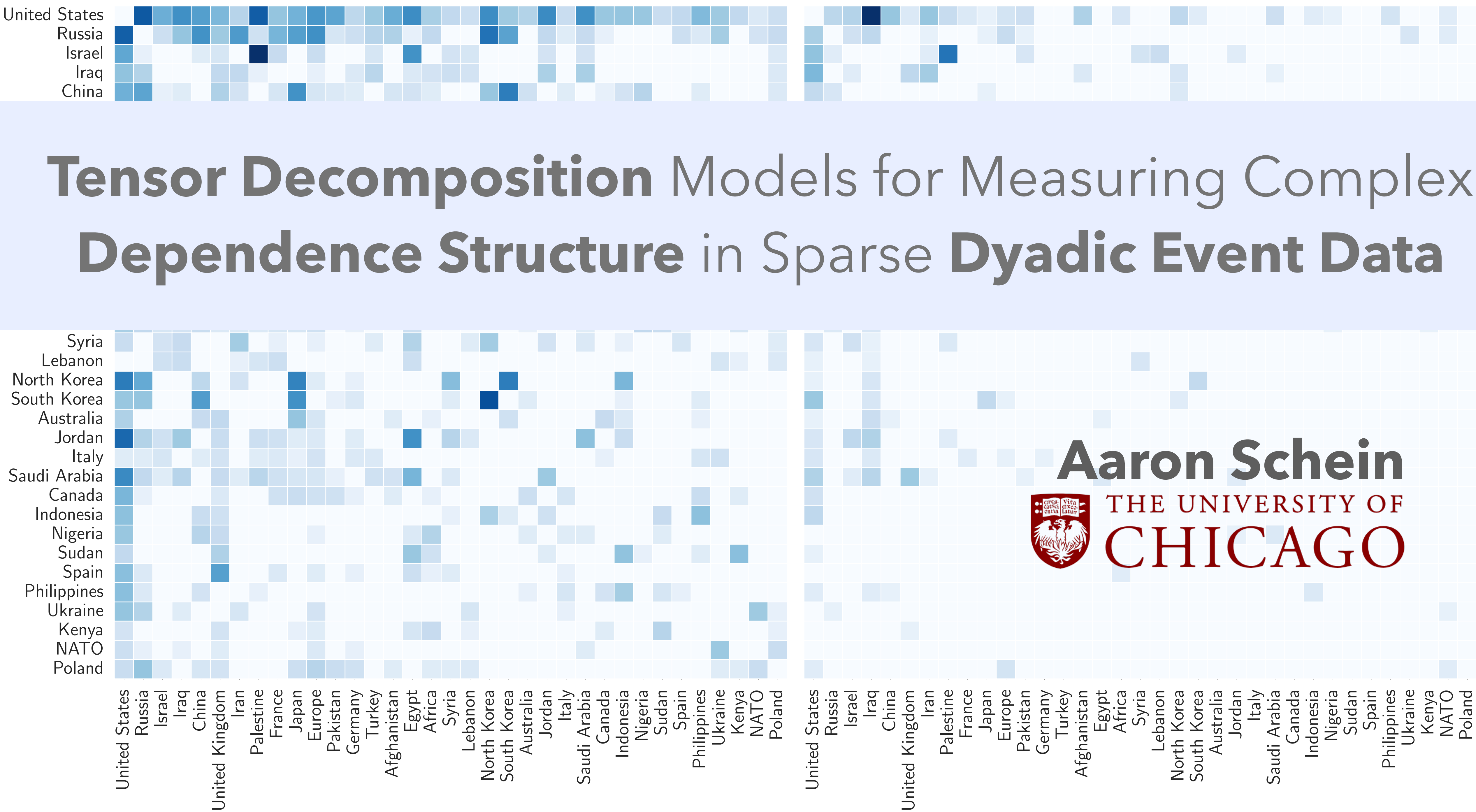
Tensor Decomposition Models for Measuring Complex **Dependence Structure** in Sparse **Dyadic Event Data**

Aaron Schein

THE UNIVERSITY OF CHICAGO



Dyadic event data

Example from Schrodt et al. [1995]:

Table 2
WEIS Coding of 1990 Iraq-Kuwait Crisis

Date	Source	Target	Type of Action
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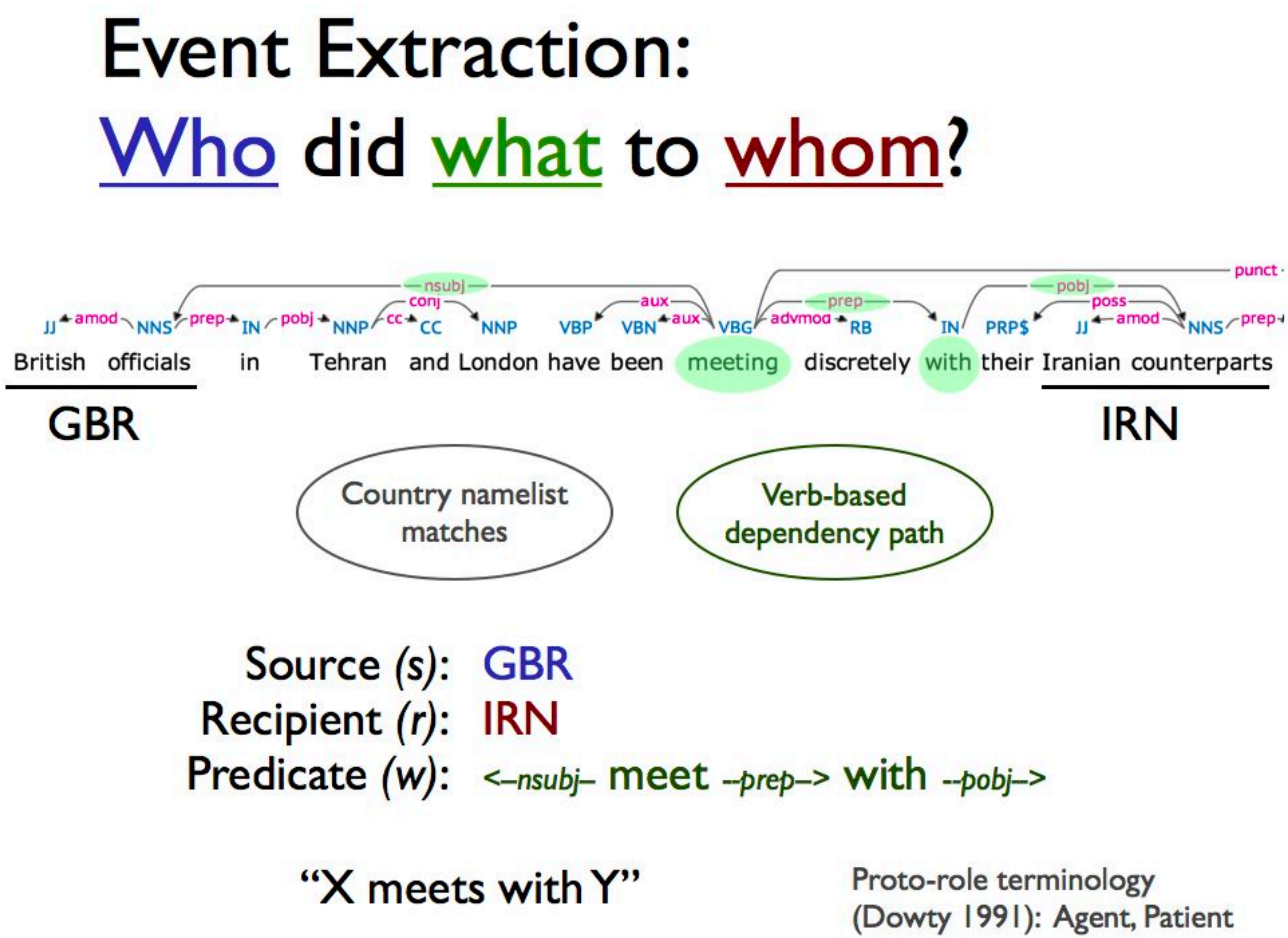
July 31, 1990: IRAQ INCREASES TROOP LEVELS ON KUWAIT BORDER
Iraq has concentrated nearly 100,000 troops close to the Kuwaiti border, more than triple the number reported a week ago, the Washington Post said in its Tuesday editions.

MACHINE-CODED EVENT DATA

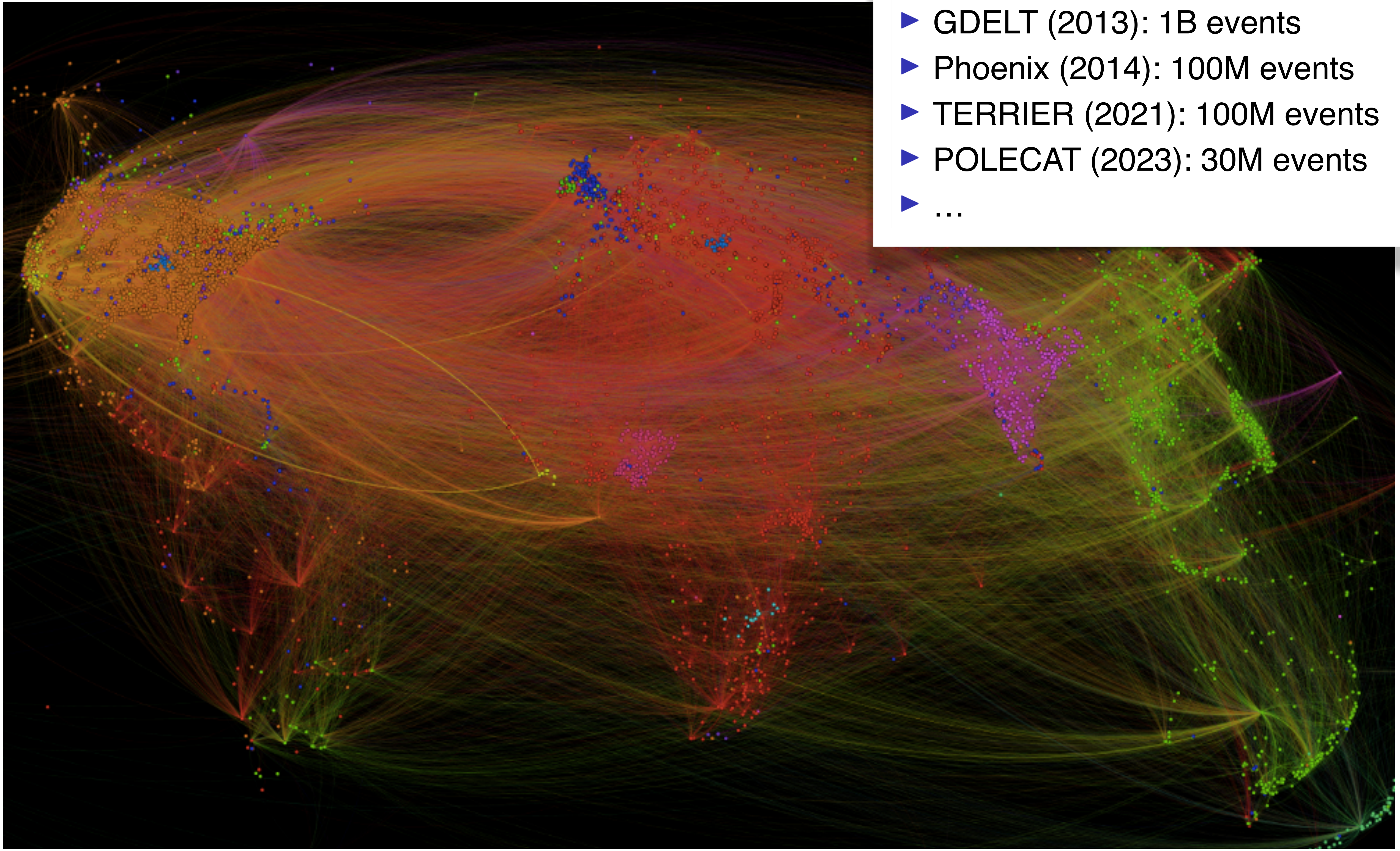
Figure from O’Connor [2013]:



MODERN DYADIC EVENT DATA SETS

Lots of progress on event **extraction**…

- ICEWS (2011): 10M events
- GDELT (2013): 1B events
- Phoenix (2014): 100M events
- TERRIER (2021): 100M events
- POLECAT (2023): 30M events
- …



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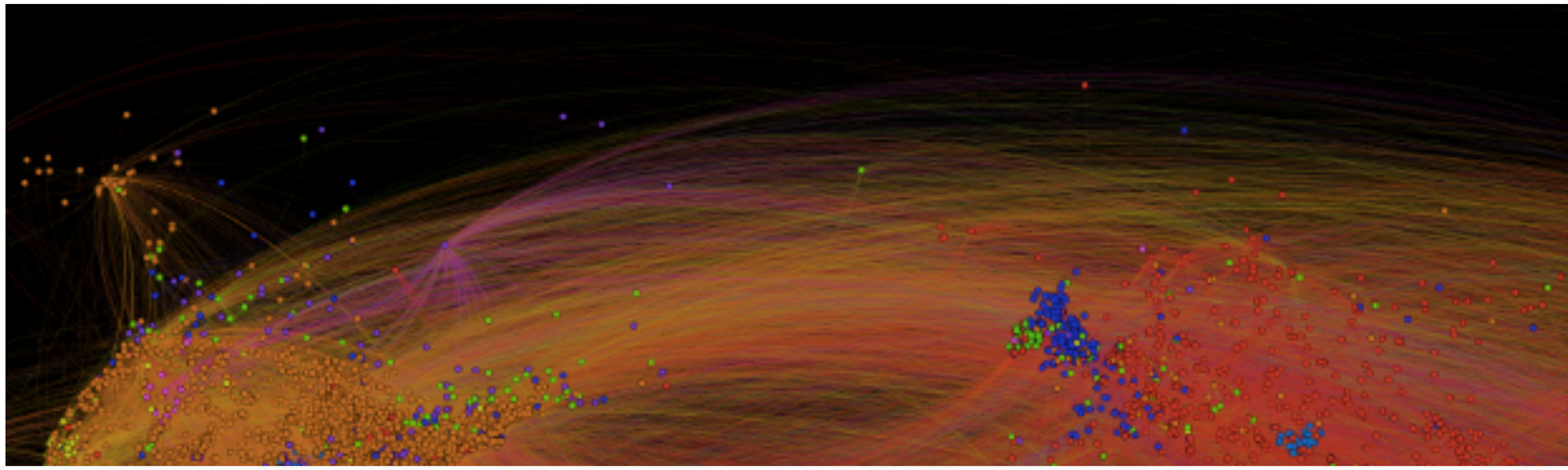

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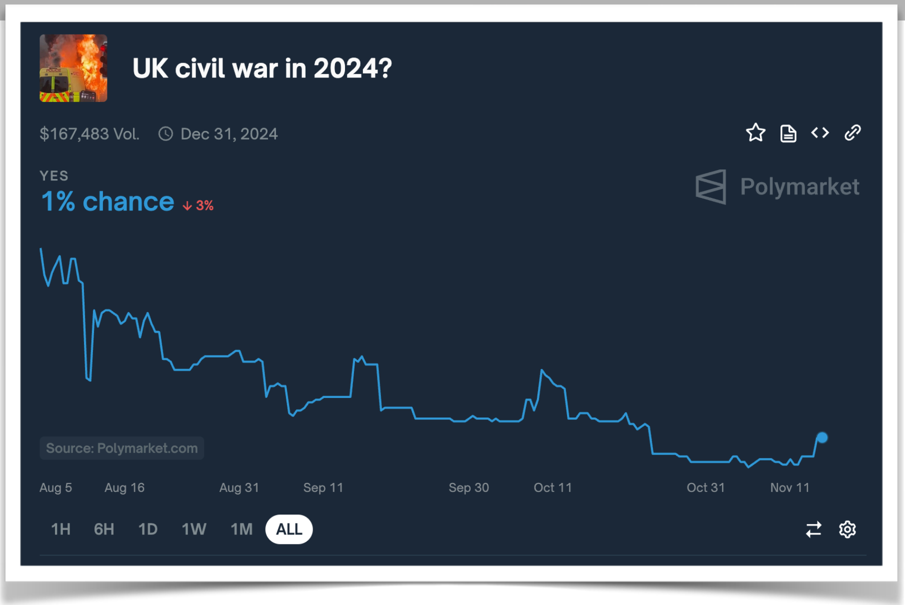
Event_Date	Source_Name	Source_Sectors	Source_Country	Event_Text	CAME(	Target_Name	Target_Sectors	Target_Country
2022-06-14	Foreign Affairs (Japan)	Executive,Forei...	Japan	Consult	40	Abul Kalam Abdul ...	Executive,Forei...	Bangladesh
2002-04-10	Business (Iraq)	Social,Business	Iraq	Sign formal agre...	57	Industry (Bahrain)	Social,Business	Bahrain
2005-01-13	Israeli Defense Forces	Military,Govern...	Israel	fight with small ...	193	Citizen (Palestinia...	General Popul...	Occupied Pal...
2001-03-01	Royal Administration (...	Government	Nepal	Engage in negot...	46	Jiang Zemin	Executive Offic...	China
2005-06-27	Foreign Affairs (Irelan...	Executive,Forei...	Ireland	Consult	40	Secretary of State ...	Government,E...	United Kingd...
2006-03-13	Civil Service (Kazakhs...	Government	Kazakhstan	Consult	40	Foreign Affairs (C...	Foreign Minist...	China
1999-08-30	Ilya Iosifovich Klebanov	Executive Offic...	Russian Fed...	Host a visit	43	Jiang Zemin	Executive,Gov...	China
2022-04-20	Nikol Pashinyan	Elite,Executive,...	Armenia	Express intent t...	36	Federal Assembly	Government,L...	Switzerland
2012-09-03	Legislature (Syria)	Government,Le...	Syria	Consult	40	Foreign Affairs (R...	Executive,Gov...	Russian Fed...
2005-07-27	Head of Government (...	Executive,Gove...	Guyana	Engage in negot...	46	Tang Jiaxuan	Elite,Governme...	China
2009-10-02	Defense / Security Mi...	Government,De...	Belarus	Consult	40	Defense / Security...	Defense / Sec...	Russian Fed...
2004-12-17	Jean-Claude Juncker	Elite,Executive ...	Luxembourg	Criticize or deno...	111	Citizen (Belgium)	Social,General ...	Belgium
2004-04-23	Vladimir Putin	Executive,Exec...	Russian Fed...	Consult	40	Leonid Kuchma	Elite,Executive,...	Ukraine
2018-07-26	Qasem Soleimani	Government,Mi...	Iran	Threaten	130	Donald Trump	Government,E...	United States

Results per page: 50

(POTENTIAL) APPLICATIONS OF EVENT DATA



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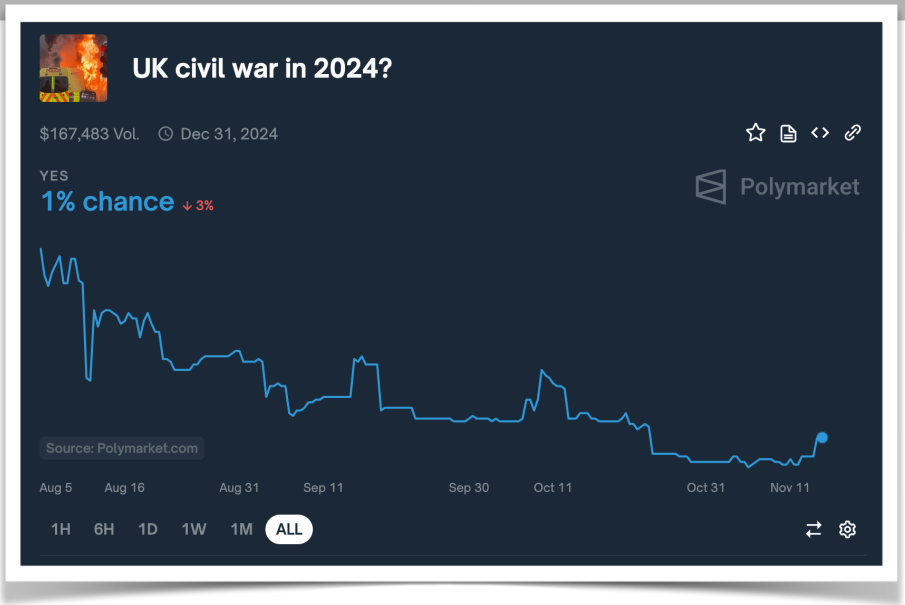
- **Predictive:**
  - e.g., forecasting civil war



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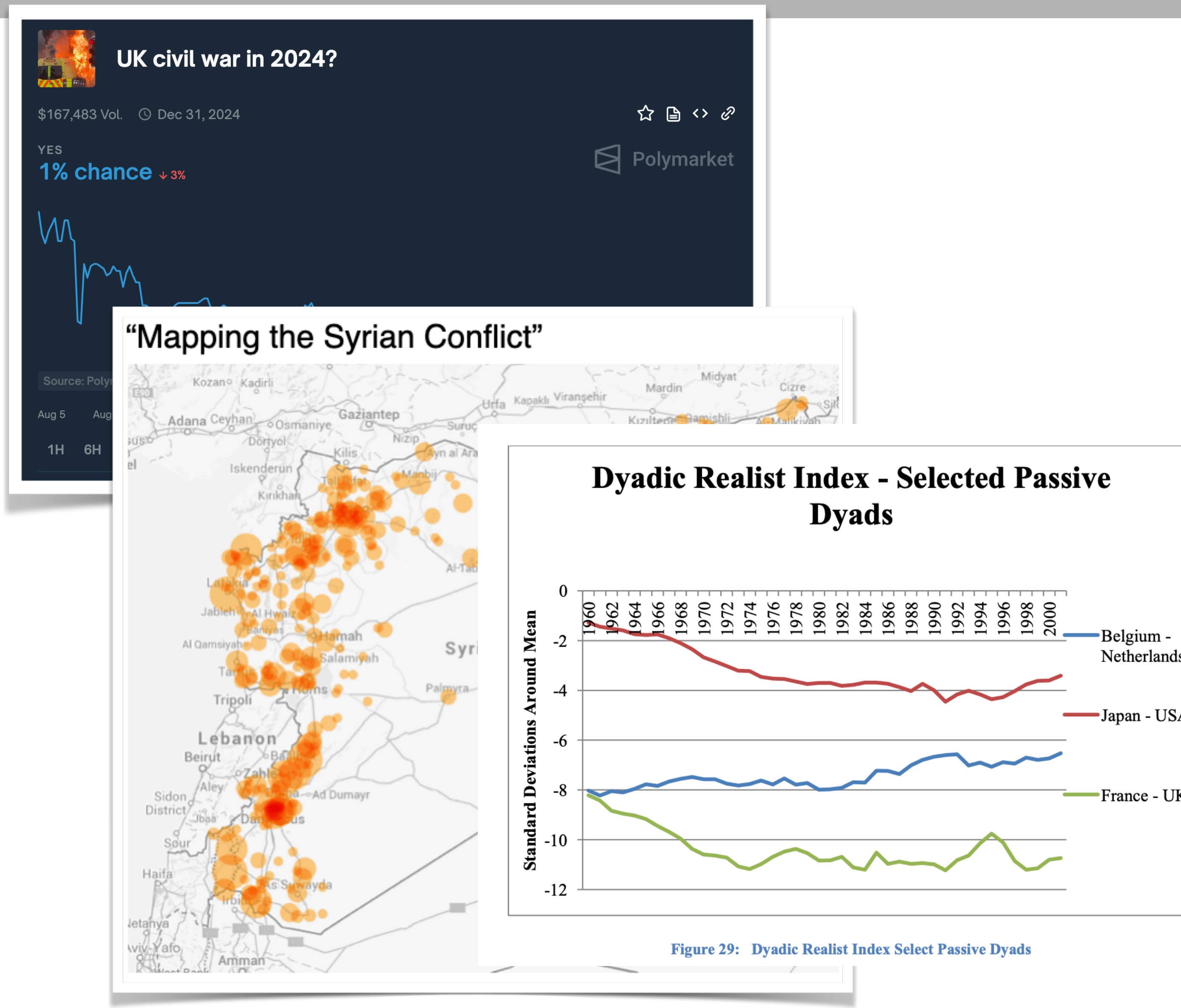

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- **Predictive:**
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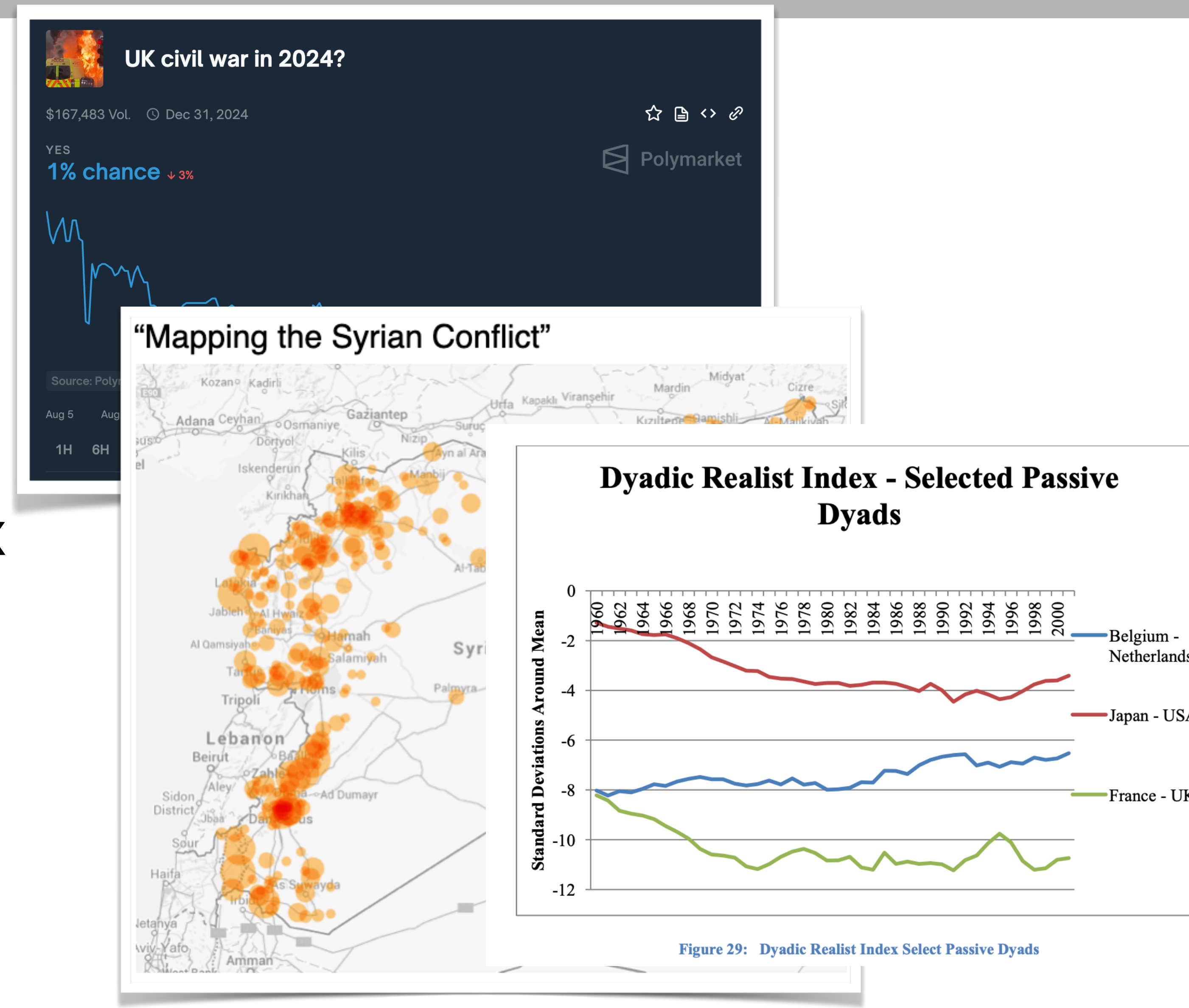
- **Descriptive**:
  - e.g., “power imbalance” index
  - e.g., conflict mapping
- **Exploratory:**
  - hypothesis generation



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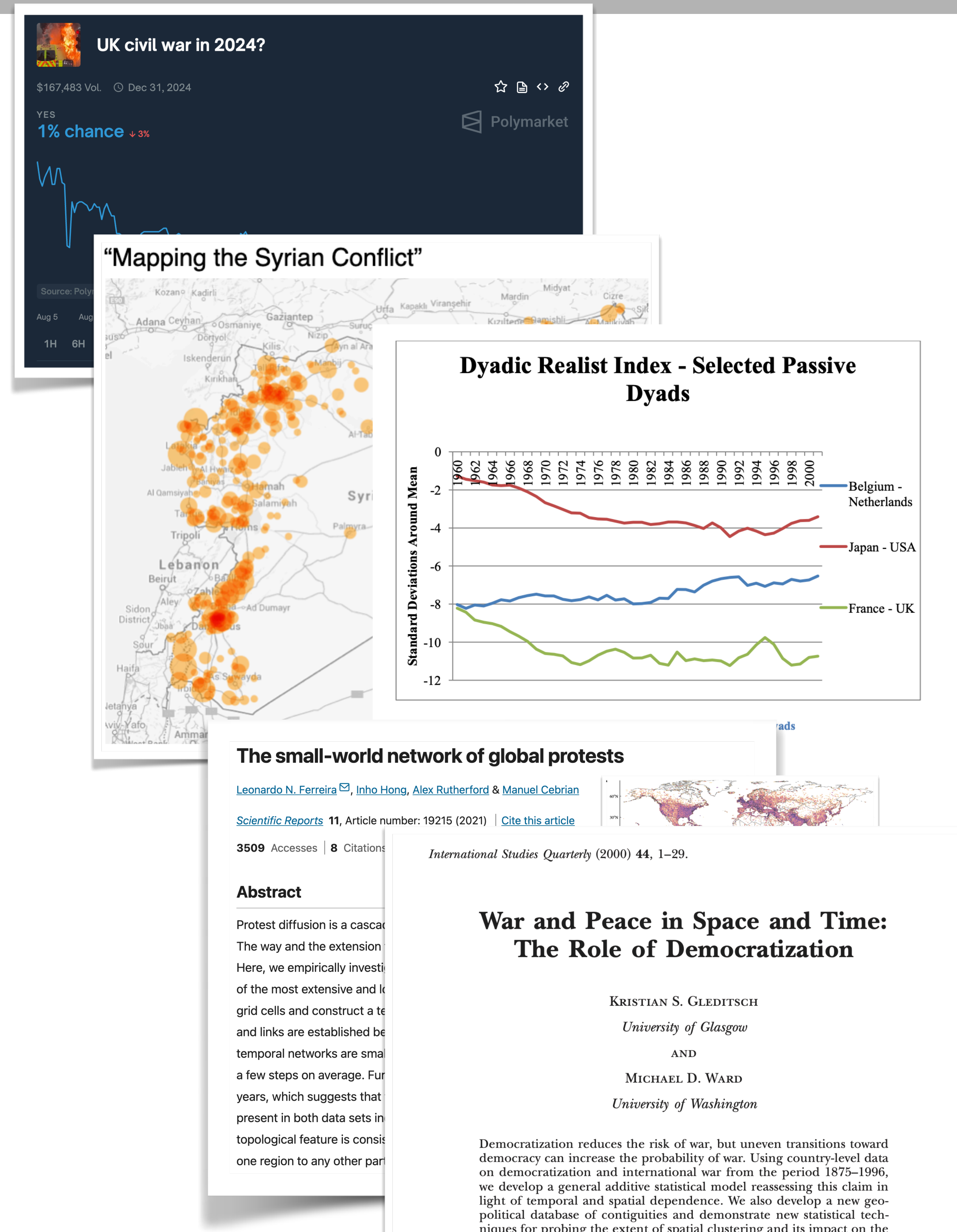

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- **Descriptive**:
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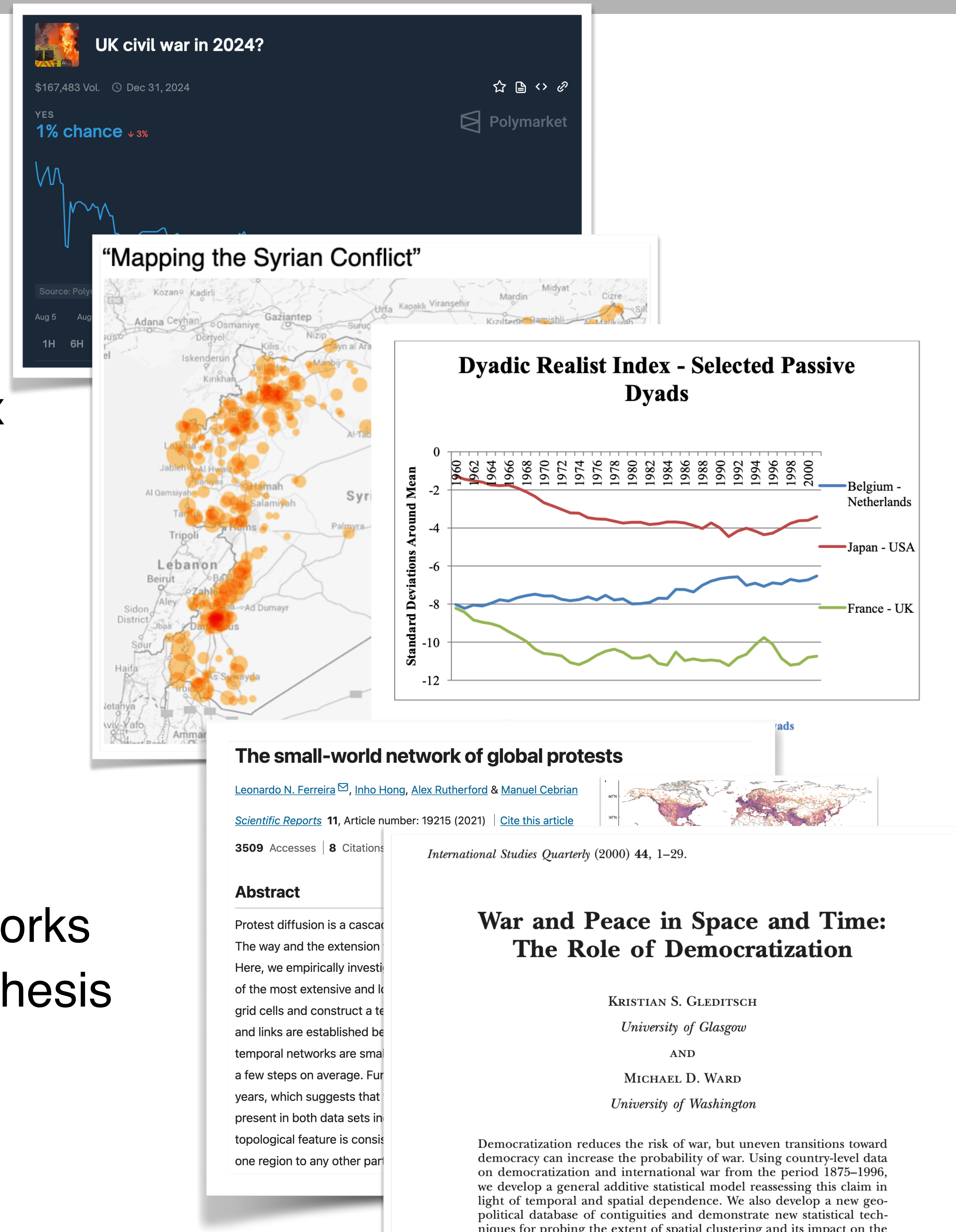
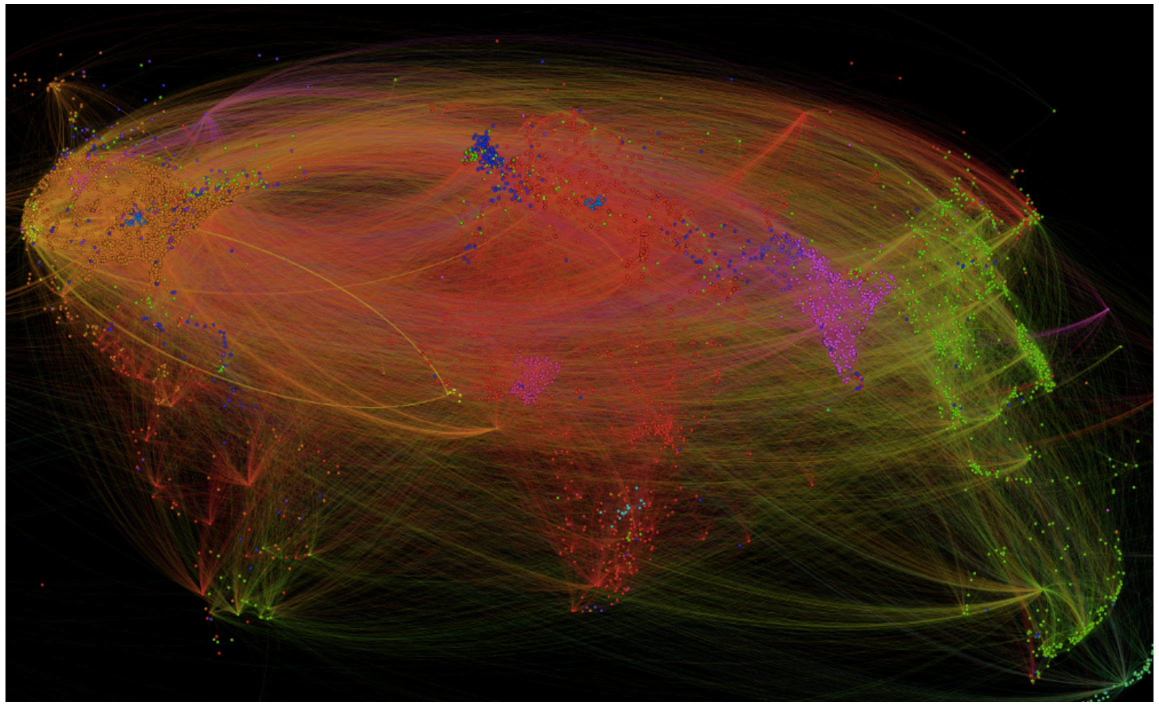
- **Exploratory:**
  - hypothesis generation
- **Explanatory**:
  - test theories / hypotheses
  - e.g., small-world protest networks
  - e.g., Democratic peace hypothesis



CHALLENGES

- **Noisy, error-prone, biased:**
  - Extraction / coding errors
  - Reporting / sourcing biases



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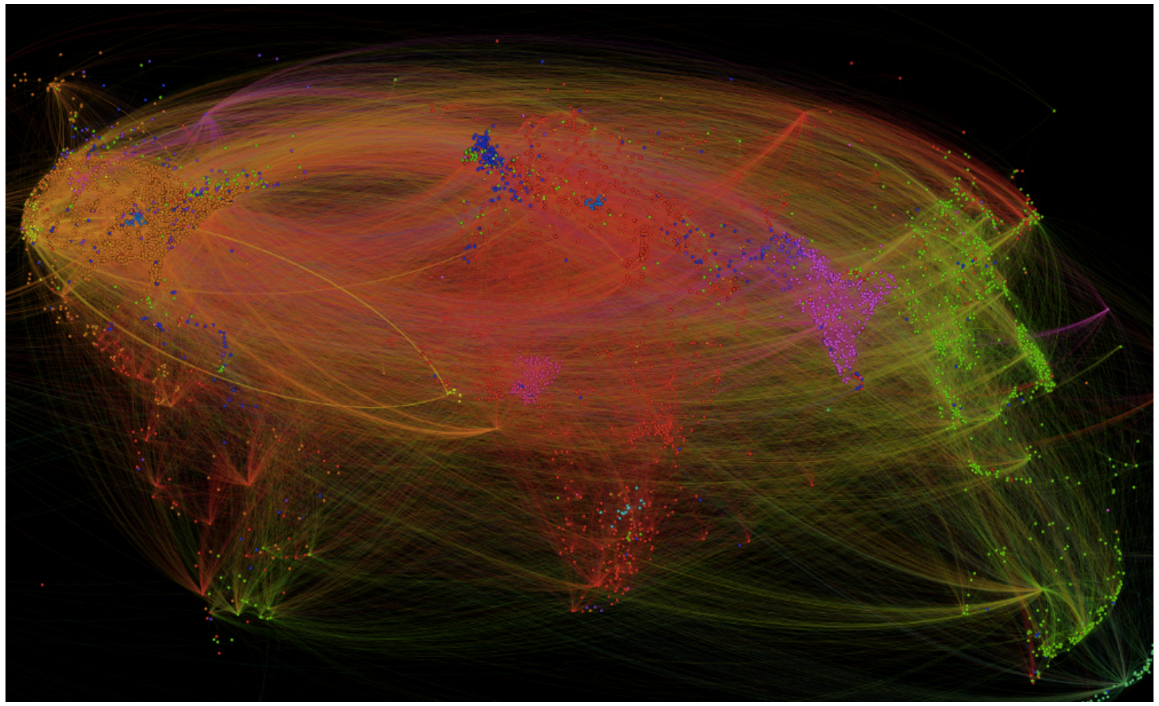
  - e.g., hierarchical coding schemes



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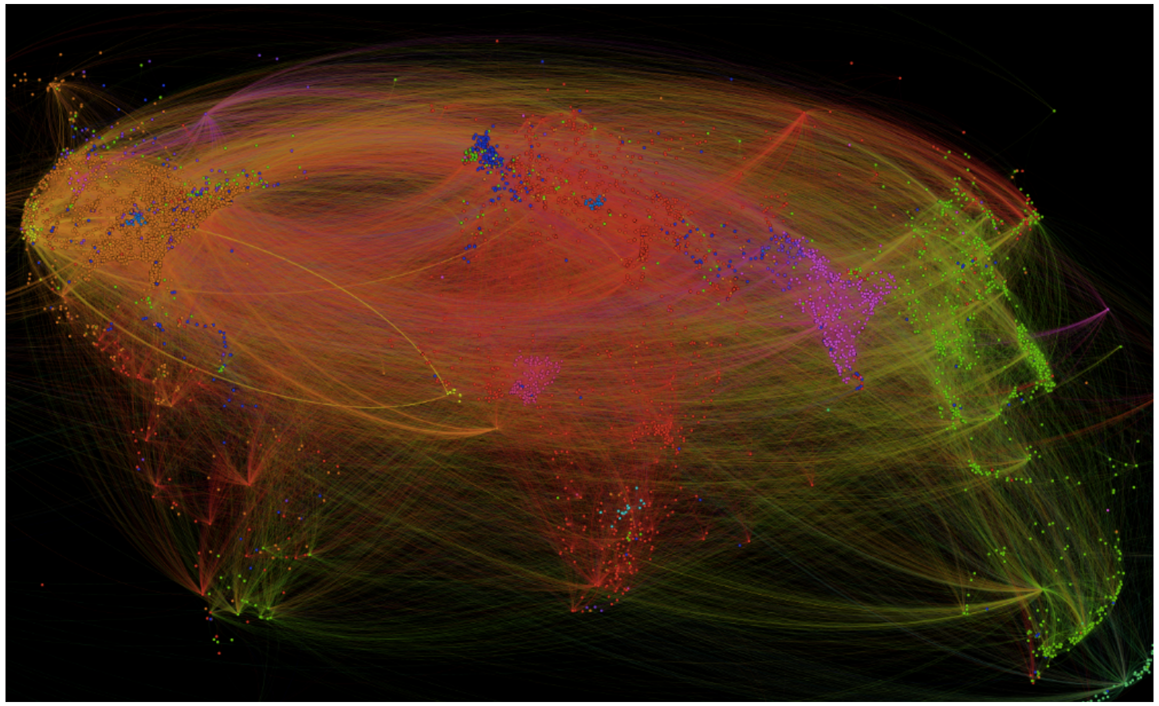

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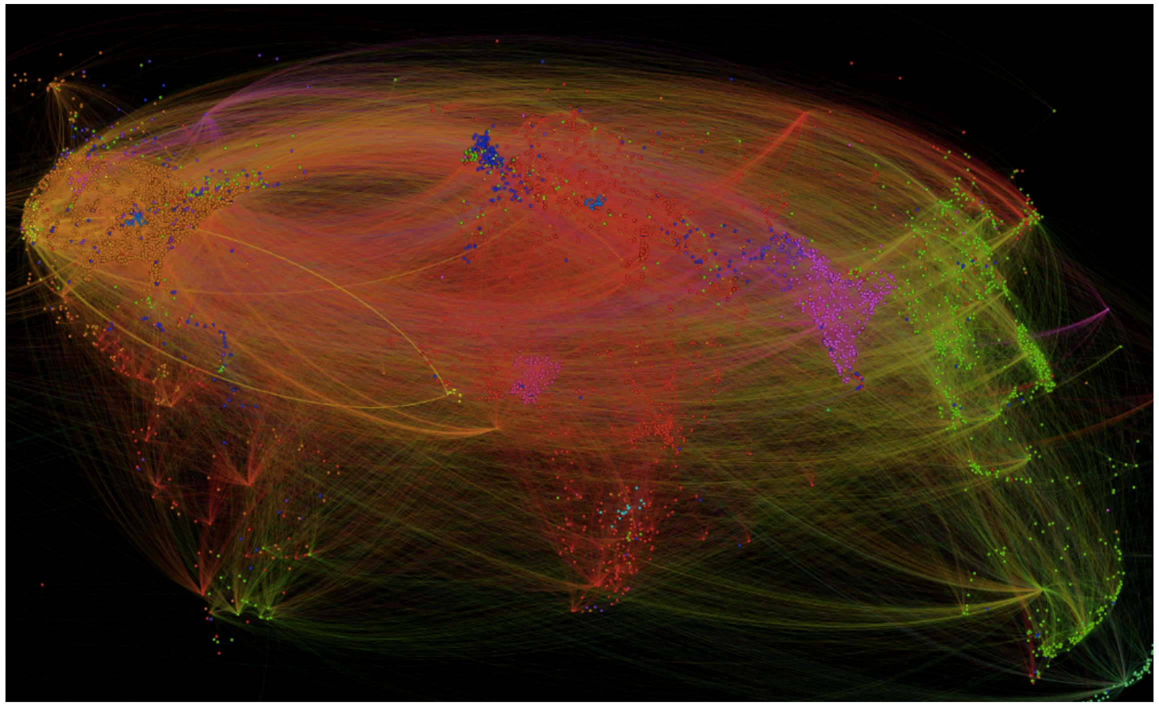

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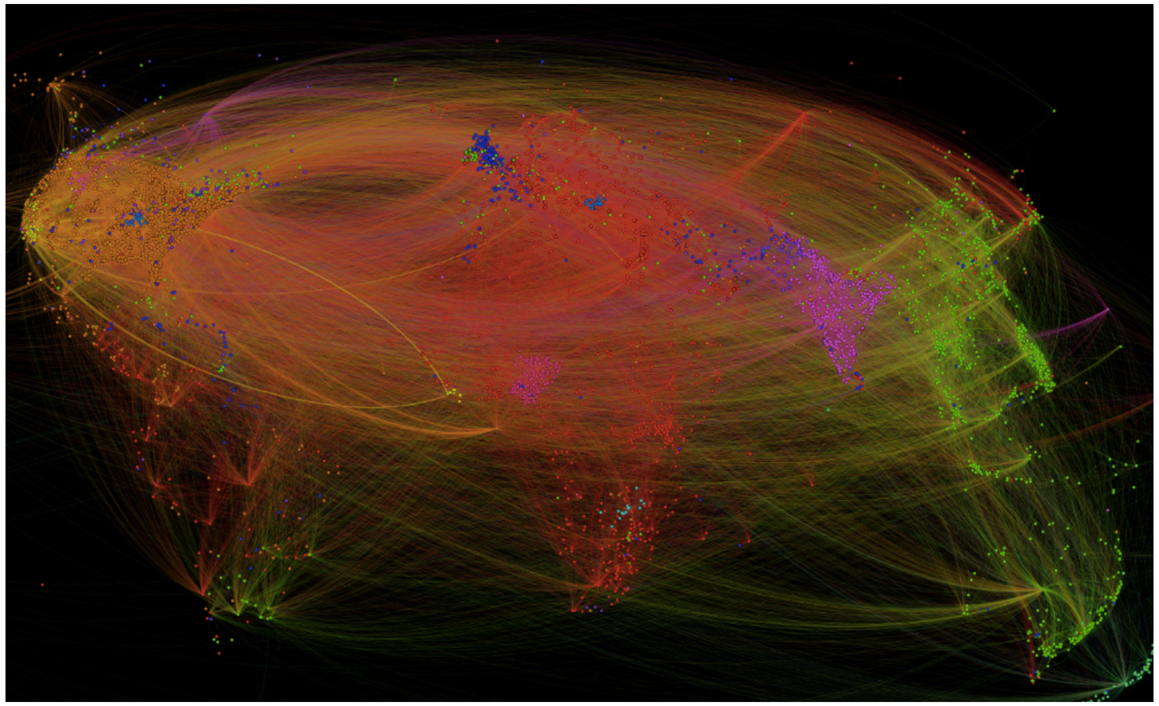

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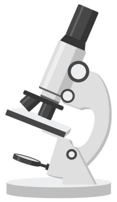


⟹ **Need: models to extract meaningful signal**

CHALLENGES

Relax, Tensors Are Here
Dependencies in International Processes☆

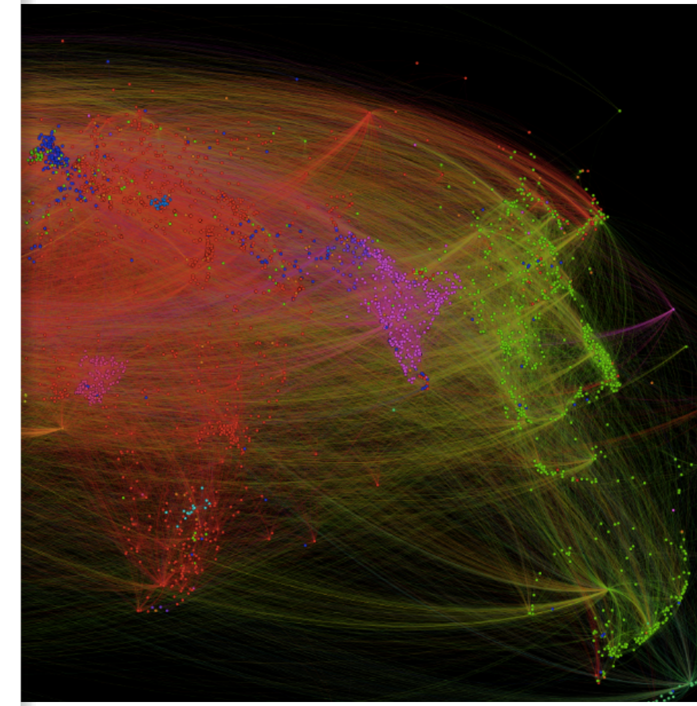
Shahryar Minhas[a], Peter D. Hoff[b], Michael D. Ward[a,*]

[a]*Department of Political Science, Duke University, Durham, NC 27701, USA*
[b]*Departments of Biostatistics & Statistics, University of Washington, Seattle, WA, USA*

Abstract

Previous models of international conflict have suffered two shortfalls. They tended not to embody dynamic changes, focusing rather on static slices of behavior over time. These models have also been empirically evaluated in ways that assumed the independence of each country, when in reality they are searching for the interdependence among all countries. We illustrate a solution to these two hurdles and evaluate this new, dynamic, network based approach to the dependencies among the ebb and flow of daily international interactions using a newly developed, and openly available, database of events among nations.

