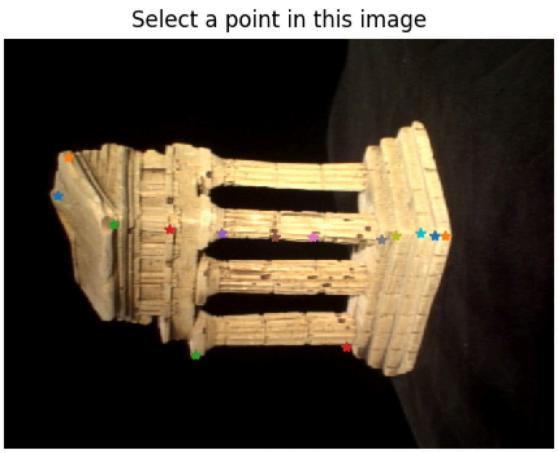
$$+\left(C^2_{14}-C^2_{34}x_{i2}\right)=0 \quad ⑤$$

$$\left(C^2_{21}-C^2_{31}y_{i2}\right)x_i+\left(C^2_{22}-C^2_{32}y_{i2}\right)y_i+\left(C^2_{23}-C^2_{33}y_{i2}\right)z_i$$
$$+\left(C^2_{24}-C^2_{34}y_{i2}\right)=0 \quad ⑥$$

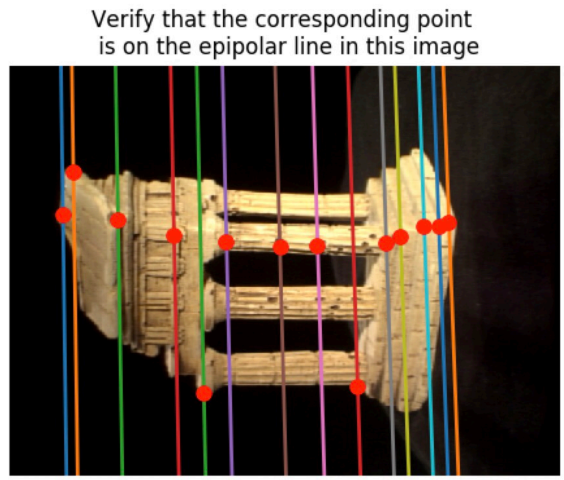
From ③ ④ ⑤ ⑥

$$\underbrace{\begin{bmatrix} C^1_{11}-C^1_{31}x_{i1} & C^1_{12}-C^1_{32}x_{i1} & C^1_{13}-C^1_{33}x_{i1} & C^1_{14}-C^1_{34}x_{i1} \\ C^1_{21}-C^1_{31}y_{i1} & C^1_{22}-C^1_{32}y_{i1} & C^1_{23}-C^1_{33}y_{i1} & C^1_{24}-C^1_{34}y_{i1} \\ C^2_{11}-C^2_{31}x_{i2} & C^2_{12}-C^2_{32}x_{i2} & C^2_{13}-C^2_{33}x_{i2} & C^2_{14}-C^2_{34}x_{i2} \\ C^2_{21}-C^2_{31}y_{i2} & C^2_{22}-C^2_{32}y_{i2} & C^2_{23}-C^2_{33}y_{i2} & C^2_{24}-C^2_{34}y_{i2} \end{bmatrix}}_{A_i} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = 0$$

Q 4.1

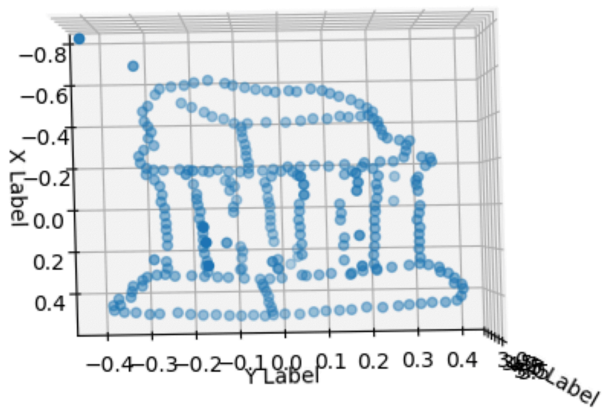


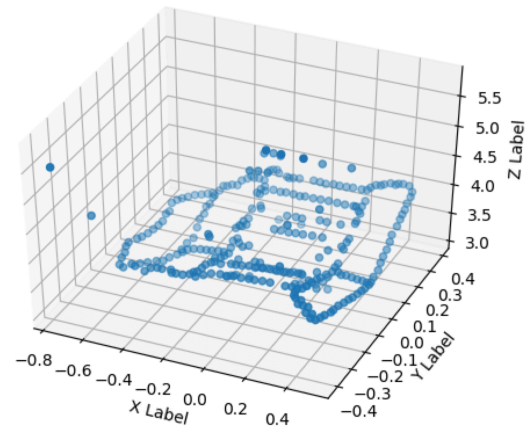
Select a point in this image



Verify that the corresponding point
is on the epipolar line in this image

Q4.2


X Label
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
Y Label
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4
Z Label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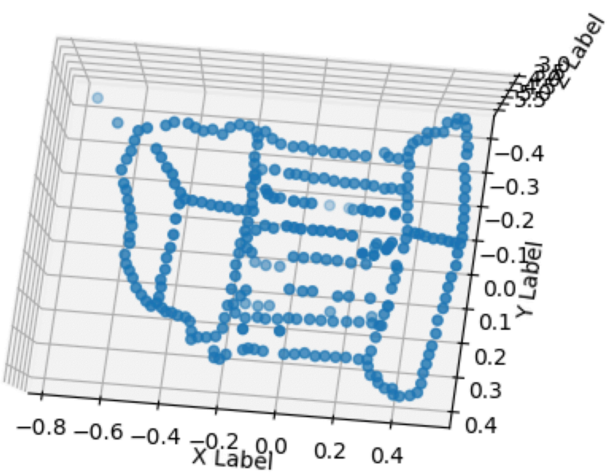
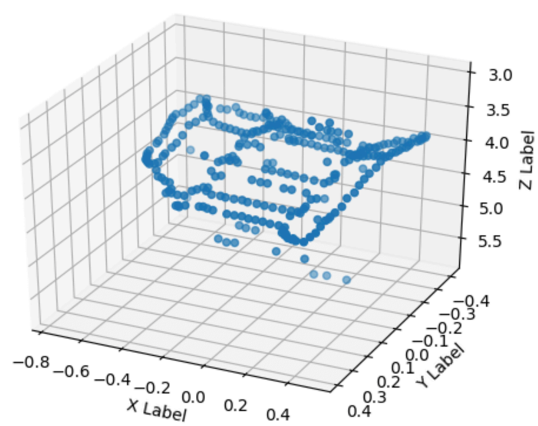
5.0
4.5
4.0
3.5
3.0
X Label
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4
Y Label
0.4 0.3 0.2 0.1 0.0 −0.1 −0.2 −0.3 −0.4


Z Label
Y Label
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
X Label
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4


Z Label
3.0
3.5
4.0
4.5
5.0
5.5
X Label
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4
Y Label
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4