

**Mathematics: analysis and approaches**  
**Higher level**  
**Paper 1**

**Topic: Derivatives**

Candidate name

75 minutes

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**Instructions to candidates**

- Write your name in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions.
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[34 marks]**.

Answer **all** questions. Answers must be written within the answer boxes provided.  
Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Using the first principle, show that the derivative of  $(x + 1)^3$  is  $3(x + 1)^2$ .

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2. [Maximum mark: 6]

Find the limit:  $\lim_{x \rightarrow 0} \frac{\sec^4 x - \cos^2 x}{x^4 - x^2}$ .

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3. [Maximum mark: 8]

Consider the curve with the equation  $x^3 + 4y^3 = xy$ .

(a) Find an expression for  $\frac{dy}{dx}$ . [3]

(b) Find the coordinates of all points where the tangent to the curve is vertical. [5]

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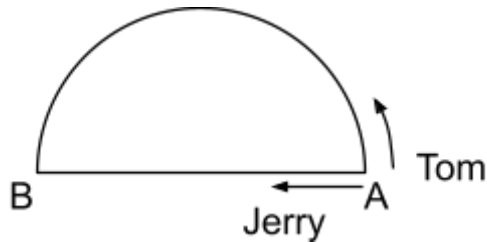
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4. [Maximum mark: 8]

Tom and Jerry travels from point  $A$  to point  $B$  along two different paths: Tom travels along the semi-circle with radius of 1 km at the speed of 1 km/h, and Jerry travels along the diameter  $AB$  at the speed of  $\frac{1}{\pi}$  km/h.



Find the rate of change of the distance between Tom and Jerry at the time when Jerry travels half way through.

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5. [Maximum mark: 7]

Consider the function  $f(x) = \sqrt{x^2 \ln(x) + 4 - x^2}$ , where  $x \in \mathbb{R}, x > 0$ .

(a) Show that the distance,  $l$ , between the origin and any point on the graph is given by  $l = \sqrt{x^2 \ln(x) + 4}$ . [1]

(b) Hence, find the  $x$ -coordinate of the point on the graph of  $f$  which is the closest to the origin. [6]

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Answer **all** questions in the answer booklet provided. Please start each question on a new page.

6. [Maximum mark: 19]

Consider the curve  $C$  defined by the equation  $e^{x+y} - 1 = x^2 + y^2$ .

(a) Show that  $\frac{dy}{dx} = \frac{2x - e^{x+y}}{e^{x+y} - 2y}$ . [5]

(b) Show that there is no point on the graph where the tangent is horizontal. [7]

(c) Show that the graph of  $C$  is symmetric about the line  $y = x$ . [3]

(d) Find the coordinates of the point on the curve  $C$  where the tangent has a gradient of -1. [4]