Keywords: Dynamic networks, time series, international crises, event data, tensor products



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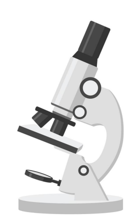

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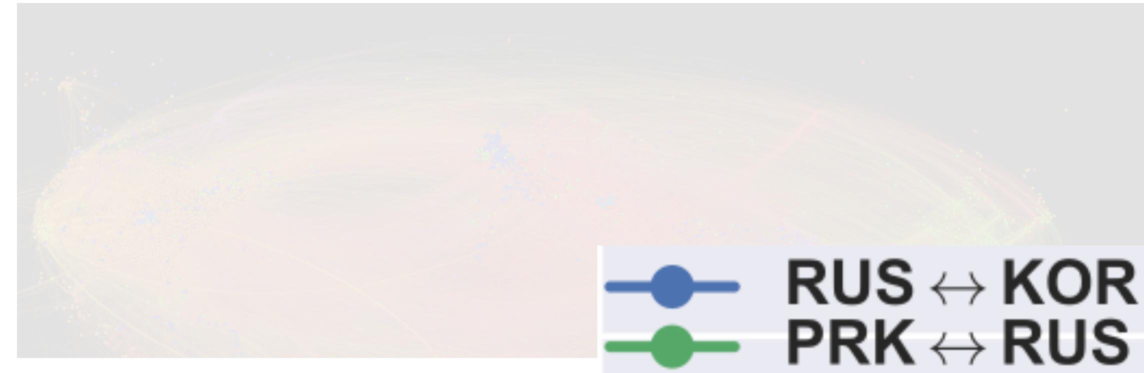

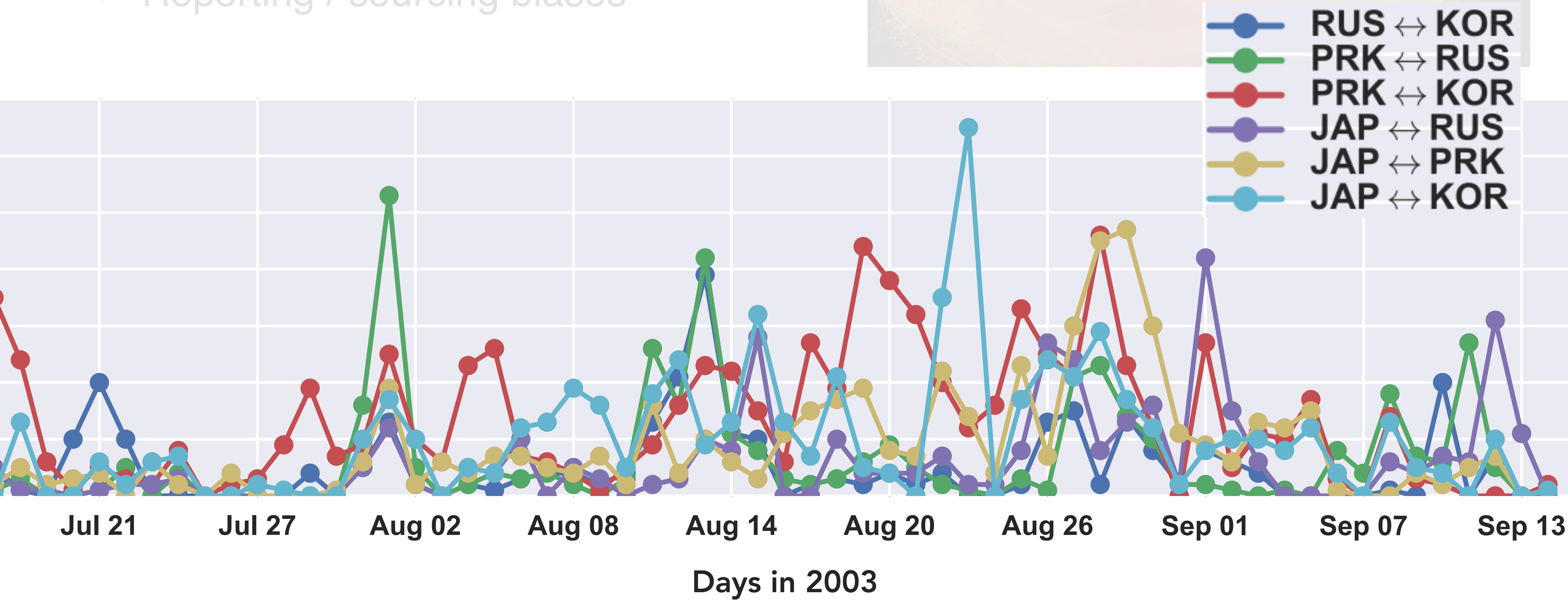

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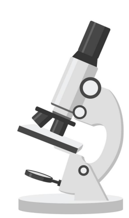

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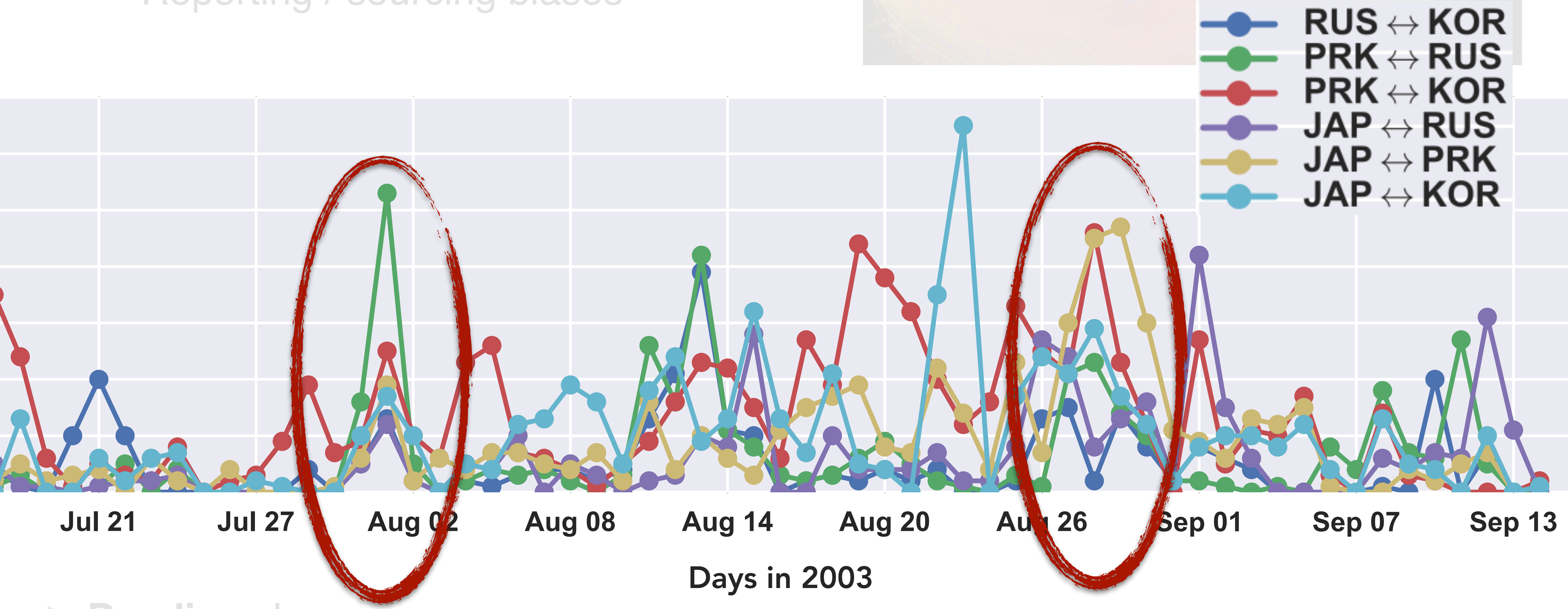

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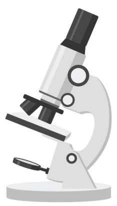

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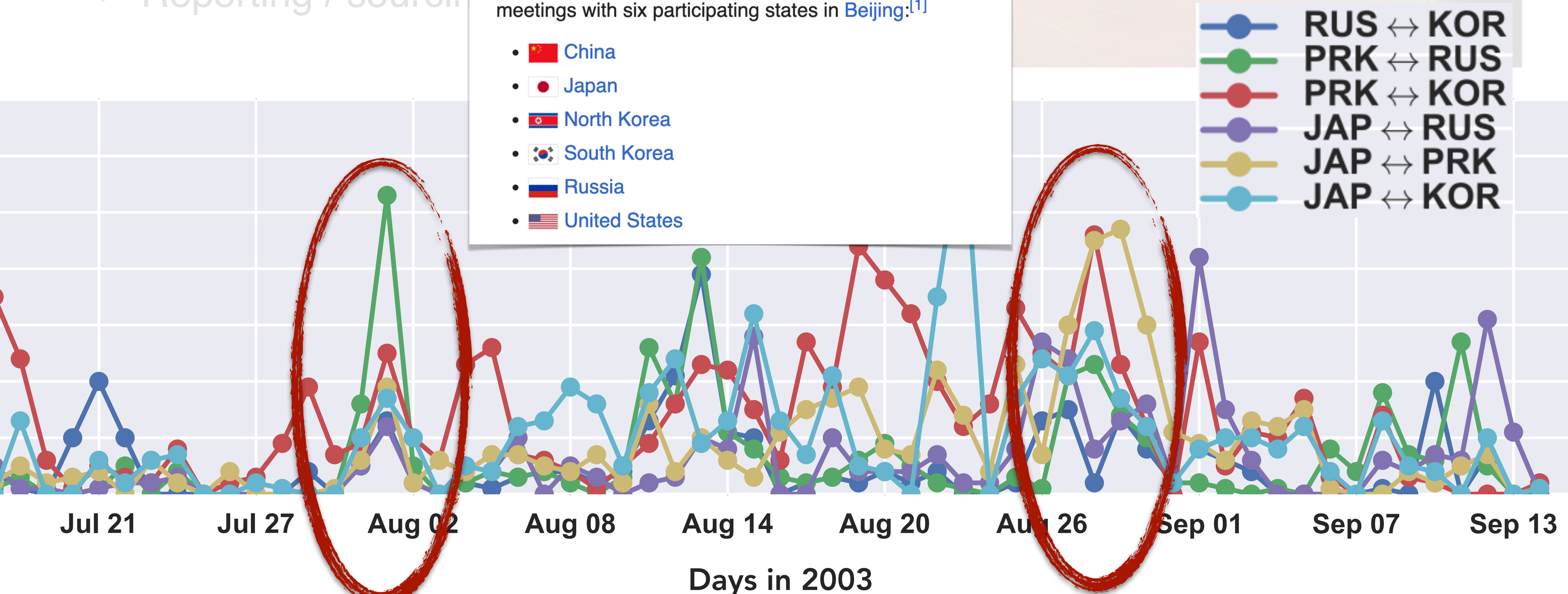
Six-party talks

Article Talk

From Wikipedia, the free encyclopedia

The **six-party talks** aimed to find a peaceful resolution to the security concerns as a result of the North Korean nuclear weapons program. There was a series of meetings with six participating states in Beijing:[1]

- China
- Japan
- North Korea
- South Korea
- Russia
- United States



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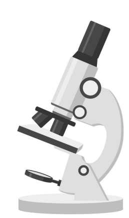

⇨ **Need: models to extract meaningful signal** **(there *is* signal)**

OUTLINE

- **Motivation:** Dyadic event data (and its challenges)

- **Non-negative (Poisson) tensor decompositions**
  - **CP** extracts "multilateral" structure [Schein et al., 2015]
  - **Tucker** extracts "communities" [Schein et al., 2016]
  - Why (allocative) Poisson factorization
  - **ALℓ_0CORE** avoids the "exponential blowup" [Hood & Schein, 2024]

Appendix (*time permitting*)

- **State-space** modeling
  - **Gamma state-space** models [Schein et al., 2016b; Schein et al., 2019]
  - **Modeling "escalation"** with a new matrix prior [Stoehr et al., 2023]

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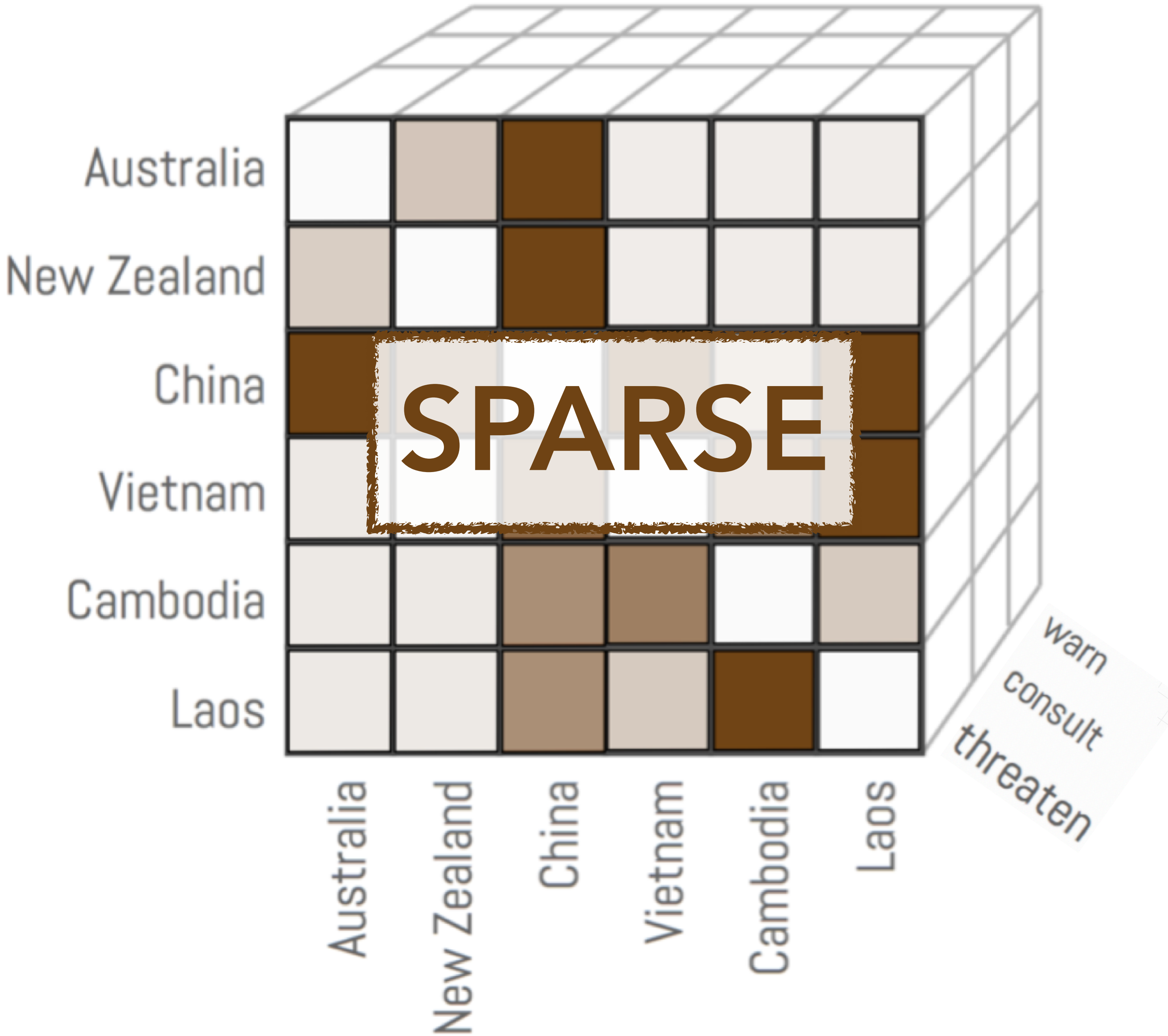
Event **type counts**

$$y^{(t)}_{i \xrightarrow{a} j} =$$

"the number of tokens of type **country *i*** took **action *a*** to **country *j*** during **time *t***"

Event **type counts**



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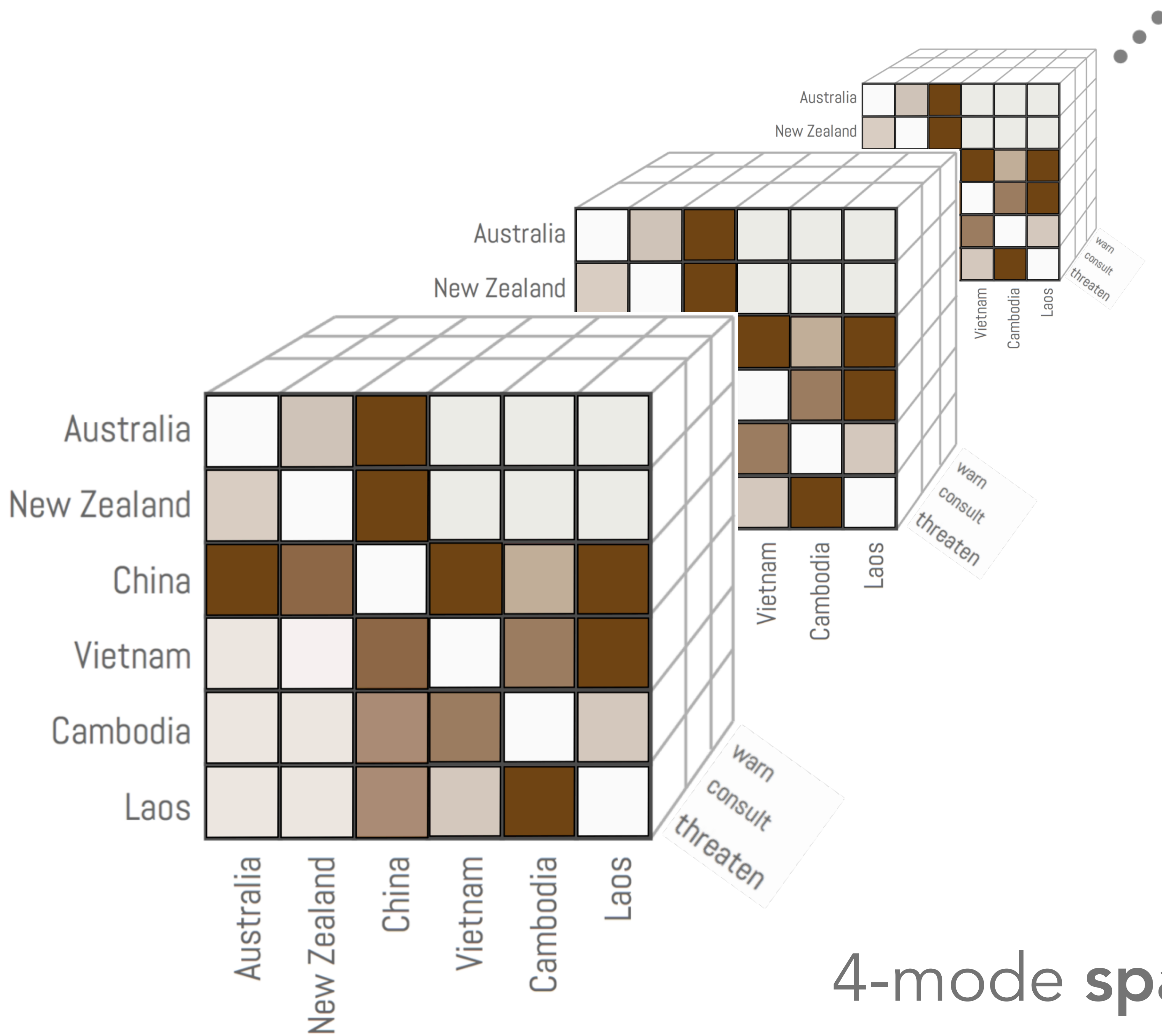
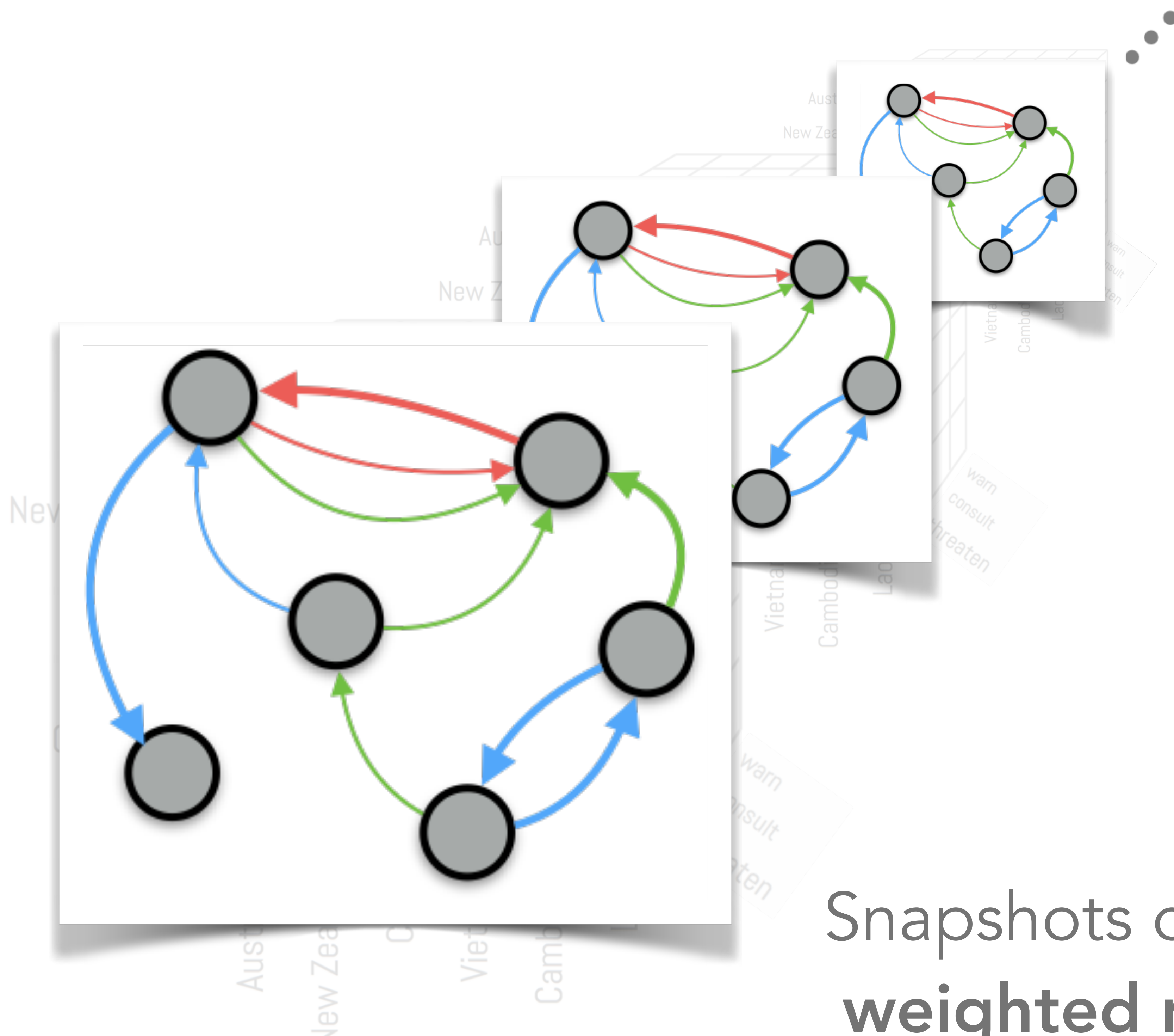


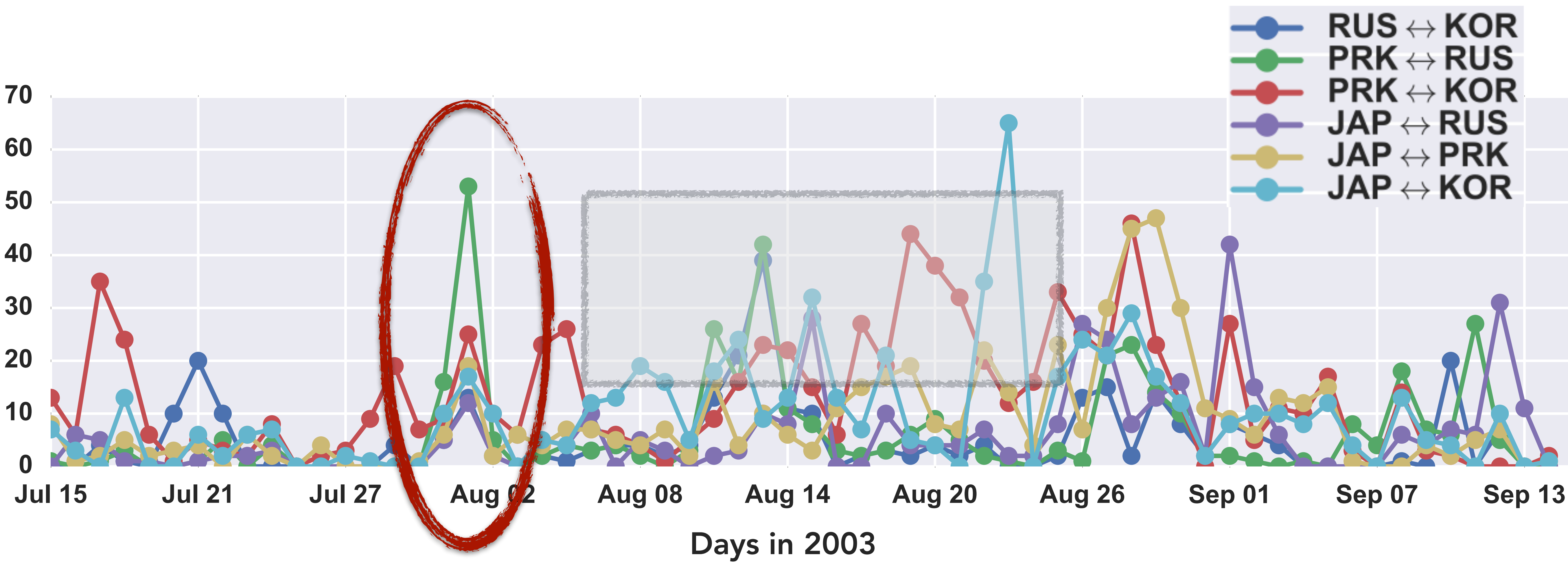
4-mode **sparse count tensor**

Event **type counts**



Snapshots of a **weighted multiplex network**

Event **type counts**



REPRESENTATIONS OF DYADIC DATA

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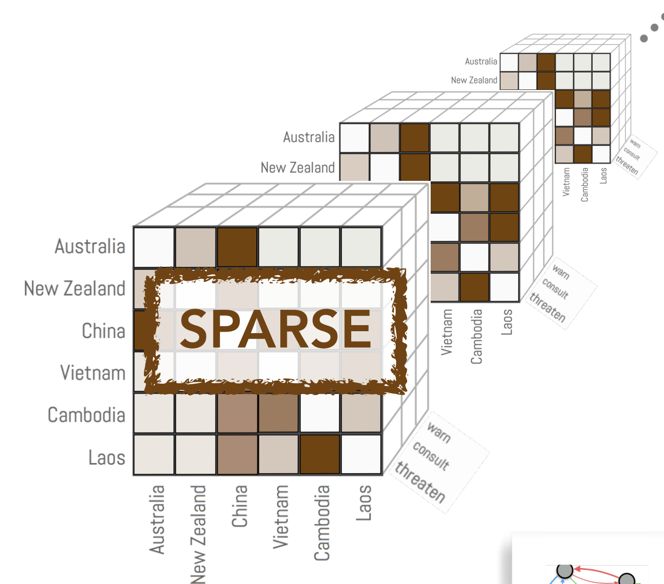
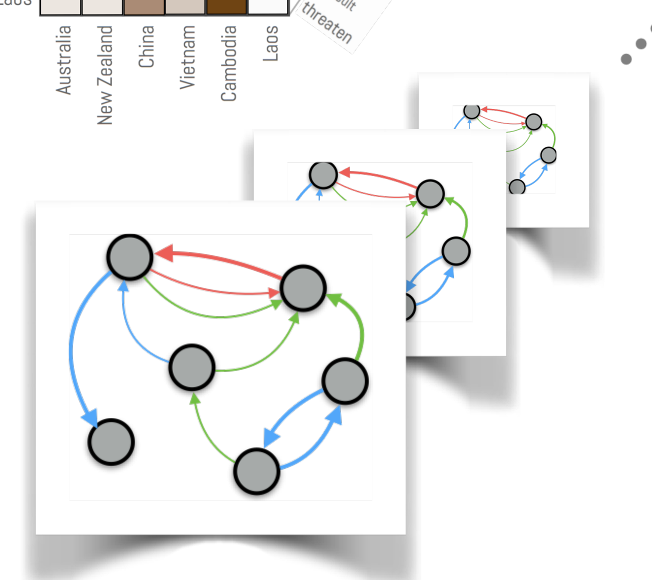
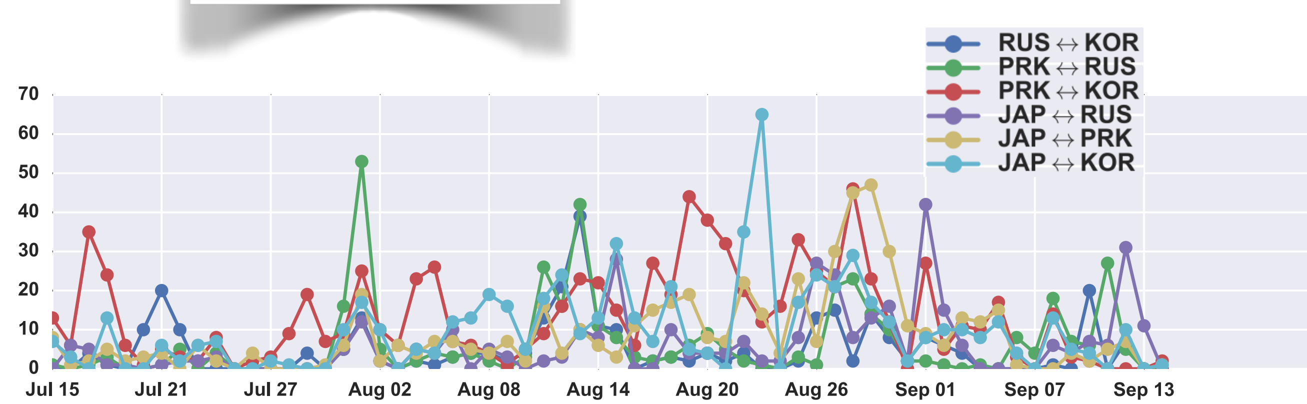
count tensor



networks



time series



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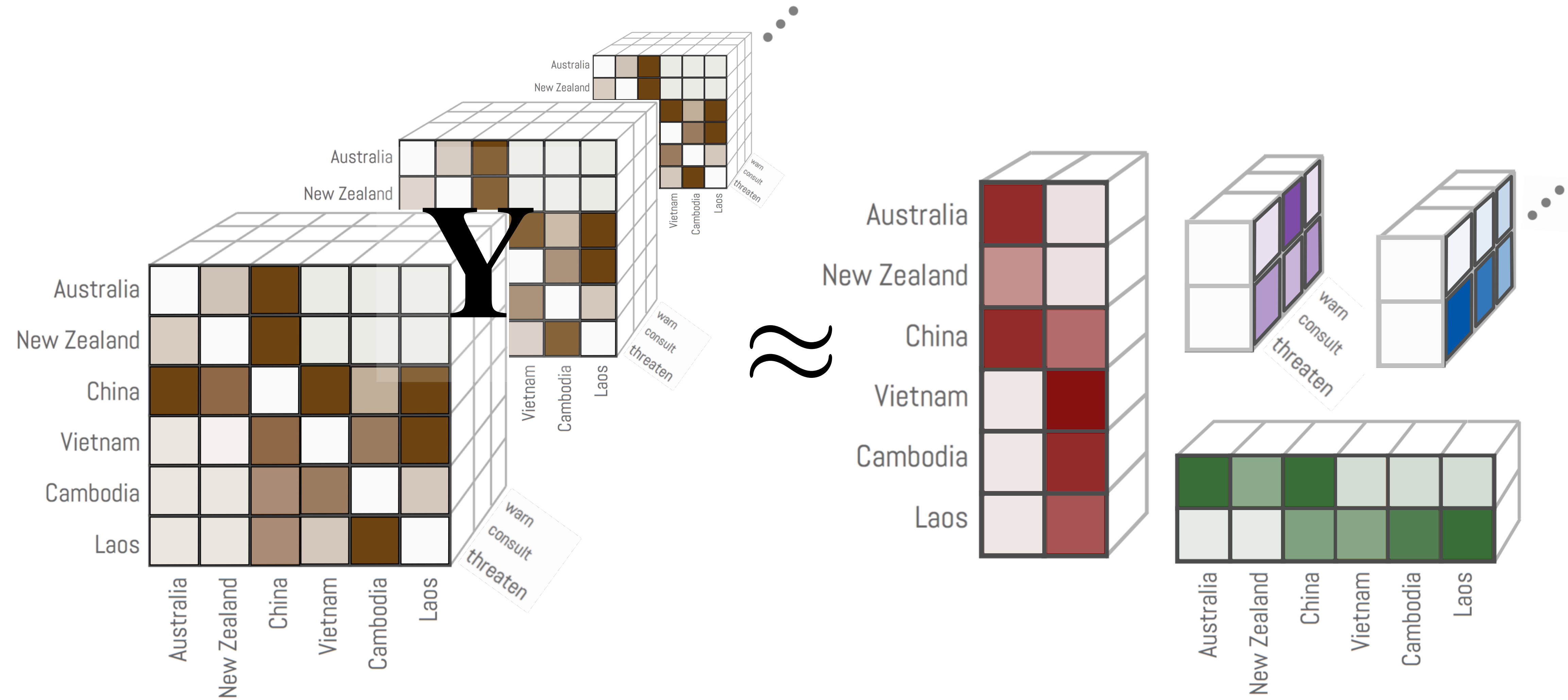

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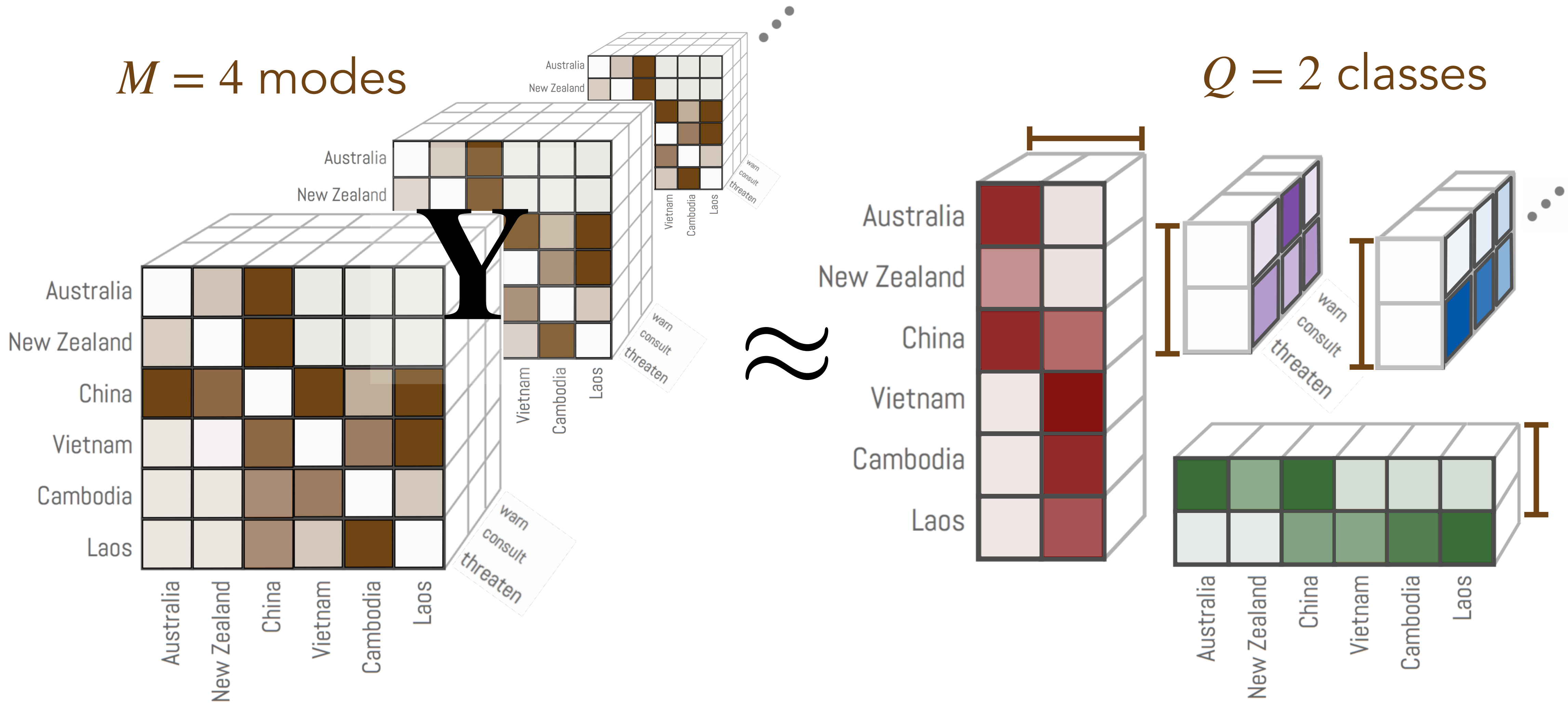

- M-mode tensor $\rightarrow$ M factor matrices
- Q latent "classes" (or "components")

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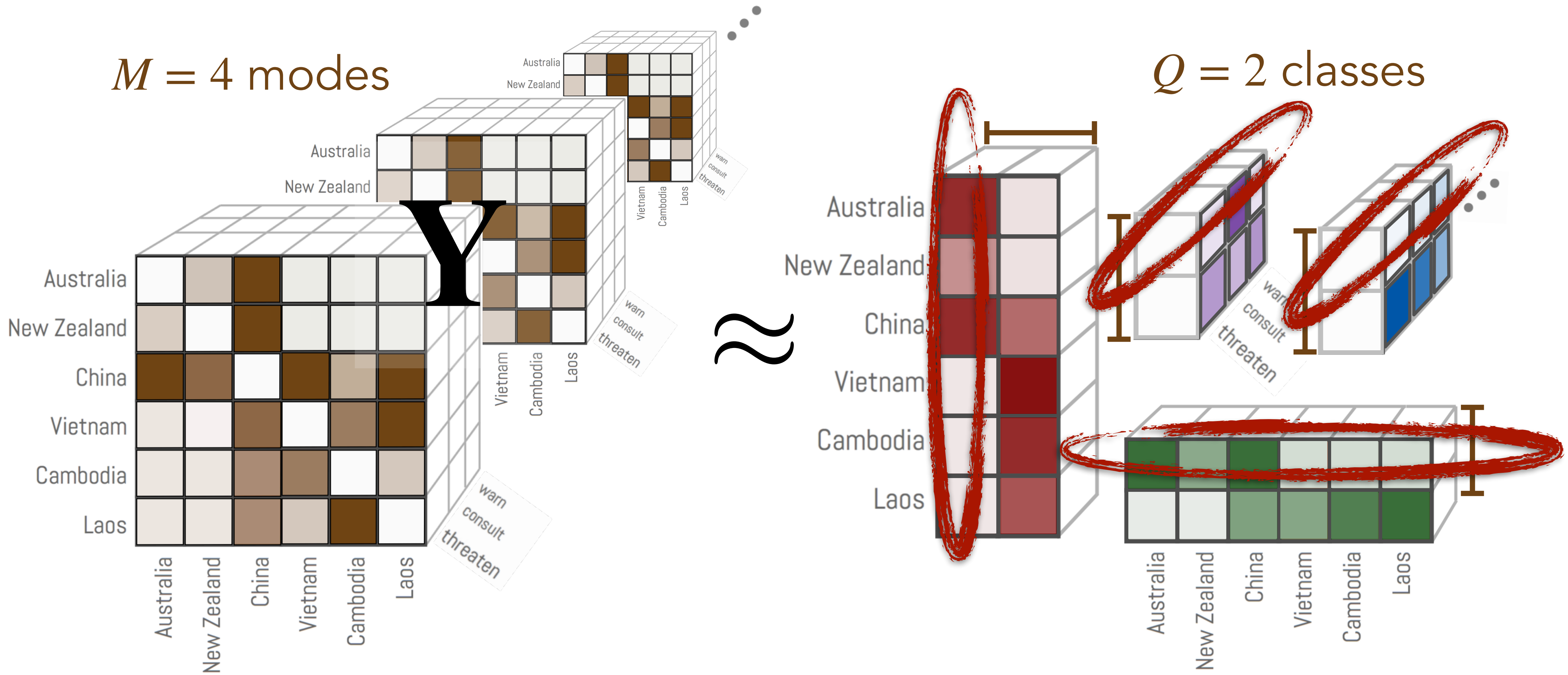

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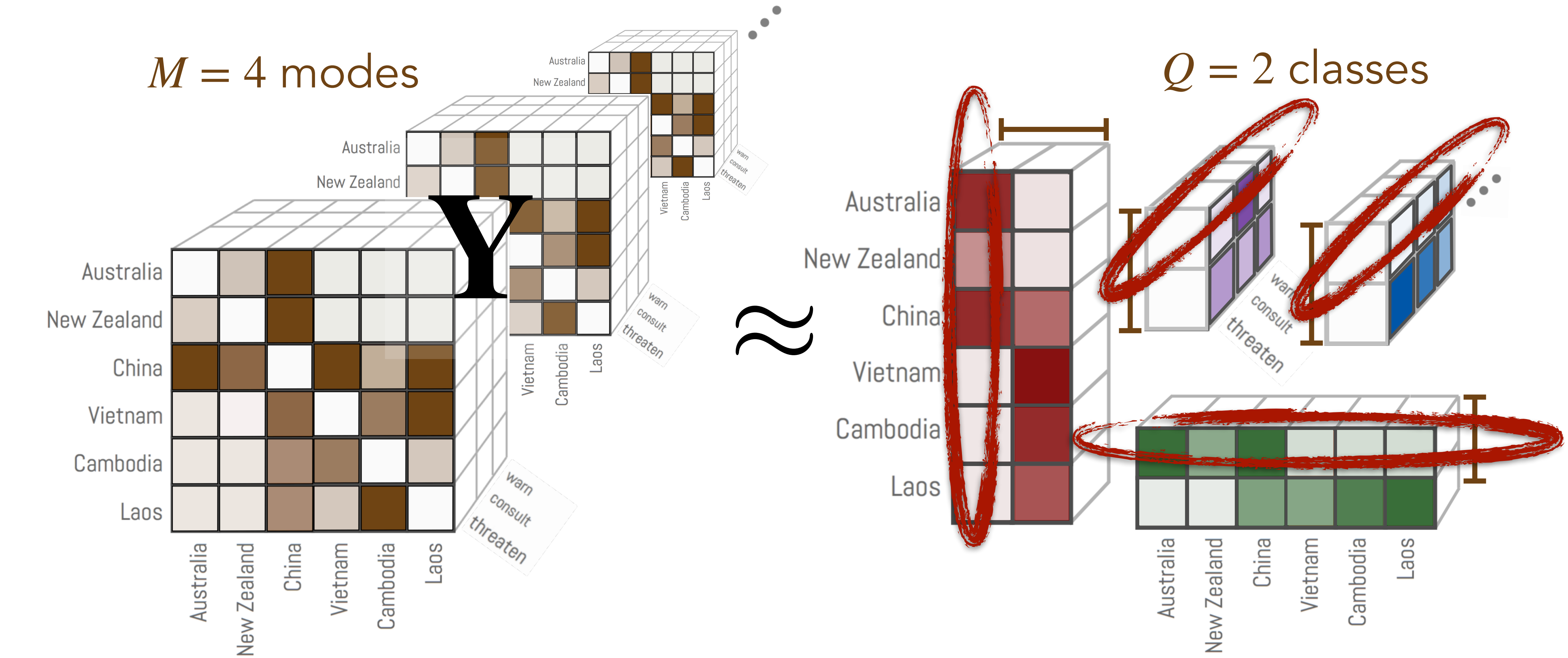
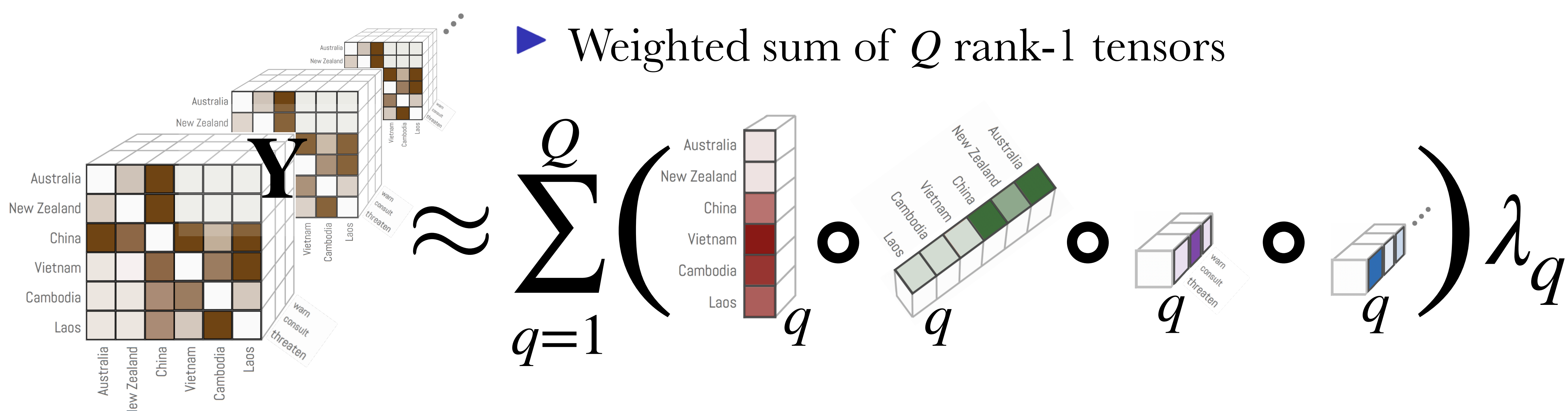
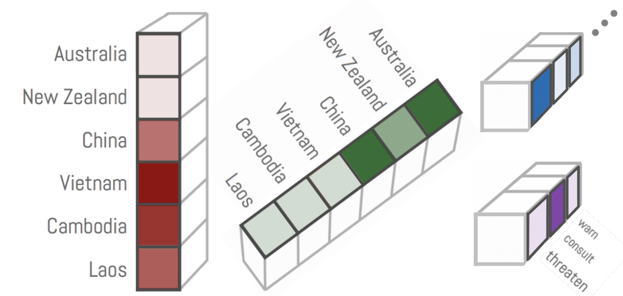
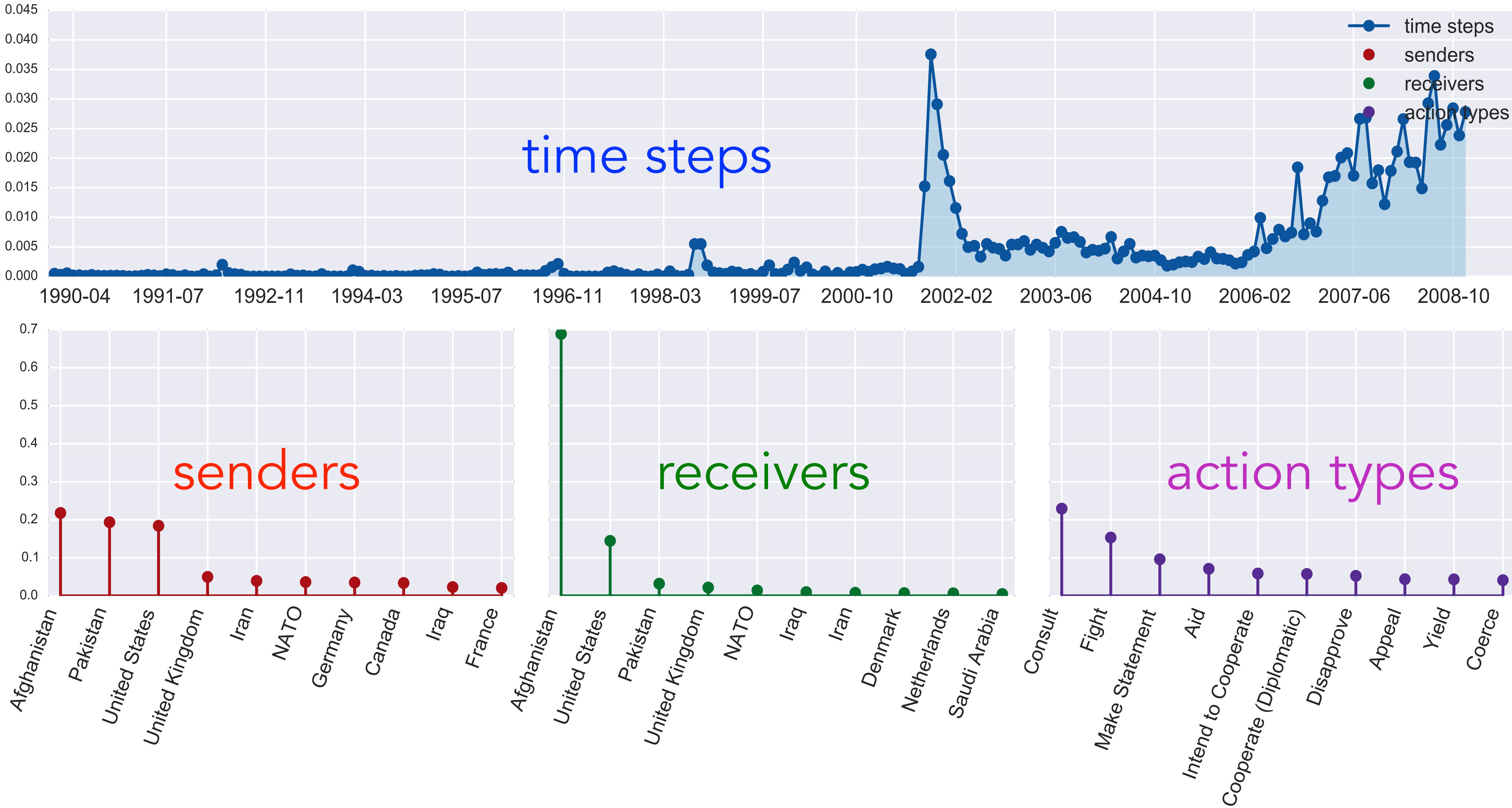


