

**Mathematics: analysis and approaches**  
**Higher level**  
**Paper 1**

Topic: Integral

Student name

75 minutes

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**Instructions to candidates**

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[55 marks]**.

Q:	1	2	3	4	5	6	7
Marks:	/7	/5	/8	/8	/7	/10	/10

Total
/55

Please **do not** write on this page.

Answers written on this page  
will not be marked.

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

By using the substitution  $u = \sin x$ , find  $\int \frac{\sin x \cos x}{\cos^2 x + \sin x + 1} dx$ . [7]

2. [Maximum mark: 5]

Find the real number  $a$  such that  $\int_a^{2a} \frac{e^x}{x} dx = \int_a^{2a} \frac{e^x}{x^2} dx$ .

[5]

- 3.** [Maximum mark: 8]

Using the substitution  $x = a \sec \theta$ , show that  $\int_{a\sqrt{2}}^{2a} \frac{dx}{x^3 \sqrt{x^2 - a^2}} = \frac{1}{24a^3}(3\sqrt{3} + \pi - 6)$ . [8]

- 4.** [Maximum mark: 8]

Using the substitution  $x = a \sec \theta$ , show that  $\int_{a\sqrt{2}}^{2a} \frac{dx}{x^3 \sqrt{x^2 - a^2}} = \frac{1}{24a^3}(3\sqrt{3} + \pi - 6)$ .

[8]

**5.** [Maximum mark: 7]

Consider the function  $f(x) = \frac{x}{x^2+2}$  for  $x \geq 0$ .

- (a) Find all axes intercepts of  $f$ . [1]

(b) Given that the region bounded by  $f(x)$ , the  $x$ -axis, and the line  $x = c$  is  $\ln 3$ , find the value of  $c$ . [6]

**6.** [Maximum mark: 10]

(a) Find  $\int (\ln x)^2$ . [5]

(b) Hence, evaluate the improper integral  $\int_0^1 (\ln x)^2$ . [5]

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 10]

Consider the family of functions  $f_n(x) = \cos^n x$ , where  $x \in \mathbb{R}, n \in \mathbb{N}$ .

- (a) Show that:

$$\int \cos^n x \, dx = \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx \text{ for } n > 1.$$

[4]

- (b) Hence, show that  $\int f_n(x) \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int f_{n-2}(x) \, dx$  for  $n > 1$ .

[2]

- (c) Hence, find an expression for  $\int \cos^4 x \, dx$ , giving your answer in the form  
 $p \cos^3 x \sin x + q \cos x \sin x + rx + c$  where  $p, q, r \in \mathbb{Q}^+$ .

[4]

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