MA589 Proj1

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1

(a)

```
eye <- function(matrix)
{
  diag(ncol(matrix))
}
```

(b)

```
hilbert <- function(n)
{
  outer(1:n,1:n,function(i,j){1/(i+j-1)})
}
```

(c) approach 1

```
library(magrittr)
tr <- function(matrix)
{
  eigen(matrix)$values%>%sum()
}
```

(c) approach 2

```
tr <- function(matrix)sum(diag(matrix))
```

(d)

```
norm2 <- function(vector)
{
  library(magrittr)
  a <- abs(vector)%>%max()
  a*sqrt(crossprod(vector/a,vector/a))
}
norm2(1e200*rep(1,100))
```

```
##         [,1]
## [1,] 1e+201
```

(e)

```
invtri <- function(umatrix)
{
  eye <- function(matrix)
  {
    diag(ncol(matrix))
  }
  tcrossprod(backsolve(umatrix,eye(umatrix)))
}
```

2

(a)

```
epsilon <- function()
{
  eps <- 1
  while((1+eps<=1)|(1+eps/2!=1))
  {
    eps <- eps/2
  }
  eps
}
```

(b)

```
lg <- function(x)
{
  log(1+exp(x))
}
lg(0)
```

```
## [1] 0.6931472
```

```
lg(-80)
```

```
## [1] 0
```

```
lg(80)
```

```
## [1] 80
```

```
lg(800)
```

```
## [1] Inf
```

(c)

```
lgmini <- function(x)
{
  if(exp(x)>epsilon()/2)
  {log(1+exp(x))}
  else
  {log(1)}
}
```

(d)

```
lgmax <- function(x)
 {
   for(i in 1:50)
   {
     if(exp(i)==exp(i)+1)
     {
       a <- i
       break
     }
   }
   if(x<a)
   {
     return(log(1+exp(x)))
   }
   else
   {
     return(x)
   }
}
lgmax(800)
```

```
## [1] 800
```

3

(a)

```
lstirling <- function(n,k)
{
  j <- c(0:k)
  S <-  sum((-1)^(k-j)*choose(k,j)*j^n)/factorial(k)
  log(S)
}
lstirling(10,1)-log(1)
```

```
## [1] 0
```

```
lstirling(10,2)-log(511)
```

```
## [1] 0
```

```
lstirling(10,3)-log(9330)
```

```
## [1] 0
```

```
lstirling(10,4)-log(34105)
```

```
## [1] 0
```

```
lstirling(10,5)-log(42525)
```

```
## [1] 0
```

```
lstirling(10,6)-log(22827)
```

```
## [1] 0
```

```
lstirling(10,7)-log(5880)
```

```
## [1] 0
```

```
lstirling(10,8)-log(750)
```

```
## [1] 0
```

```
lstirling(10,9)-log(45)
```

```
## [1] 0
```

```
lstirling(10,10)-log(1)
```

```
## [1] 0
```

(b)

```
lstirling <- function(n,k)
{
  j <- c(0:k)
  S <-  sum((-1)^(k-j)*choose(k,j)*j^n)/factorial(k)
  log(S)
}
```

```
#lstirling(100,99)
k <- 99
n <- 100
j <- c(0:99)
S <-  sum((-1)^(k-j)*choose(k,j)*j^n)/factorial(k)
#lstirling(100,99) returns NaNs
#S(100,99) is -6.243607e+38<0, so log(S) is not available

#lstirling(100,100)
k <- 100
n <- 100
j <- c(0:100)
S <-  sum((-1)^(k-j)*choose(k,j)*j^n)/factorial(k)
#lstirling(100,100) returns NaNs
#S(100,100) is -7.63941e+37<0, so log(S) is not available
```

(c)

```
lstirling <- function(n,k)
{
  S <- function(x,y)
  {
    if(x==y)
    {
      return(1)
    }
    else if((y==0)|(x==0))
    {
      return(0)
    }
    else
      return(y*S(x-1,y)+S(x-1,y-1))
  }
  log(S(n,k))
}
lstirling(100,99)
```

```
## [1] 8.507143
```

```
lstirling(100,100)
```

```
## [1] 0
```