- Each "class" q associated with M "factors"
- Weighted sum of Q rank-1 tensors

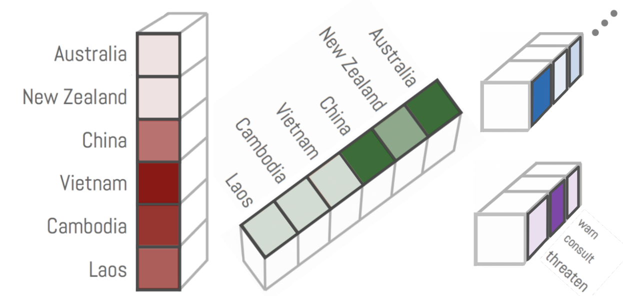


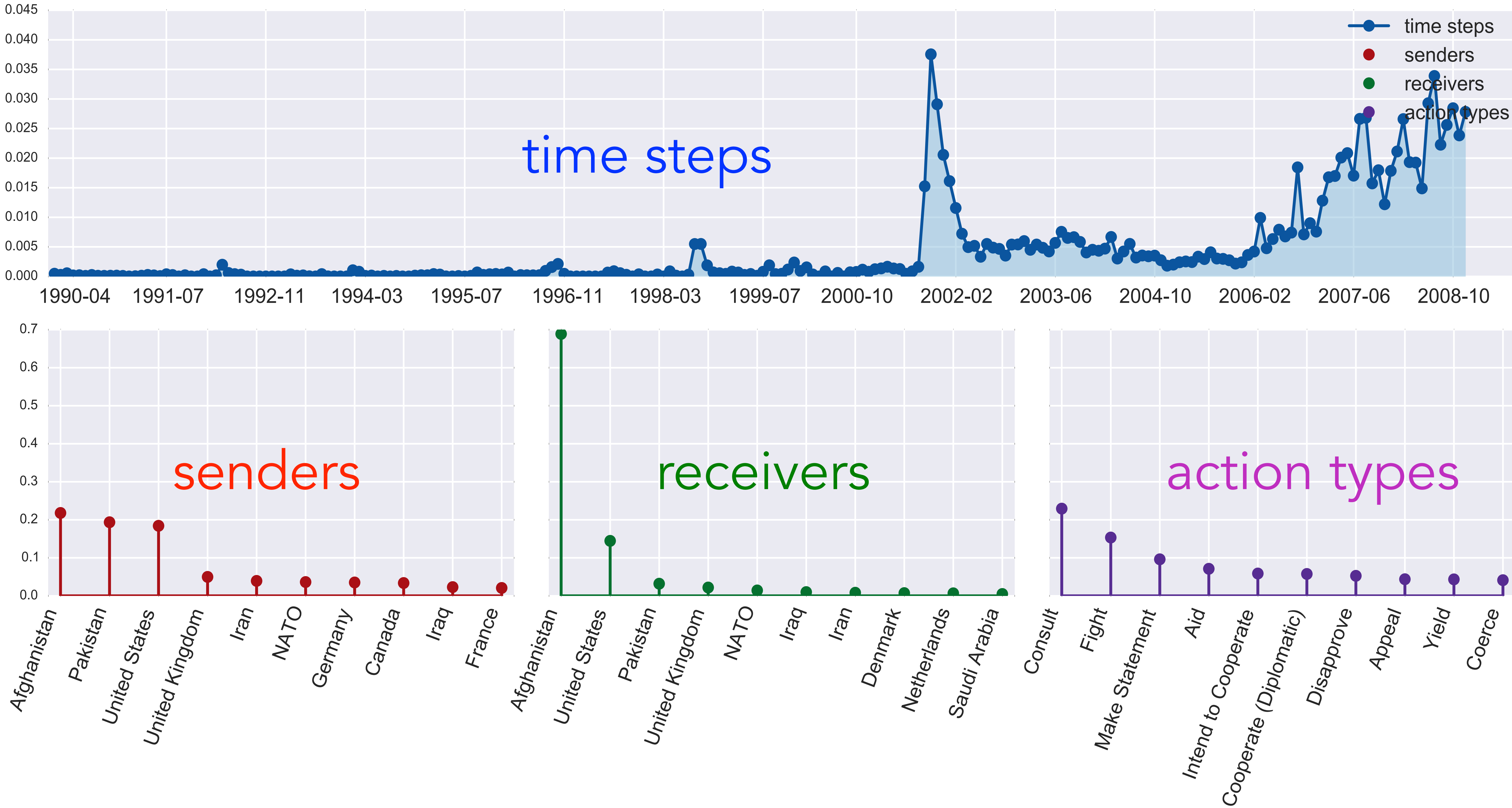
EXAMPLE CLASS



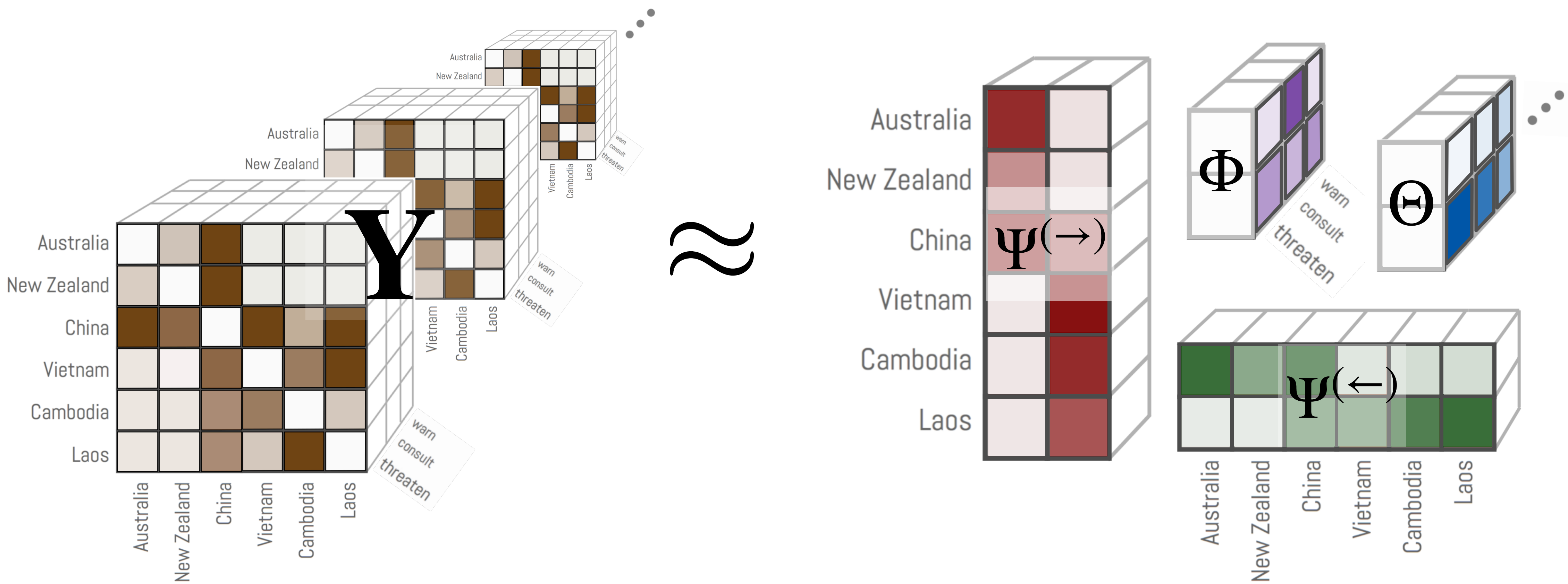


EXAMPLE CLASS: War in Afghanistan





CANONICAL POLYADIC (CP) DECOMPOSITION



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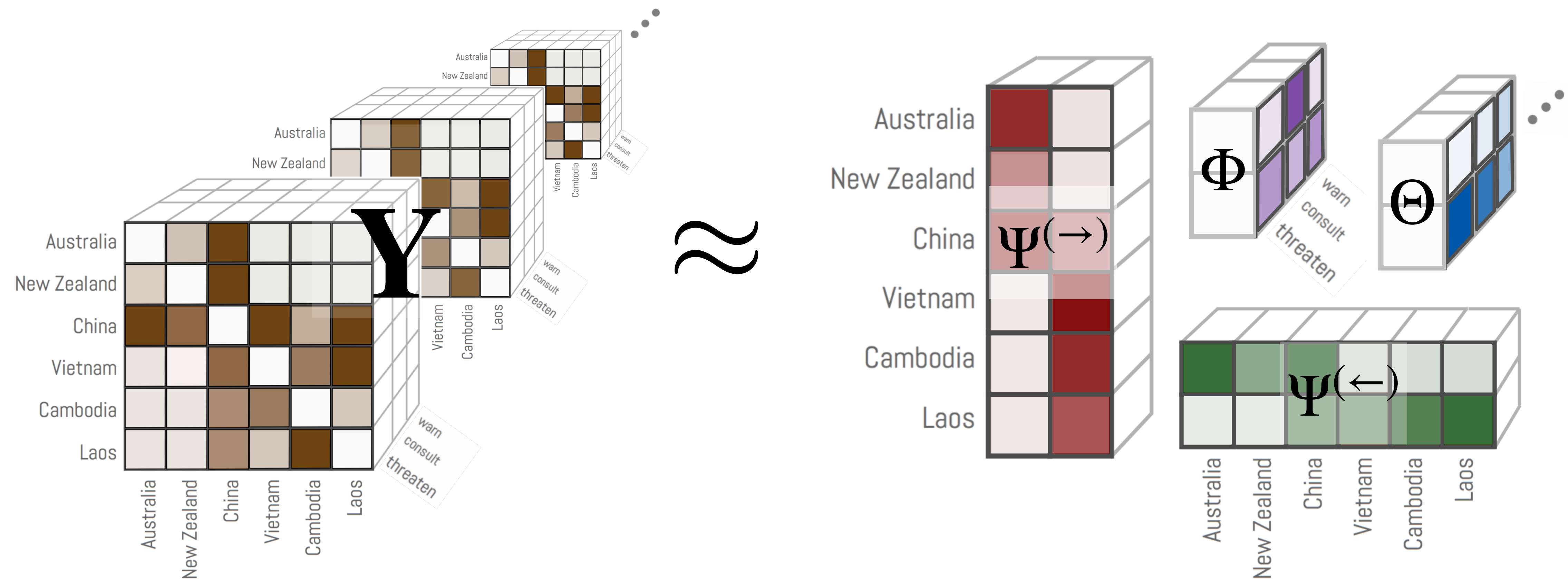
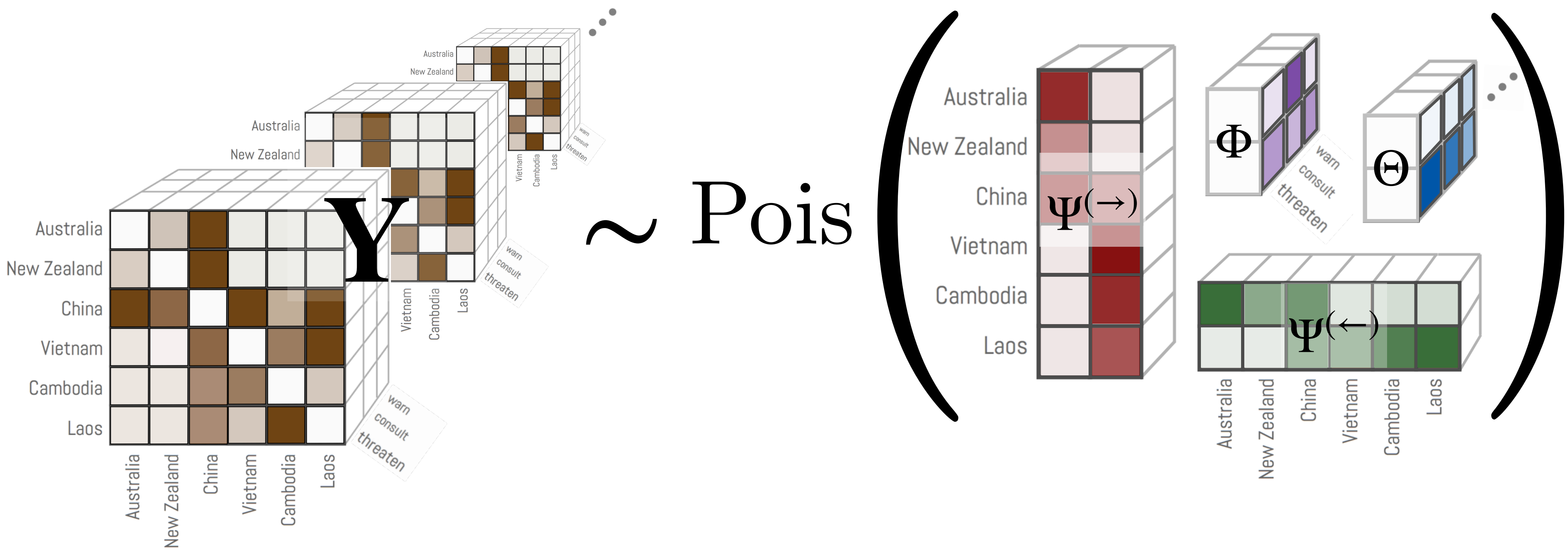


$$y^{(t)}_{i \xrightarrow{a} j} \approx \sum_{q=1}^{Q} \psi^{(\rightarrow)}_{iq}\, \psi^{(\leftarrow)}_{jq}\, \phi_{aq}\, \theta^{(t)}_{q}\, \lambda_q$$

POISSON CP DECOMPOSITION



$$y_{i \xrightarrow{a} j}^{(t)} \sim \mathrm{Pois}\left(\sum_{q=1}^{Q} \psi_{iq}^{(\rightarrow)}\, \psi_{jq}^{(\leftarrow)}\, \phi_{aq}\, \theta_q^{(t)}\, \lambda_q\right)$$

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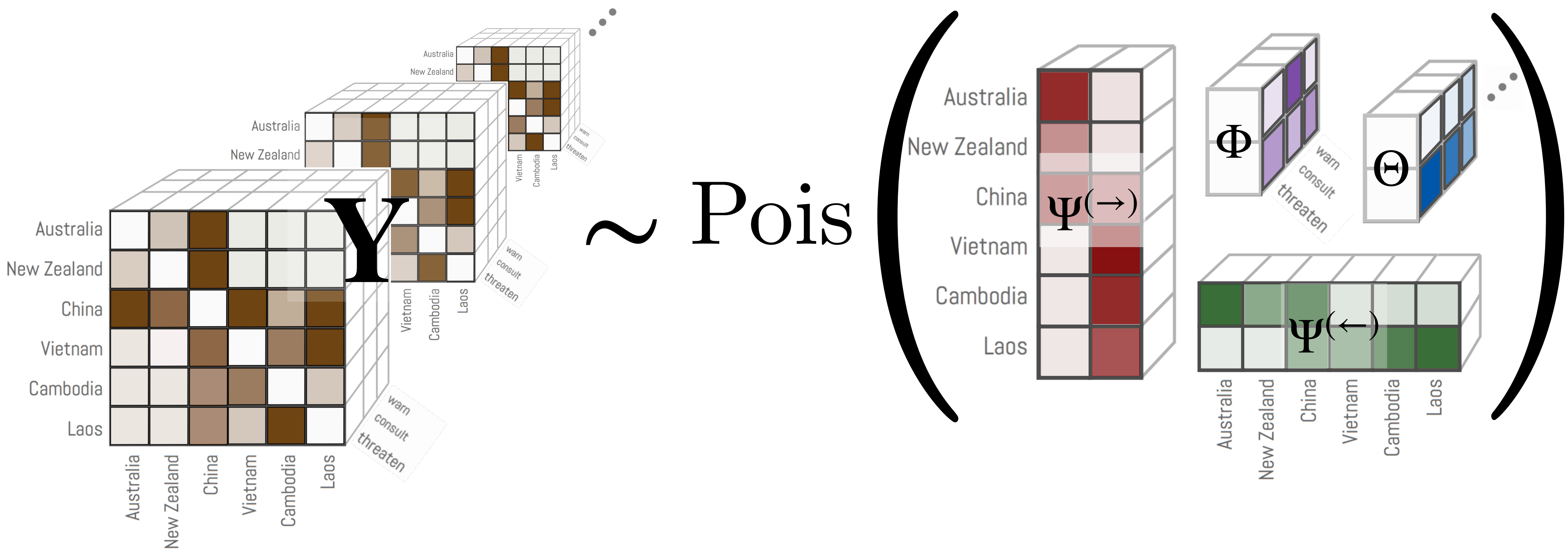

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non-negative

"parts-based" representation [Lee and Seung, 1999]

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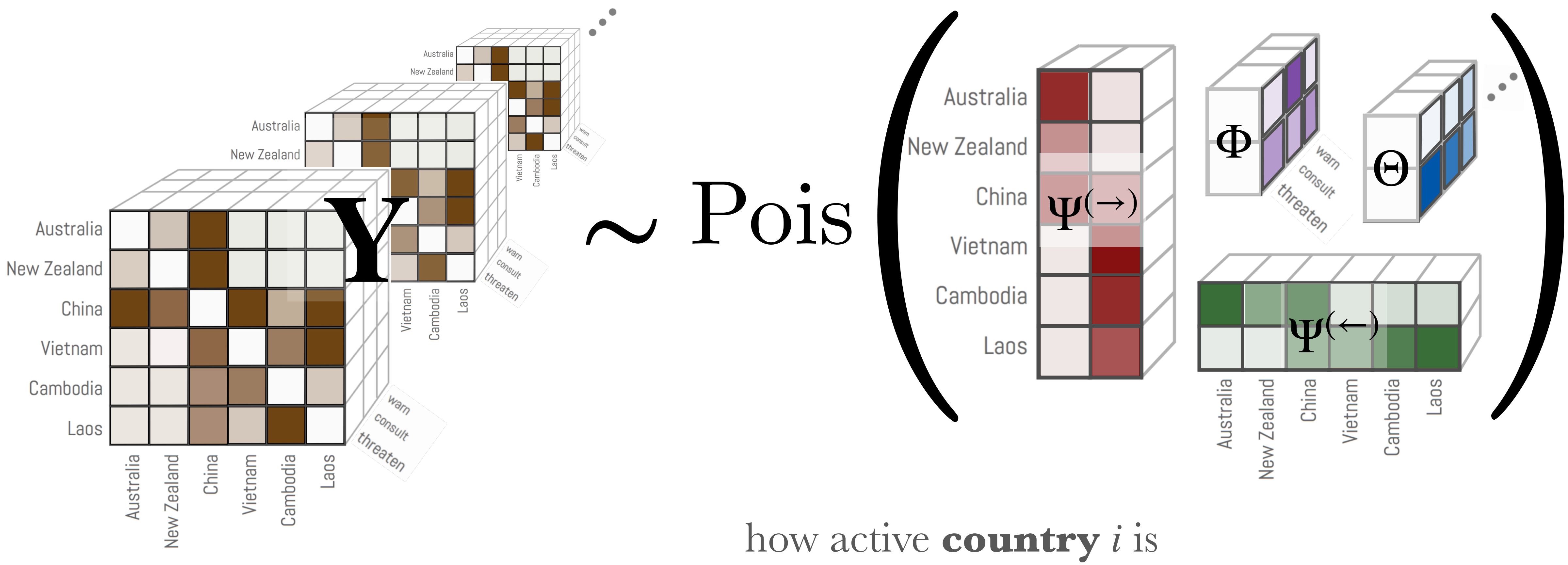

how active **country** i is *as a sender* in **class** q

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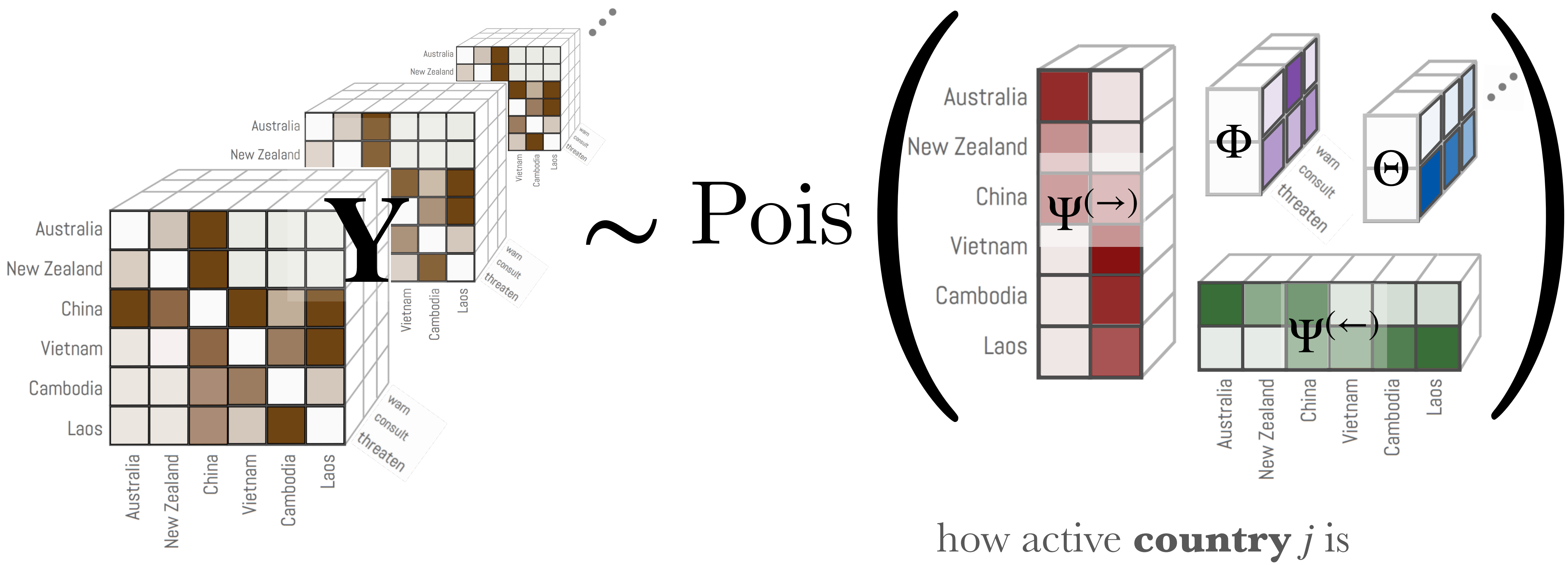

how active **country** j is
as a receiver in **class** q

$$y^{(t)}_{i \xrightarrow{a} j} \sim \mathrm{Pois}\left(\sum_{q=1}^{Q} \psi^{(\rightarrow)}_{iq}\; \psi^{(\leftarrow)}_{jq}\; \phi_{aq}\, \theta^{(t)}_{q}\; \lambda_q\right)$$

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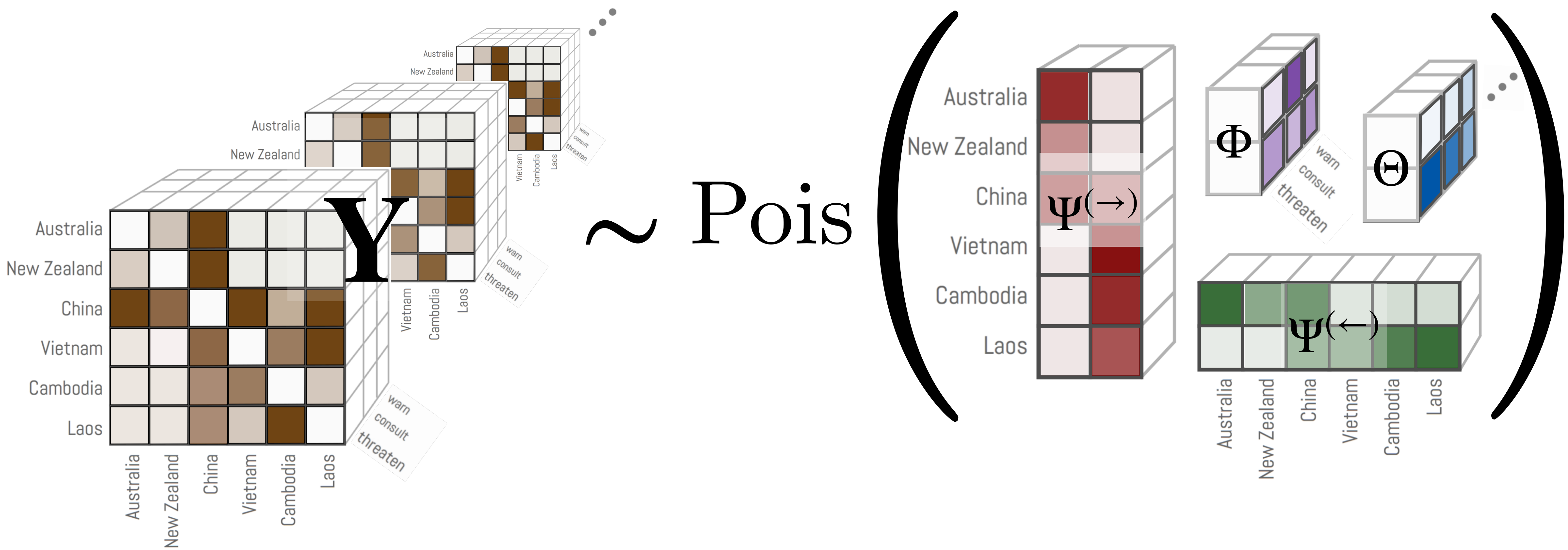

how prevalent **action** a is in **class** q

$$y^{(t)}_{i \xrightarrow{a} j} \sim \mathrm{Pois}\left(\sum_{q=1}^{Q} \psi^{(\rightarrow)}_{iq}\, \psi^{(\leftarrow)}_{jq}\, \phi_{aq}\, \theta^{(t)}_{q}\, \lambda_q\right)$$

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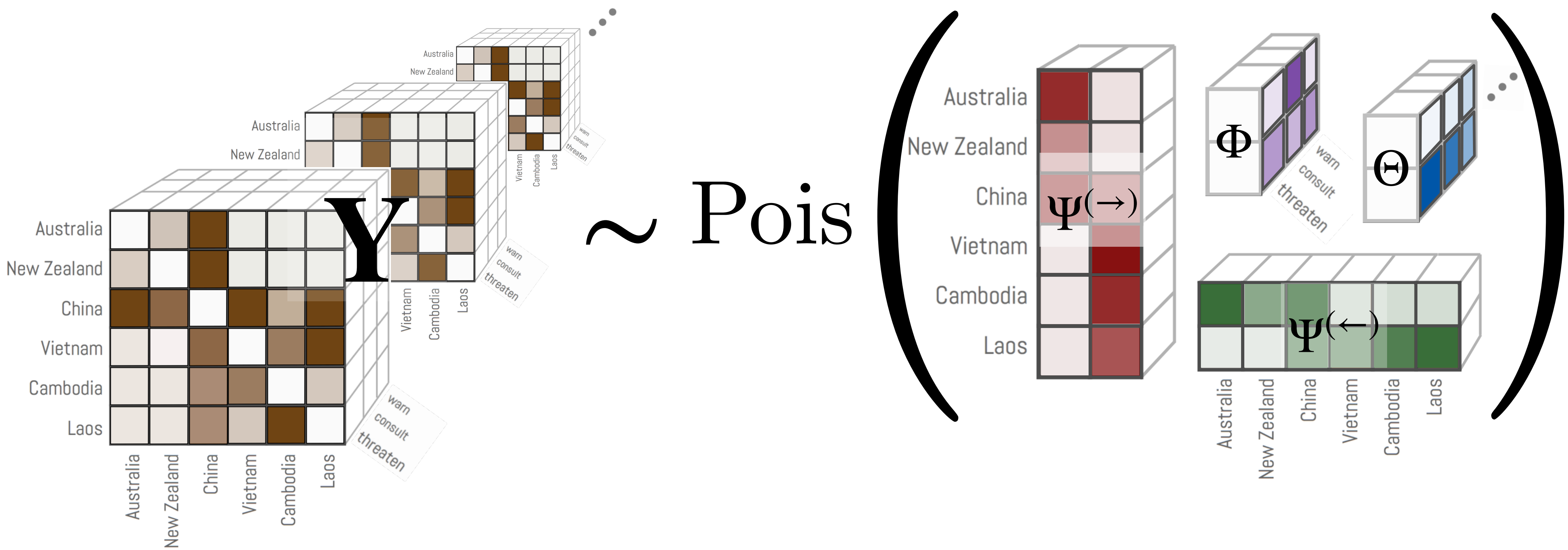

how active **class** q is
at **time** t

$$y^{(t)}_{i \xrightarrow{a} j} \sim \text{Pois}\left(\sum_{q=1}^{Q} \psi^{(\rightarrow)}_{iq}\, \psi^{(\leftarrow)}_{jq}\, \phi_{aq}\, \theta^{(t)}_{q}\, \lambda_q\right)$$

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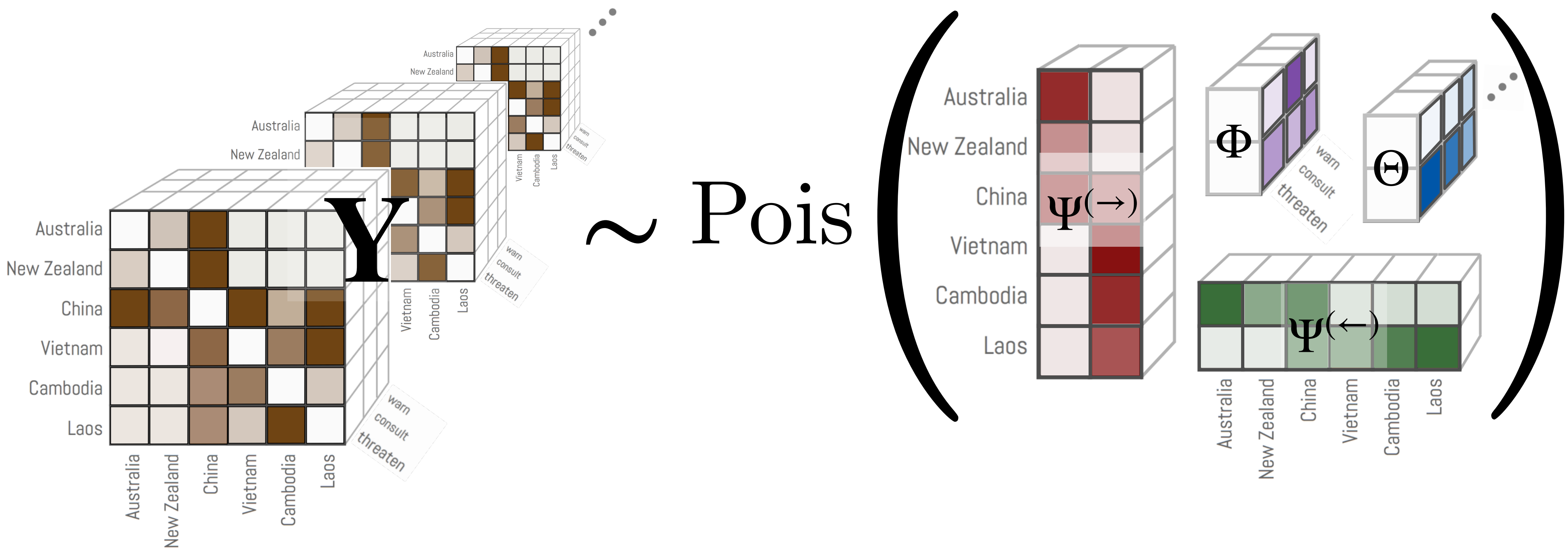

how active **class** q is overall

$$y^{(t)}_{i \xrightarrow{a} j} \sim \mathrm{Pois}\left(\sum_{q=1}^{Q} \psi^{(\rightarrow)}_{iq}\, \psi^{(\leftarrow)}_{jq}\, \phi_{aq}\, \theta^{(t)}_{q}\, \lambda_q\right)$$

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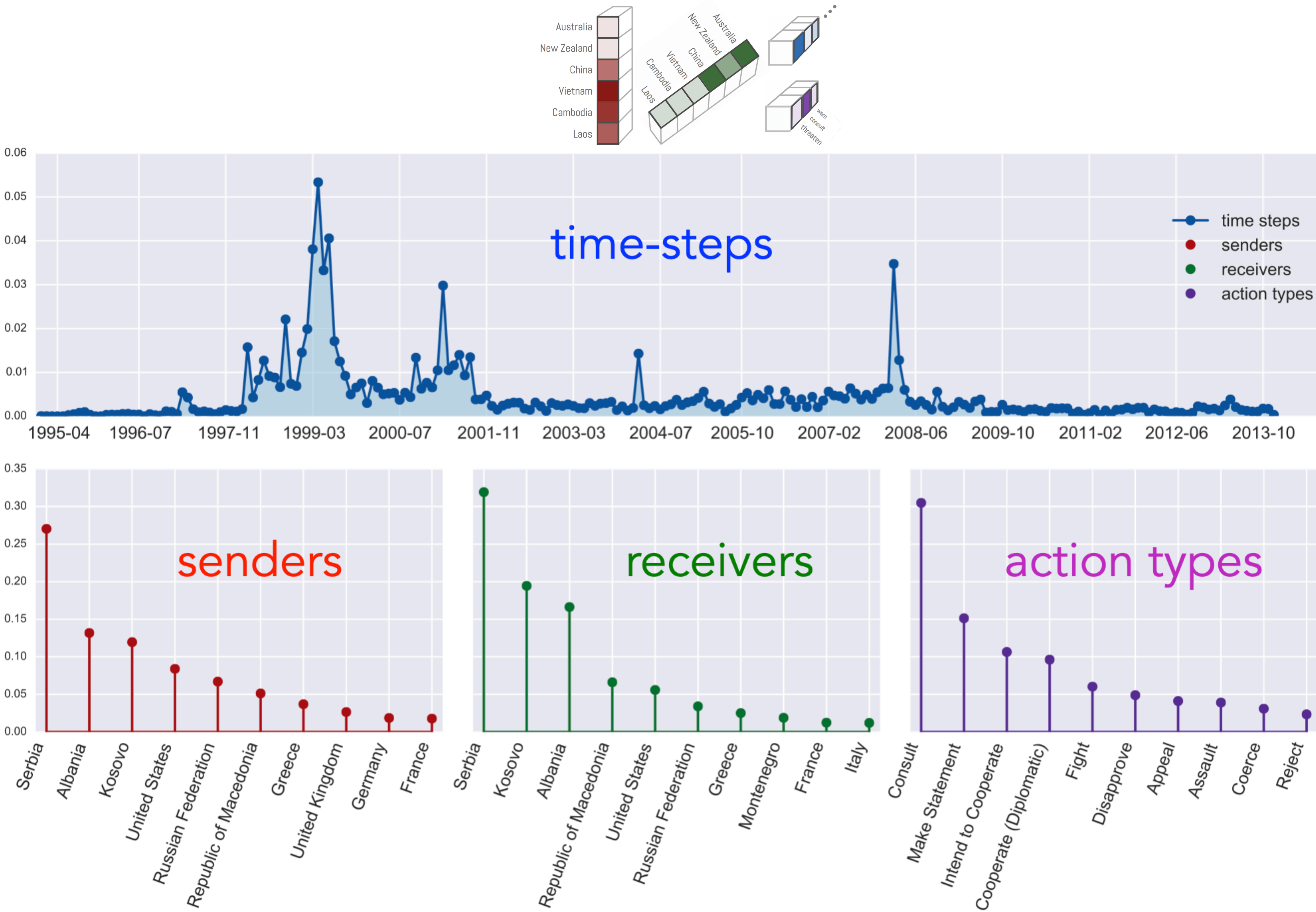
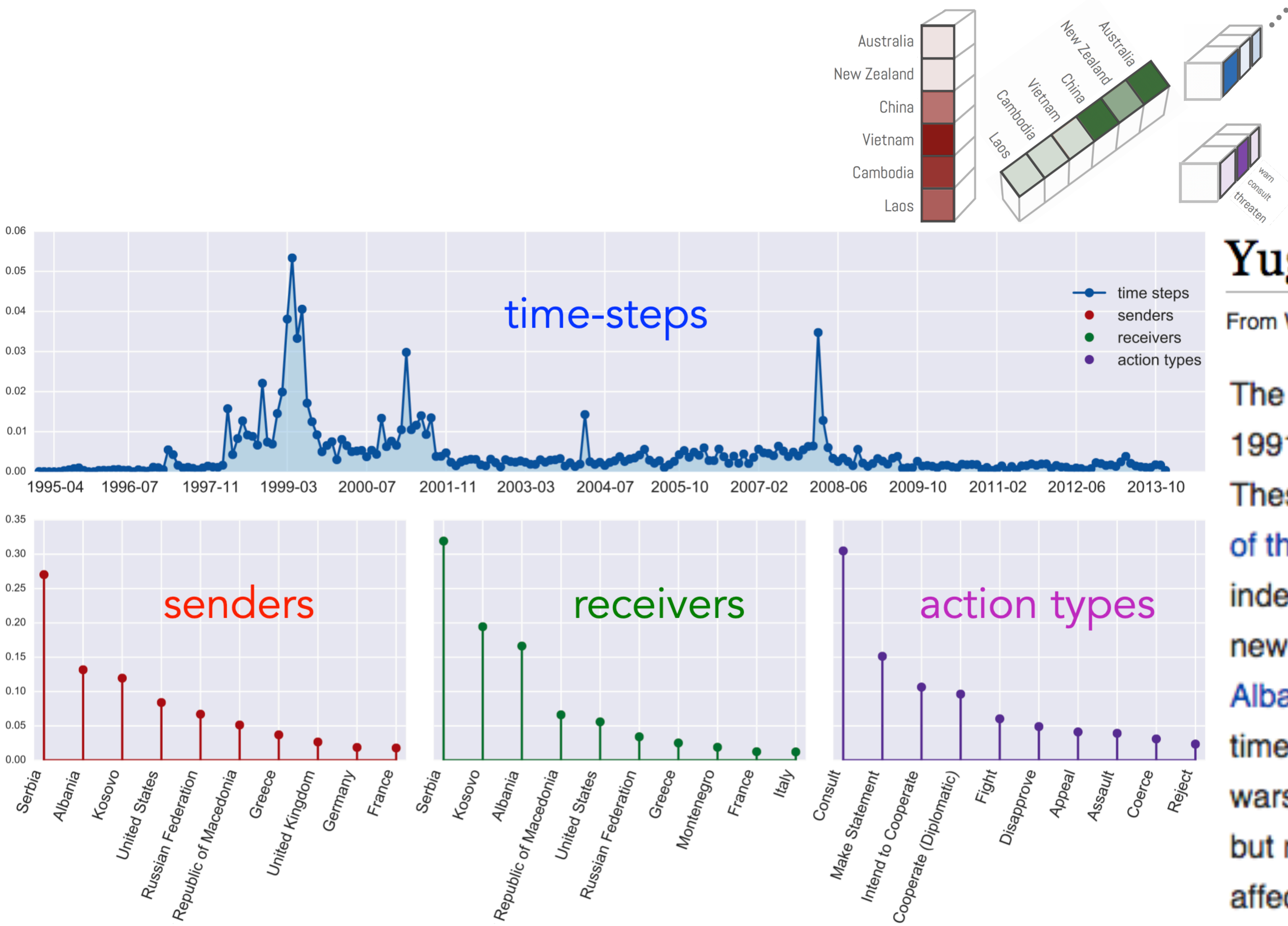
EXAMPLE CLASS:



EXAMPLE CLASS: Yugoslav Wars



Yugoslav Wars

From Wikipedia, the free encyclopedia

The **Yugoslav Wars** were ethnic conflicts fought from 1991 to 2001 inside the territory of the former Yugoslavia. These wars accompanied and/or facilitated the breakup of the country, when its constituent republics declared independence, but the issues of ethnic minorities in the new countries (chiefly Serbs in central parts and Albanians in the southeast) were still unresolved at the time the republics were recognized internationally. The wars are generally considered to be a series of separate but related military conflicts which, occurred in, and affected most of the former Yugoslav republics:[2][3][4]

- War in Slovenia (1991)
- Croatian War of Independence (1991–1995)
- Bosnian War (1992–1995)
- Kosovo War (1998–1999), including the NATO bombing of Yugoslavia
- Insurgency in the Preševo Valley (1999–2001)
- Insurgency in the Republic of Macedonia (2001)

Classes almost always describe easily recognizable **multilateral** relations / events

OUTLINE

- **Motivation:** Dyadic event data (and its challenges)

- **Non-negative (Poisson) tensor decompositions**
  - **CP** extracts "multilateral" structure [Schein et al., 2015]
  - **Tucker** extracts "communities" [Schein et al., 2016]
  - Why (allocative) Poisson factorization
  - **ALℓ_0CORE** avoids the "exponential blowup" [Hood & Schein, 2024]

Appendix (*time permitting*)

- **State-space** modeling
  - **Gamma state-space** models [Schein et al., 2016b; Schein et al., 2019]
  - **Modeling "escalation"** with a new matrix prior [Stoehr et al., 2023]

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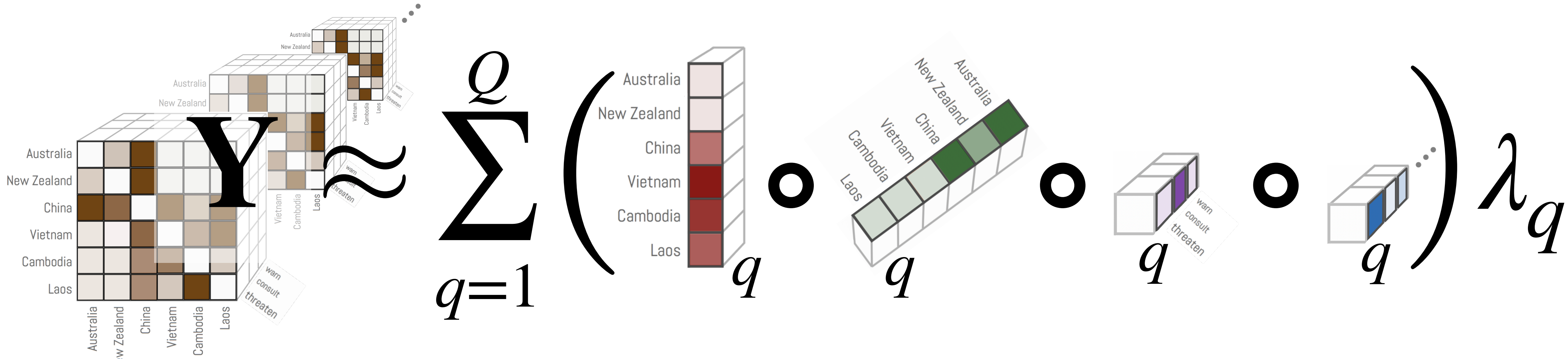
TUCKER DECOMPOSITION

- CP is unappealing if mode dimensions are very different
- e.g., (country x country x action x month) for 1995-2013 ICEWS data

  = **(250 x 250 x 20 x 216)**
- Q = 100 classes; each must learn a different 20-dimensional action factor



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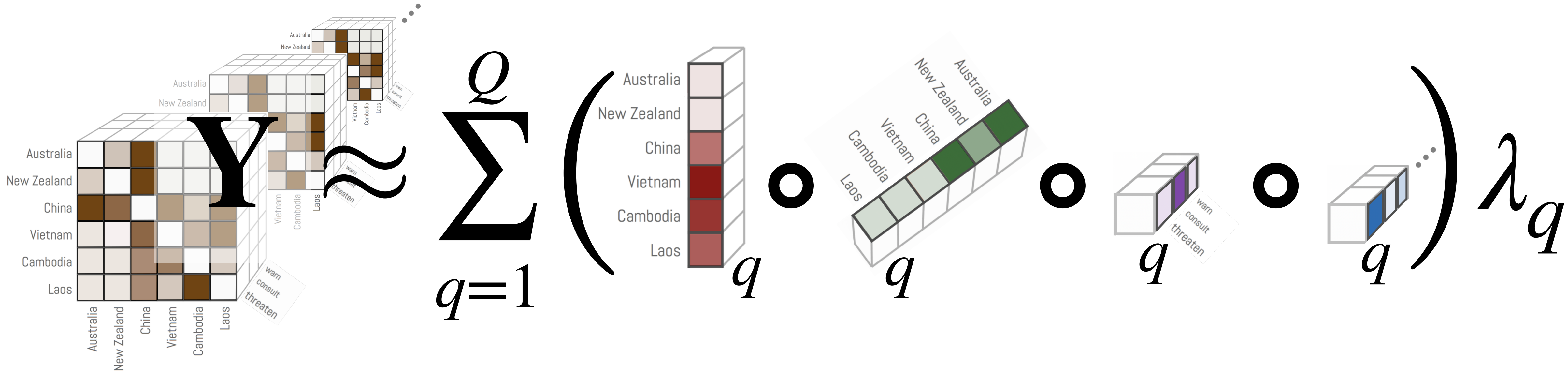
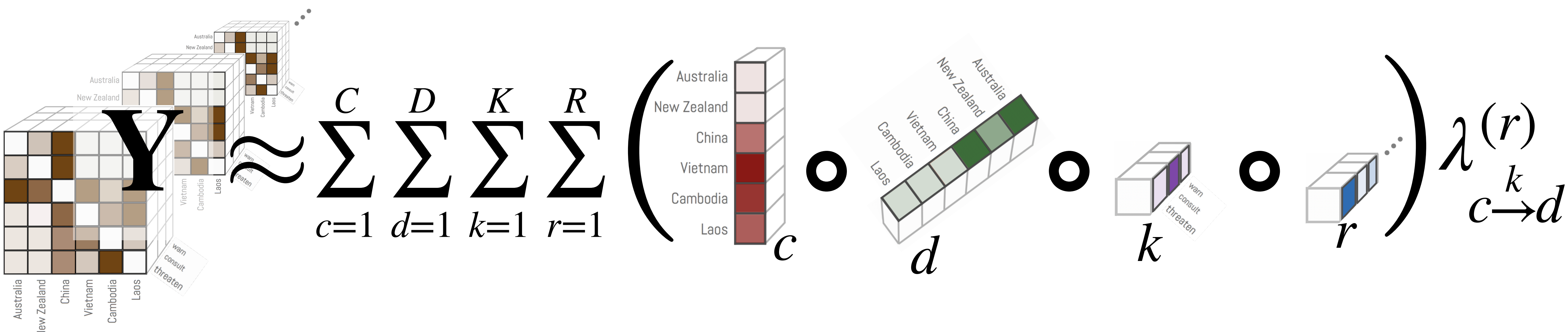


$$\mathbf{Y} \approx \sum_{q=1}^{Q} \left(q \circ q \circ q \circ q \right) \lambda_q$$

- Tucker decomposition fixes this…

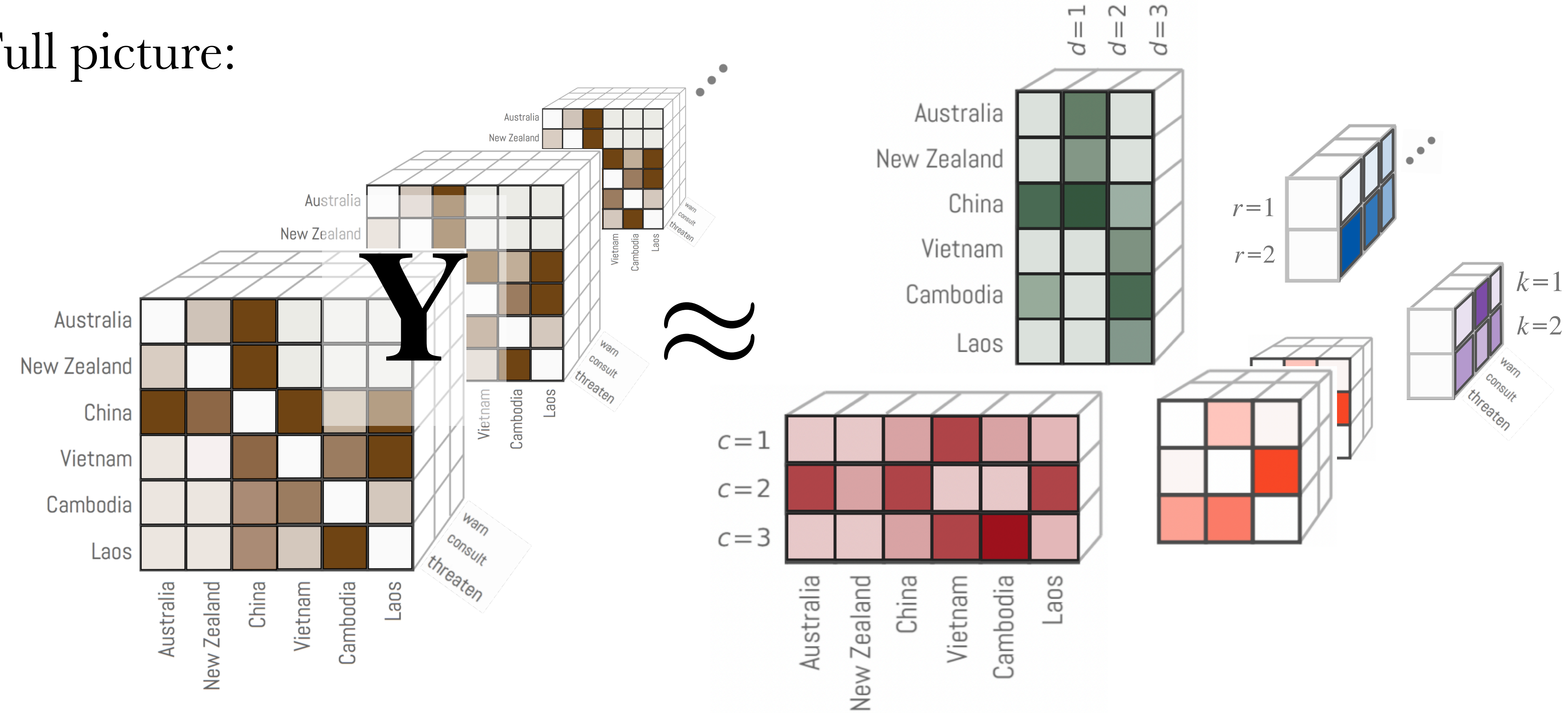


$$\mathbf{Y} \approx \sum_{c=1}^{C} \sum_{d=1}^{D} \sum_{k=1}^{K} \sum_{r=1}^{R} \left(c \circ d \circ k \circ r \right) \lambda^{(r)}_{c \xrightarrow{k} d}$$

- Every class is now a unique combination (c, d, k, r) of **shared factors**

TUCKER DECOMPOSITION

- Full picture:



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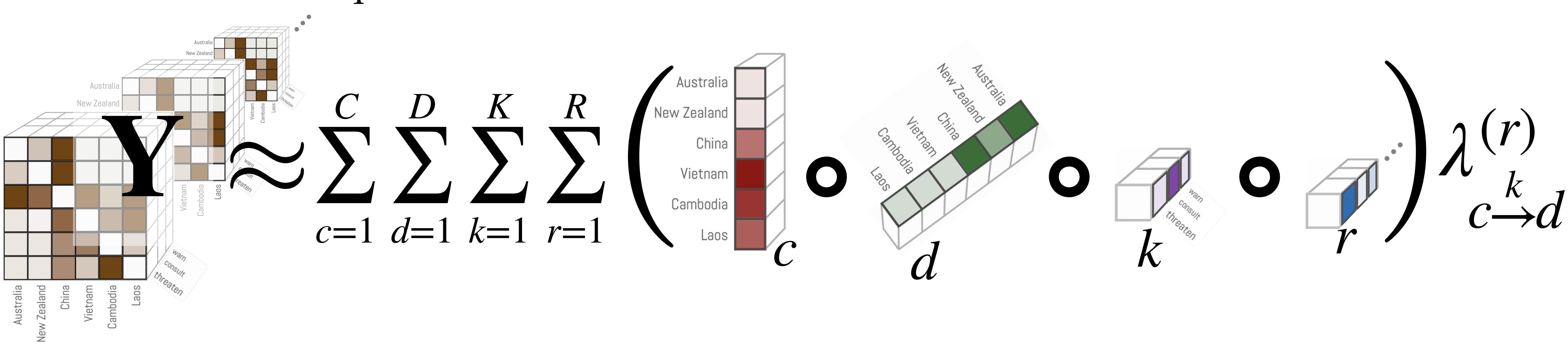

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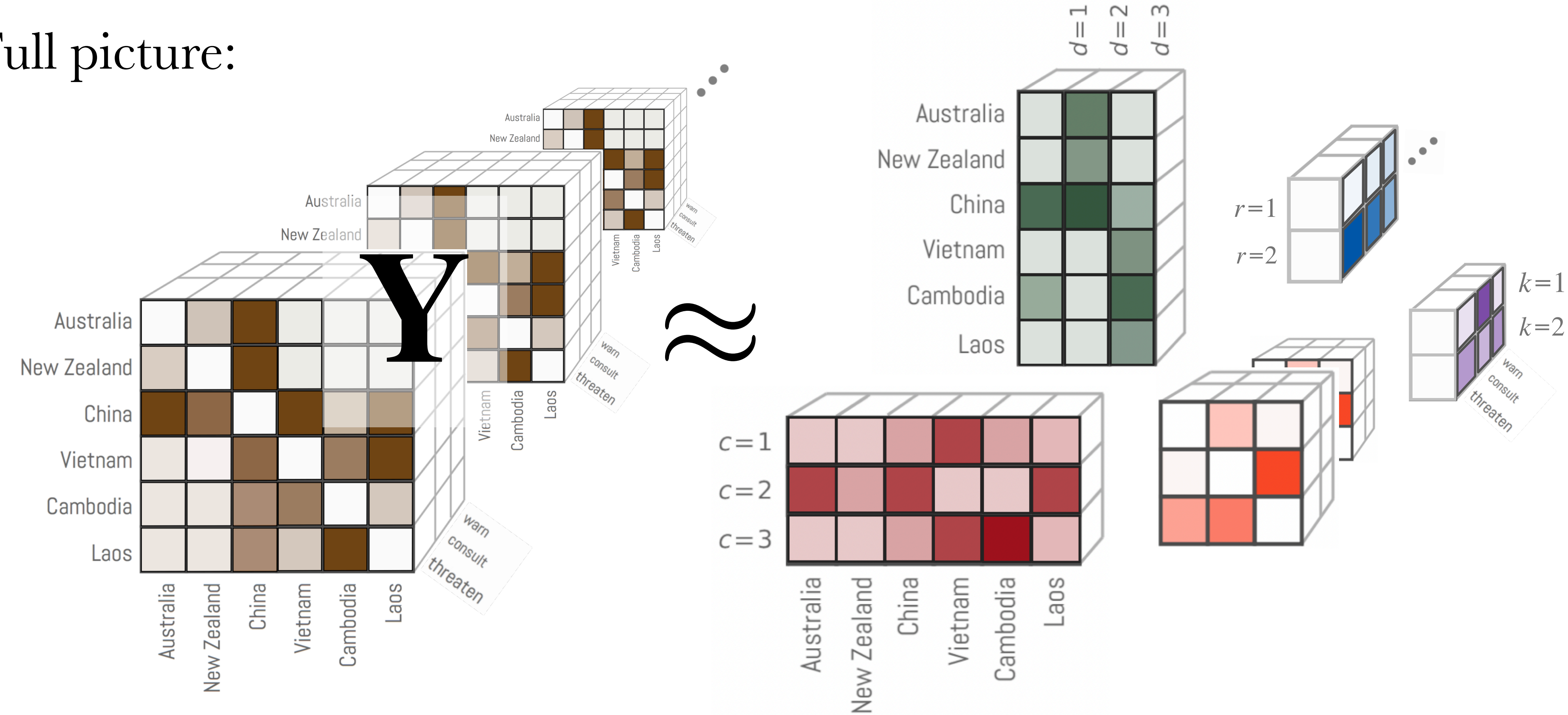
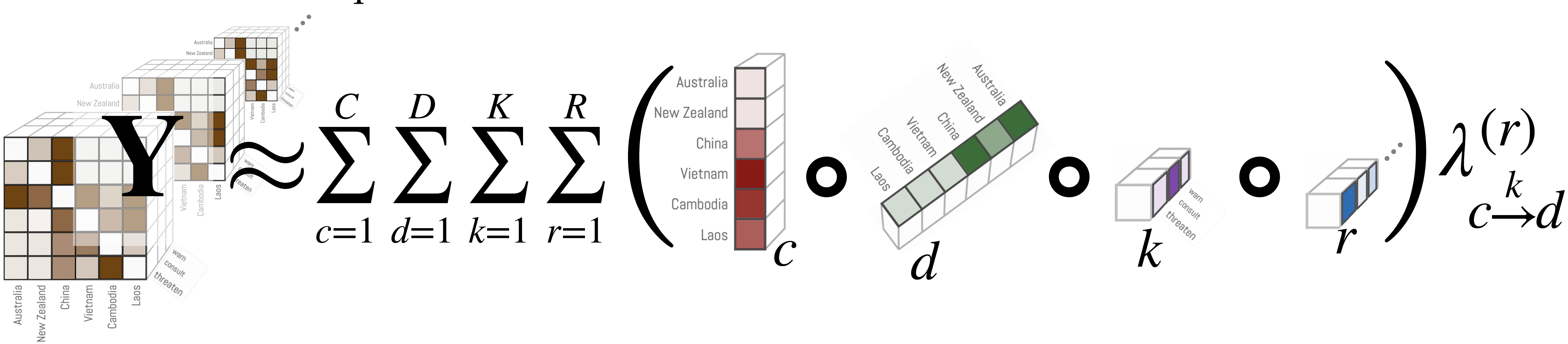


