Conditional Probability: Intermediate: Takeaways

Concepts

- Given events A and B:
  - P(A) means finding the probability of A
  - P(A|B) means finding the conditional probability of A (given that B occurs)
  - P(A ∩ B) means finding the probability that both A and B occur
  - P(A ∪ B) means finding the probability that A occurs or B occurs (this doesn't exclude the situation where both A and B occur)
- For any events A and B, it's true that:

$$P(A|B) = 1 - P(A^C|B)$$
$$P(A^C|B) = 1 - P(A|B)$$

- The order of conditioning is important, so P(A|B) is different P(B|A).
- If event A occurs and the probability of B remains unchanged (and vice versa), then events A and B are said to be **statistically independent** (although the term "independent" is used more often). Mathematically, statistical independence between A and B implies that:

$$P(A) = P(A|B)$$
$$P(B) = P(B|A)$$
$$P(A \cap B) = P(A) \cdot P(B)$$

- If events events A and B are **statistically dependent**, it means that the occurrence of event A changes the probability of event B and vice versa. In mathematical terms, this means that:

$$P(A) \neq P(A|B)$$
$$P(B) \neq P(B|A)$$
$$P(A \cap B) \neq P(A) \cdot P(B)$$

- If three events A, B, C are **mutually independent**, then two conditions must hold: they should be pairwise independent, but also independent together. If any of these two conditions doesn't hold, then the events are not mutually independent.
- The multiplication rule for dependent events:

$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C \mid A \cap B)$$

- The mutliplication rule for independent events:

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$P(A \cap B \cap \ldots \cap Y \cap Z) = P(A) \cdot P(B) \cdot \ldots \cdot P(Y) \cdot P(Z)$$

Resources

- An intuitive approach to understanding independent events
- An easy intro to some basic conditional probability concepts
- A brief reminder on set complements