4

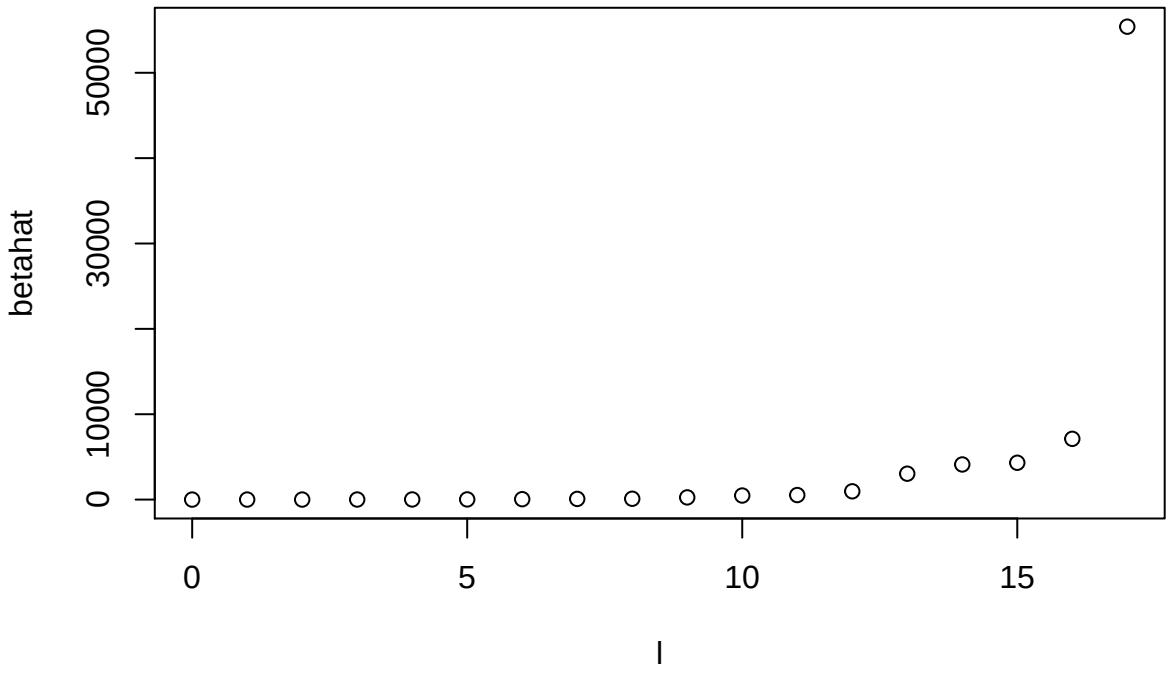
```
hilbert <- function(n)
{
  outer(1:n,1:n,function(i,j){1/(i+j-1)})
```

```
}
norm2 <- function(vector)
{
  library(magrittr)
  a <- abs(vector)%>%max()
  a*sqrt(crossprod(vector/a,vector/a))
}
X <- hilbert(7)
y <- rep(1,7)

#(a)
q <- qr(X)
norm2(backsolve(qr.R(q),qr.qty(q,y)))
```

```
##          [,1]
## [1,] 52421.81
```

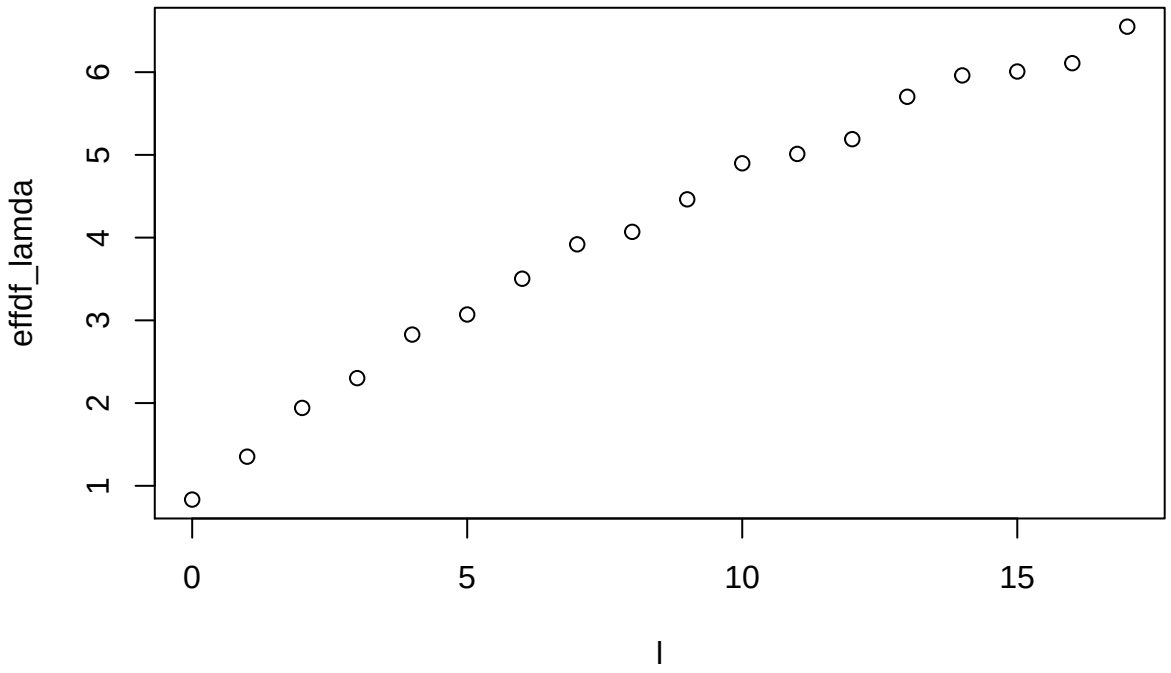
```
#(b)
eye <- function(matrix)
{
  diag(ncol(matrix))
}
ridgenorms <- function(lamda,X,y)
{
  YY <- c(y,rep(0,nrow(X)))
  XX <- rbind(X,sqrt(lamda)*eye(X))
  q <- qr(XX)
  norm2(backsolve(qr.R(q),qr.qty(q,YY)))
}
l <- 0:17
lamda <- 10^(-l)
betahat <- sapply(lamda,ridgenorms,X,y)
plot(l,betahat)
```




```
#(c)
invtri <- function(umatrix)
{
  eye <- function(matrix)
  {
    diag(ncol(matrix))
  }
  tcrossprod(backsolve(umatrix,eye(umatrix)))
}
library(magrittr)
tr <- function(matrix)
{
  eigen(matrix)$values%>%sum()
}

effdf <- function(lamda,X)
{
  R <- qr.R(qr(X))
  tr(invtri(chol(eye(R)+lamda*invtri(R))))
}

#(d)
l <- 0:17
lamda <- 10^(-l)
effdf_lamda <- sapply(lamda,effdf,X)
plot(l,effdf_lamda)
```


effdf_lamda
1
2
3
4
5
6
0
5
10
15
l