- Tucker decomposition fixes this… different number of factors in each mode



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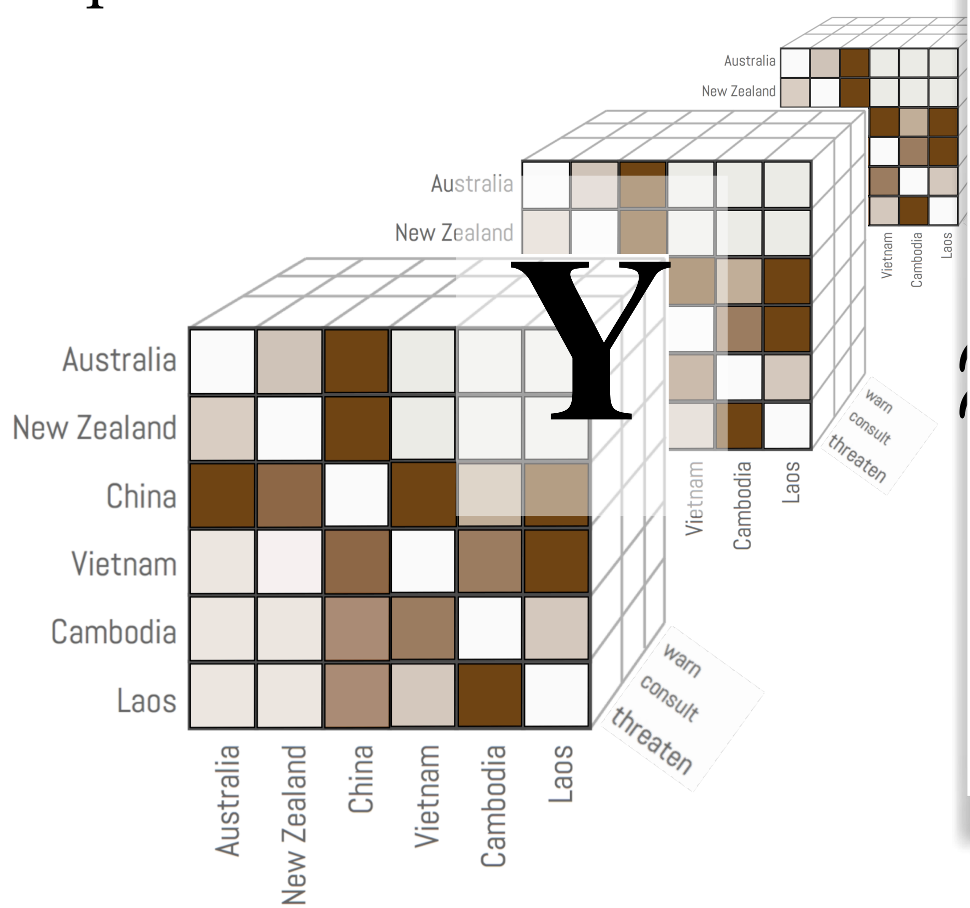
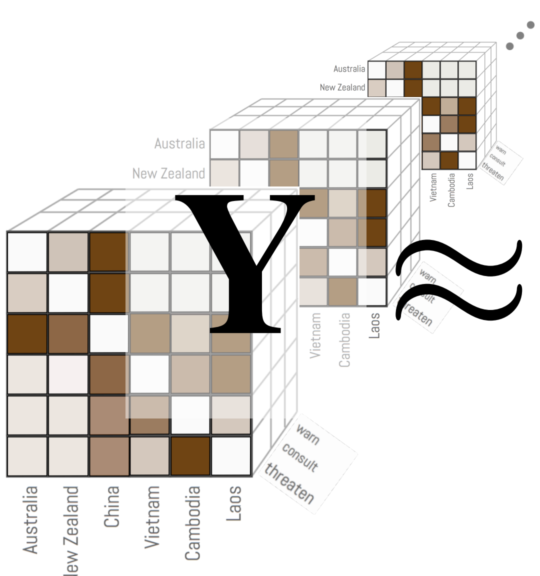


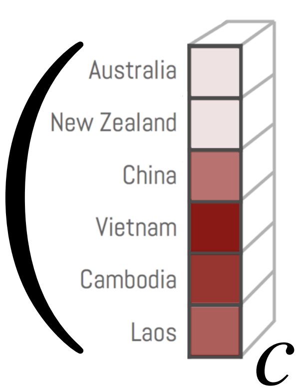
Convergent lines of research on Tucker for multilayer networks / dyadic event data:

- Tucker (regression) for dyadic event data: […] Minhas, Hoff, & Ward (2015), Hoff (2015), Hoff (2016), […]
- Non-negative Tucker for dyadic events: Schein et al. (2016)
- Non-negative Tucker for multilayer networks: De Bacco et al. (2017), Aguiar et al. (2023)

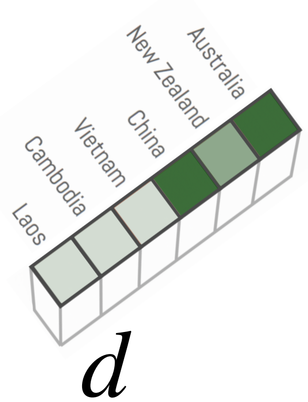
- Tucker decomposition fixes this… different number of factors in each mode



$$\approx \sum_{c=1}^{C}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{r=1}^{R}\Big($$



$\circ$



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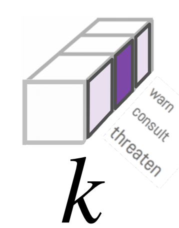

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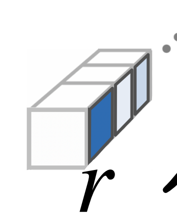

$$\Big)\lambda^{(r)}_{c\xrightarrow{k}d}$$

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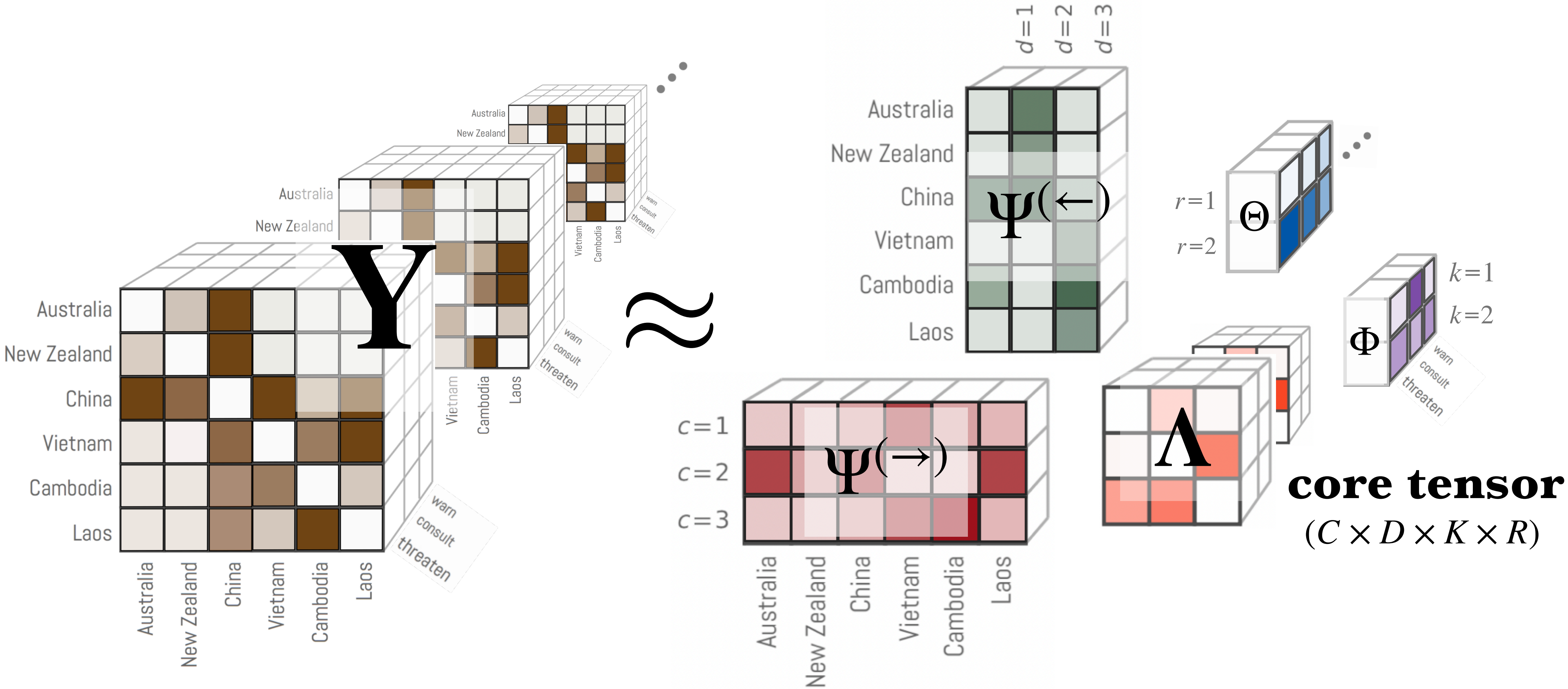

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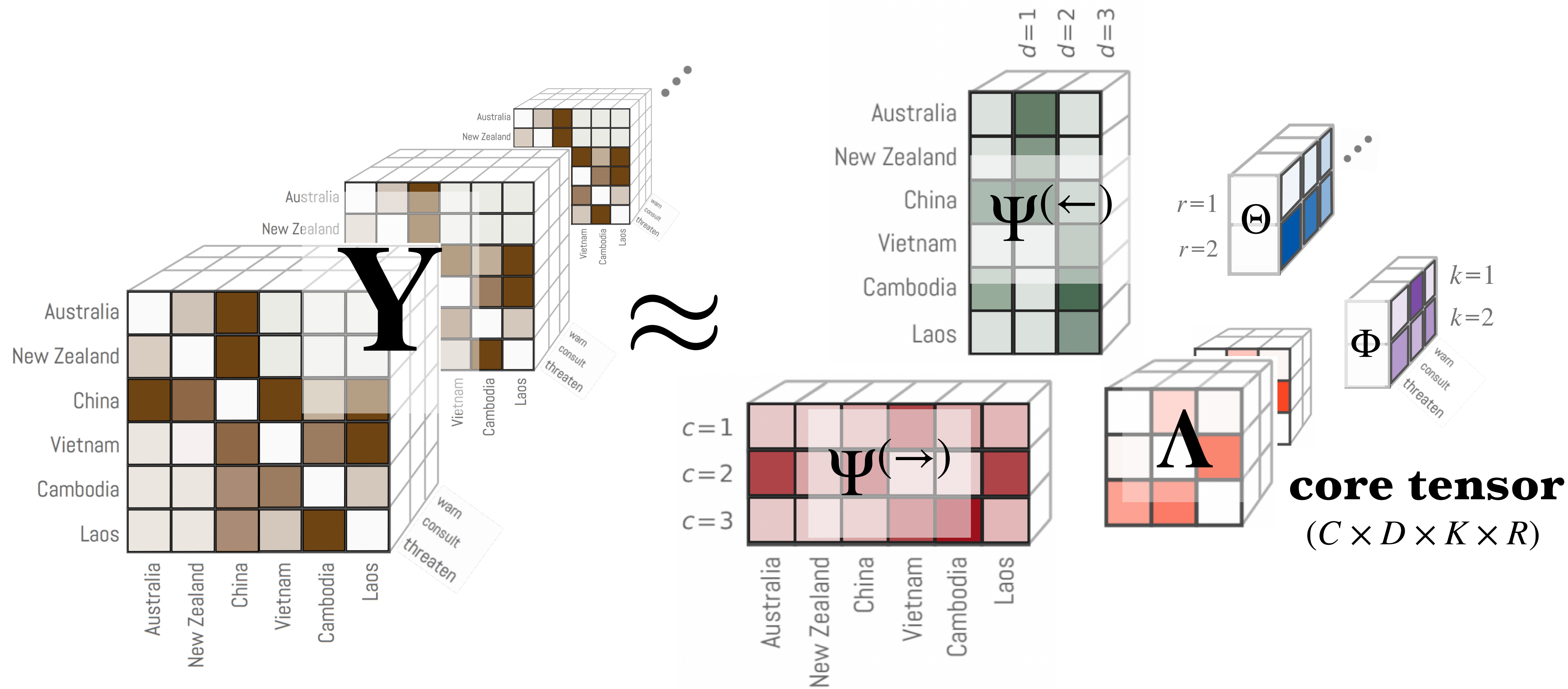

- **Optional:** $\Psi^{(\rightarrow)} = \Psi^{(\leftarrow)} = \Psi$

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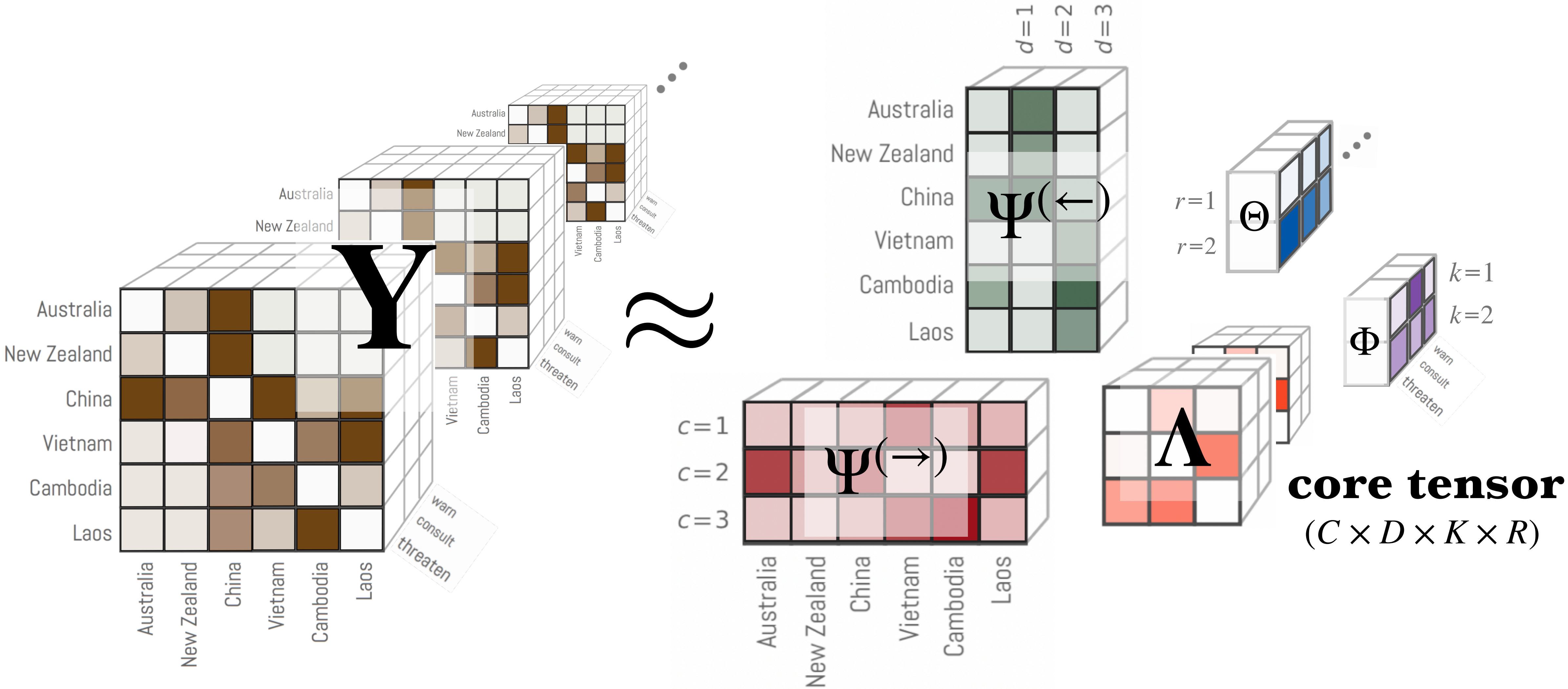

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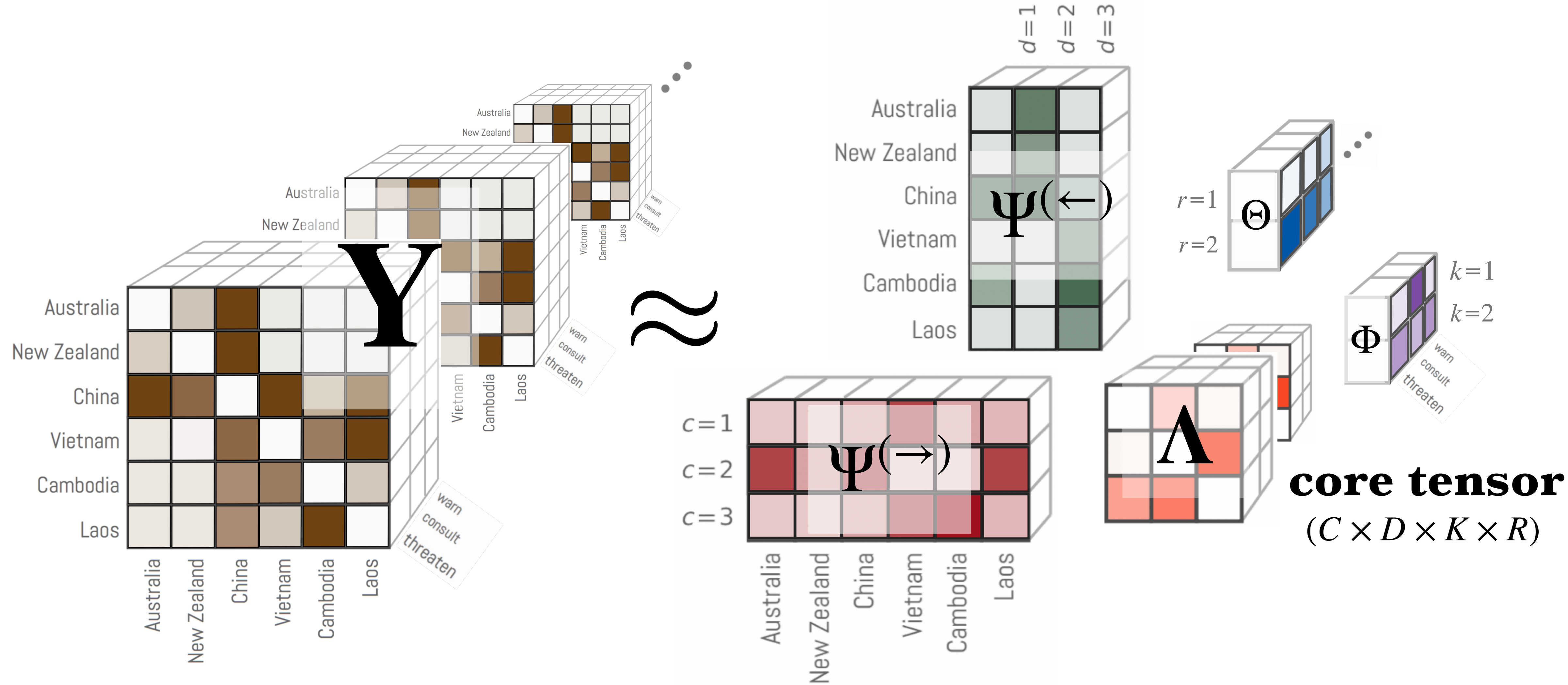

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non-negative

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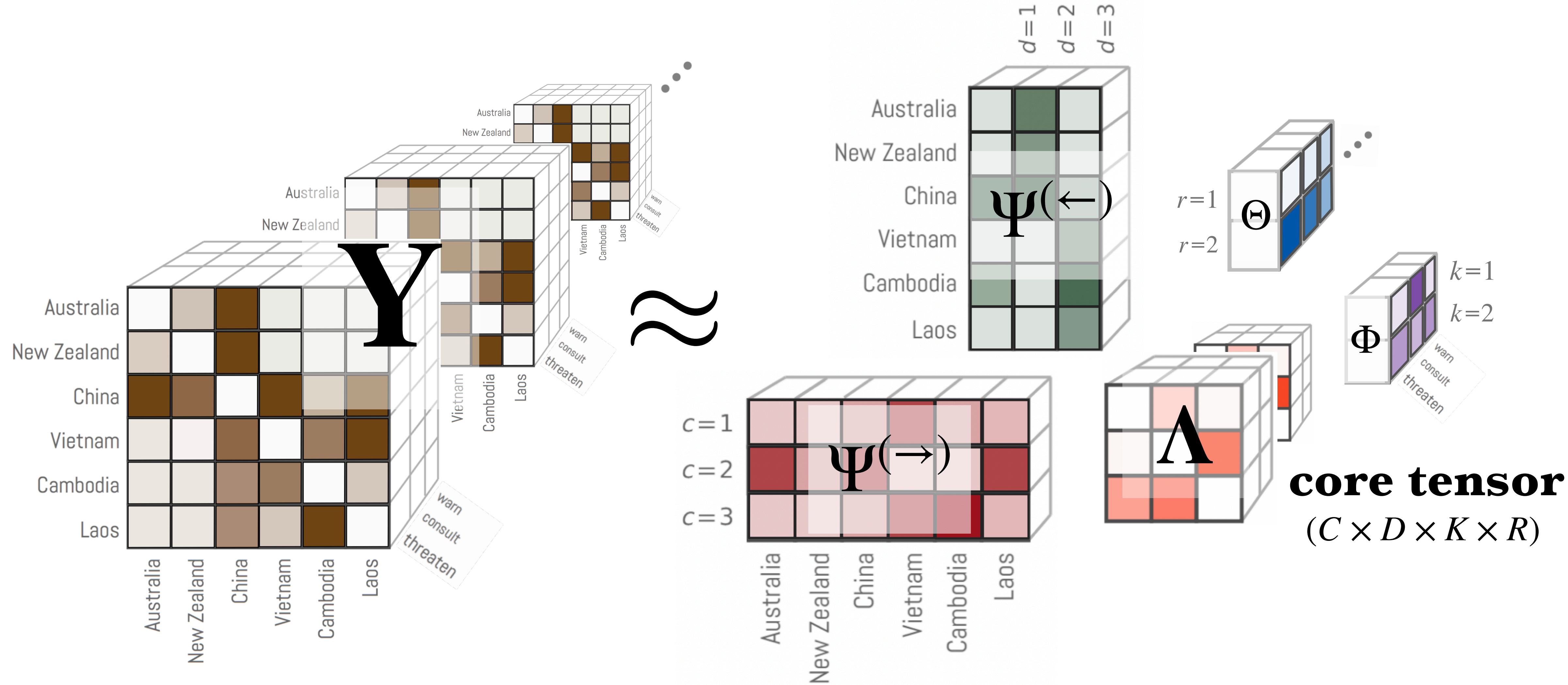

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how active **country** i is
in (sender) **community** c

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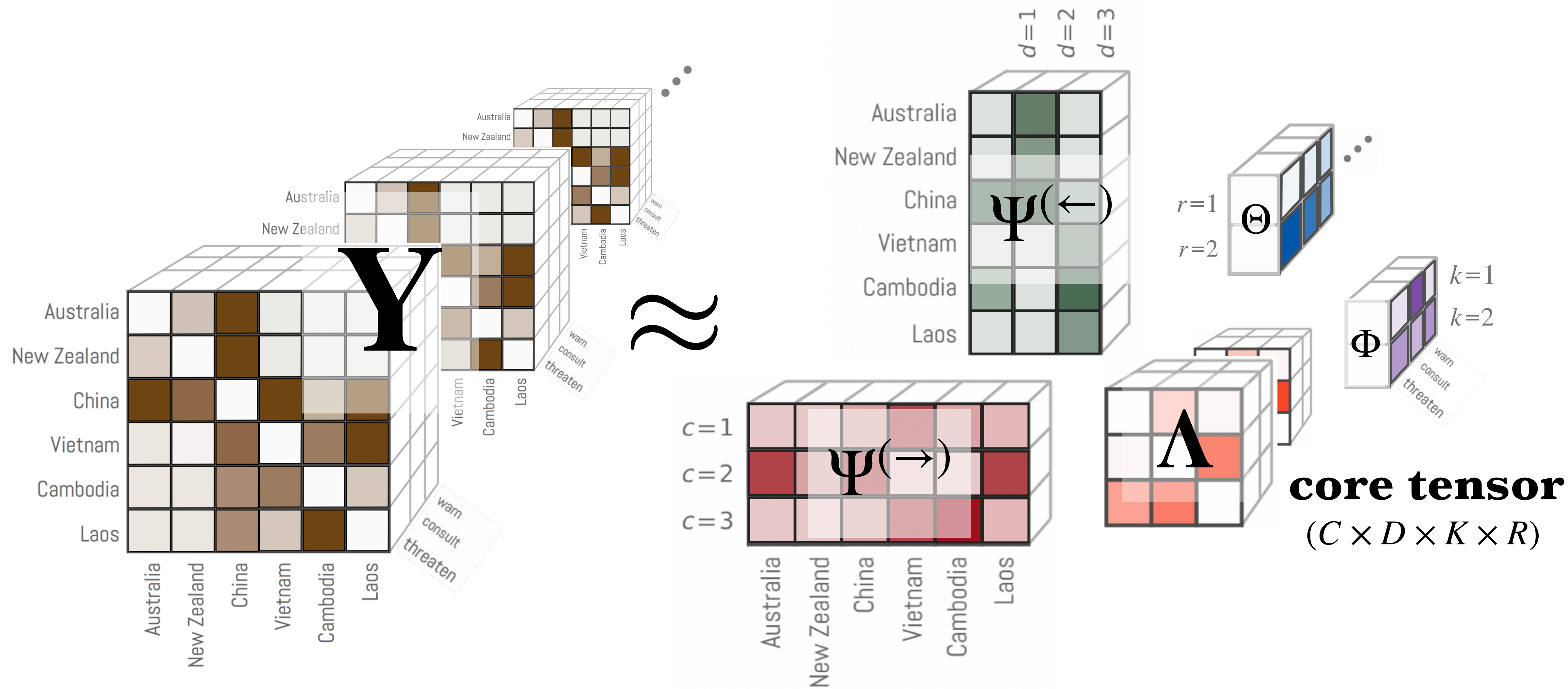

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how active **country** j is
in (receiver) **community** d

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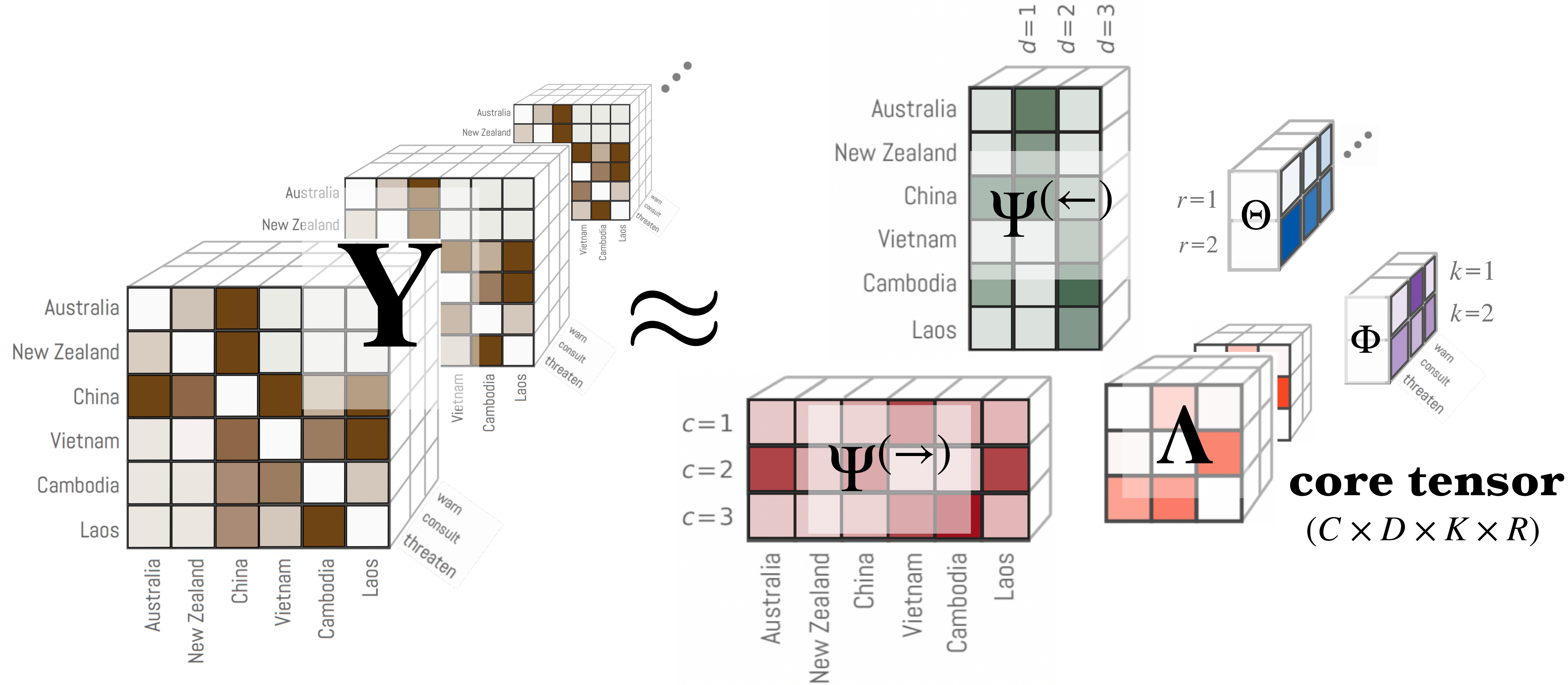

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how prevalent **action** a is in **topic** k

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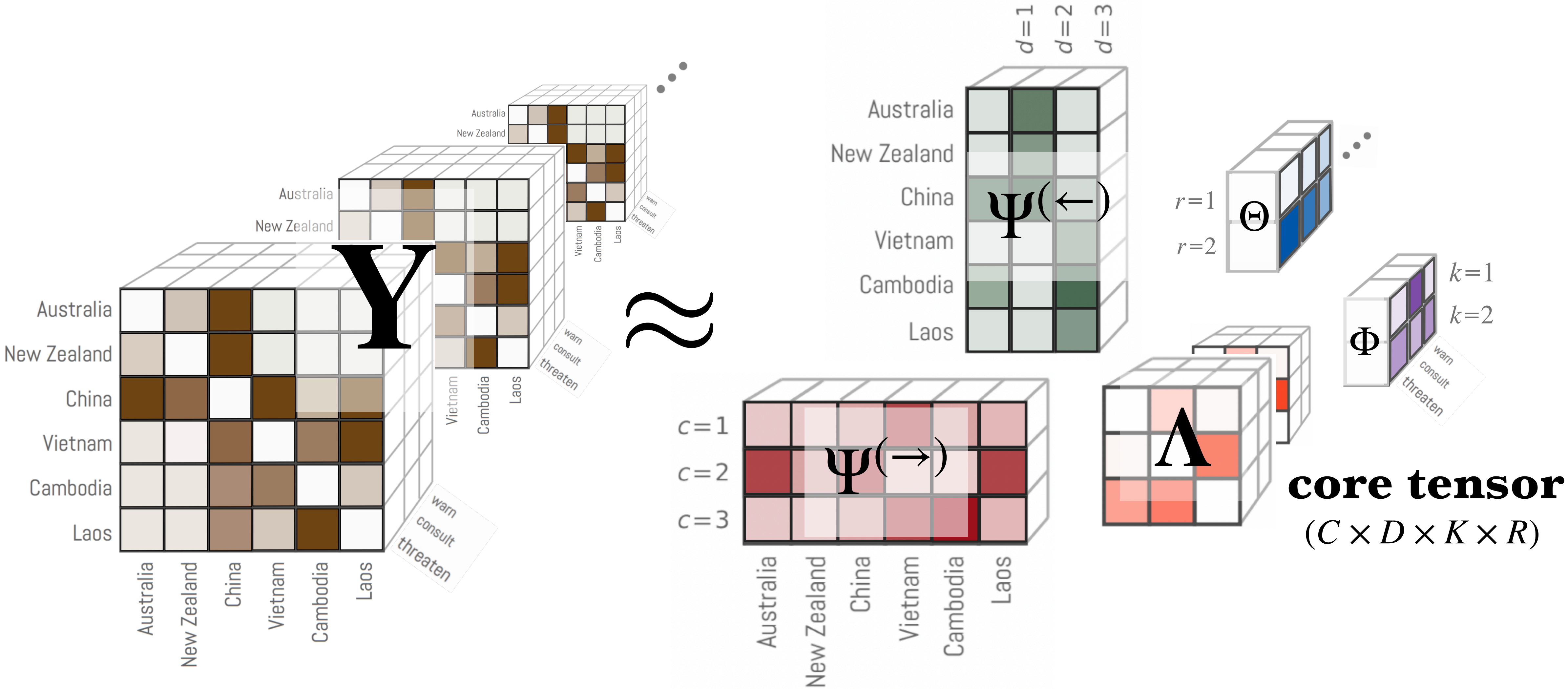

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how active **regime** r is at **time** t

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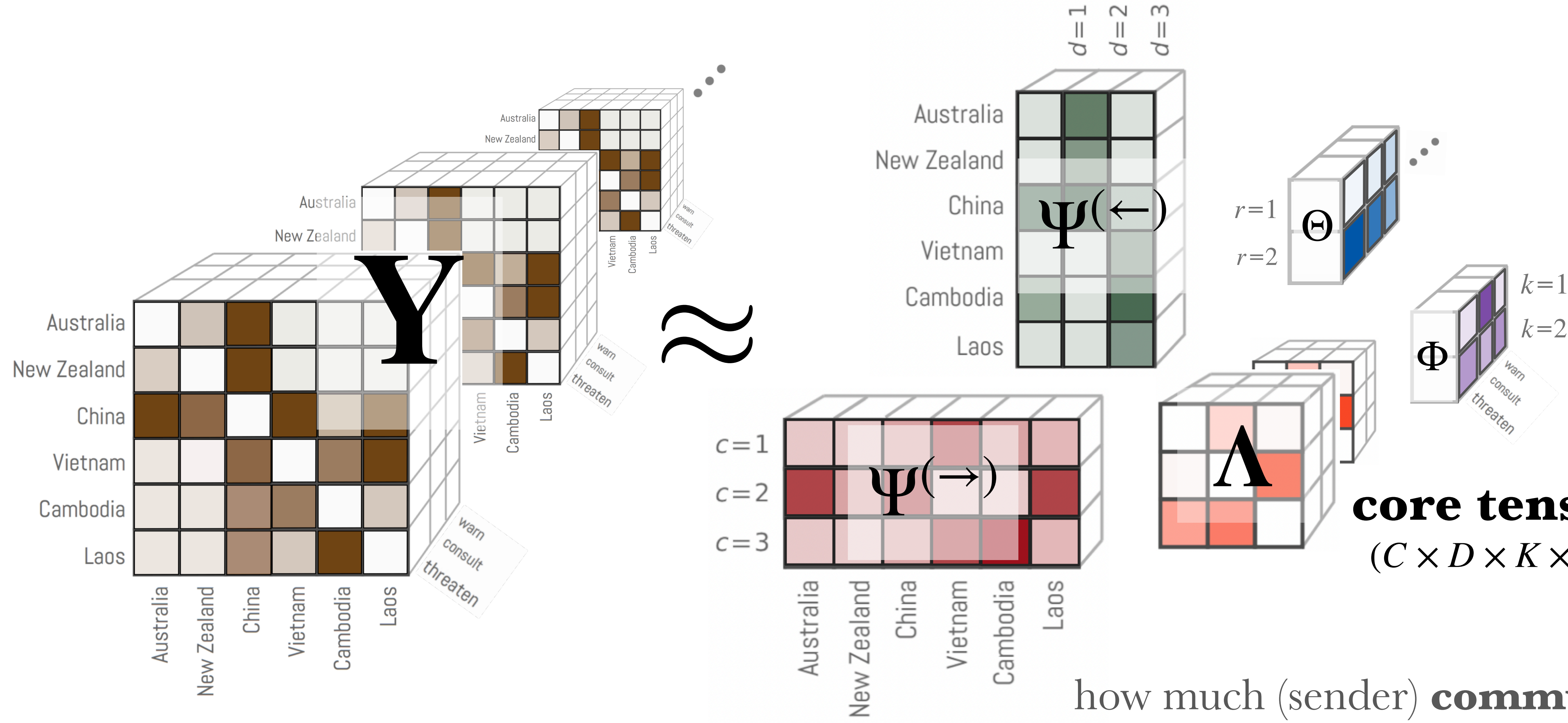

core tensor
$(C \times D \times K \times R)$

how much (sender) **community** c
takes **topic** k
toward (receiver) **community** d
under **regime** r

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$$y_{i \xrightarrow{a} j}^{(t)} \sim \text{Pois}\left(\sum_{c=1}^{C}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{r=1}^{R} \psi_{ic}^{(\rightarrow)}\, \psi_{jd}^{(\leftarrow)}\, \phi_{ak}\, \theta_r^{(t)}\, \lambda_{c \xrightarrow{k} d}^{(r)}\right)$$

EXAMPLE RESULTS

- GDELT tensor (1995-2000), monthly: (150 x 150 x 20 x 60)
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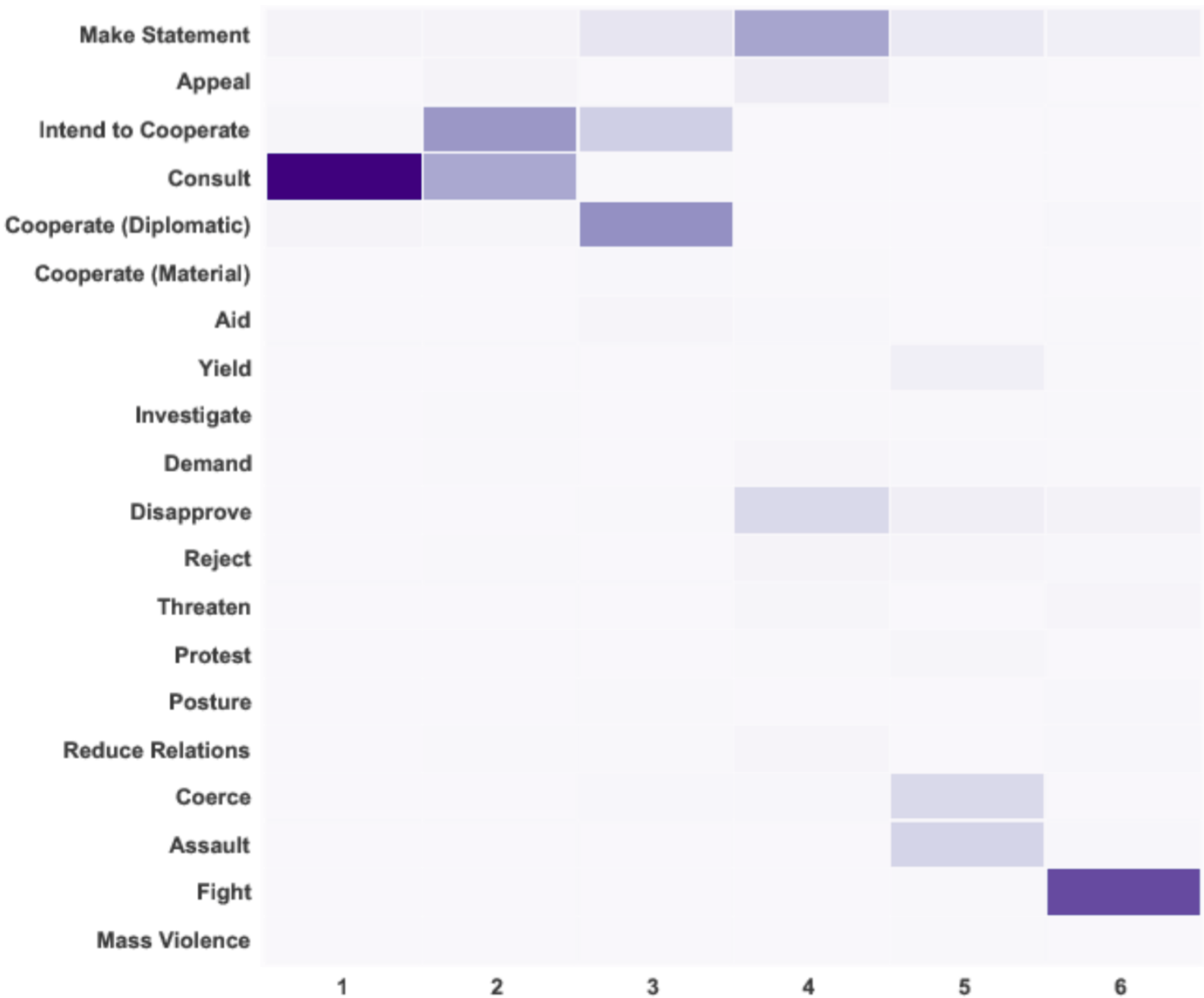

ϕ_{ak}

topics of **actions**

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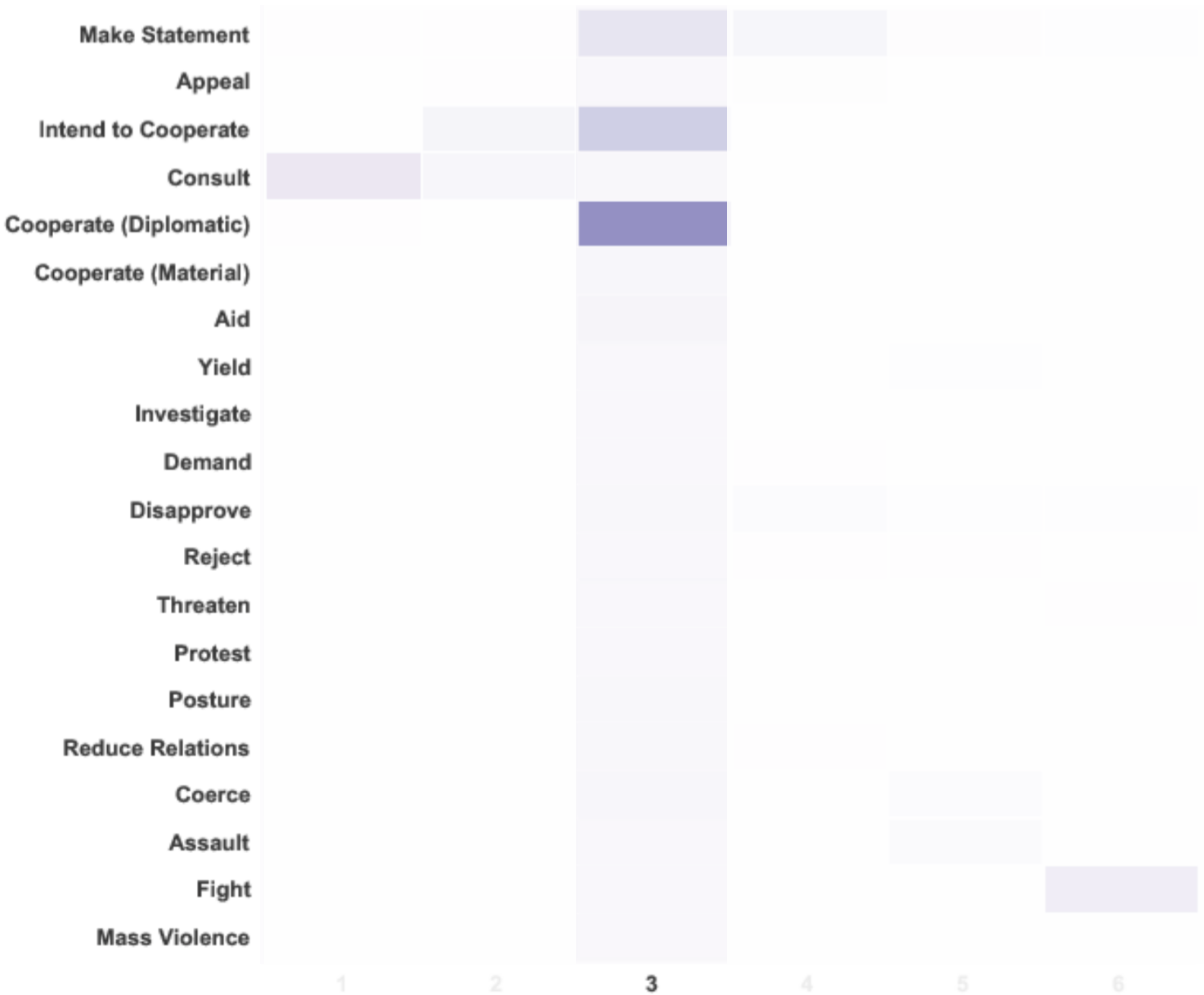

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$$\phi_{ak}$$

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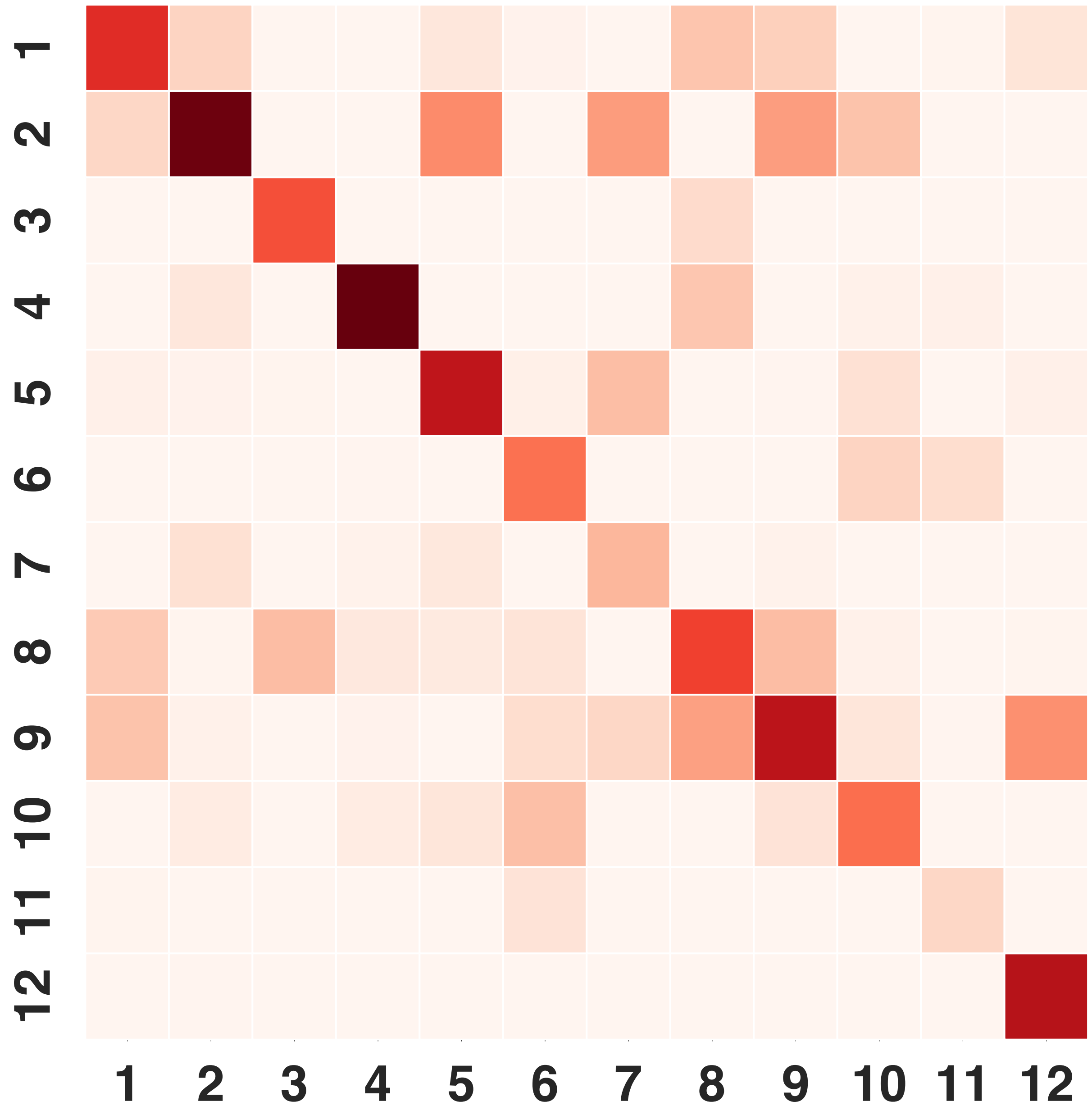
$$\lambda^{k}_{c \rightarrow d}$$



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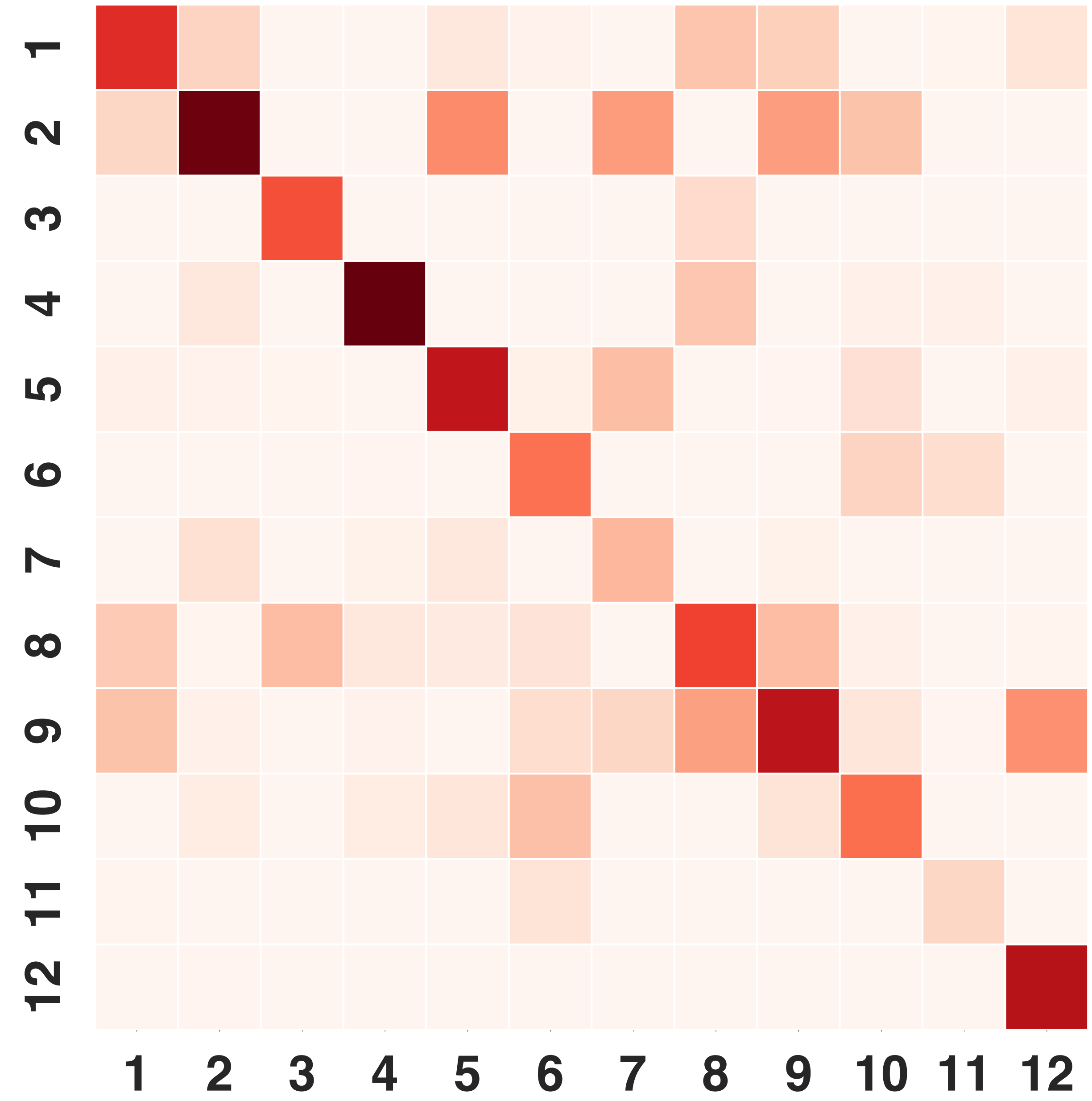
$$\lambda_{c \rightarrow d}^{k}$$

Slice of core tensor for
"Lip service" topic k



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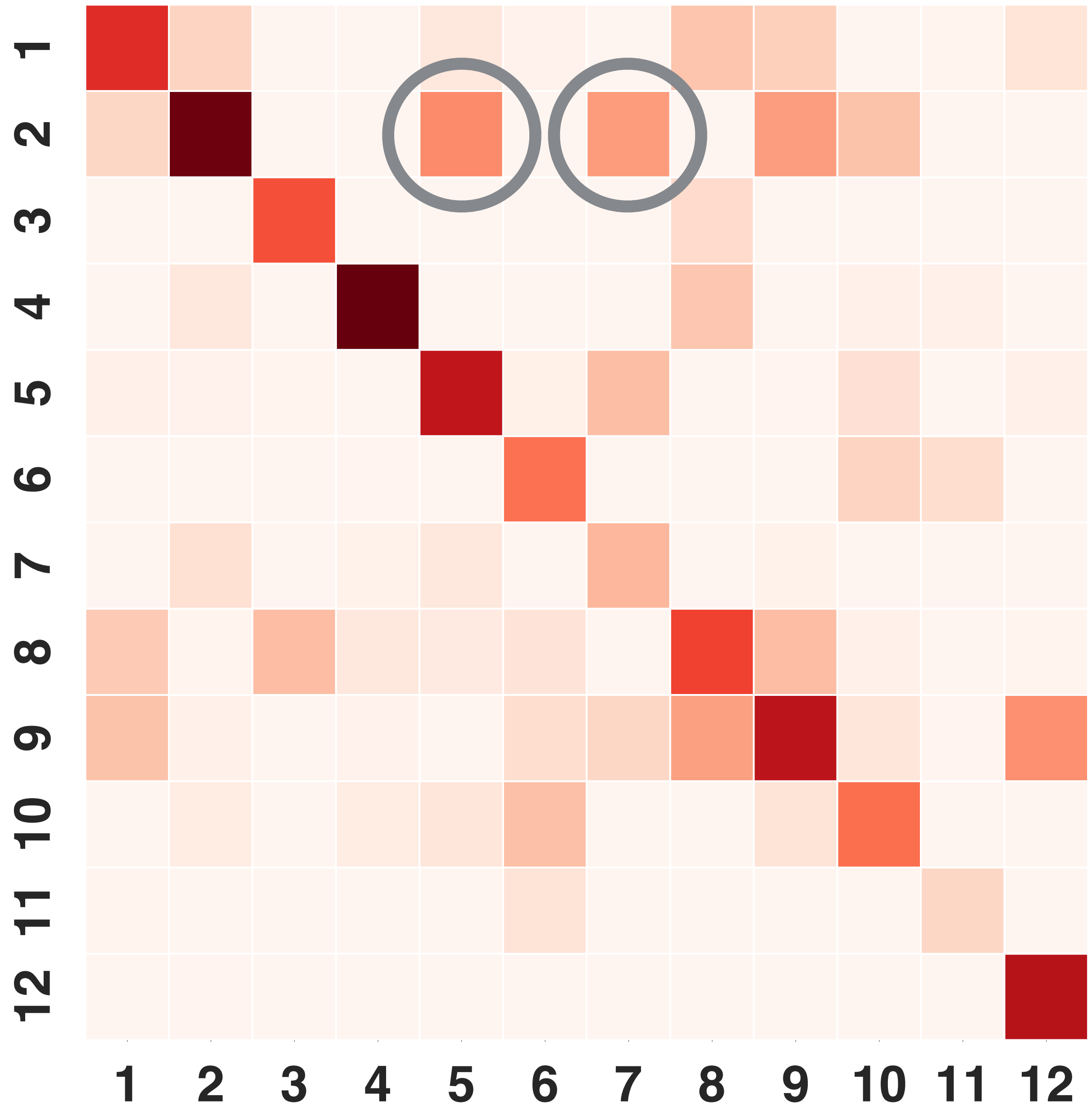
Slice of core tensor for "Lip service" topic k



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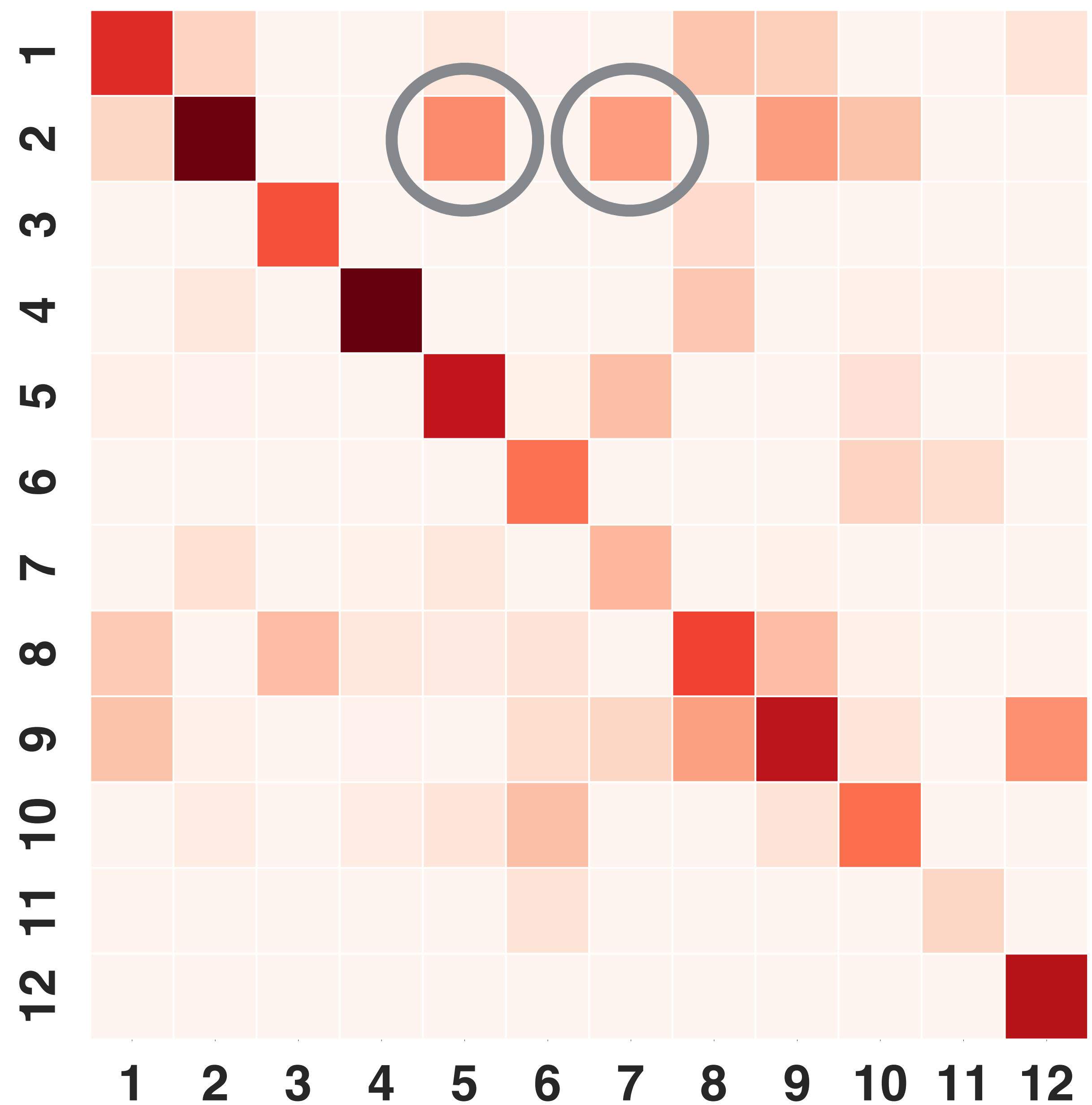

Country-community matrix Ψ

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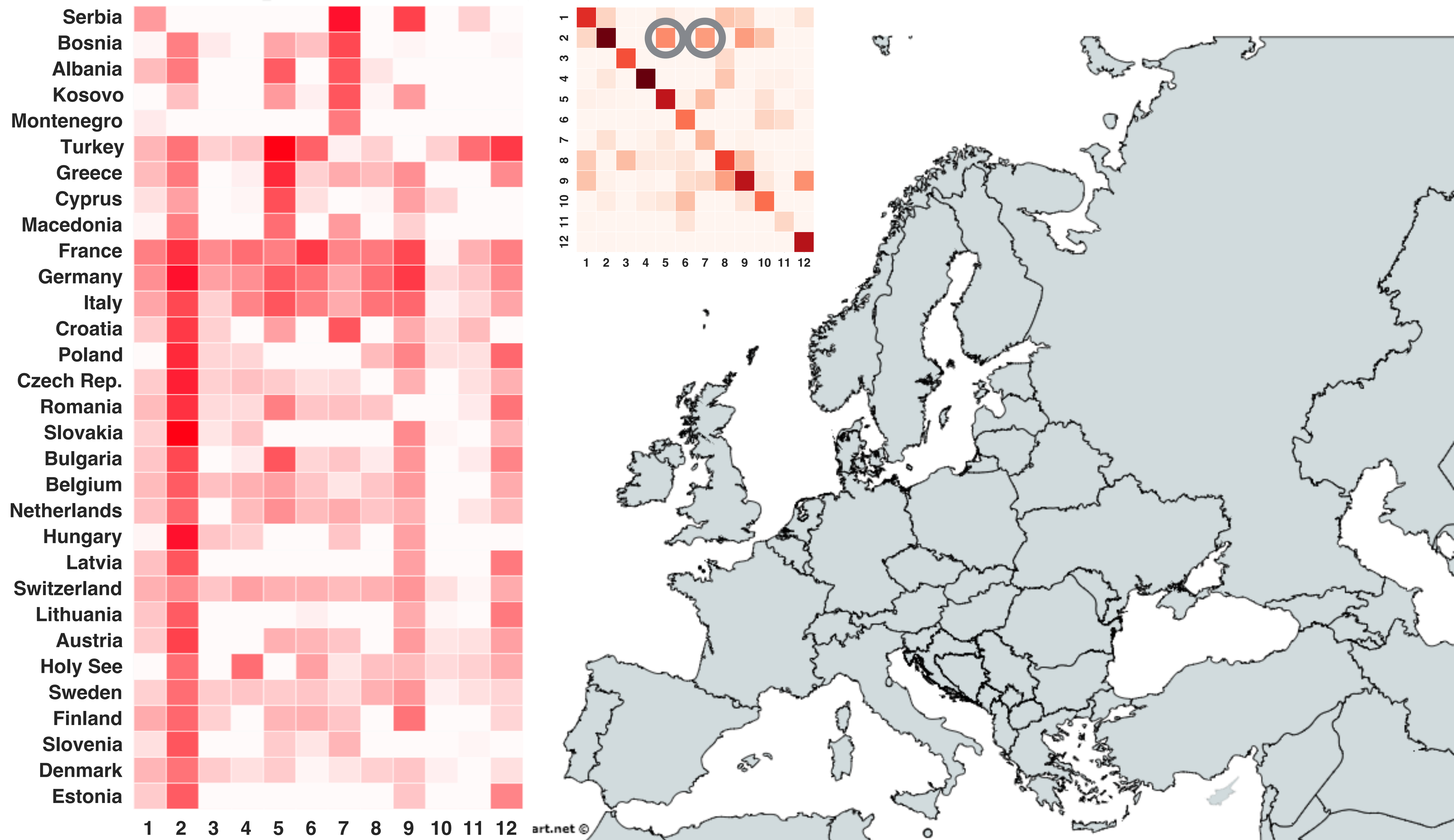

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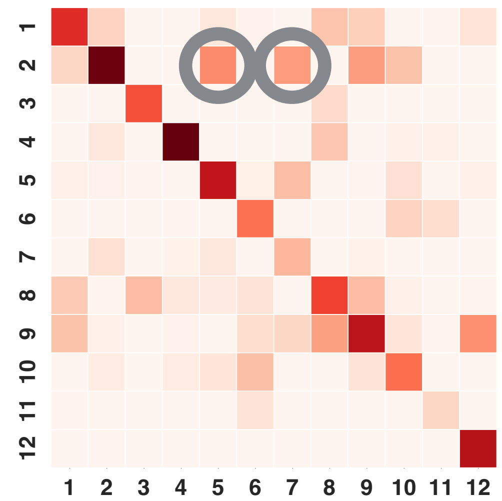

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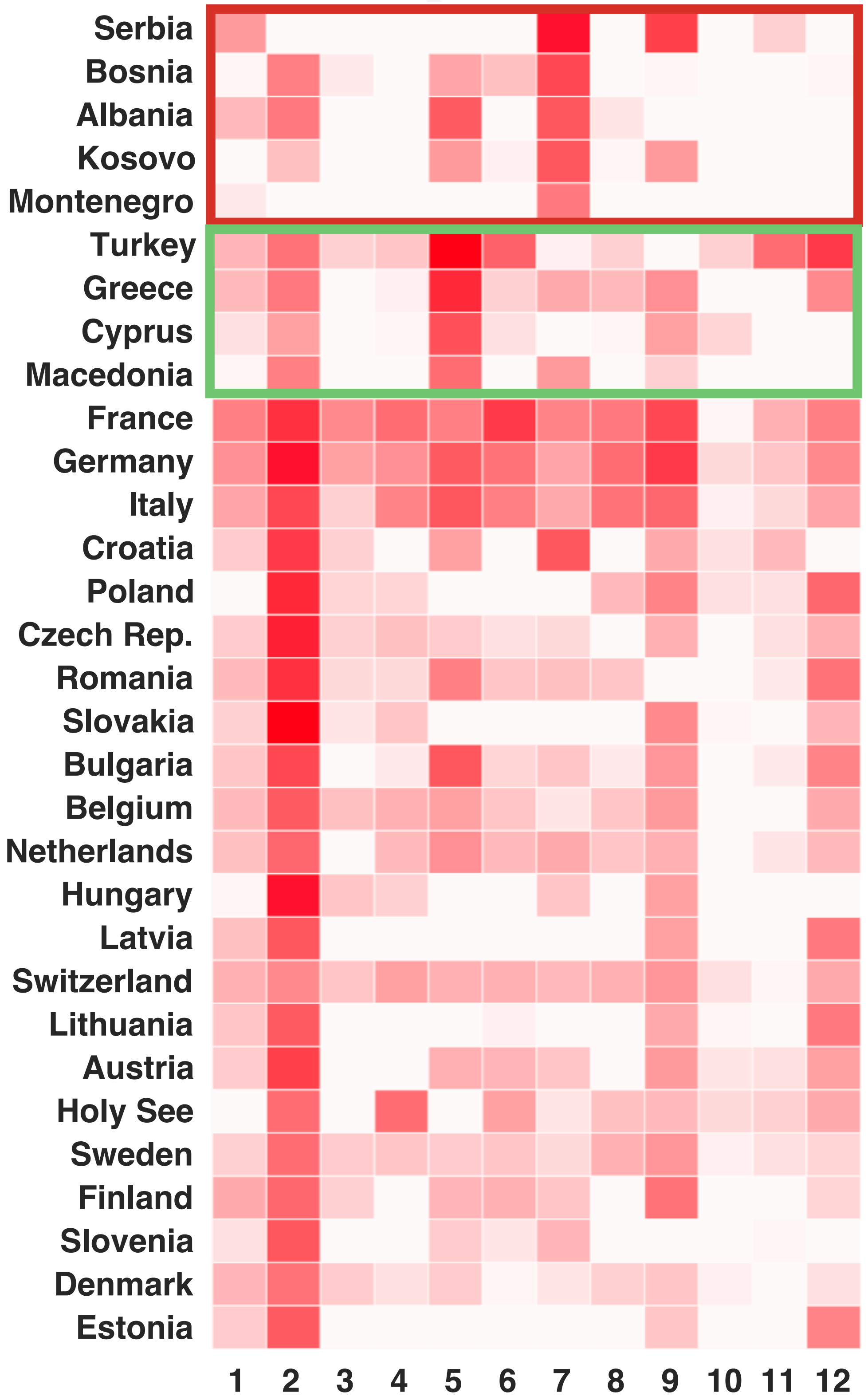

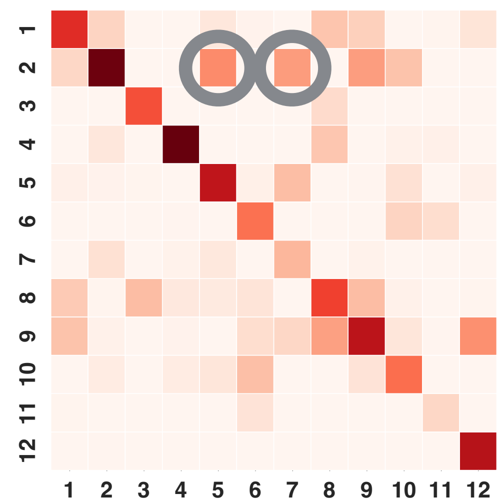

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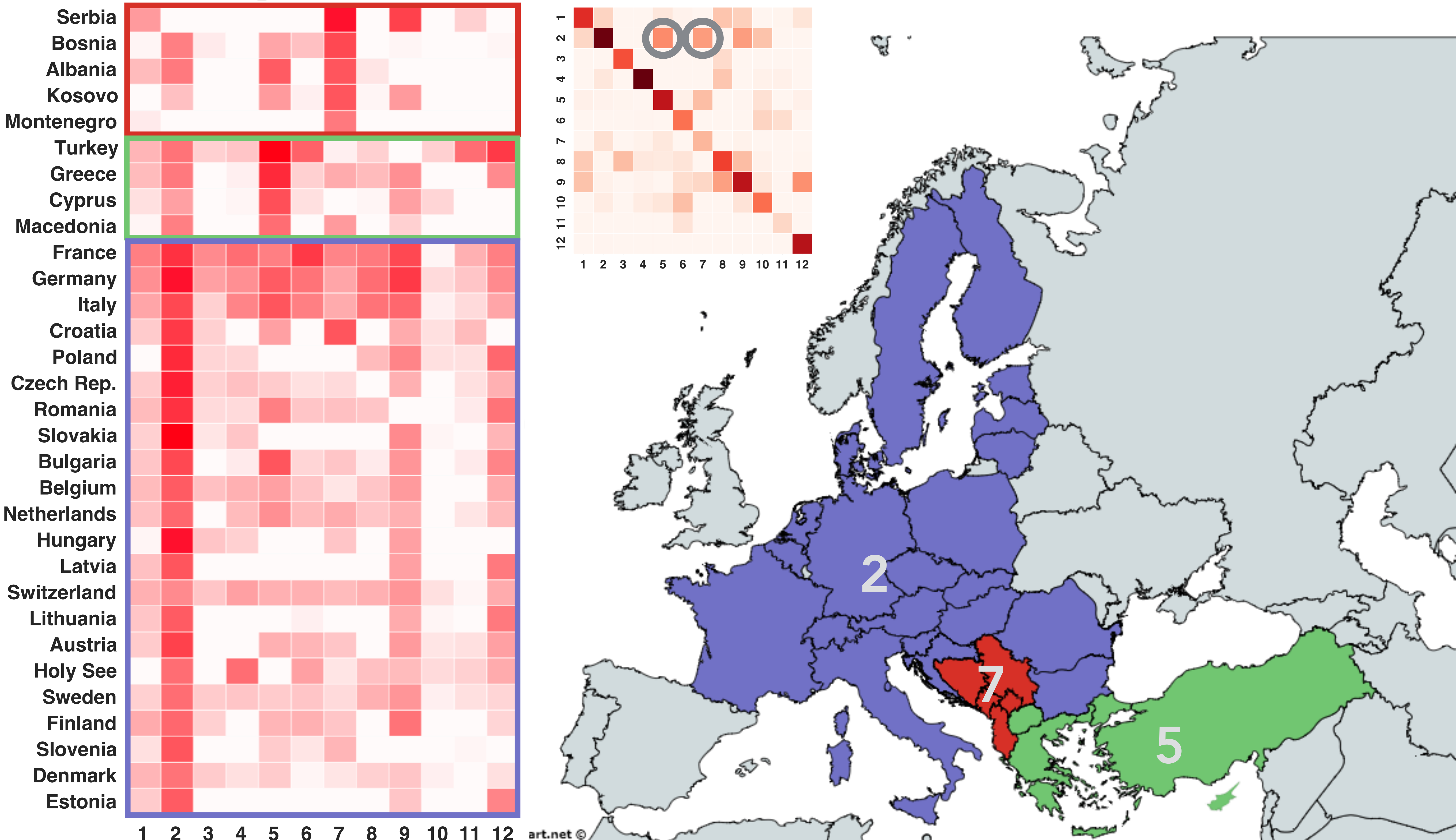

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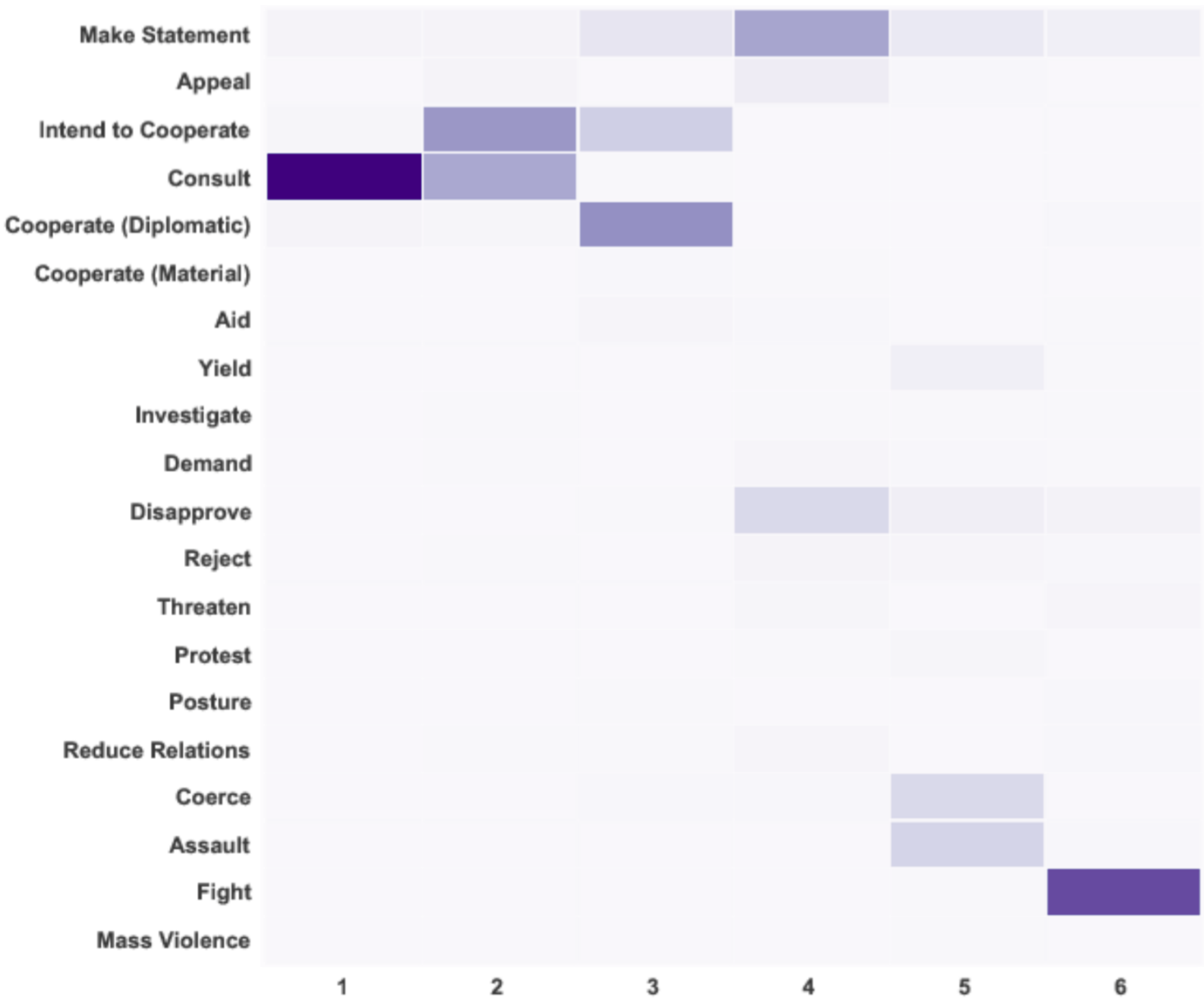

ϕ_{ak}

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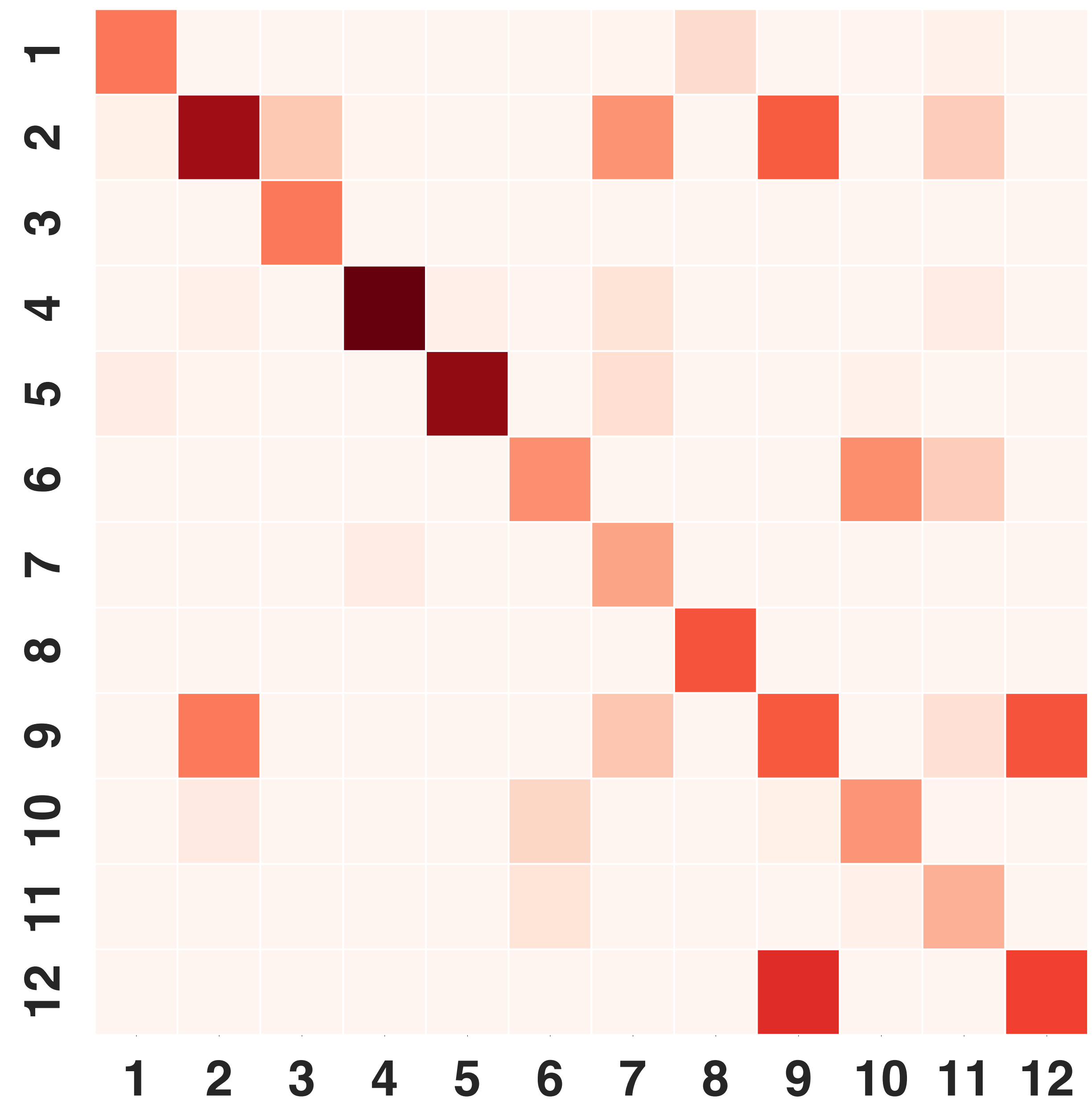

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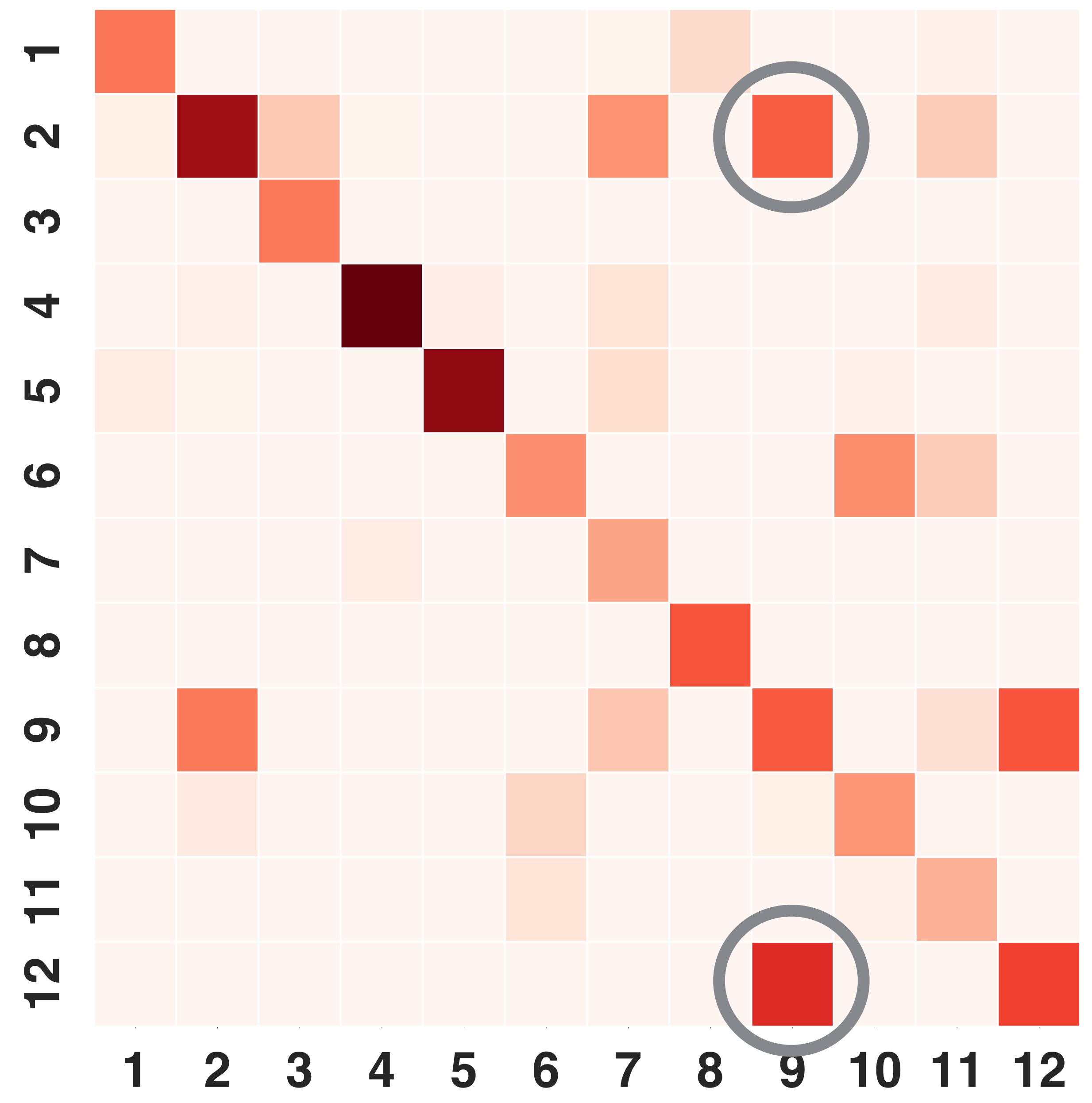

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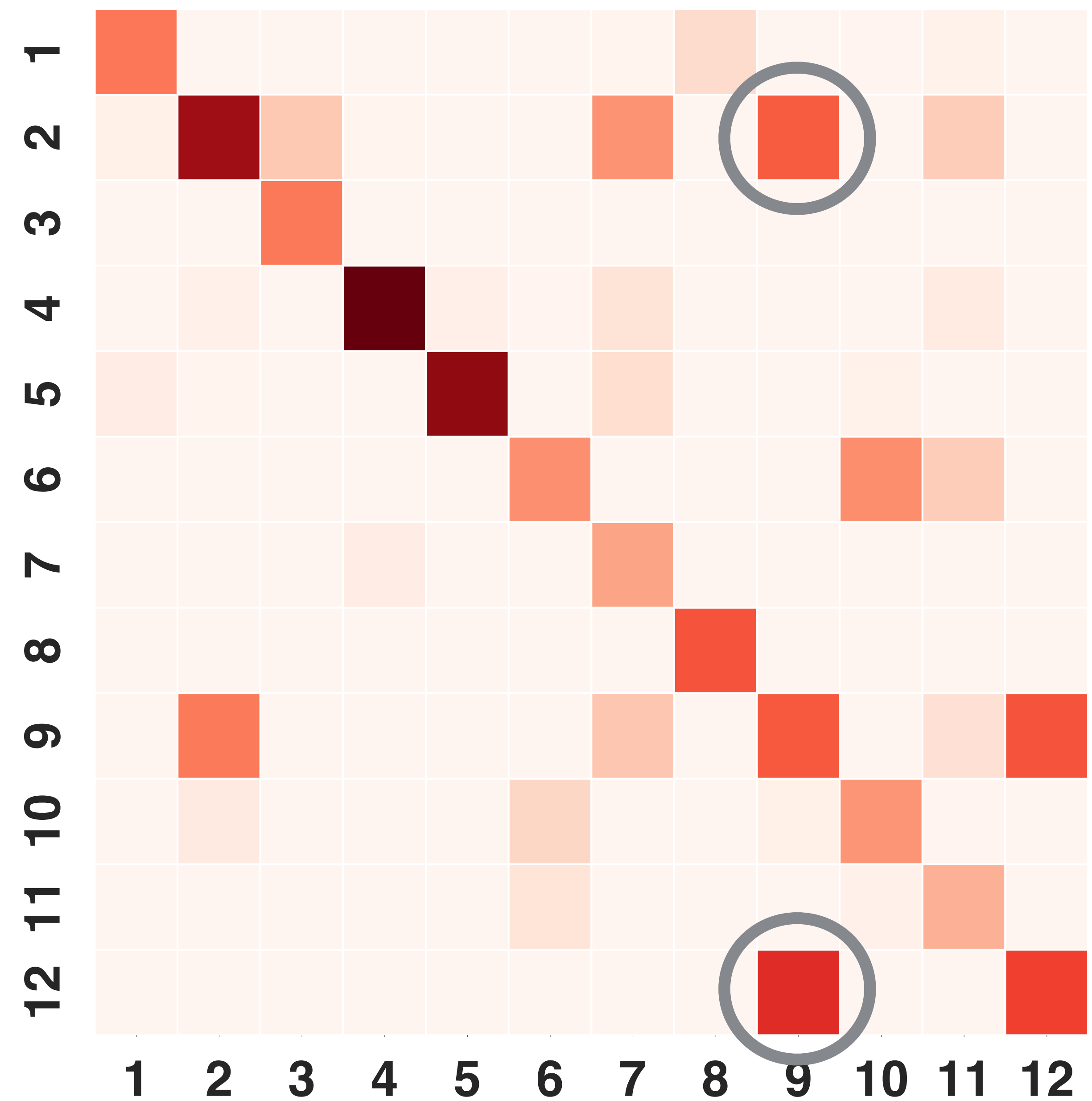

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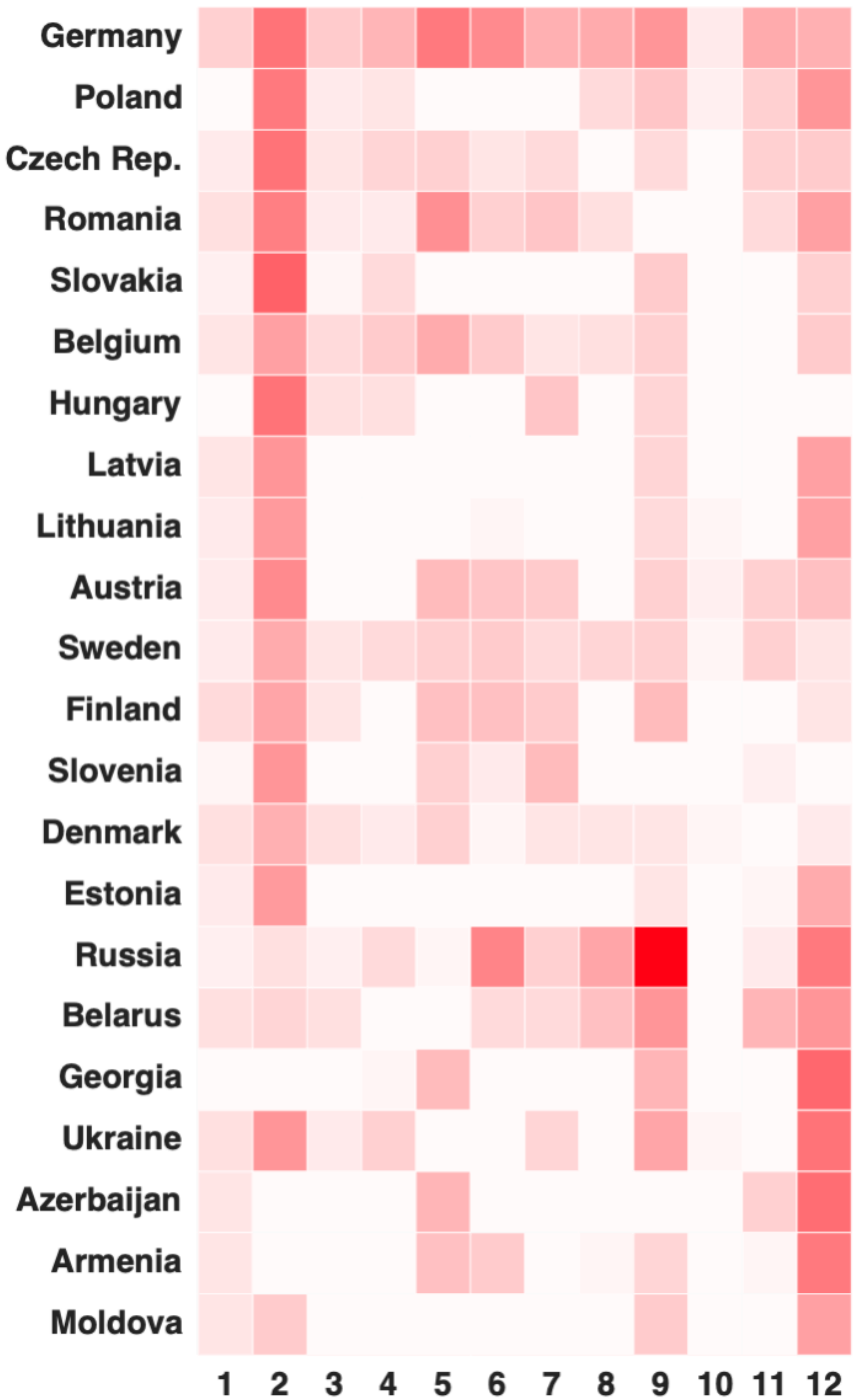

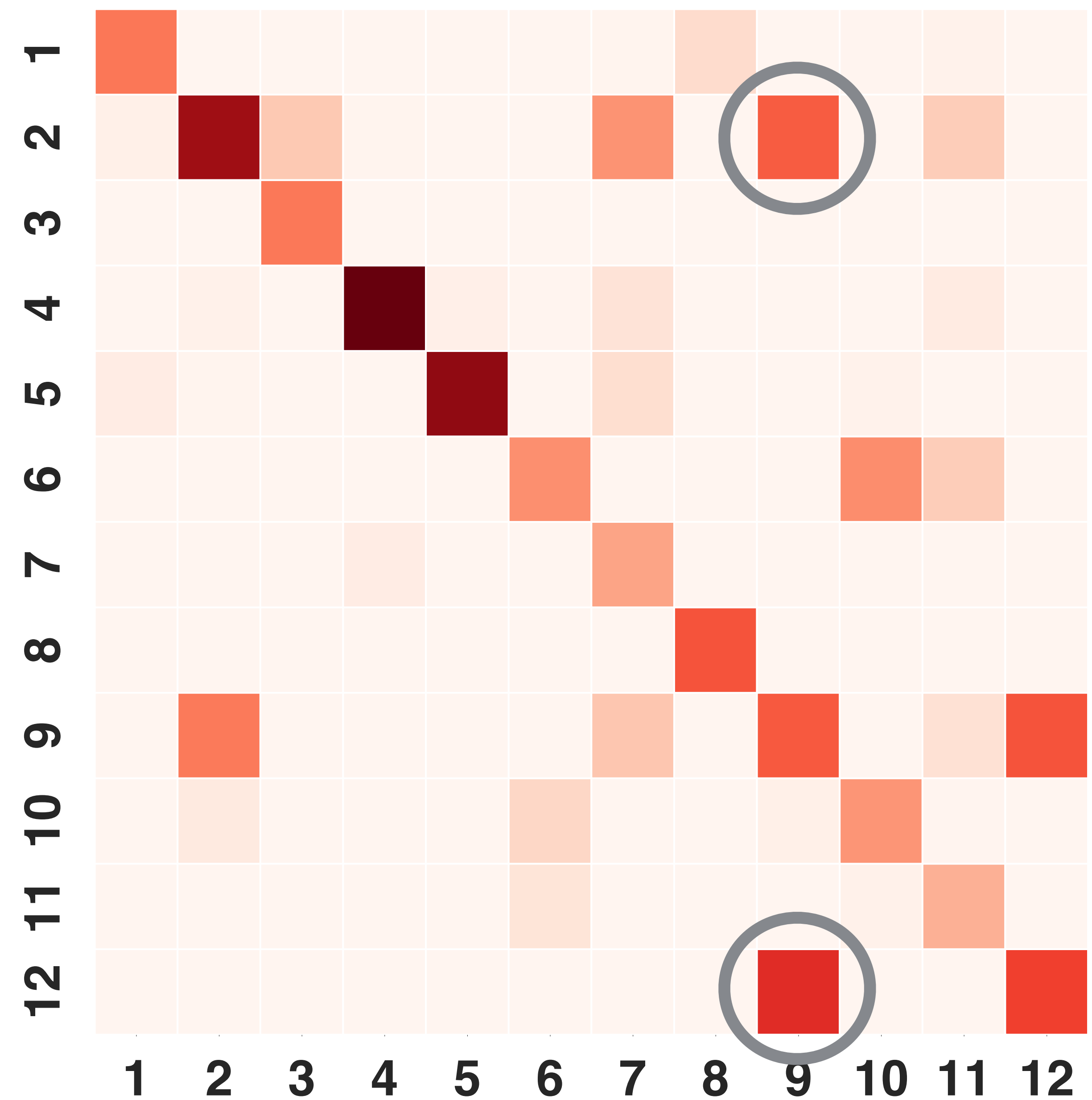

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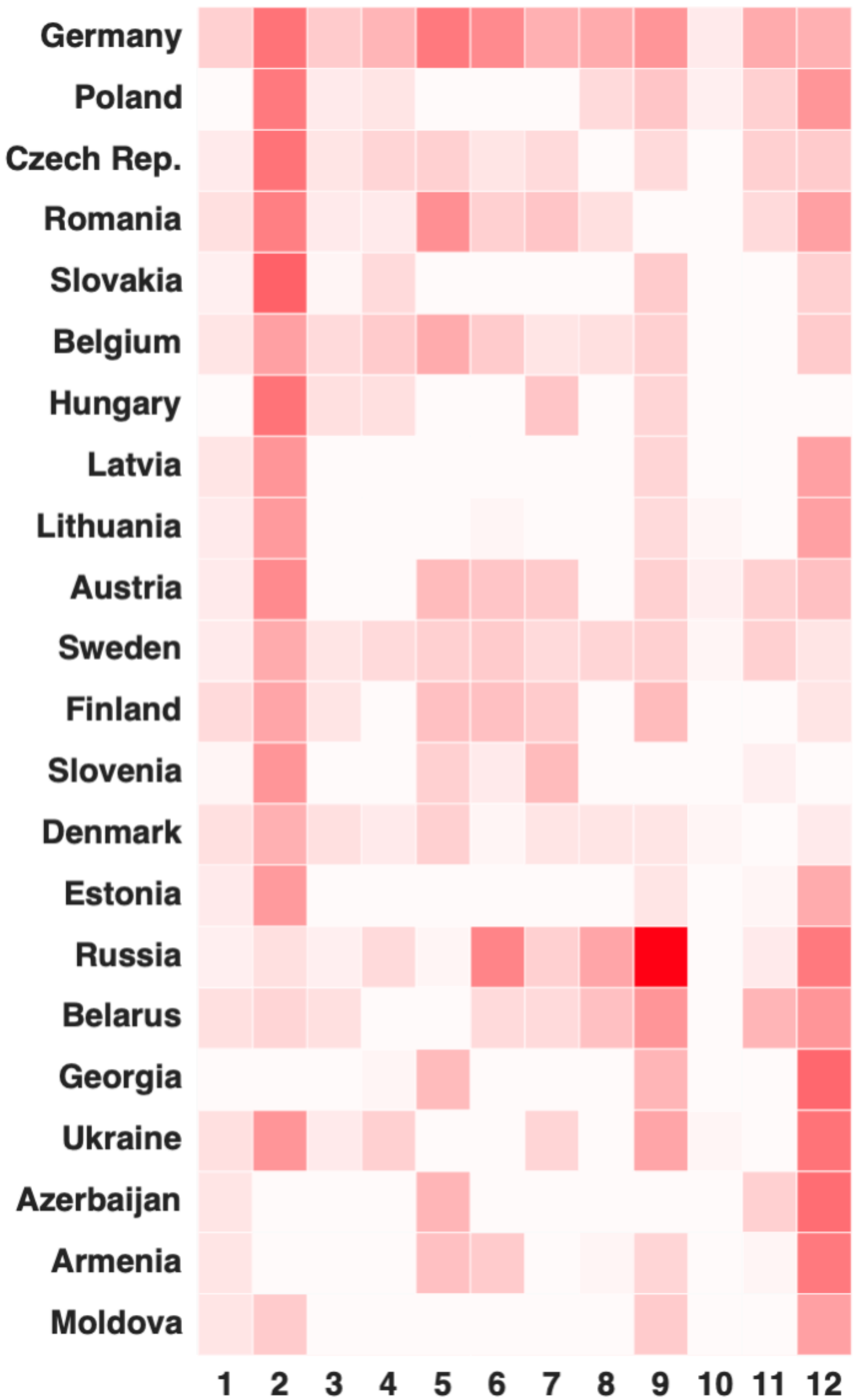

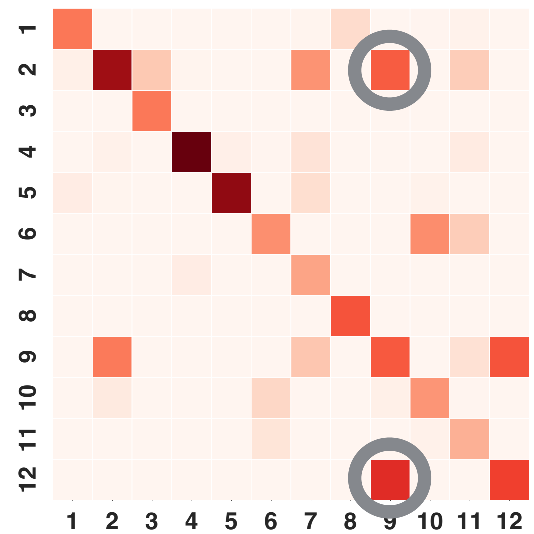

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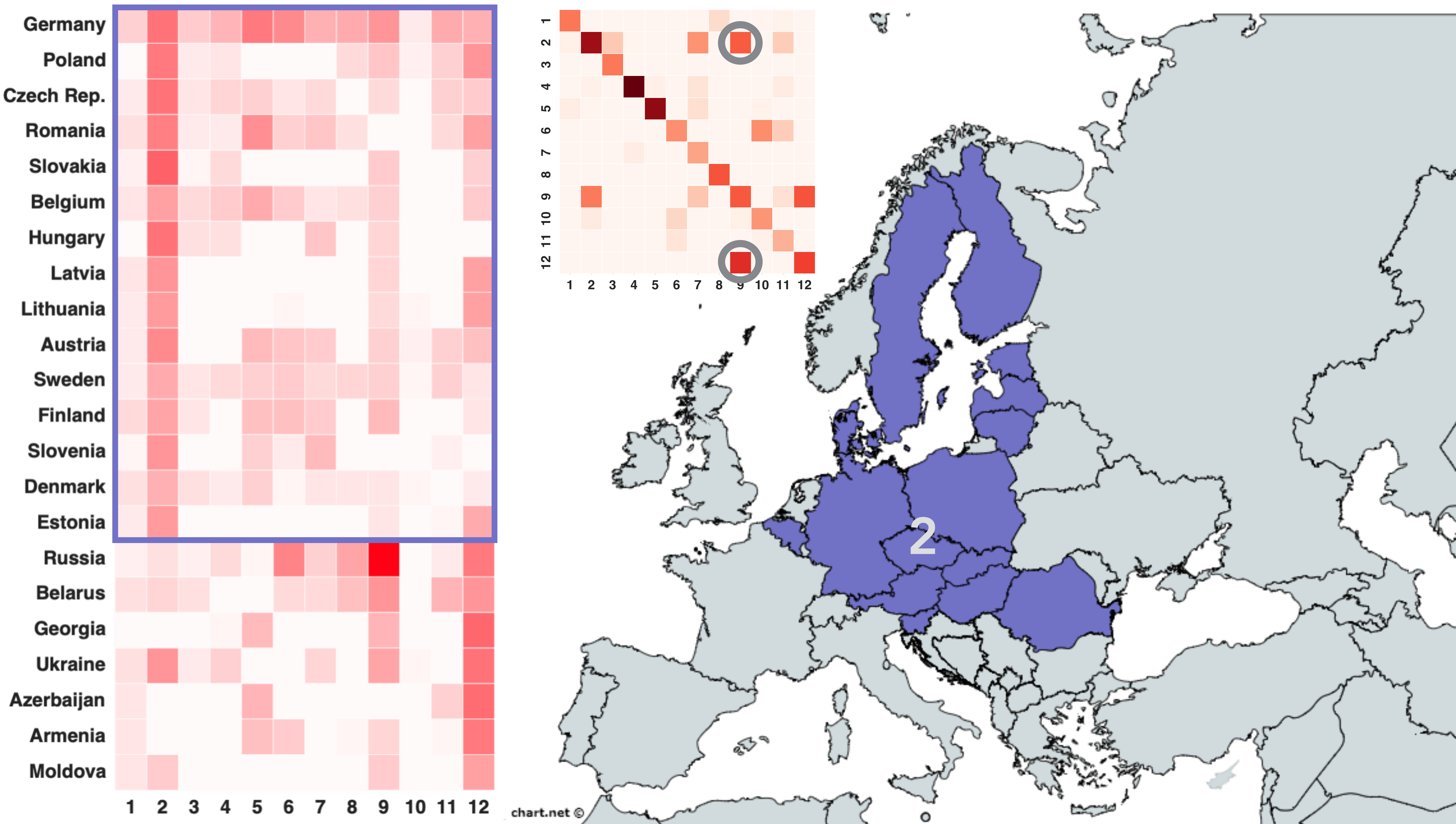

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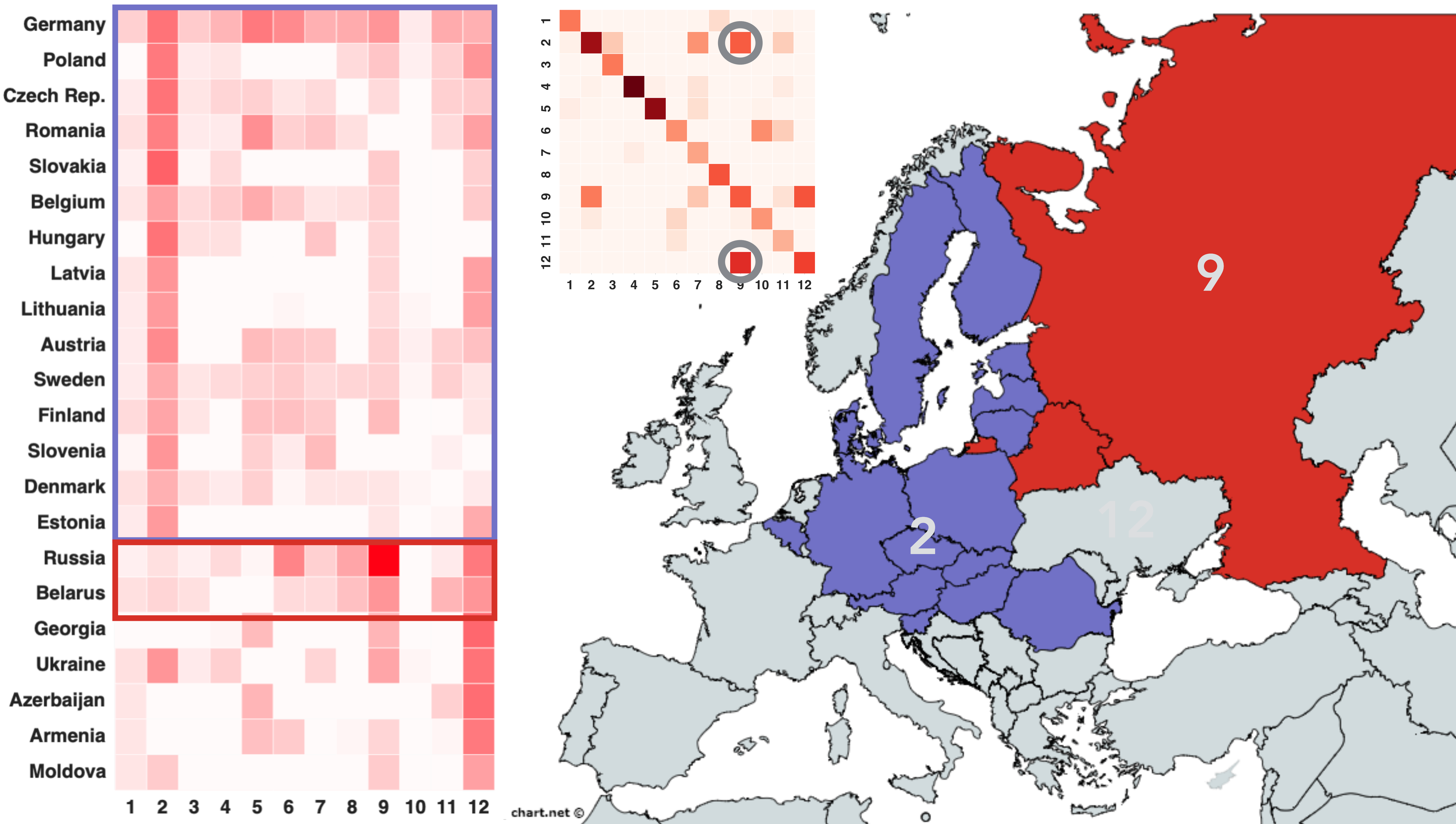

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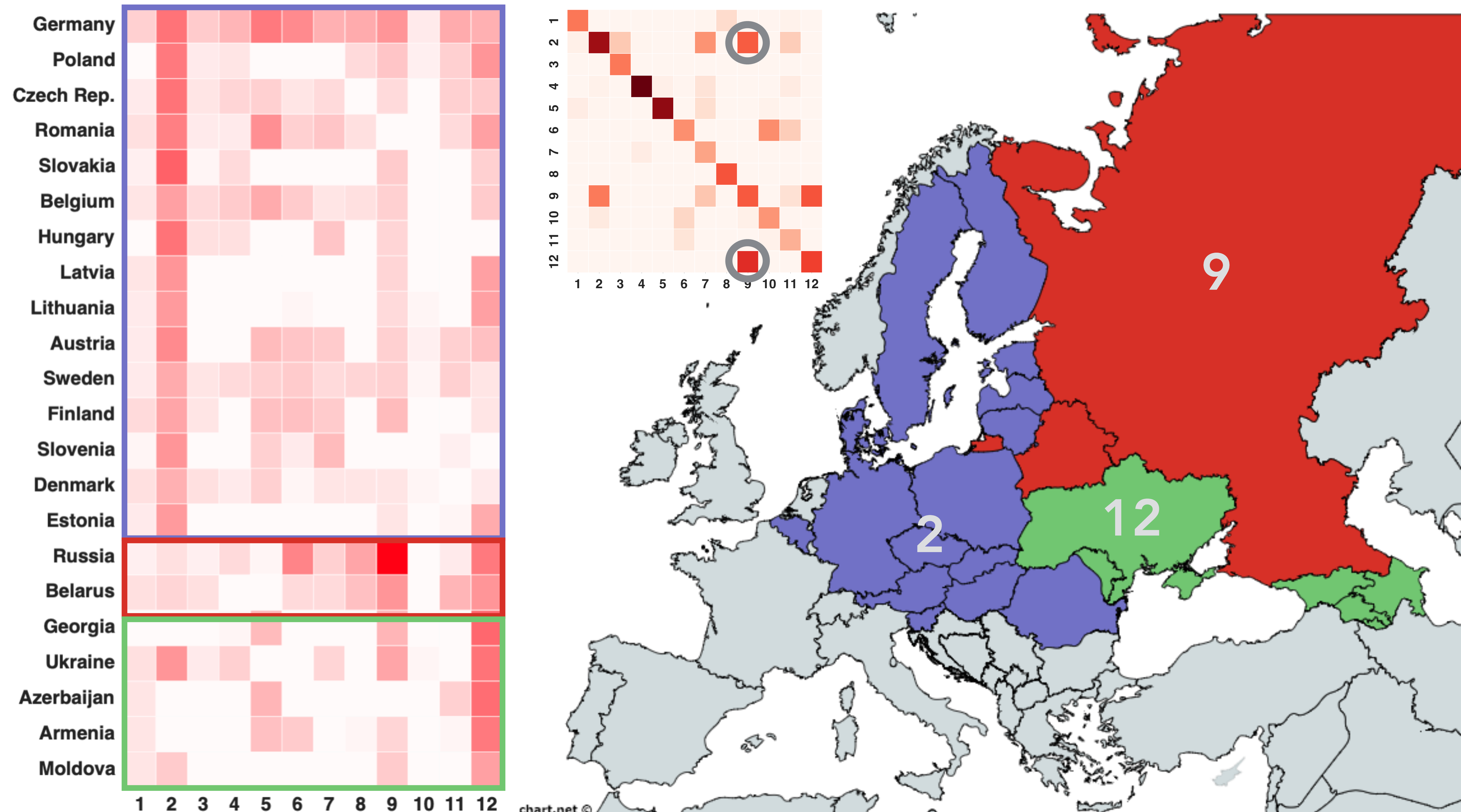

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  - **CP** extracts "multilateral" structure [Schein et al., 2015]
  - **Tucker** extracts "communities" [Schein et al., 2016]
  - Why (allocative) Poisson factorization
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All previous models were an instance of:

$$y_{\mathbf{i}} \sim \mathrm{Pois}(\mu_{\mathbf{i}}) \qquad \mathbf{i} \equiv (\mathbf{i}_1, \ldots, \mathbf{i}_M) \qquad \mu_{\mathbf{i}} = \sum_{\boldsymbol{\kappa}} \mu_{\mathbf{i},\boldsymbol{\kappa}}$$

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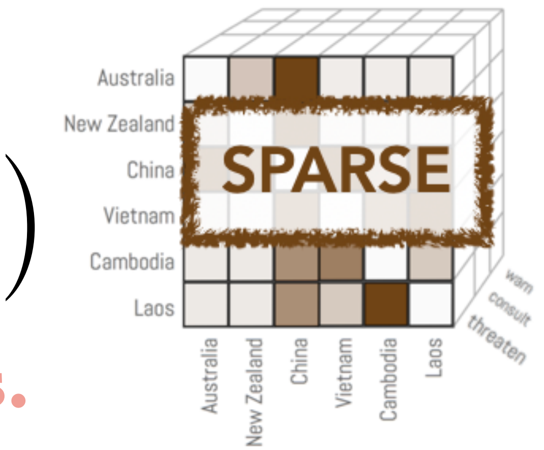

Scales with **only the non-zeros.**

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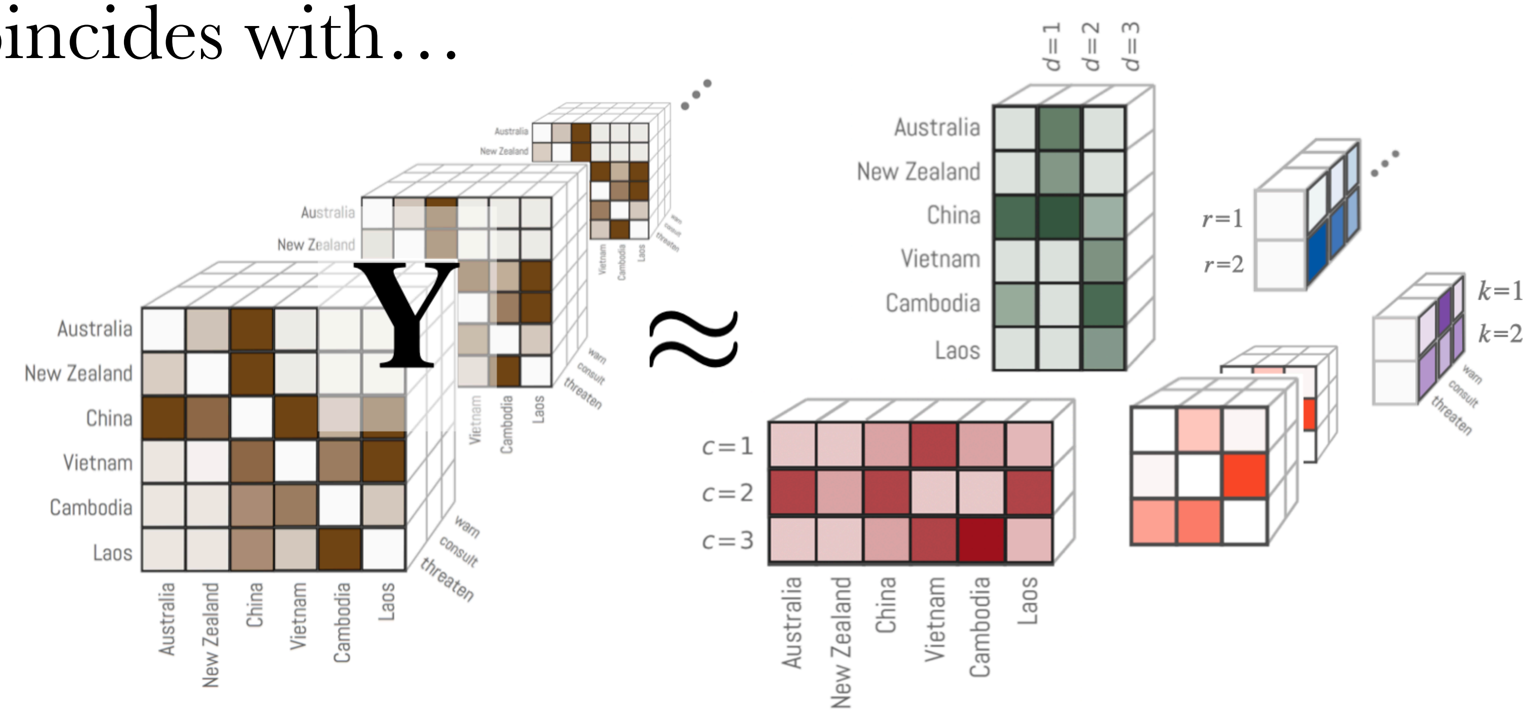

…**latent class allocation** of its tokens

Table 2
WEIS Coding of 1990 Iraq-Kuwait Crisis

Date	Source	Target	Type of Action
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900725	IRQ	EGY	ASSURE
900727	IRQ	KUW	WARN
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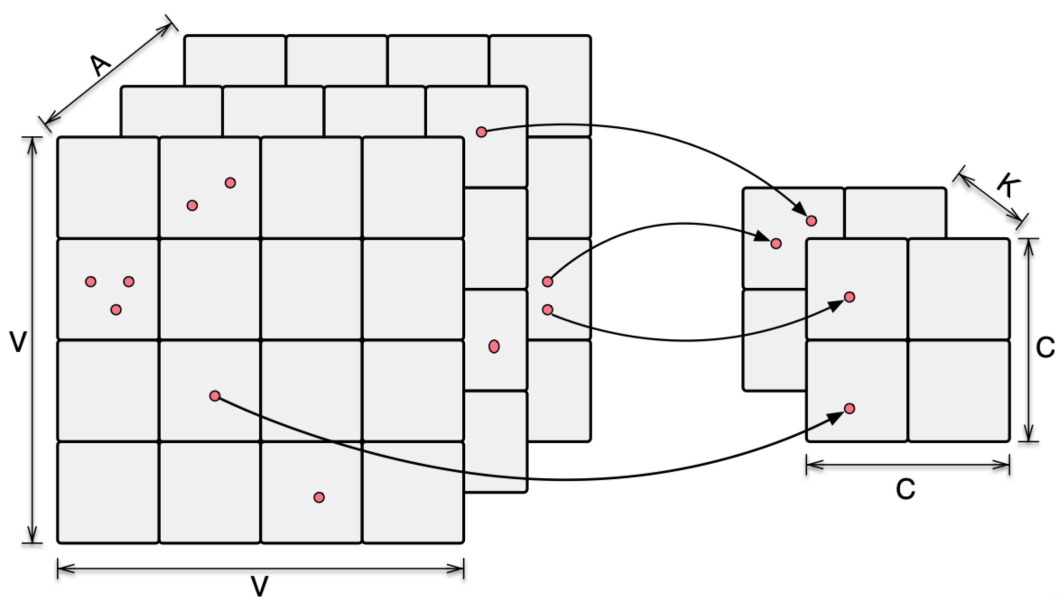

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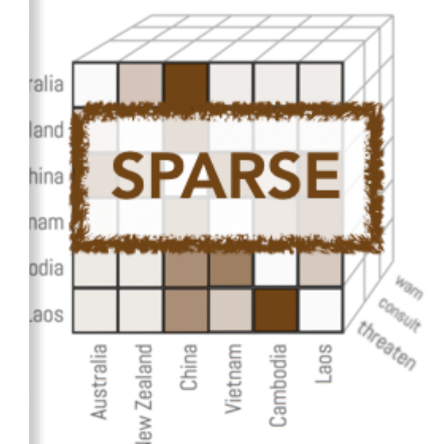

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Folk theorem of statistical computing [Gelman, 2008]:

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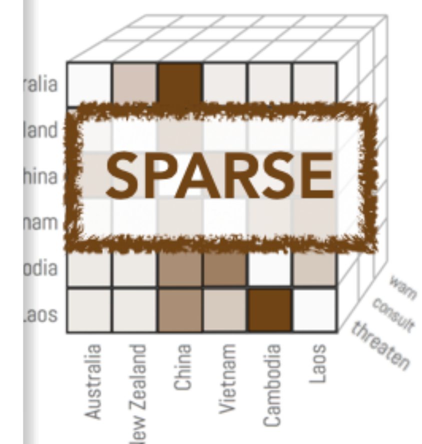

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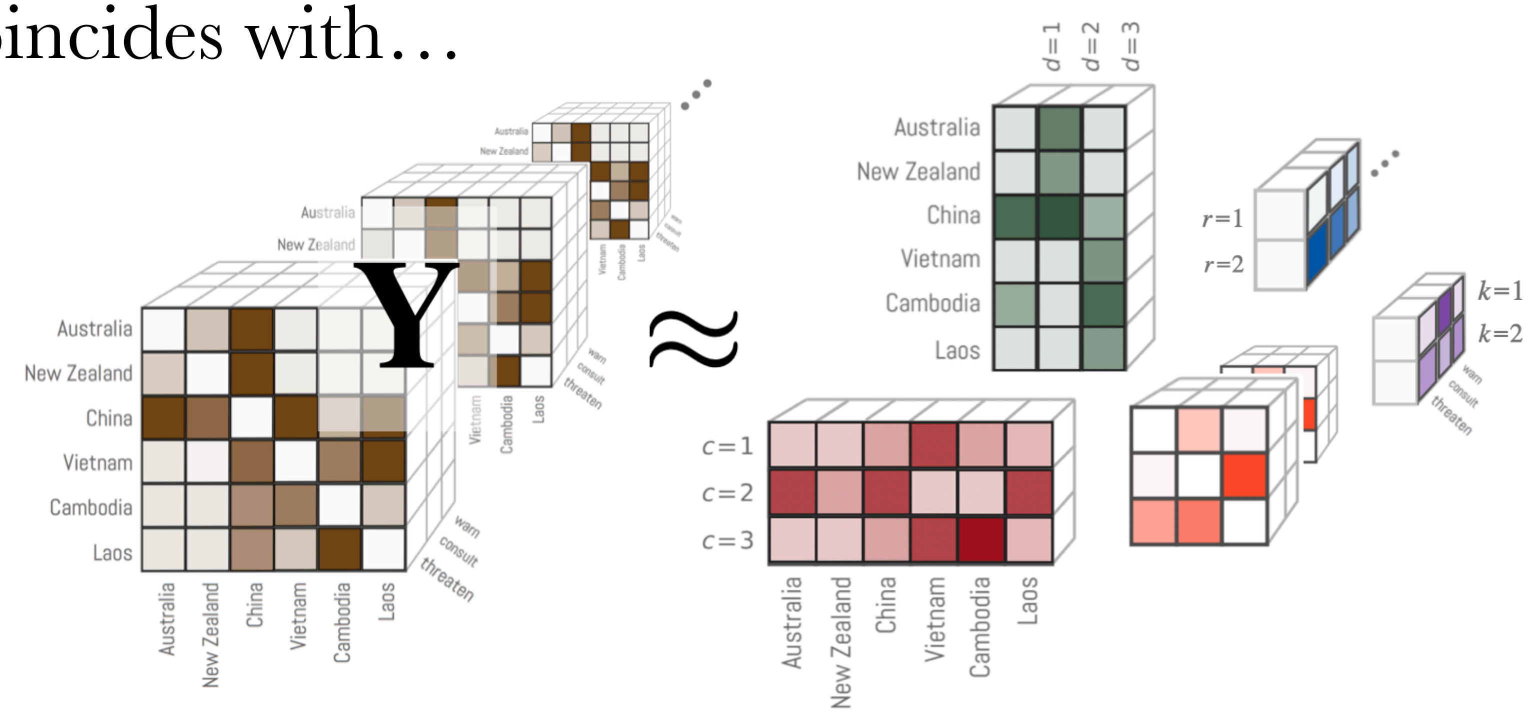

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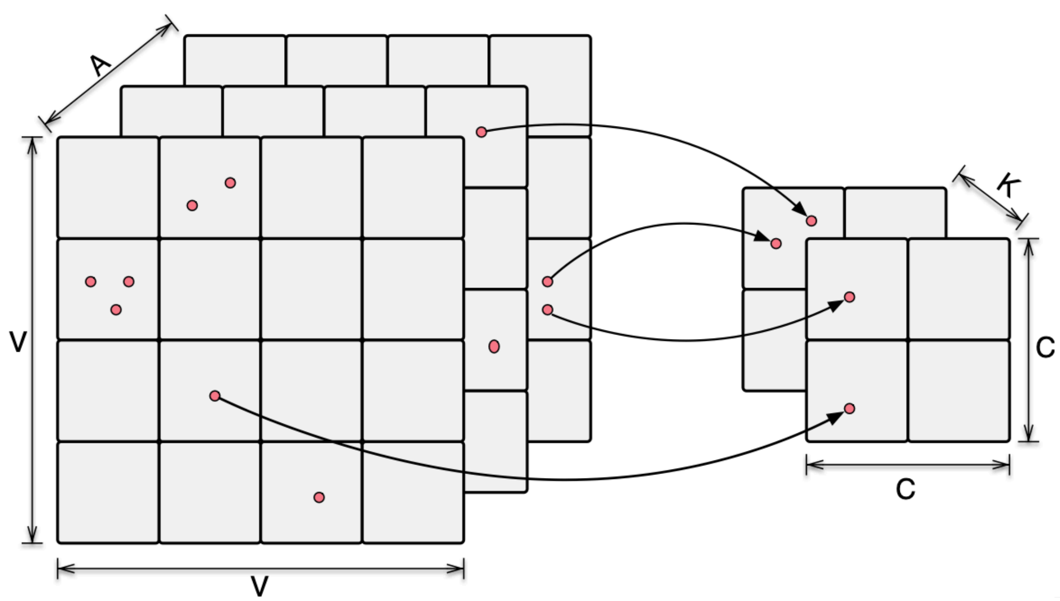

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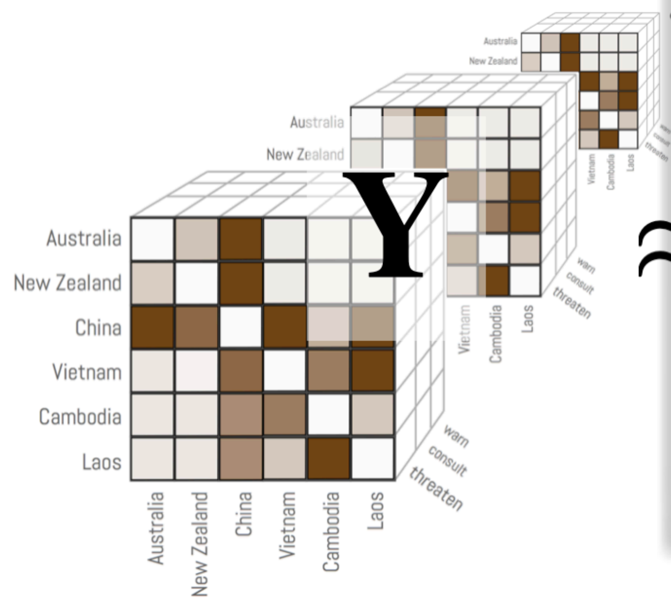

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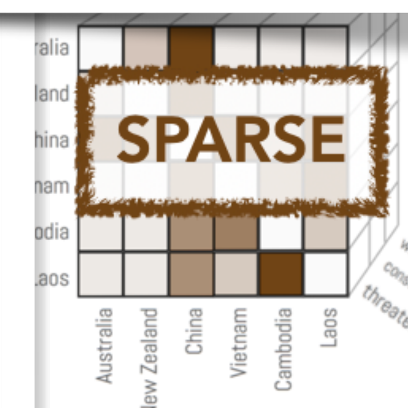

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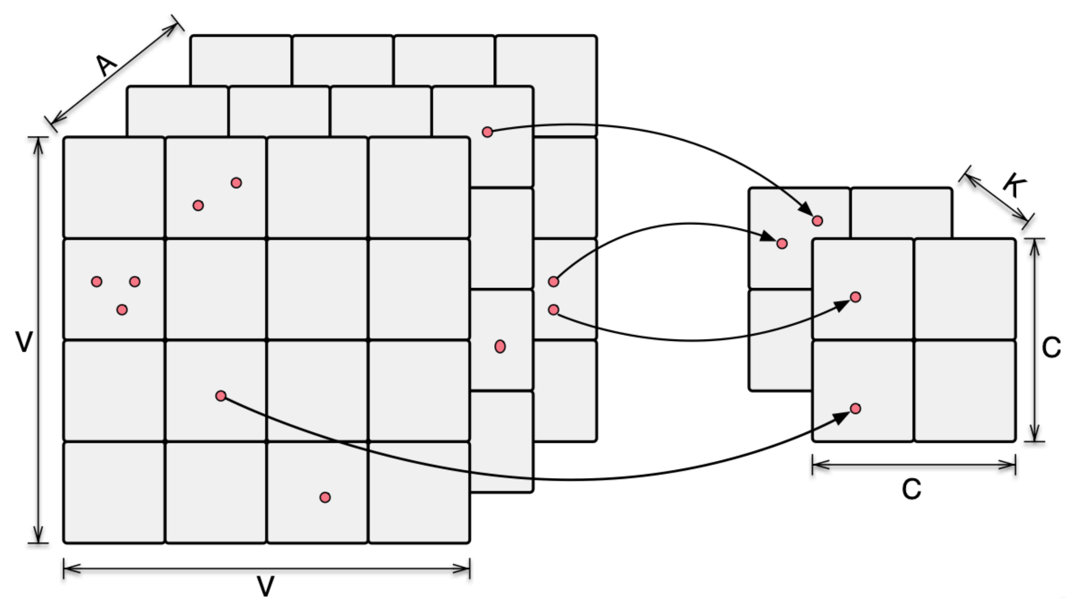

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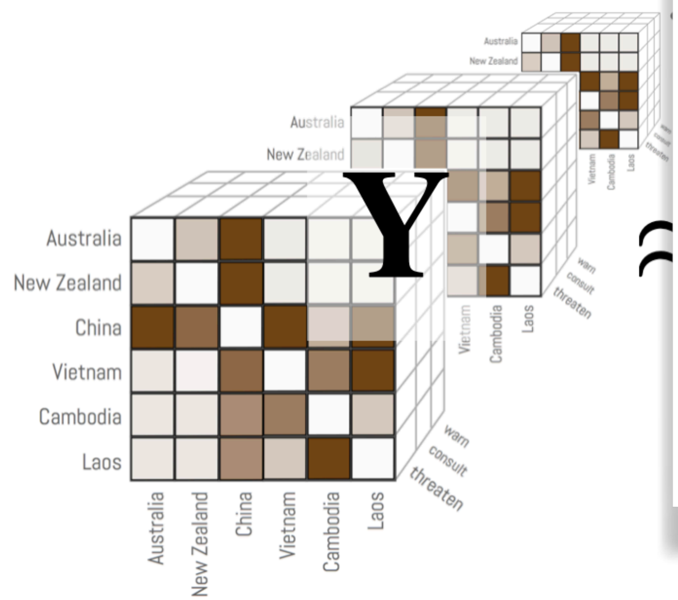

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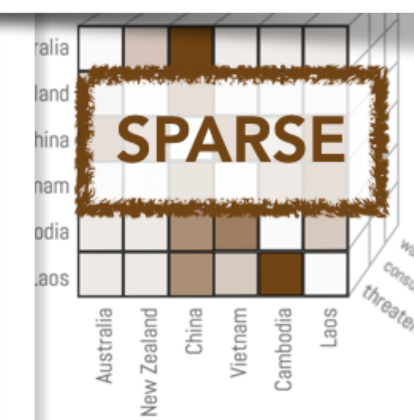

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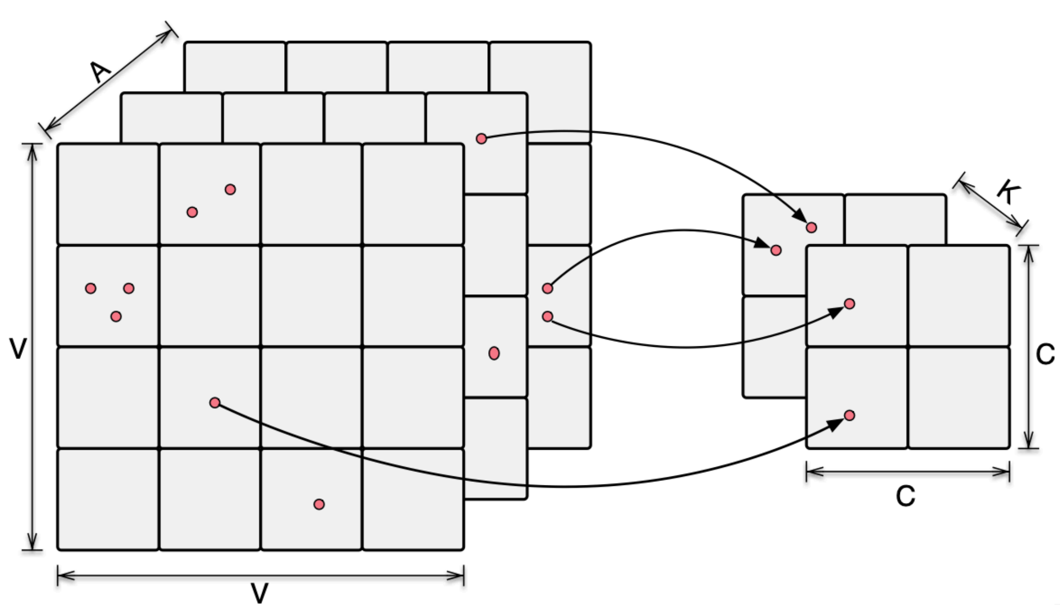
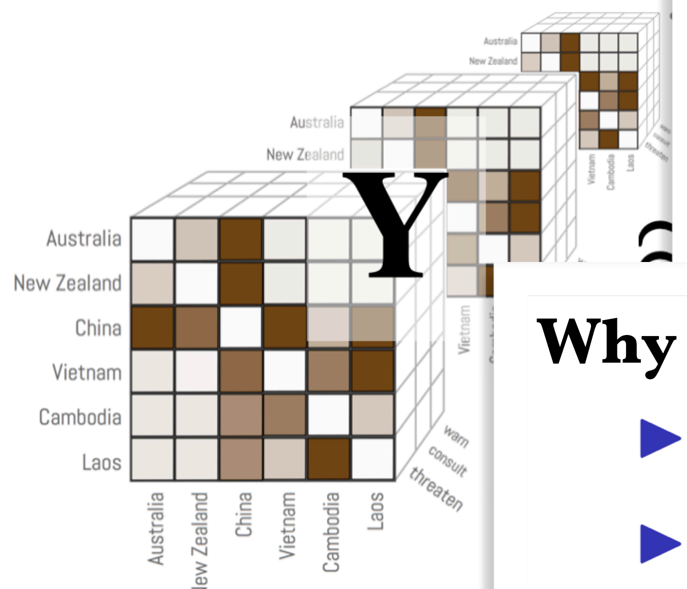


ALLOCATIVE POISSON FACTORIZATION

"Latent source" representation [Dunson & Herring, (2005); Cemgil (2009)]:

Decomposition of a t

coincides with…



…latent class all

Table 2
WEIS Coding of 1990 Iraq-Kuwait C

Date	Source	Target
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Equivalence to models specified explicitly for tokens:

- Tensor decomposition for contingency table analysis: […] [Dunson & Xing, 2012], [Zhou et al., 2015], [Johndrow, Bhattacharya & Dunson, 2017] […]
- Latent class models: […] [Gu & Dunson, 2023] […]

Why this is particularly important in this setting:

- ICEWS (1995-2023): **20M tokens**
- (country x country x action x month)

  =(250 x 250 x 20 x 336)

  ≈ **450 million entries**
- (country x country x action x day x publisher)

  =(250 x 250 x 20 x 10,220 x 200)

  ≈ **2.5 trillion entries**
- **When we add columns, or increase granularity, we get enormous tensors, but inference is still $\mathcal{O}$(#tokens)**

Folk theorem of statistical computing [Gelman, 2008]:
"When you have computational problems, often there's a problem with your model."

"Late

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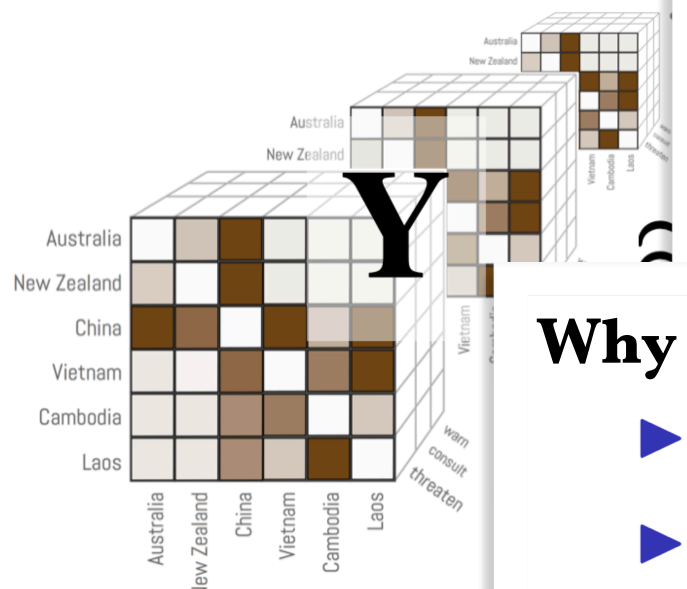

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Allocative Poisson factorization…

$$y_{\mathbf{i}} \sim \text{Pois}(\underbrace{\textstyle\sum \ldots}_{\text{non-negative}})$$

…means non-negative priors.

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More challenging for building complex priors, e.g.,

$$\boldsymbol{\theta}^{(t)} \sim P(\boldsymbol{\theta}^{(t)} \,|\, \boldsymbol{\theta}^{(t-1)})$$

Often requires new tools (e.g., augmentation schemes).

OUTLINE

- **Motivation:** Dyadic event data (and its challenges)

- **Non-negative (Poisson) tensor decompositions**
  - **CP** extracts “multilateral” structure [Schein et al., 2015]
  - **Tucker** extracts “communities” [Schein et al., 2016]
  - Why (allocative) Poisson factorization
  - **ALℓ_0CORE** avoids the “exponential blowup” [Hood & Schein, 2024]

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  - **Gamma state-space** models [Schein et al., 2016b; Schein et al., 2019]
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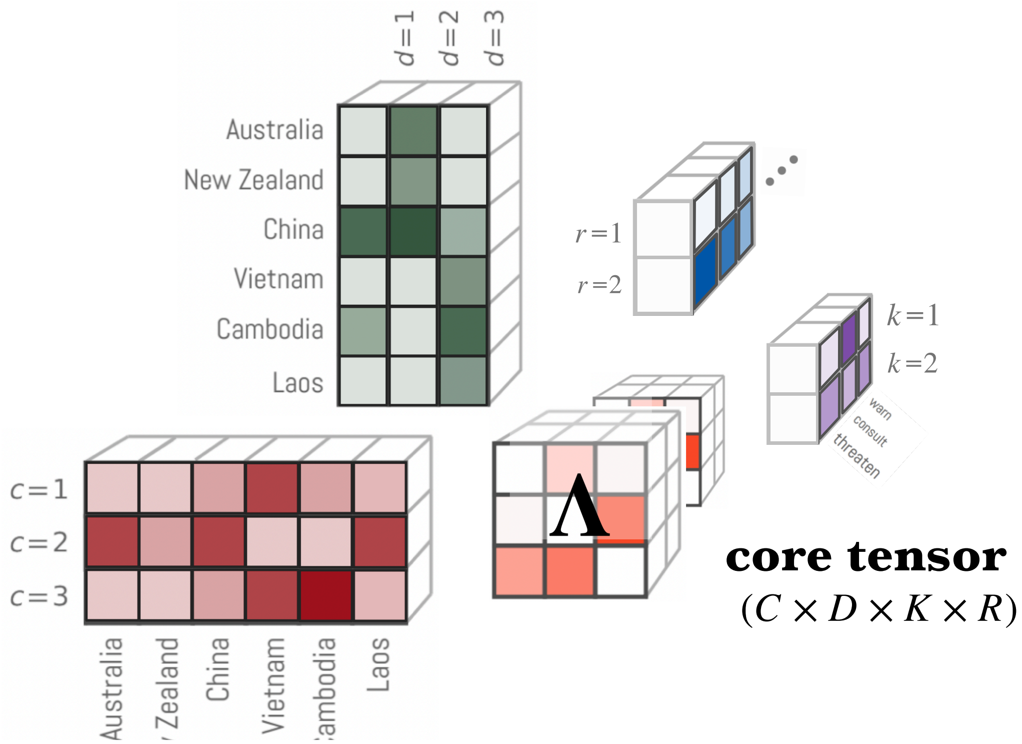
TUCKER: APPEALING

Tucker's **conceptual appeal**

- Different number of factors in each mode
- Mode-specific interpretation of factors
  - e.g., "communities" of countries
  - e.g., "topics of actions
- Factor sharing across classes; less redundancy



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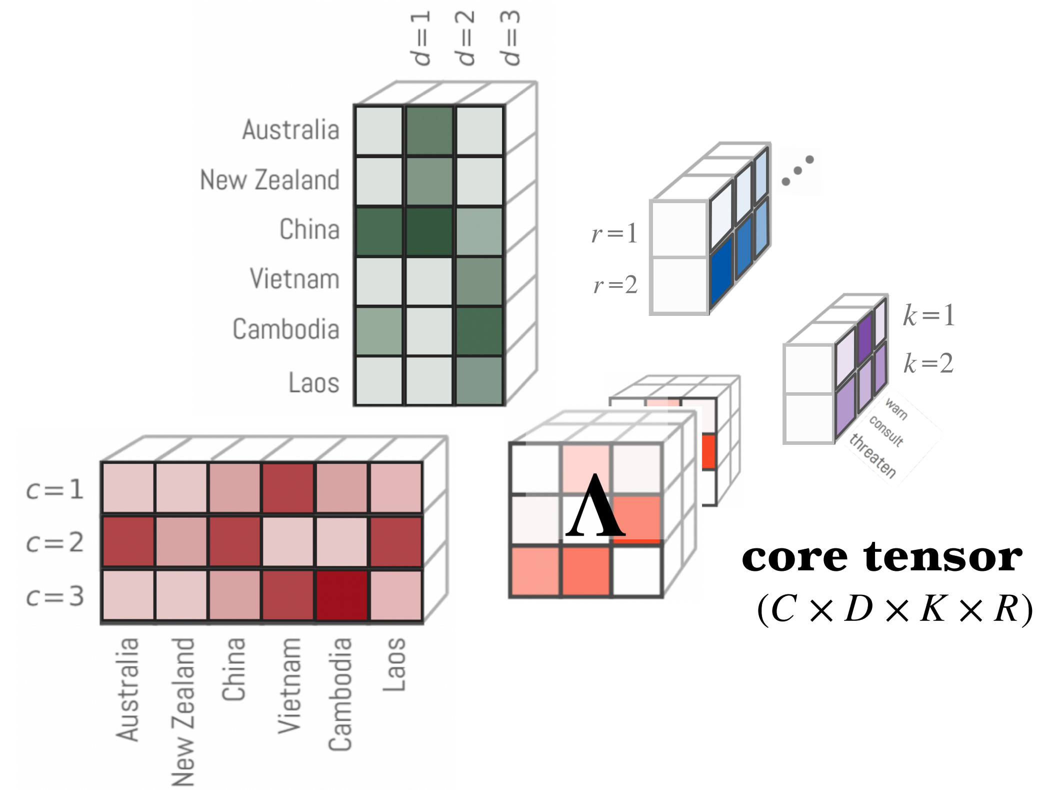

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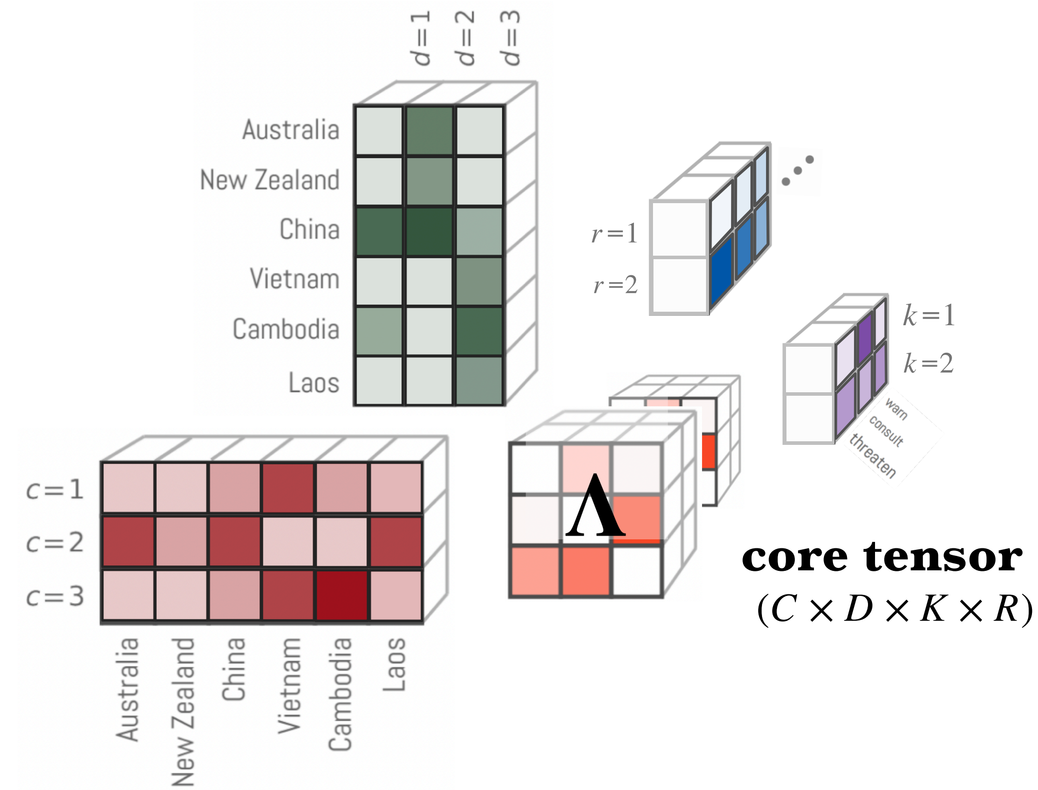

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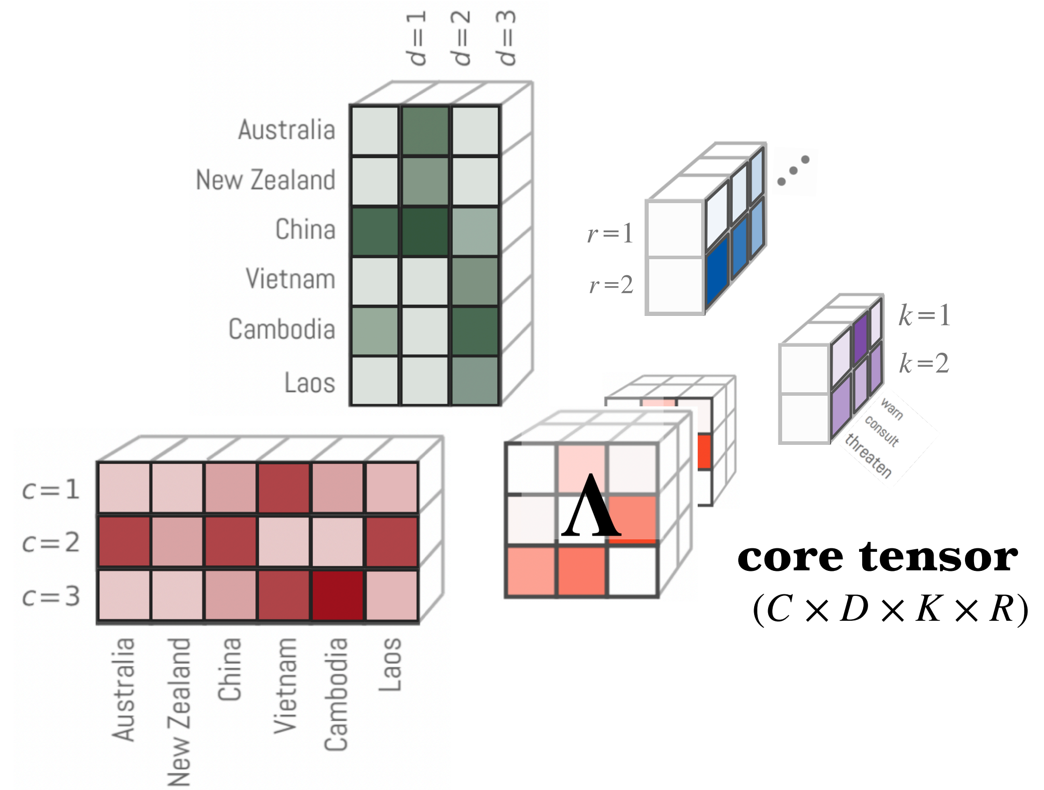

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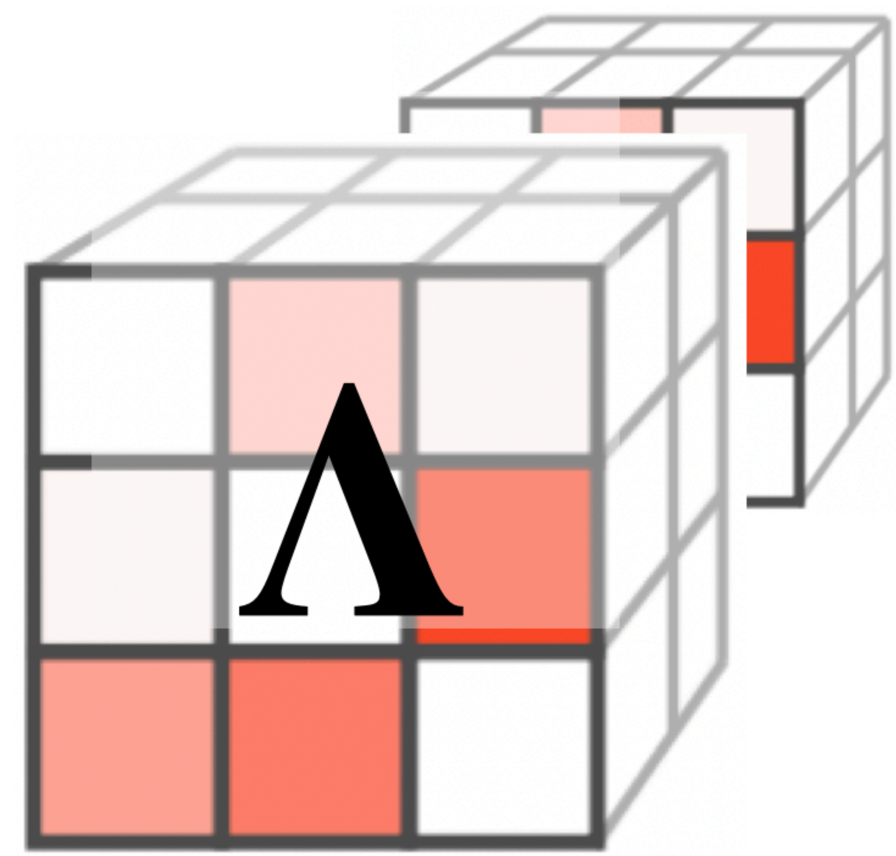

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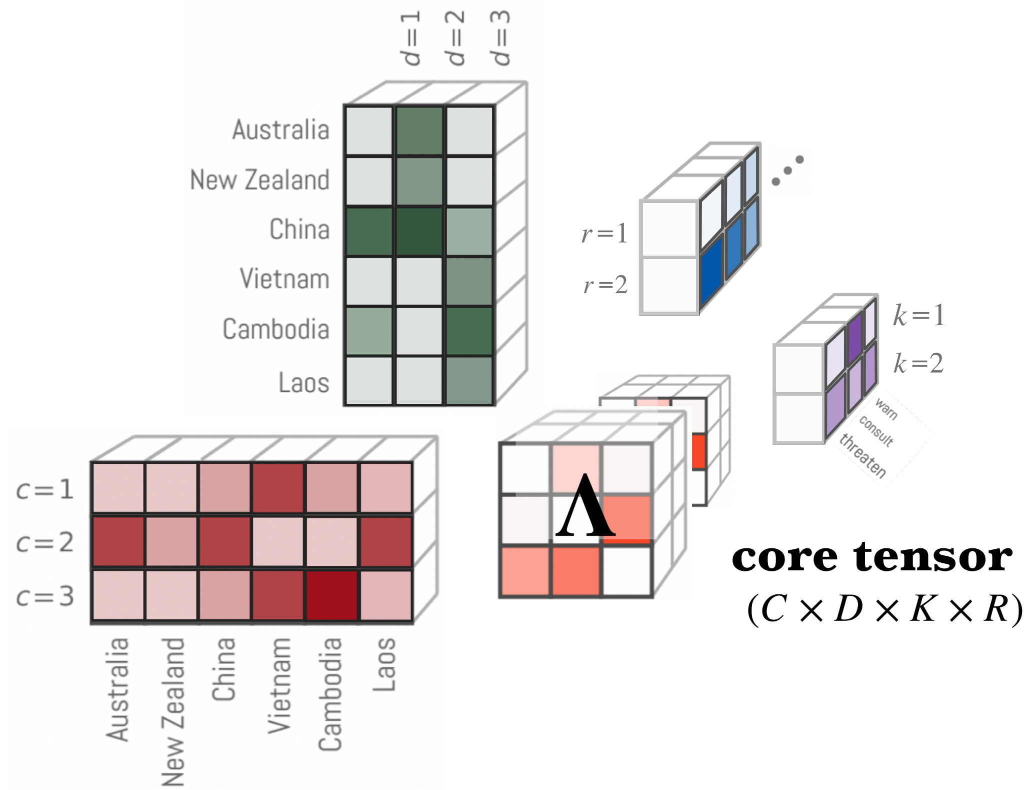

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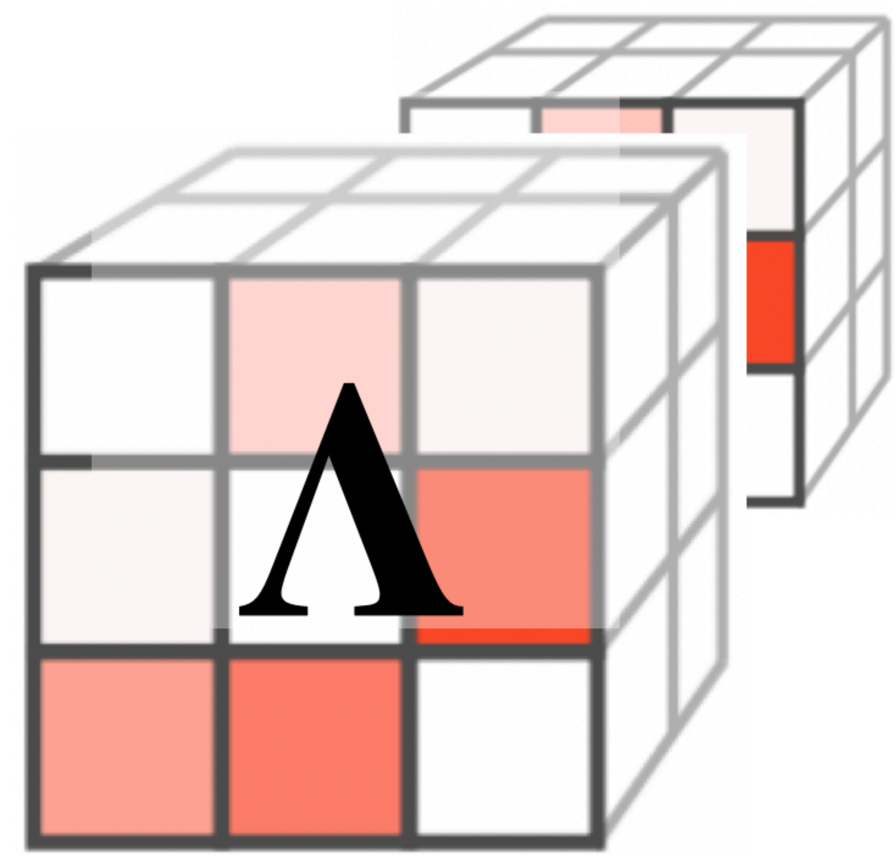


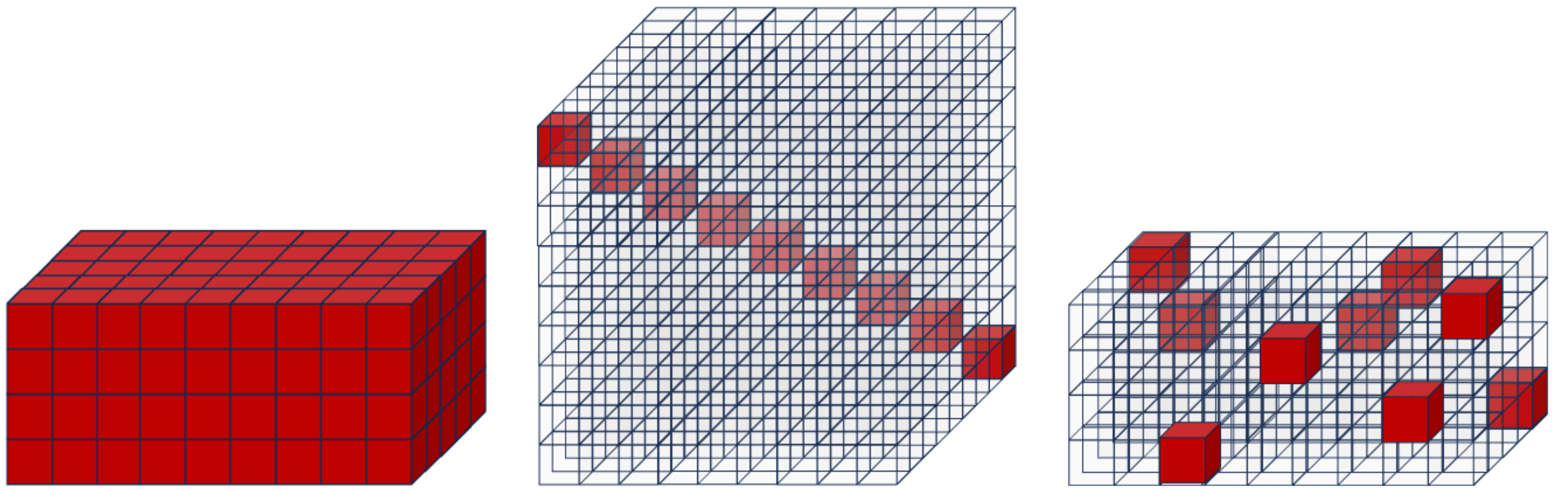
Poisson Tucker better, but still bad:

- $\mathcal{O}(||\mathbf{Y}||_0 \cdot |\mathbf{\Lambda}|)$

ALℓ_0CORE [Hood & Schein, 2024]

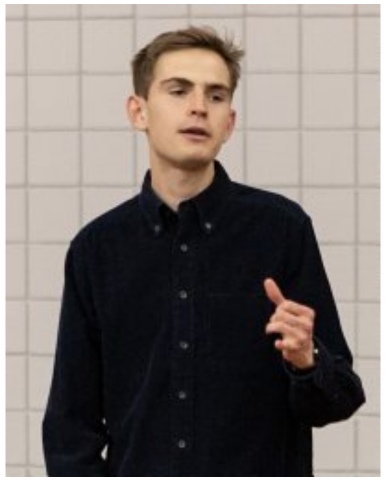
The basic idea: an ℓ_0-constrained core:

$$||\mathbf{\Lambda}||_0 \leq Q$$



(a) Tucker (b) CP (c) ALℓ_0CORE

Lead author:



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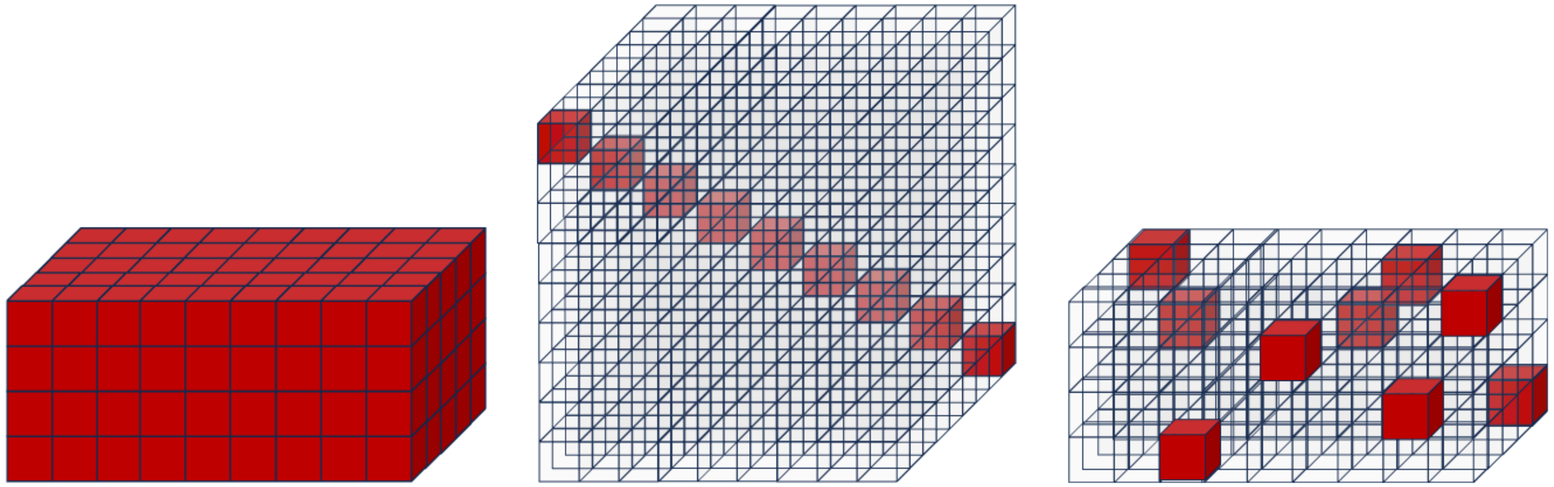

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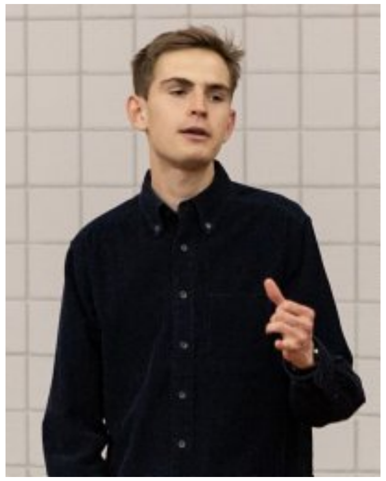

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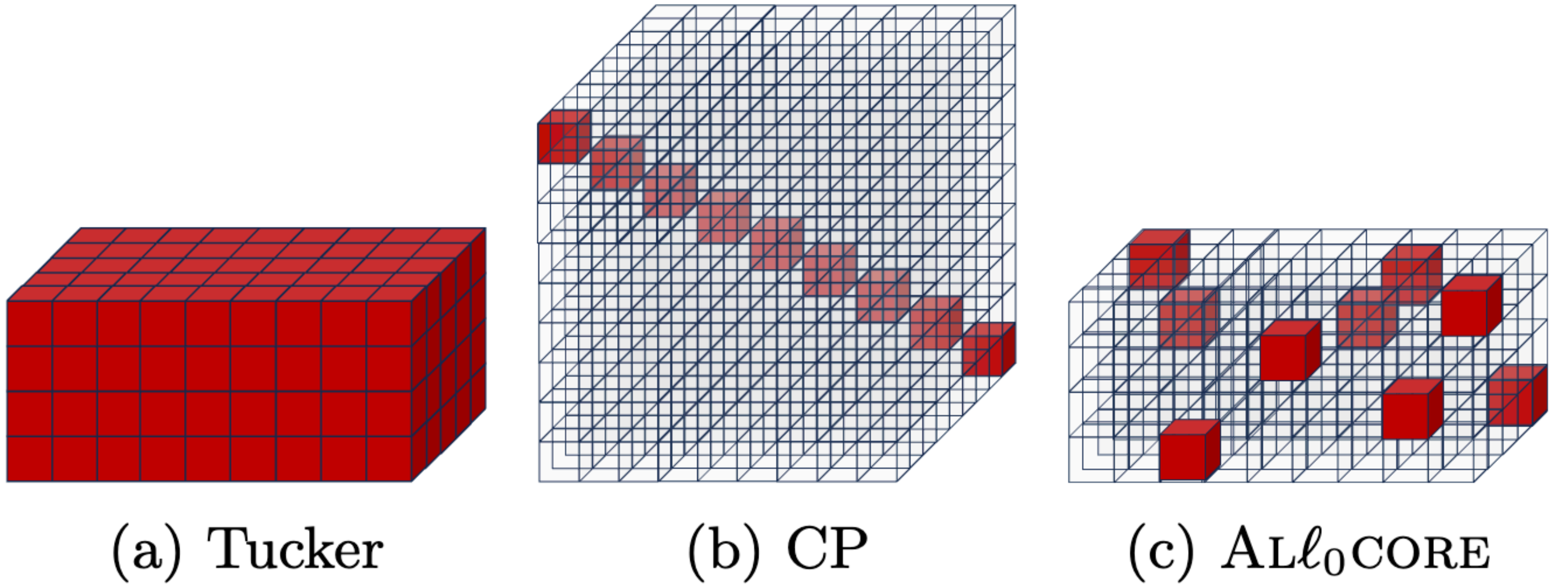

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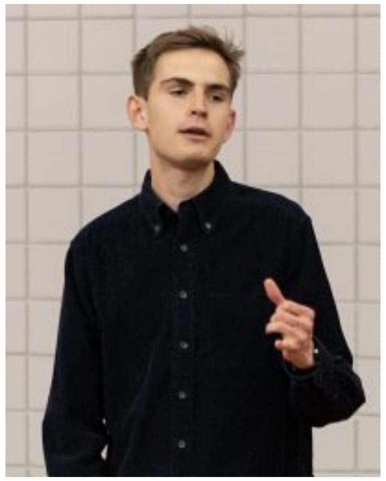

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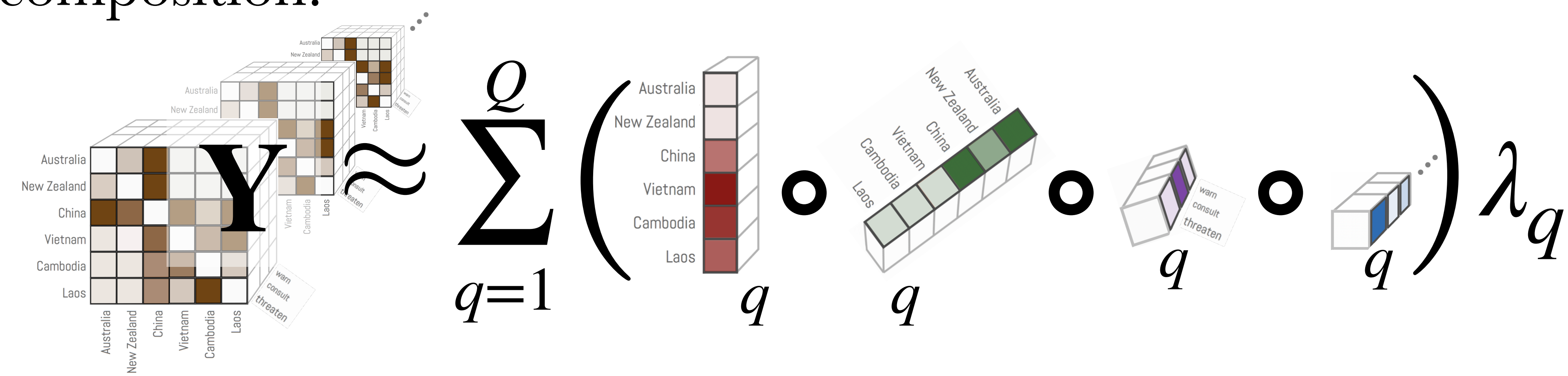
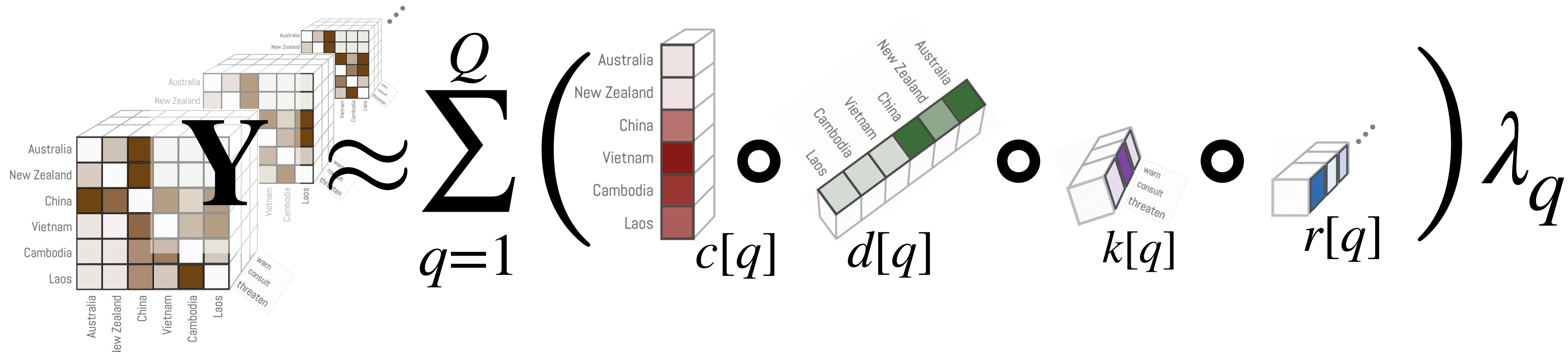
  - $\kappa_q \equiv \left(c[q], d[q]\, a[q], r[q]\right)$
- Inference involves **allocating non-zeros** across the core
- ALℓ_0CORE = "allocated ℓ_0-constrained core"

COMPARISON OF ALL THREE

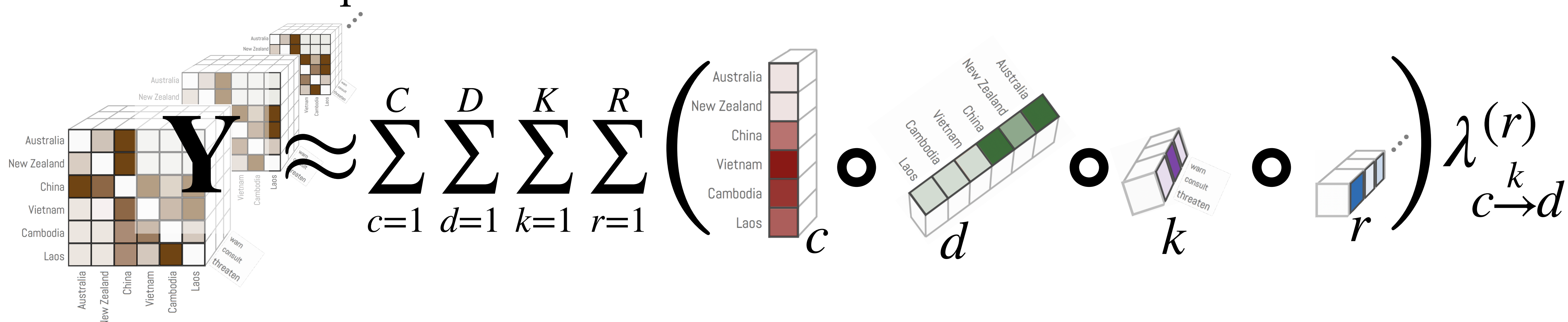
- CP decomposition:



- ALℓ_0CORE decomposition:



- Tucker decomposition



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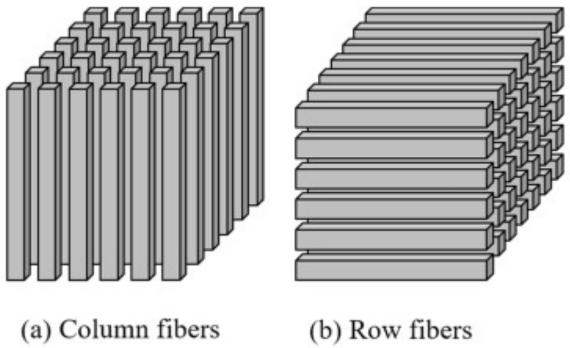

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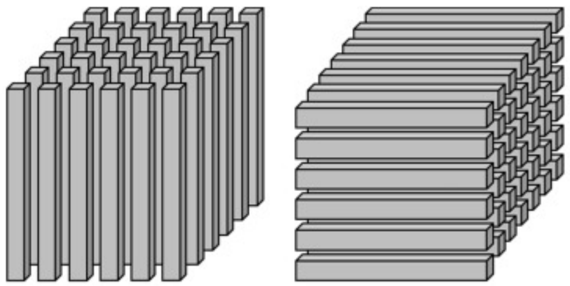
- (We can do even better with specific priors, but this works surprisingly well.)

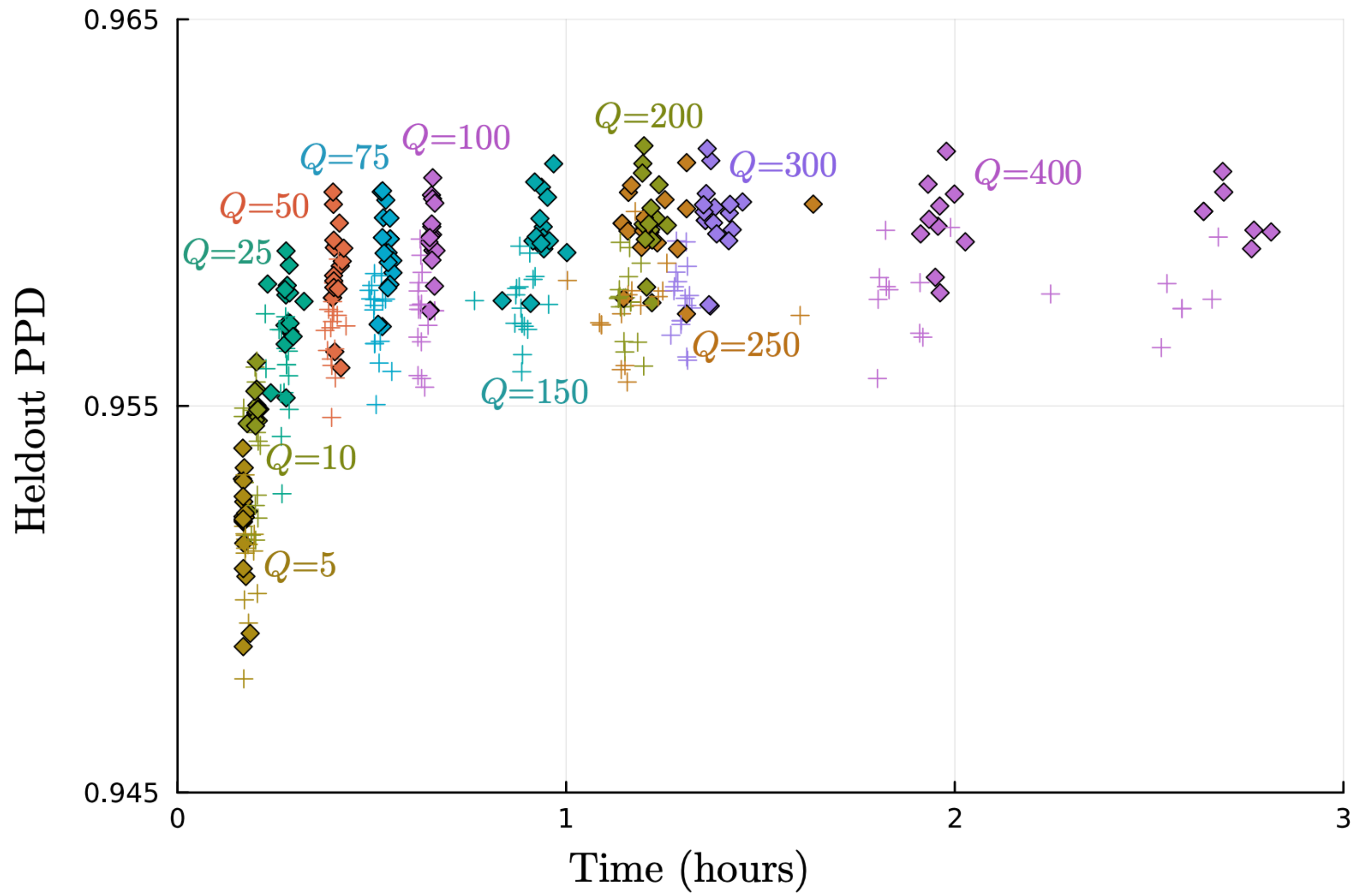


(a) Column fibers (b) Row fibers

Experiments

How dense does the core need to be?

- Consider two core shapes: small $(20 \times 20 \times 6 \times 3)$ and large $(50 \times 50 \times 6 \times 10)$
- Run $\text{A}\text{L}\ell_0\text{CORE}$ with $Q \in \{5, \ldots, 400\}$
- (Note: $Q = 400$ is 5% and 0.25% dense for the small and large cores, respectively)



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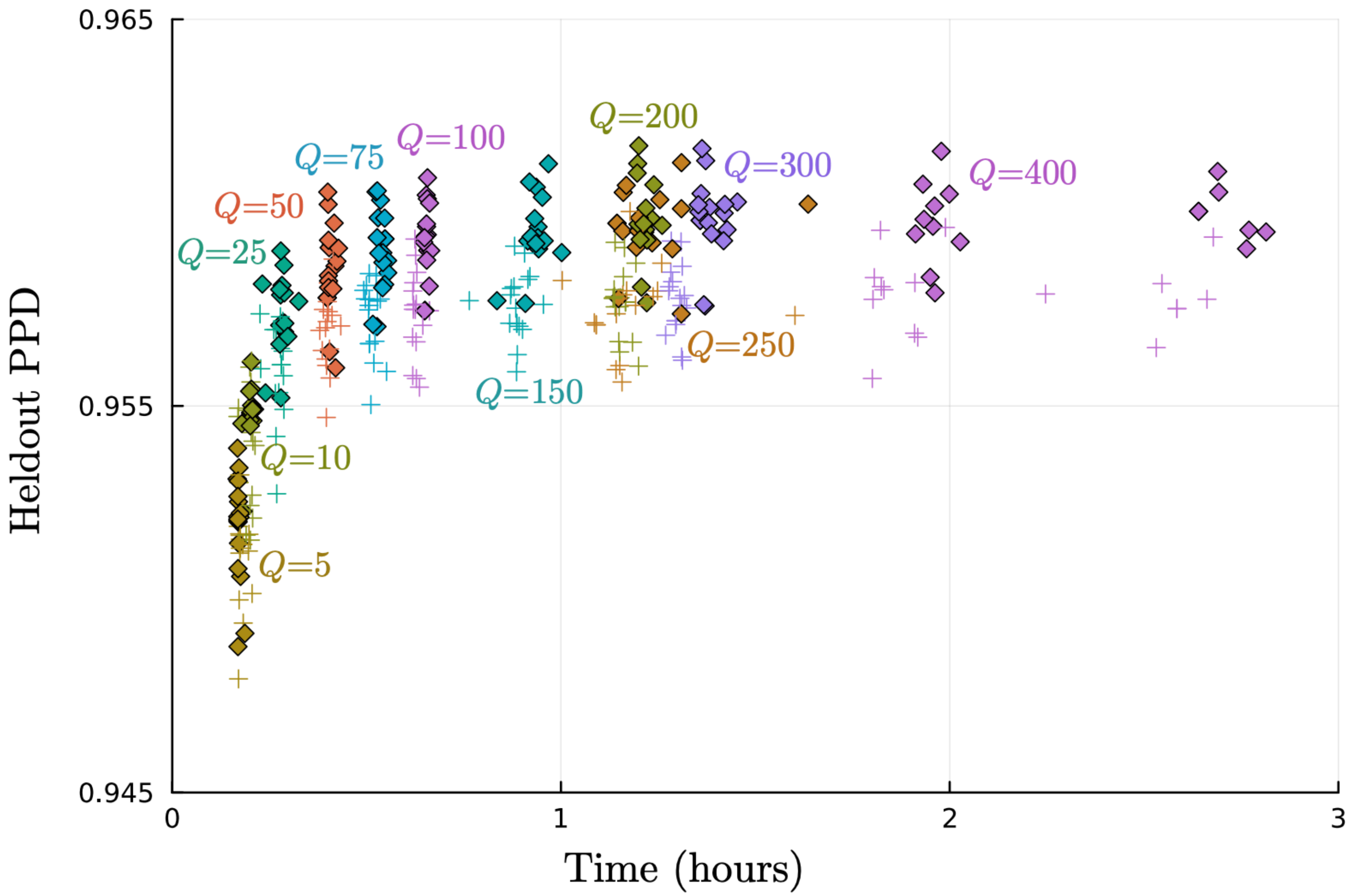

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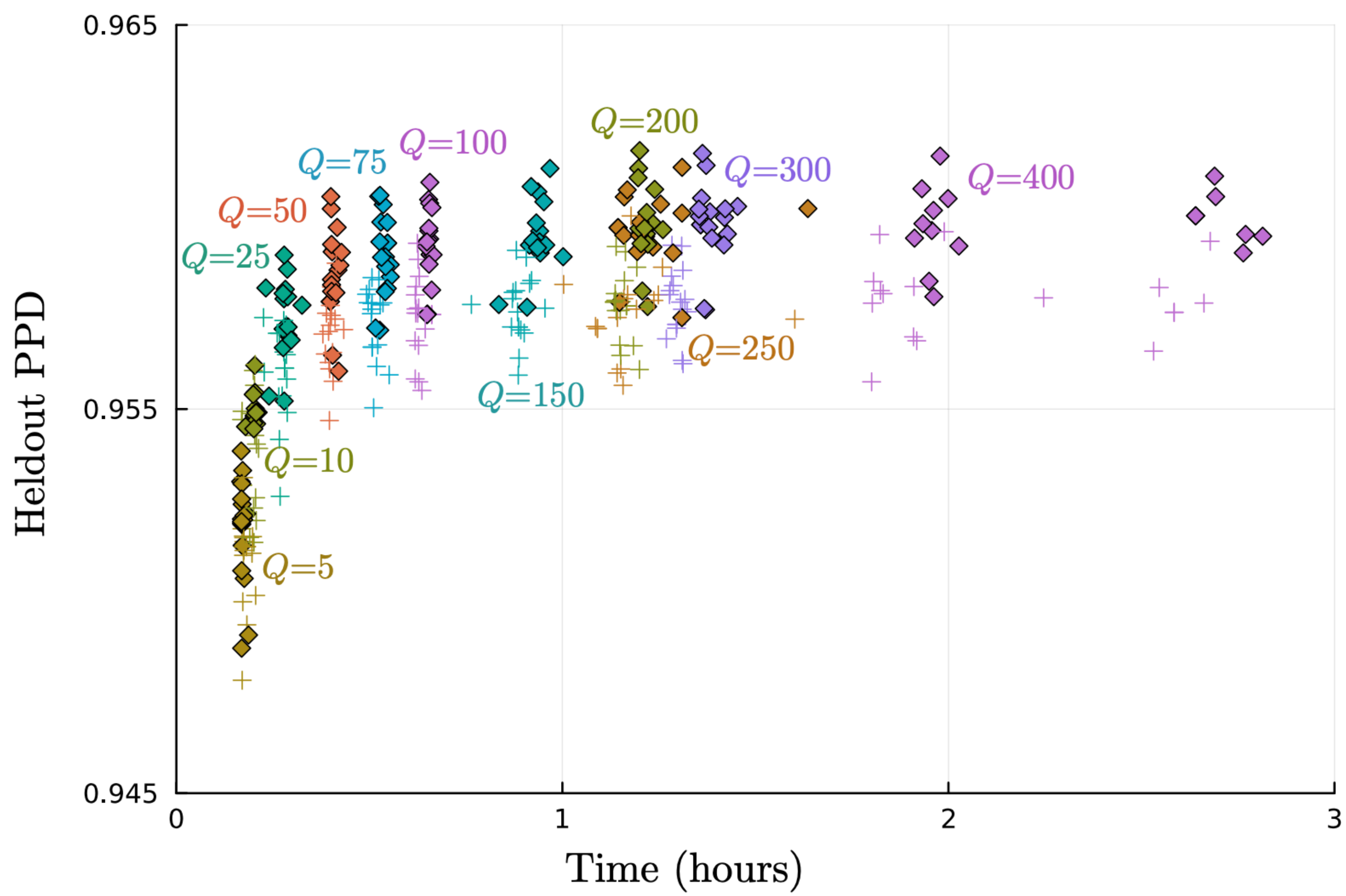

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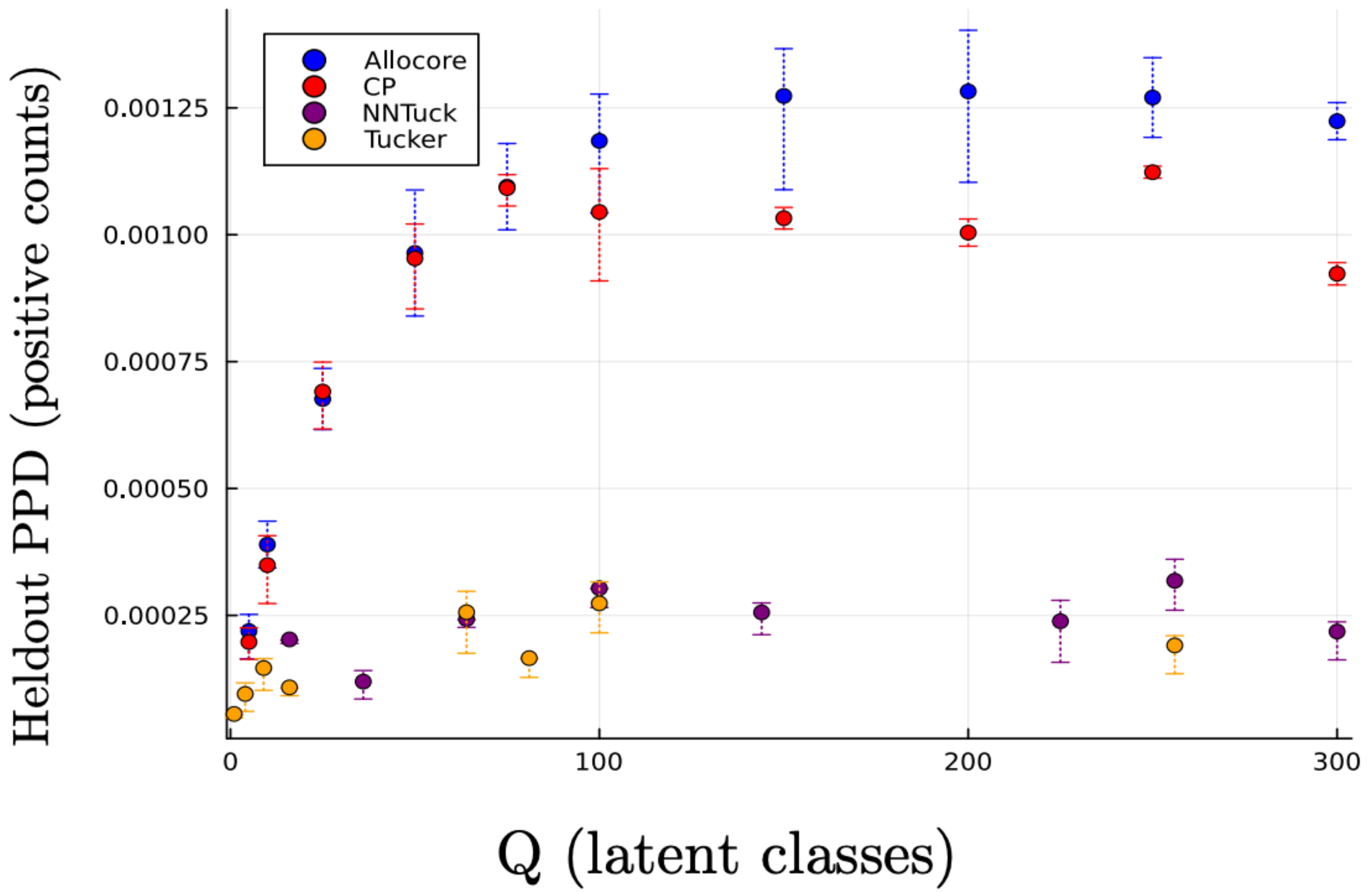
- EM with NNTUCK [Aguiar et al., 2023] would take an estimated **1.5 days**

EXPERIMENTS

How does it compare to CP and full Tucker with the **same number of parameters**?

- Set Q and use "**canonical**" $\textsc{Al}\ell_0\textsc{core}$ $(K_1 = \ldots K_M = Q)$
- Use CP decomposition with Q classes
- Use full Tucker ($\textsc{NNTuck}$) with a core size such that $|\boldsymbol{\Lambda}| \approx Q$



More parameter sharing than CP, and larger latent space than full Tucker.

POISSON ALℓ_0CORE

how active **country** *i* is
in (sender) **community** *c[q]*

$$y^{(t)}_{i \xrightarrow{a} j} \sim \text{Pois}\left(\sum_{q=1}^{Q} \psi^{(\rightarrow)}_{i,c[q]}\, \psi^{(\leftarrow)}_{j,d[q]}\, \phi_{a,k[q]}\, \theta^{(t)}_{r[q]}\, \lambda_q\right)$$

POISSON ALℓ_0CORE

how active **country** *j* is
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POISSON ALℓ_0CORE

how prevalent **action** a is
in **topic** $k[q]$

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how active **regime** $r[q]$ is at **time** t

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POISSON ALℓ_0CORE

allocated **non-zero value** q

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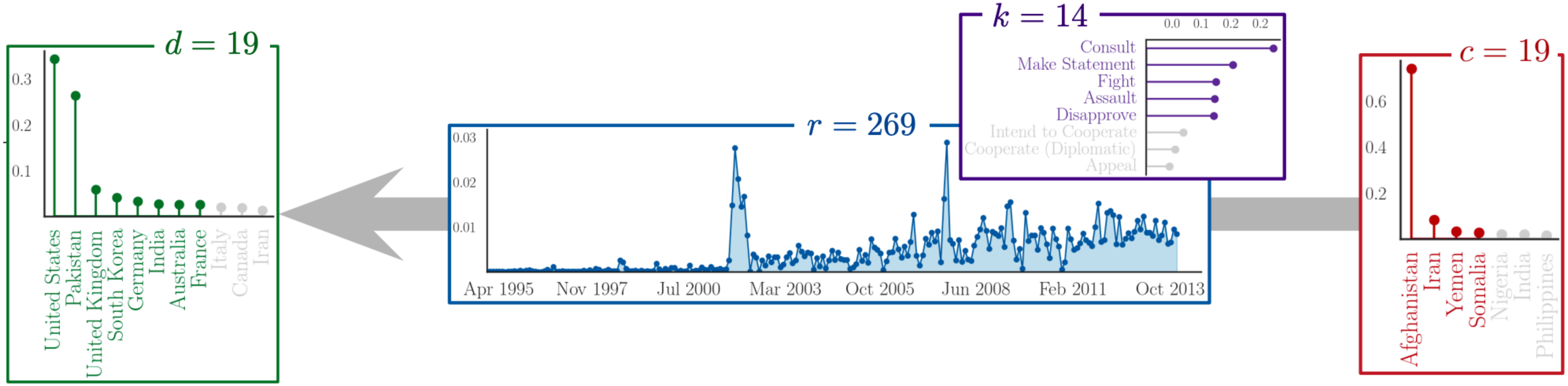
Example of inferred latent structure

Inferred non-zero core elements

rank	$\lambda^{(r)}_{c \xrightarrow{k} d}$	c	d	k	r
1	11.03	2	2	10	252
2	10.36	31	31	9	81
3	10.36	9	9	12	159
4	10.13	33	33	9	233
5	9.58	37	37	12	87
6	9.36	12	12	2	162
7	8.9	43	43	10	43
8	8.57	41	41	2	41
9	8.35	15	15	10	115
10	8.13	11	11	16	61
	...		...		
20	6.65	49	49	9	49
21	6.61	14	14	2	14
22	6.55	23	23	9	273
23	6.35	46	46	9	196
24	6.25	22	22	2	222
25	**6.24**	**40**	**40**	**19**	**40**
26	**6.23**	**19**	**19**	**14**	**269**
27	6.14	43	43	10	143
28	**6.14**	**6**	**6**	**19**	**106**
29	5.92	32	32	9	132
30	5.85	47	47	12	62
31	5.77	24	24	14	124
32	5.76	9	9	12	59
33	5.67	13	13	10	252
34	5.62	25	25	9	25
35	5.57	27	27	9	177
36	**5.53**	**17**	**17**	**14**	**67**
37	5.42	30	30	9	30
38	5.36	5	5	10	255
39	5.34	3	3	16	3
40	5.24	10	10	10	210
	...		...		

- ICEWS tensor (1995-2013), monthly: (250 x 250 x 20 x 228)
- Core size: (50 x 50 x 20 x 300) → 15 million (possible) classes
- $Q = 400$

Example of inferred latent structure



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6	9.36	12	12	2	162
7	8.9	43	43	10	43
8	8.57	41	41	2	41
9	8.35	15	15	10	115
10	8.13	11	11	16	61
	...		...		
20	6.65	49	49	9	49
21	6.61	14	14	2	14
22	6.55	23	23	9	273
23	6.35	46	46	9	196
24	6.25	22	22	2	222
25	**6.24**	**40**	**40**	**19**	**40**
26	**6.23**	**19**	**19**	**14**	**269**
27	6.14	43	43	10	143
28	**6.14**	**6**	**6**	**19**	**106**
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34	5.62	25	25	9	25
35	5.57	27	27	9	177
36	**5.53**	**17**	**17**	**14**	**67**
37	5.42	30	30	9	30
38	5.36	5	5	10	255
39	5.34	3	3	16	3
40	5.24	10	10	10	210
	...		...		

Example of inferred latent structure

Inferred nor

rank	$\lambda^{(r)}_{c \xrightarrow{k} d}$				
1	11.03				
2	10.36				
3	10.36				
4	10.13				
5	9.58				
6	9.36				
7	8.9				
8	8.57				
9	8.35				
10	8.13				
	…				
20	6.65				
21	6.61				
22	6.55				
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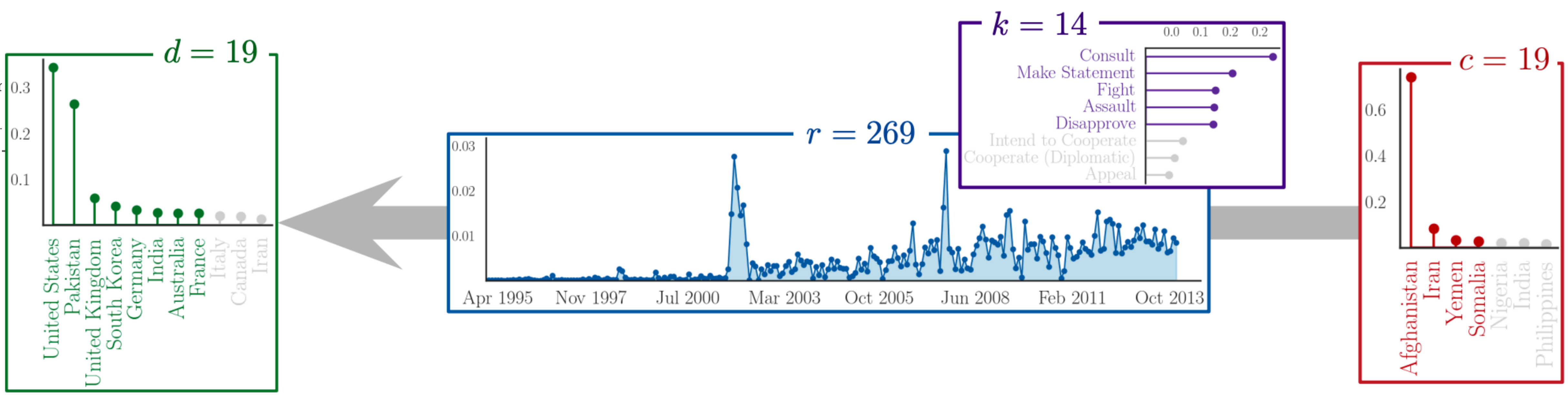

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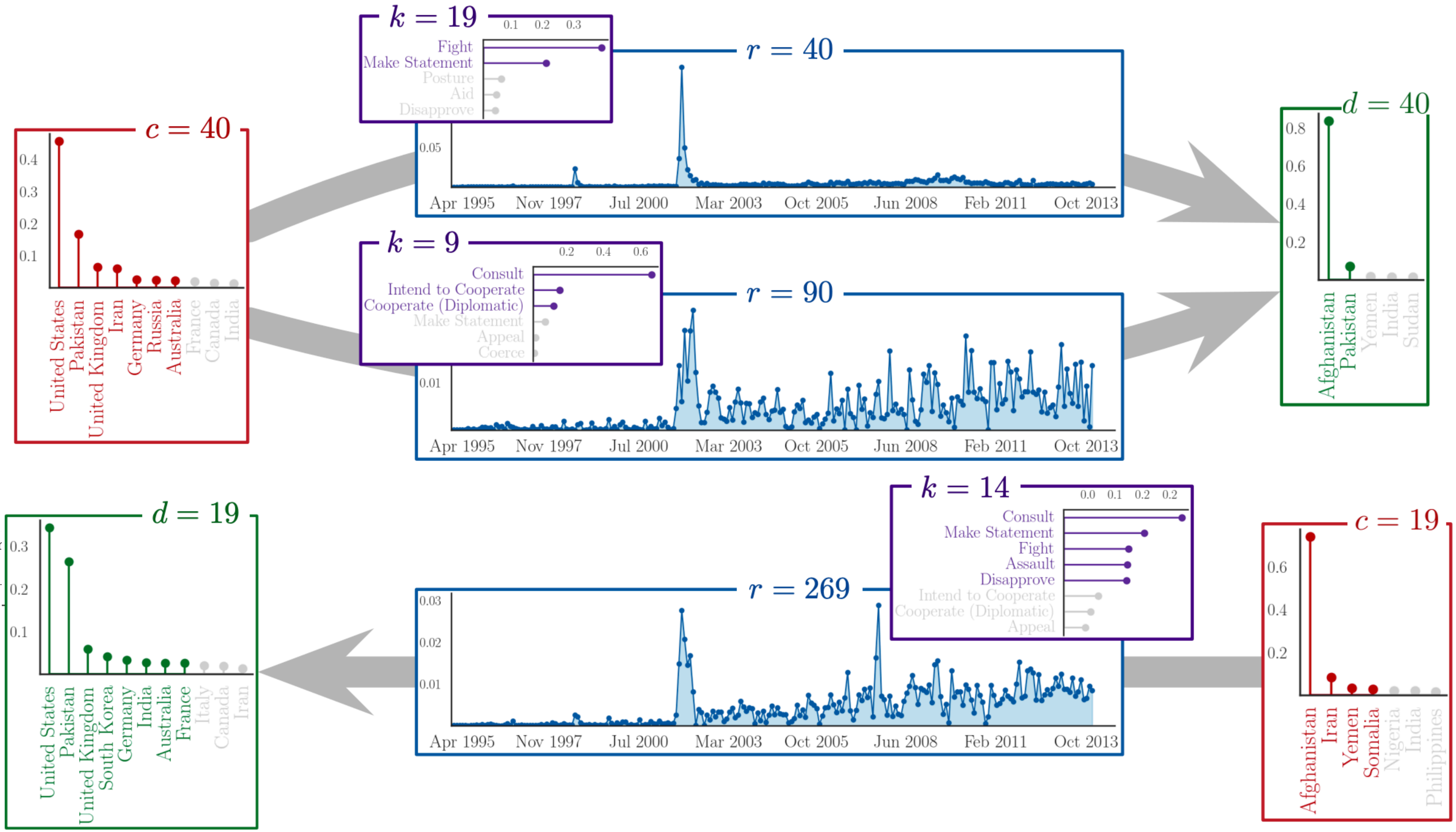

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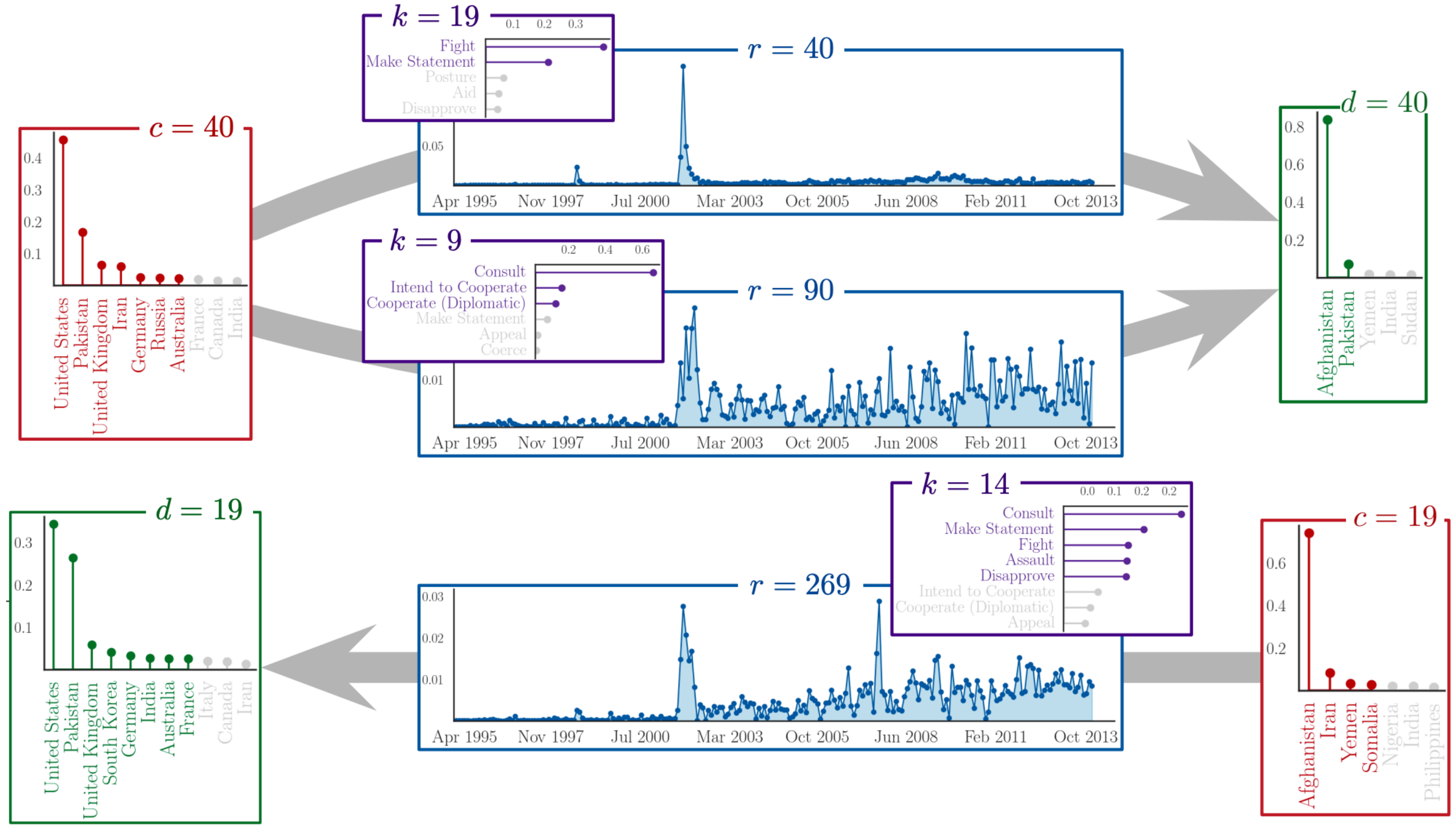

Inferred non-zero core elements

rank	$\lambda^{(r)}_{c \xrightarrow{k} d}$	c	d	k	r
1	11.03	2	2	10	252
2	10.36	31	31	9	81
3	10.36	9	9	12	159
4	10.13	33	33	9	233
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6	9.36	12	12	2	162
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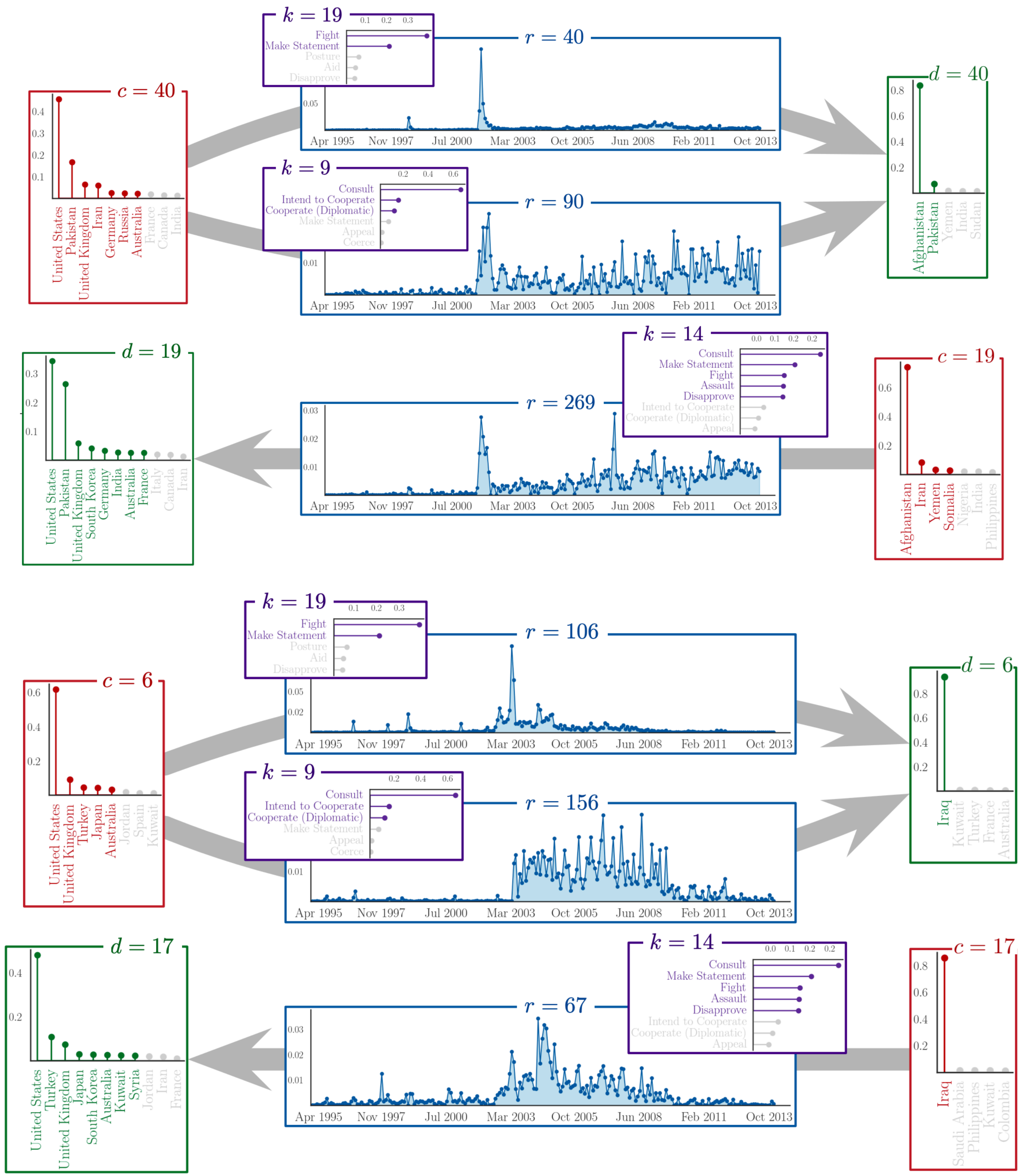

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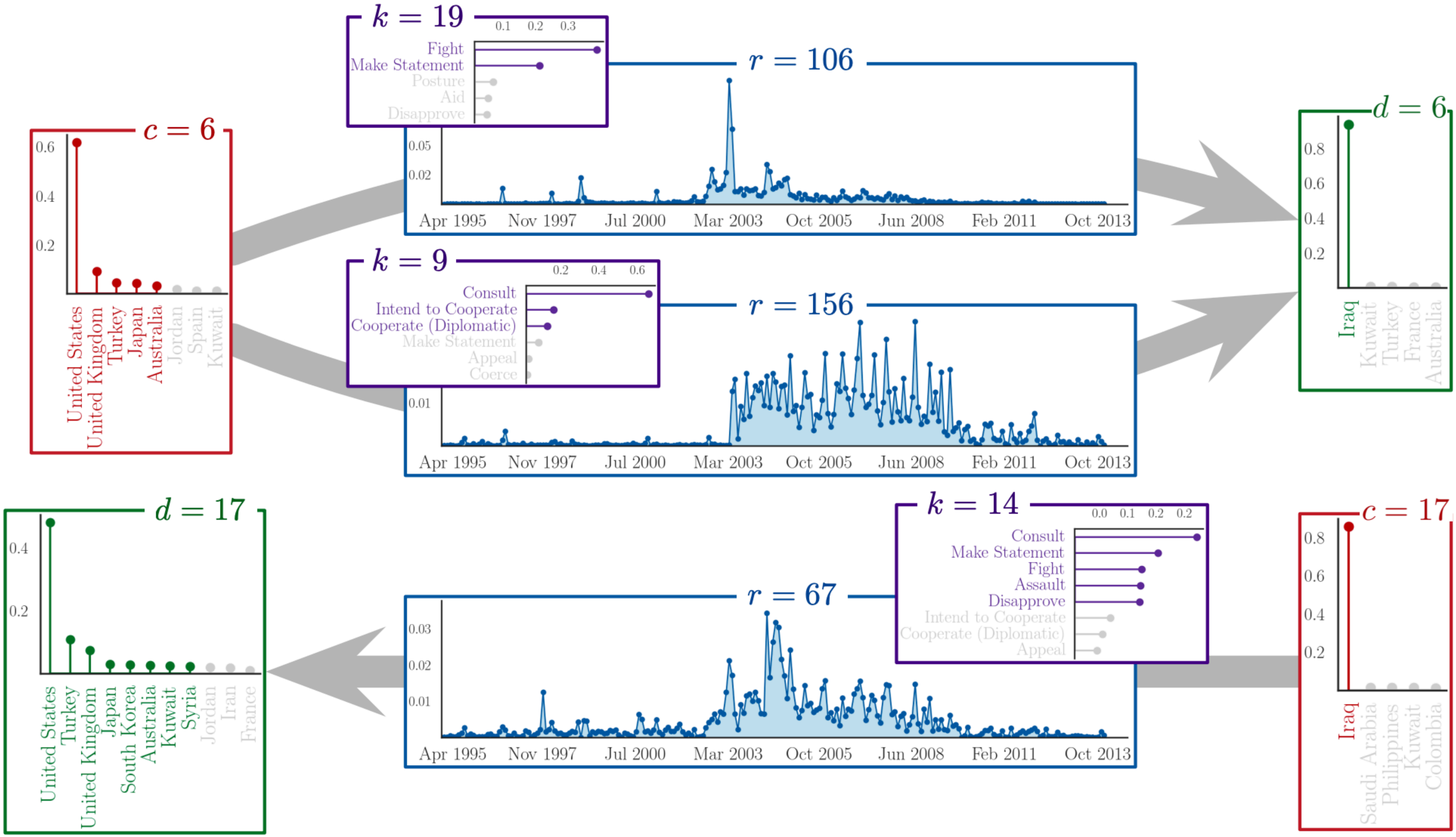

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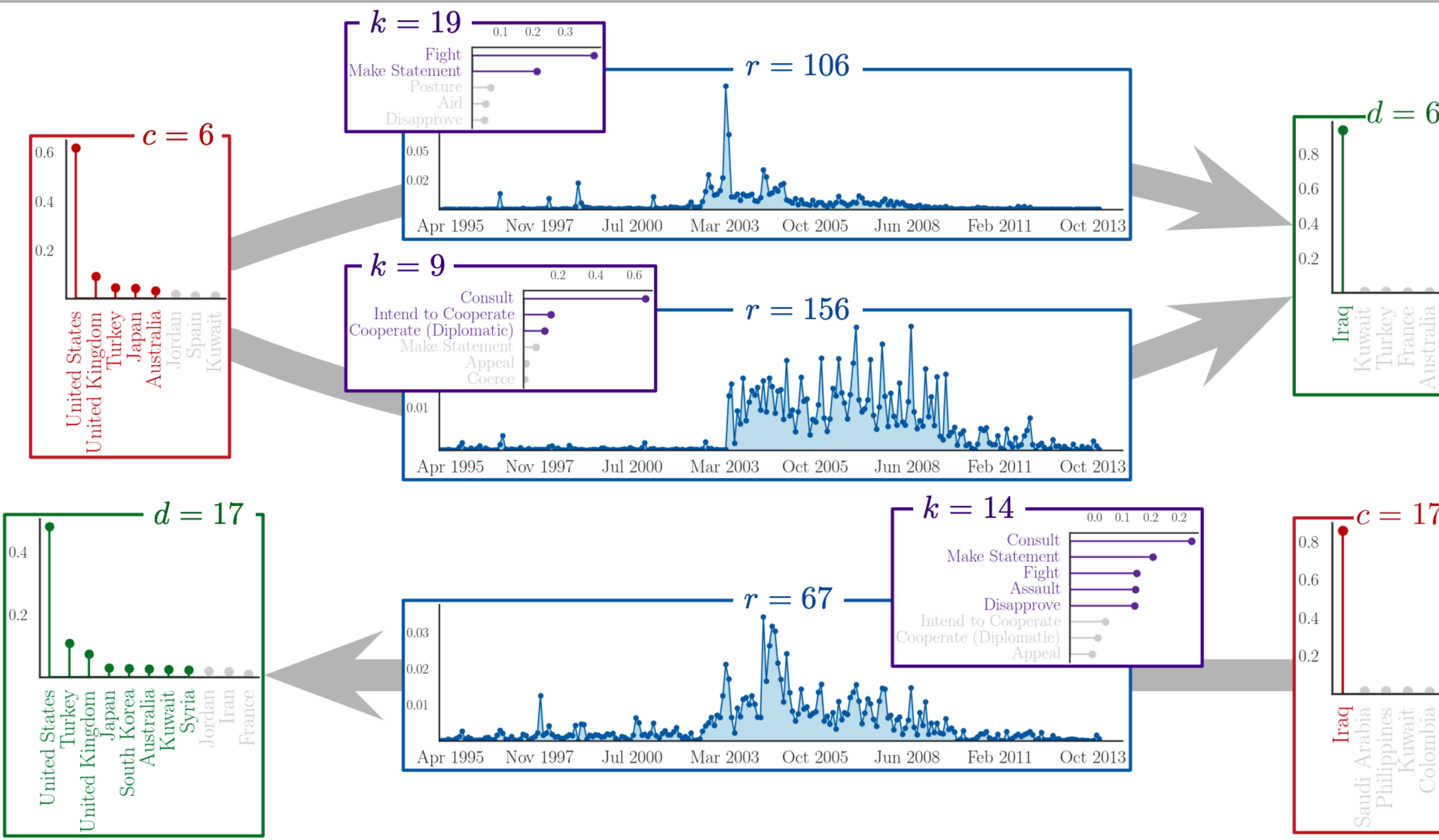

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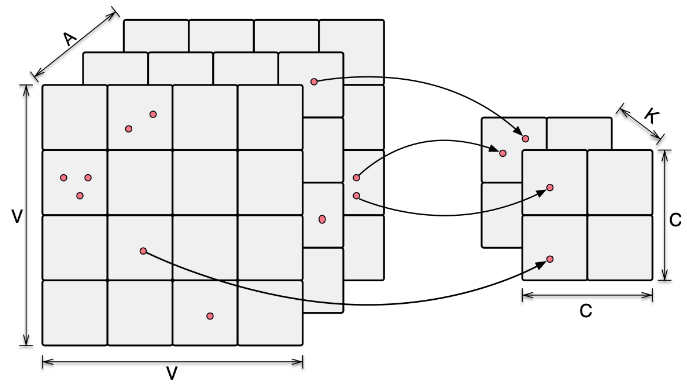
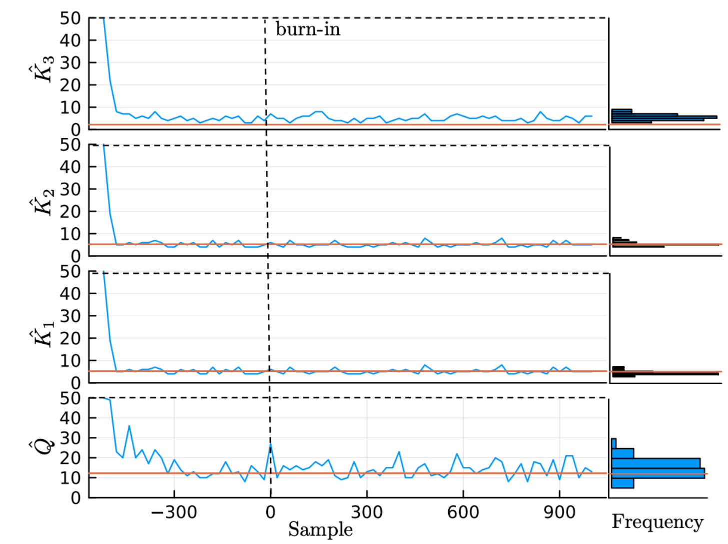
Same pattern found for **five** major wars in the observation period!

DISCUSSION / FUTURE

- **Sparse token allocation** as a principle (observed or latent)



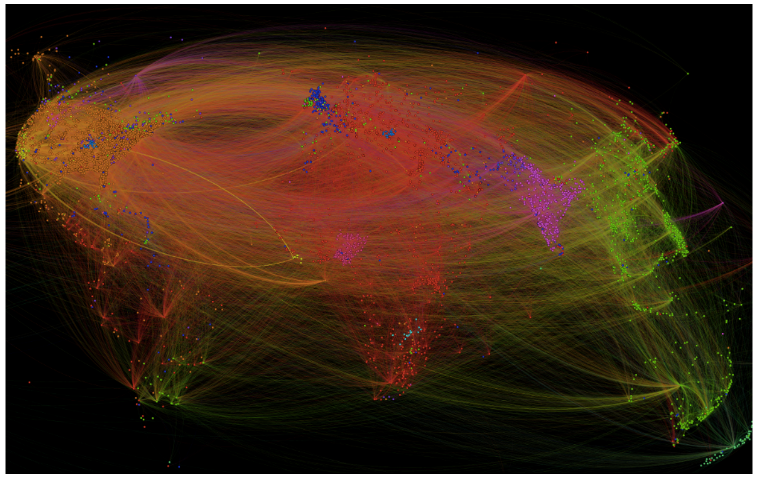
- Allocated **sparsity in factor matrices**; different motivation:
  - Interpretability (e.g., thresholding)
  - Identifiability (e.g., anchor features)



- Didn't talk about: **inferring** Q

- More on dyadic events:
  - Modeling everything by **publisher**
  - Modeling everything by **data set**

- One current extension / focus: **hypergraphs**
  - e.g., degree-5 networks → HUGE core tensors



BETTER INFERENCE

- **Dependent** prior for locations and values:

$$\mathbf{\Pi} \sim \text{Dirichlet}(1,\ldots,1)$$

$$\kappa_q \sim \text{Categorical}(\mathbf{\Pi}) \quad \text{for} \quad q = 1\ldots Q$$

$$\lambda_{\boldsymbol{\kappa}} \sim \begin{cases} \text{Gamma}(m_\kappa 1) & \text{if } m_\kappa > 0 \\ \delta_0 & \text{otherwise} \end{cases}$$

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Sum-proportion independence of gammas [Lukacs 1956]

"Delta Dirichlet" distribution [Lewy 1996]

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$$y_{\mathbf{i},\boldsymbol{\kappa}} \sim \text{Pois}\left(\lambda_{\boldsymbol{\kappa}} \cdots\right) \xrightarrow{\text{“Latent source” representation}} \begin{array}{l} y_{\bullet} \sim \text{Pois}\left(\lambda_{\bullet} \cdots\right) \\ \left(y_{\boldsymbol{\kappa}}\right)_{\boldsymbol{\kappa}} \sim \text{Multinomial}\left(y_{\bullet}, \tilde{\boldsymbol{\Lambda}}\right) \end{array}$$

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"Latent source" representation

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$$\boxed{\begin{array}{ll} \mathbf{M} \sim \text{DirMult}(Q,1,\ldots,1) & \lambda_\bullet \sim \text{Gamma}(Q,1) \\ \left(y_{\boldsymbol{\kappa}}\right)_{\boldsymbol{\kappa}} \sim \text{DeltaDirMult}\left(y_\bullet, \mathbf{M}\right) & y_\bullet \sim \text{Pois}\left(\lambda_\bullet \cdots\right) \end{array}}$$

BETTER INFERENCE

- Update to $\lambda_\bullet$ is conjugate
- New(?) form of "quasi"-conjugacy:

$$(\mathbf{M} \mid -) = \mathbf{1}(\mathbf{Y} > 0) + \mathbf{M}'$$

$$\mathbf{M}' \sim \text{DirMult}(Q - ||\mathbf{Y}||_0, 1 + \mathbf{Y})$$

- We can sample $\mathbf{M}'$ exactly using an urn scheme that is $\mathcal{O}(Q)$

$$\mathbf{M} \sim \text{DirMult}(Q, 1, \ldots, 1) \qquad \lambda_\bullet \sim \text{Gamma}(Q, 1)$$

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OUTLINE

- **Motivation:** Dyadic event data (and its challenges)

- **Non-negative (Poisson) tensor decompositions**
  - **CP** extracts "multilateral" structure [Schein et al., 2015]
  - **Tucker** extracts "communities" [Schein et al., 2016]
  - Why (allocative) Poisson factorization
  - **ALℓ_0CORE** avoids the "exponential blowup" [Hood & Schein, 2024]

Appendix (*time permitting*)

- **State-space** modeling
  - **Gamma state-space** models [Schein et al., 2016b; Schein et al., 2019]
  - **Modeling "escalation"** with a new matrix prior [Stoehr et al., 2023]

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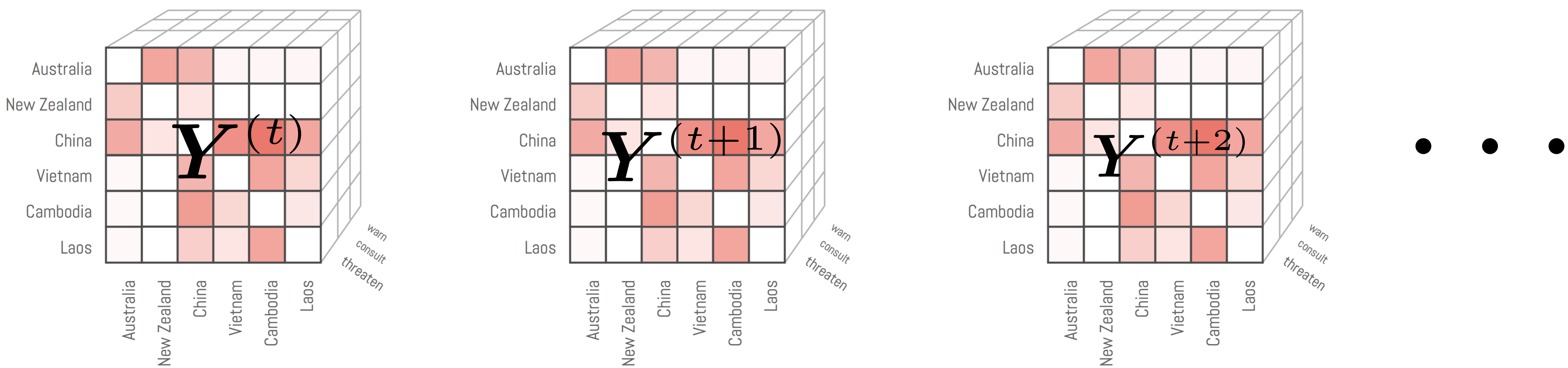
Building in **temporal structure**

dynamic tensor



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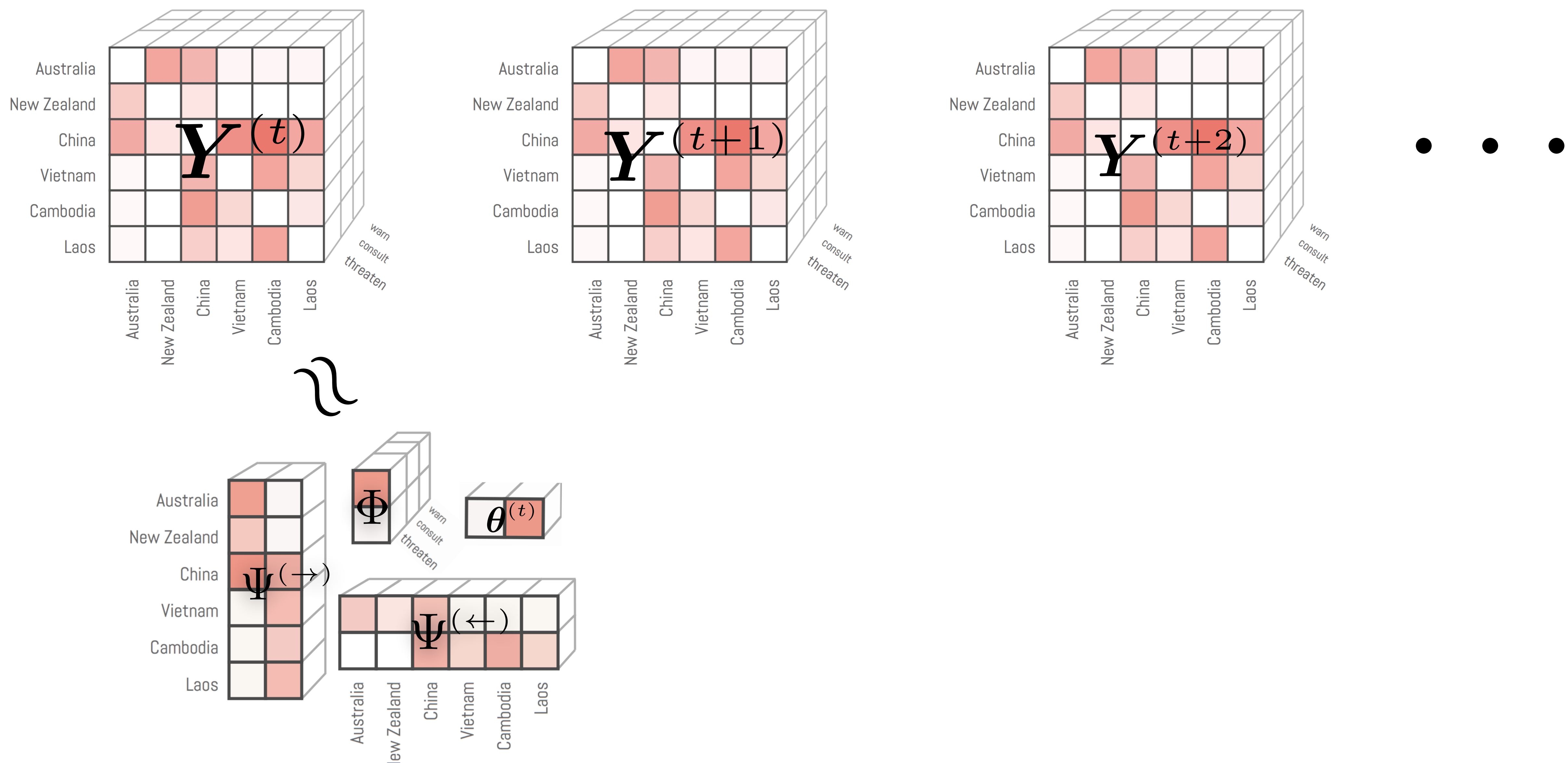

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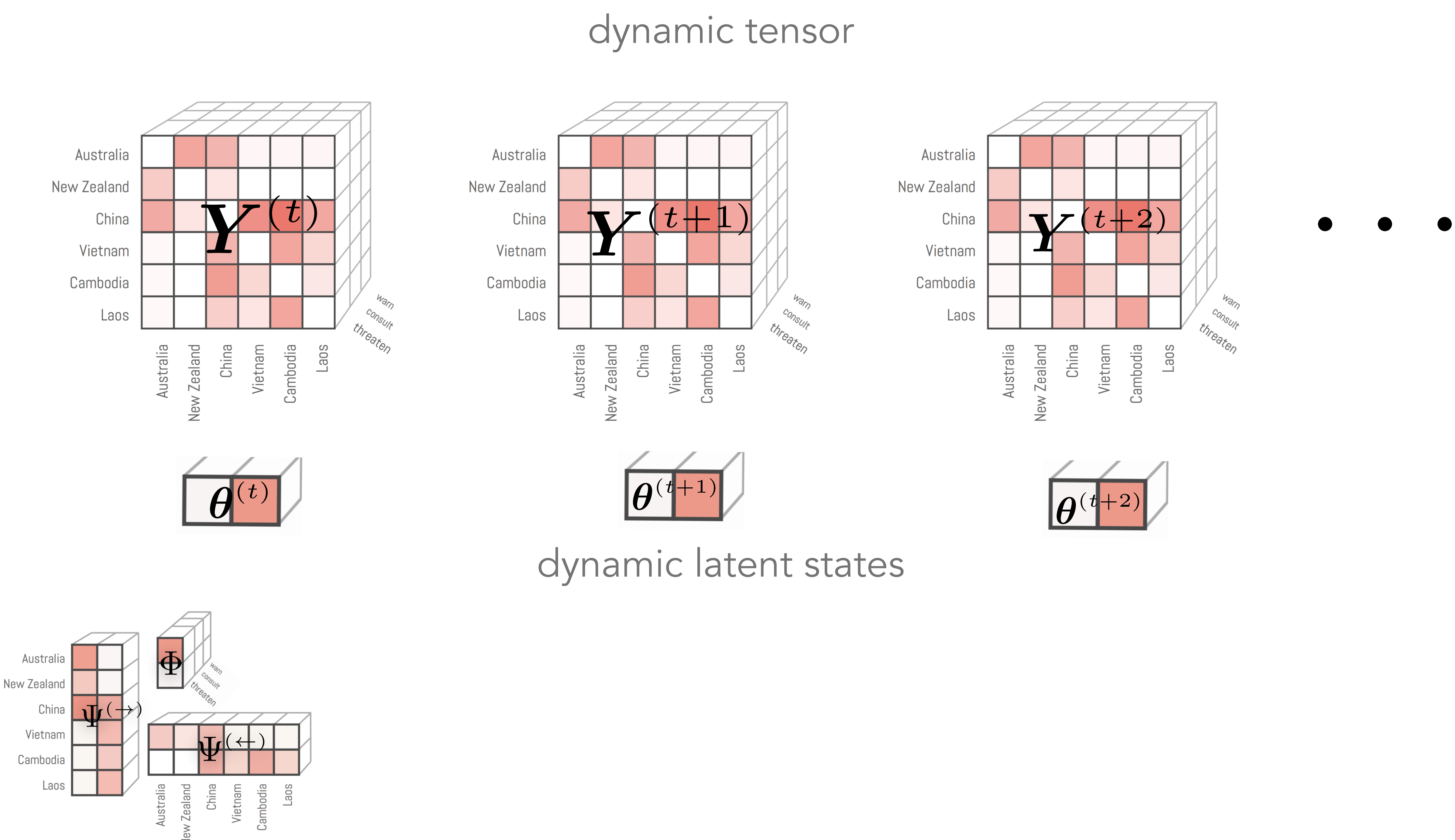

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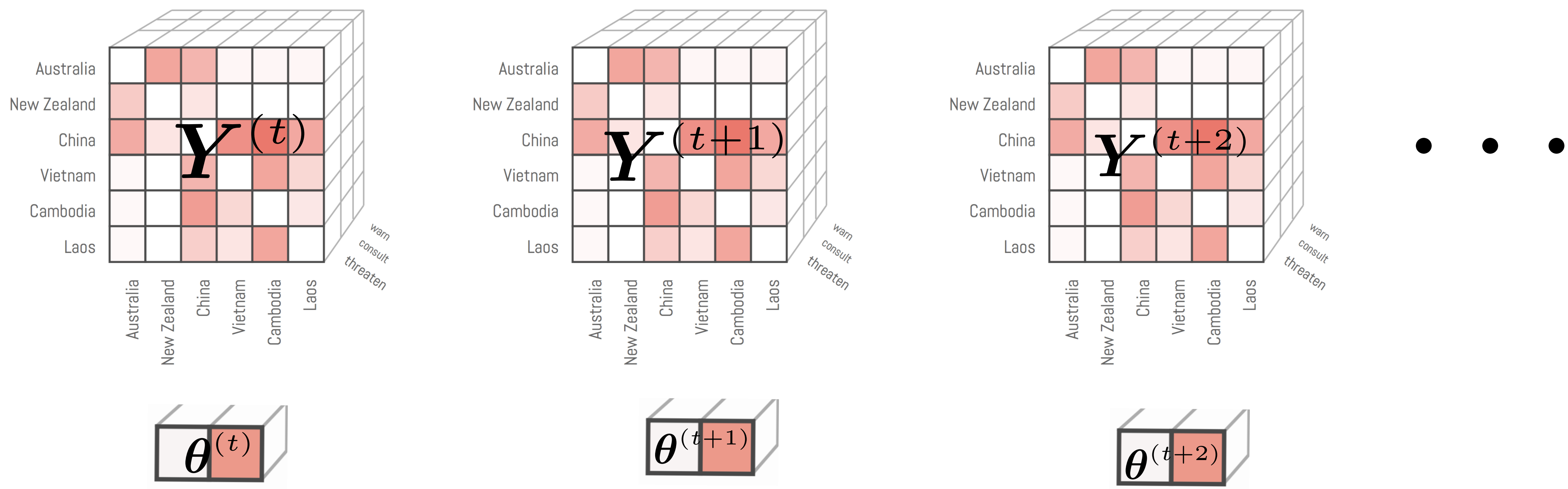
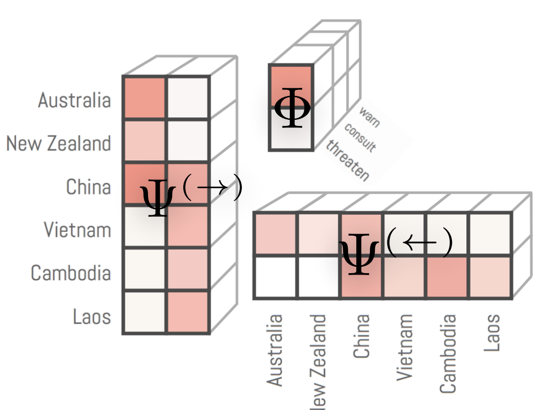


dynamic latent states



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$$\boldsymbol{\theta}^{(t)} \sim P(\boldsymbol{\theta}^{(t)} \mid \boldsymbol{\theta}^{(t-1)})$$

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$$y_{\mathbf{i}} \sim \text{Pois}\Big(\sum \cdots\Big)$$

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Poisson LDS [Macke et al., 2011]

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Gaussian priors**:**

toolkit *of approximate Bayesian inference*

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OUTLINE

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Poisson—gamma dynamical systems

[NeurIPS '16]

$$\theta_k^{(t)} \sim \mathrm{Gam}\left(\tau_0 \sum_{k'=1}^{K} \pi_{kk'} \theta_{k'}^{(t-1)},\ \tau_0\right)$$

Poisson—gamma dynamical systems

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how active component *k'*
is at time step *t-1*

Poisson—gamma dynamical systems

[NeurIPS '16]

$$\theta_k^{(t)} \sim \mathrm{Gam}\left(\tau_0 \sum_{k'=1}^{K} \pi_{kk'} \theta_{k'}^{(t-1)},\ \tau_0\right)$$

the **probability** of transitioning
from component k' into k

Poisson—gamma dynamical systems

[NeurIPS '16]

$$\theta_k^{(t)} \sim \text{Gam}\left(\tau_0 \sum_{k'=1}^{K} \pi_{kk'} \theta_{k'}^{(t-1)},\ \tau_0\right)$$

$$\boxed{\sum_{k=1}^{K} \pi_{kk'} = 1}$$

the **probability** of transitioning from component *k′* into *k*

Poisson—gamma dynamical systems

[NeurIPS '16]

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Poisson—gamma dynamical systems

[NeurIPS '16]

$$\theta_k^{(t)} \sim \mathrm{Gam}\left(\tau_0 \sum_{k'=1}^{K} \pi_{kk'} \theta_{k'}^{(t-1)},\ \tau_0\right)$$

concentration
hyperparameter

Poisson—gamma dynamical systems

[NeurIPS '16]

$$\mathbb{E}\left[\theta_k^{(t)}\right] = \frac{\tau_0 \sum_{k'=1}^{K} \pi_{kk'} \theta_{k'}^{(t-1)}}{\tau_0}$$

Poisson—gamma dynamical systems

[NeurIPS '16]

$$\mathbb{E}\left[\theta_k^{(t)}\right] = \frac{\cancel{\tau_0} \sum_{k'=1}^{K} \pi_{kk'} \theta_{k'}^{(t-1)}}{\cancel{\tau_0}}$$

Poisson—gamma dynamical systems

[NeurIPS '16]

$$\mathbb{E}\left[\theta_k^{(t)}\right] = \sum_{k'=1}^{K} \pi_{kk'}\theta_{k'}^{(t-1)}$$

Poisson—gamma dynamical systems

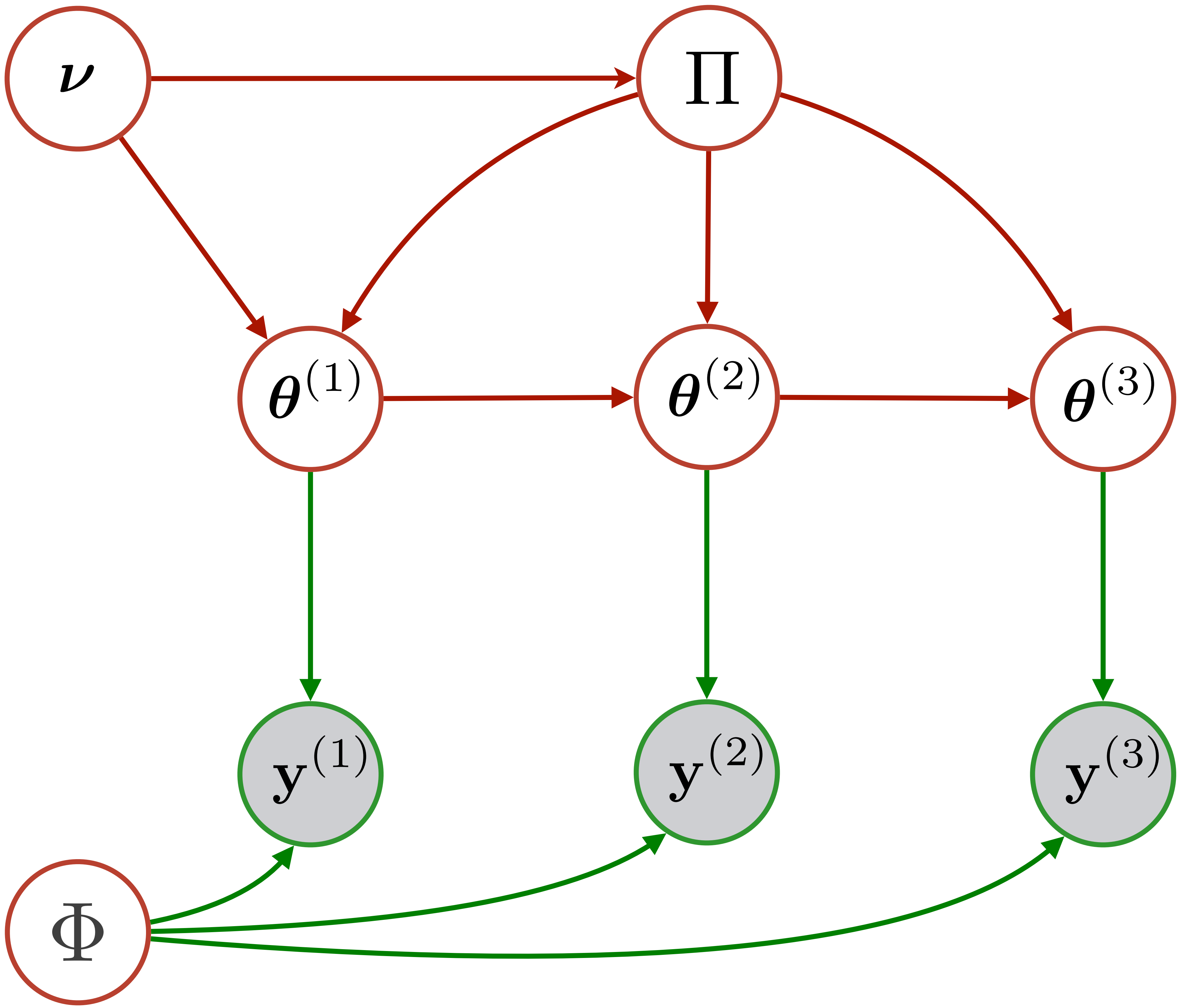
[NeurIPS '16]

$$\mathbb{E}\left[\boldsymbol{\theta}^{(t)}\right] = \Pi \boldsymbol{\theta}^{(t-1)}$$

$$\mathbb{E}\left[\boldsymbol{y}^{(t)}\right] = \Phi \boldsymbol{\theta}^{(t)}$$

(matches linear dynamical systems)

Technical challenge

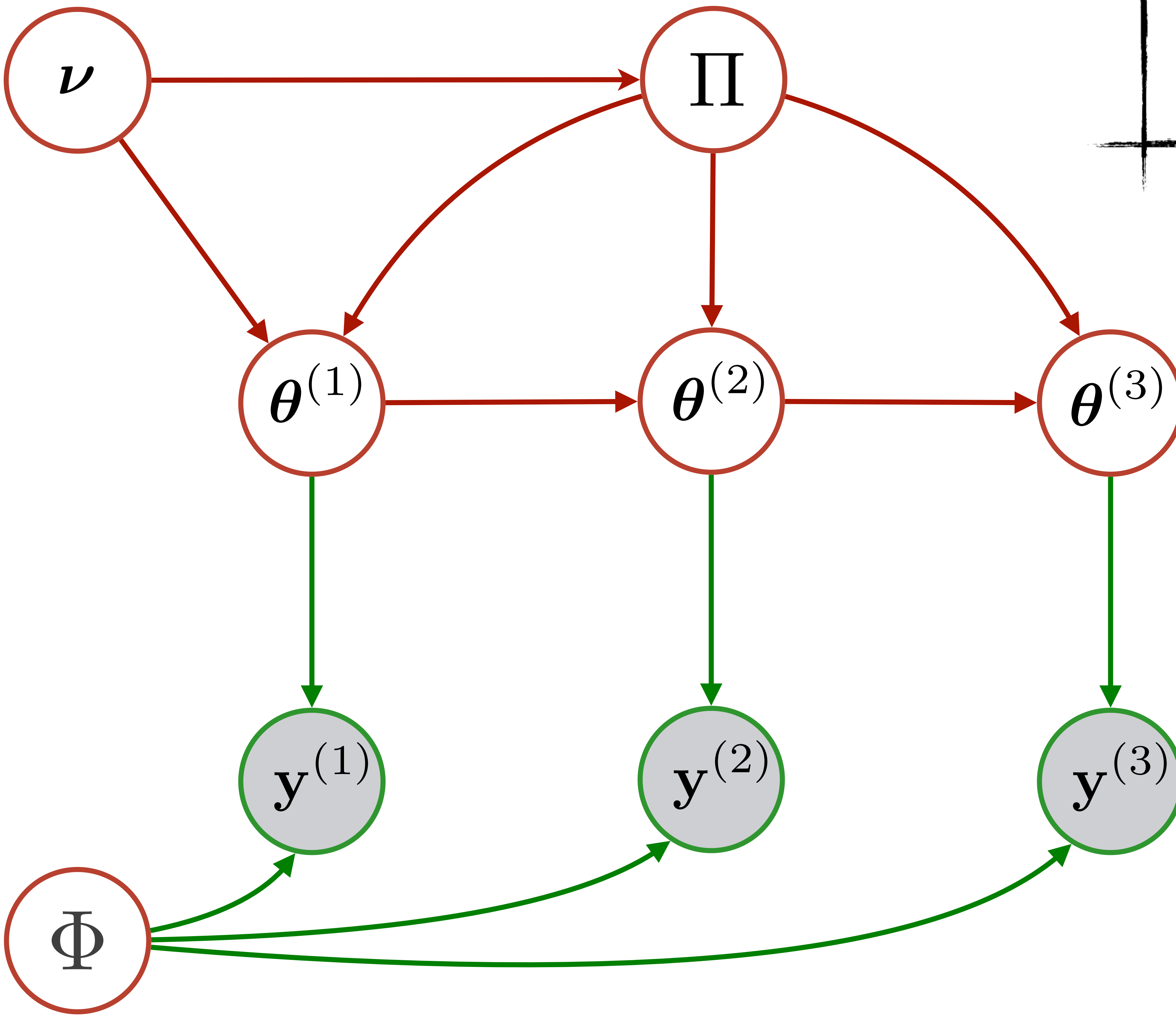


Technical challenge

Legend

Poisson/Multinomial

Gamma/Dirichlet



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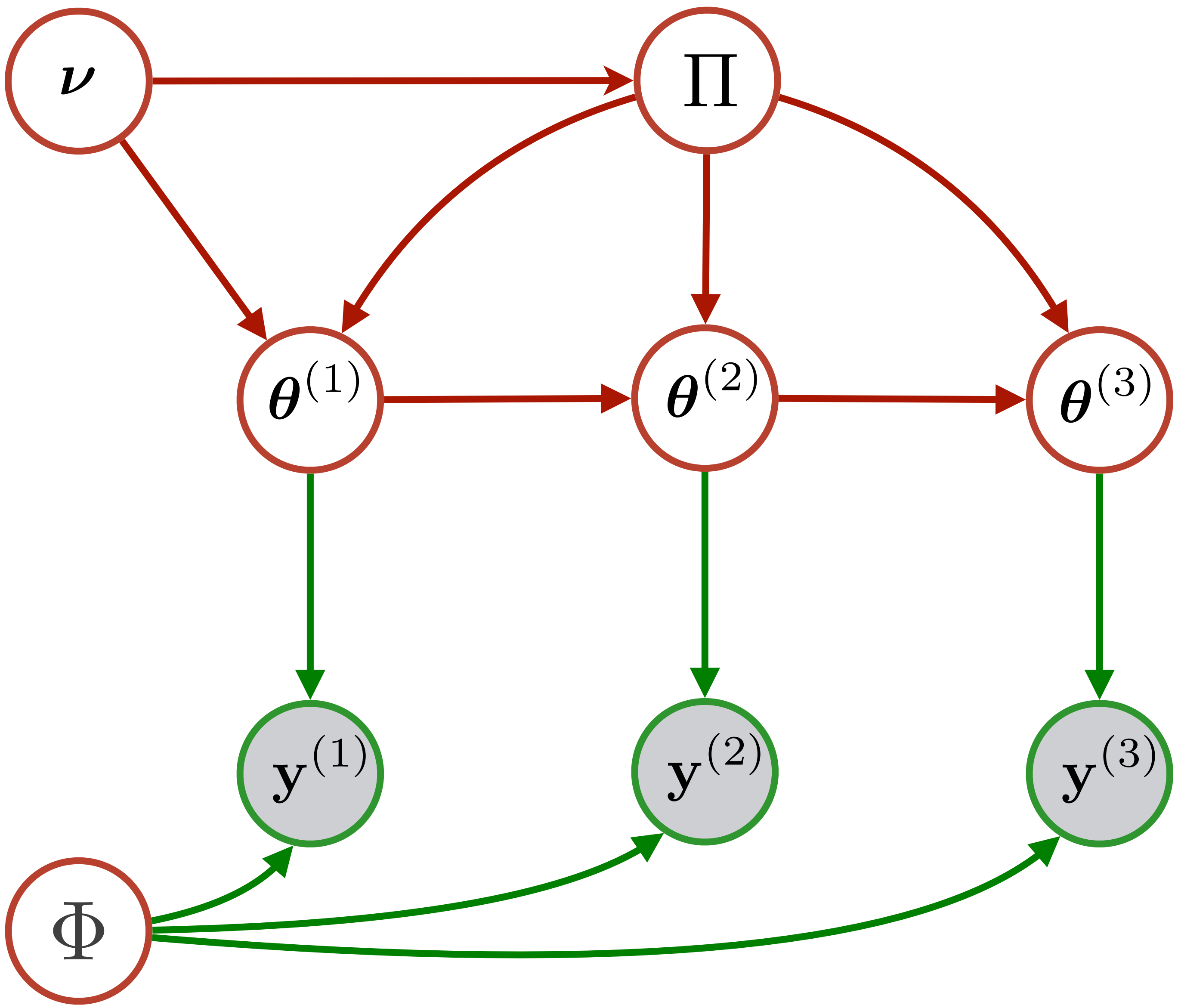

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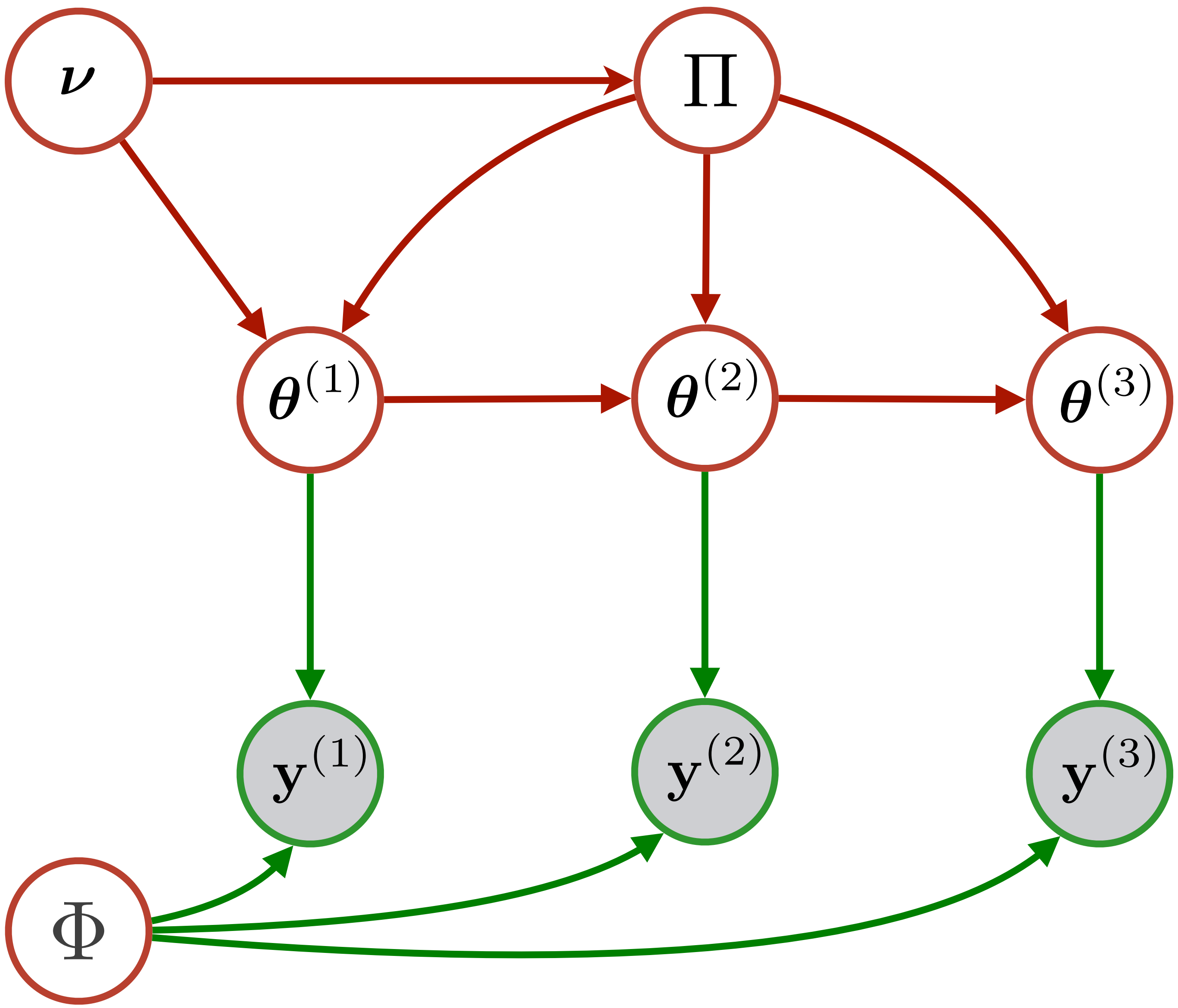


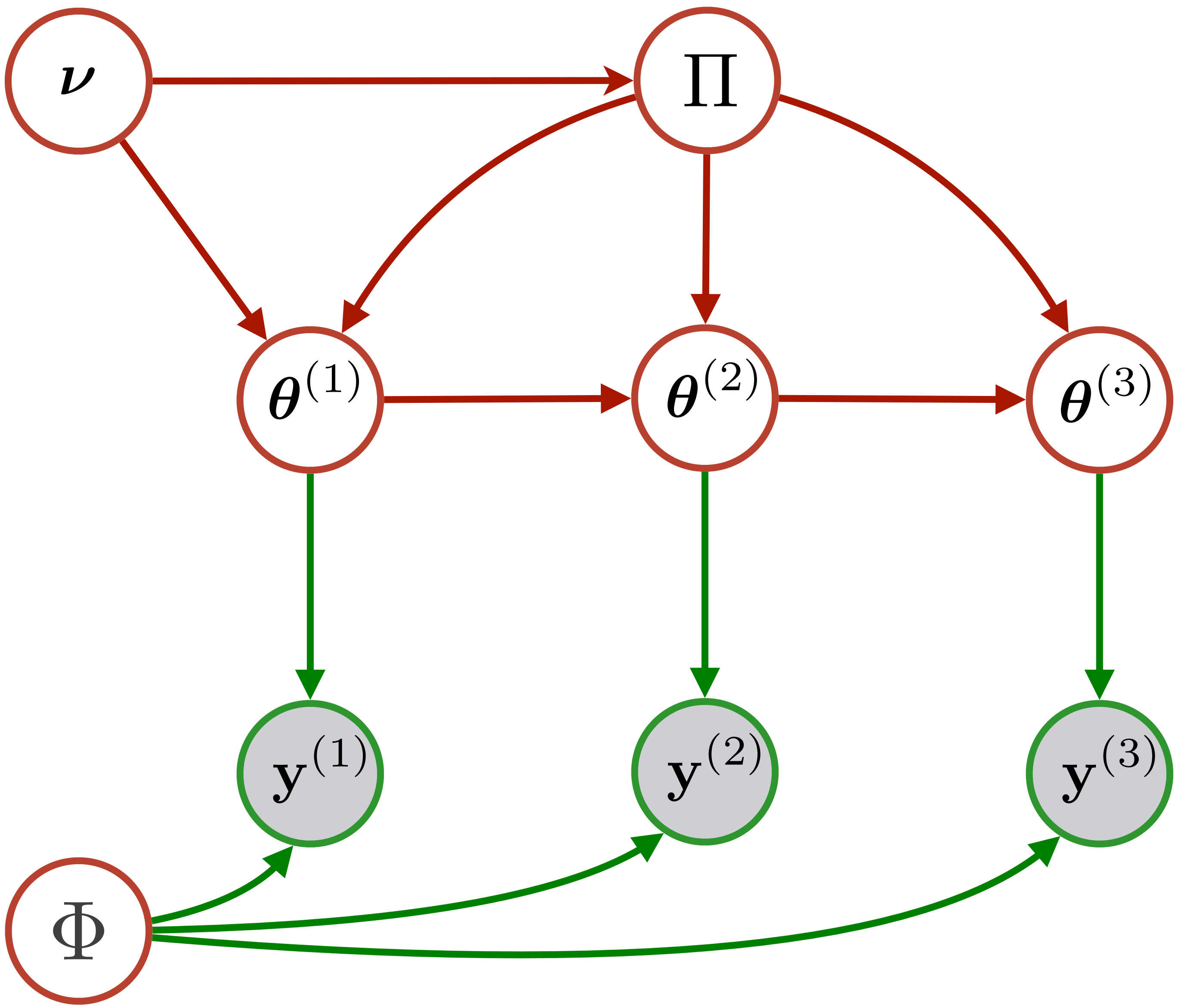
$$P(\Phi \mid Y)$$

Technical challenge

Legend

Poisson/Multinomial

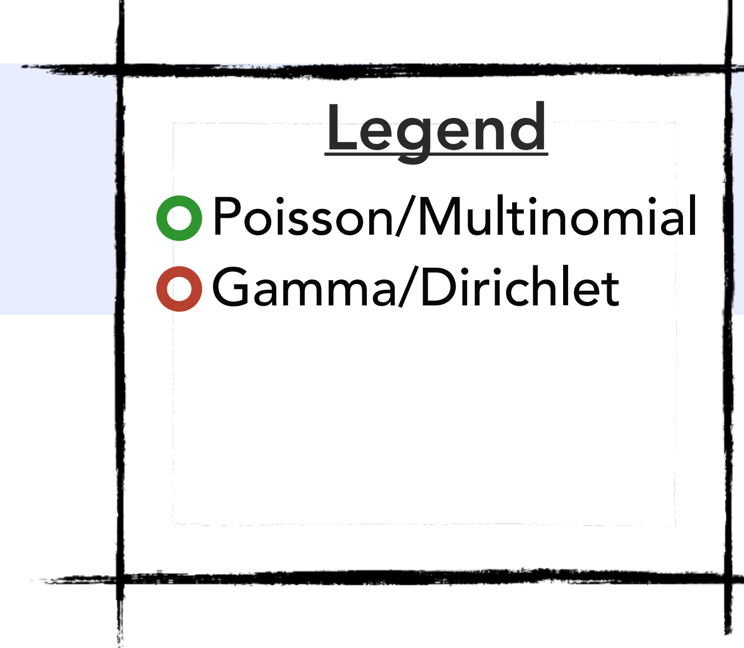
Gamma/Dirichlet

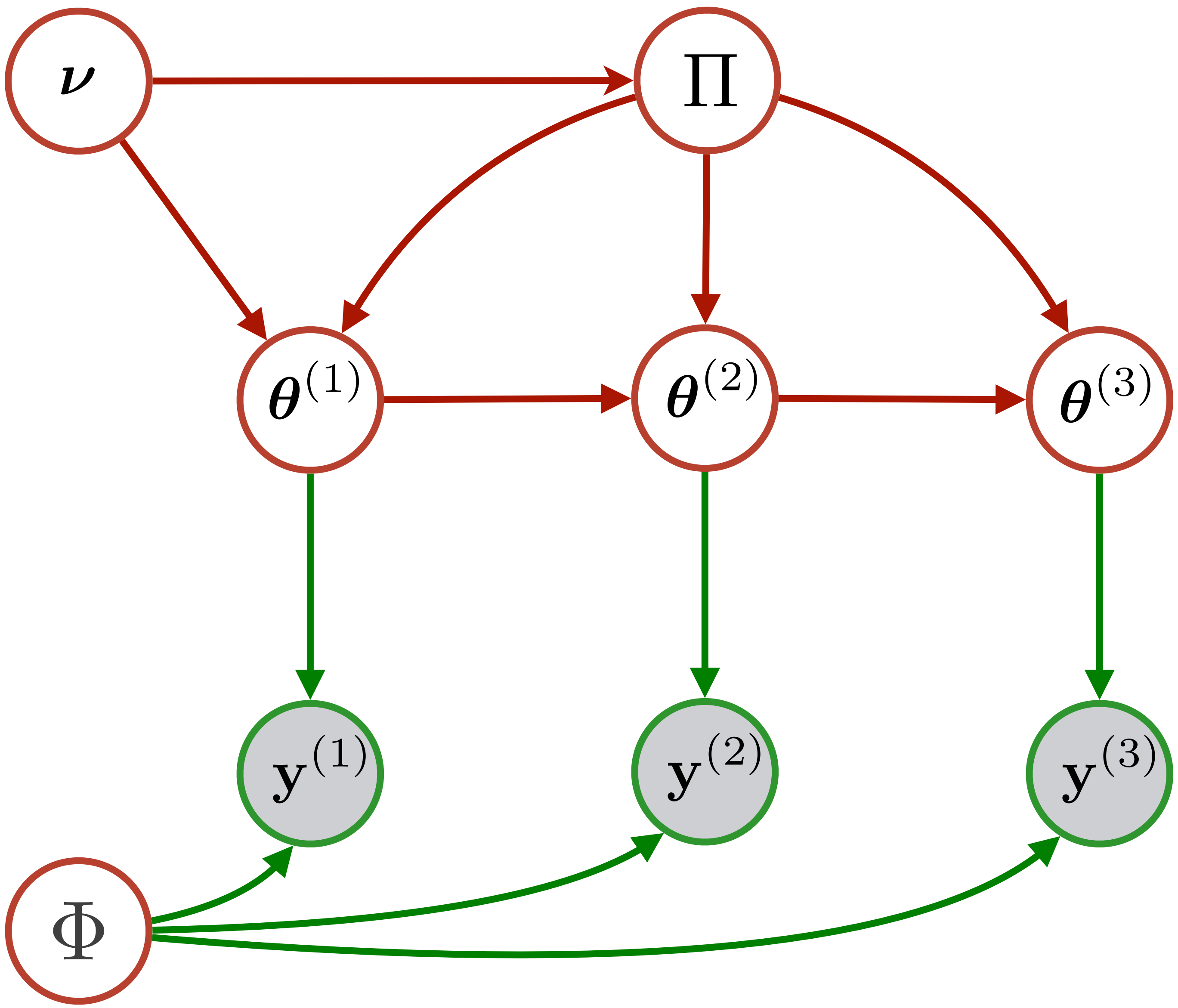


$$P(\Phi \mid Y) \checkmark$$

(conditional posterior has closed form)

Technical challenge





$$\Phi \sim P(\Phi \mid Y) \checkmark$$

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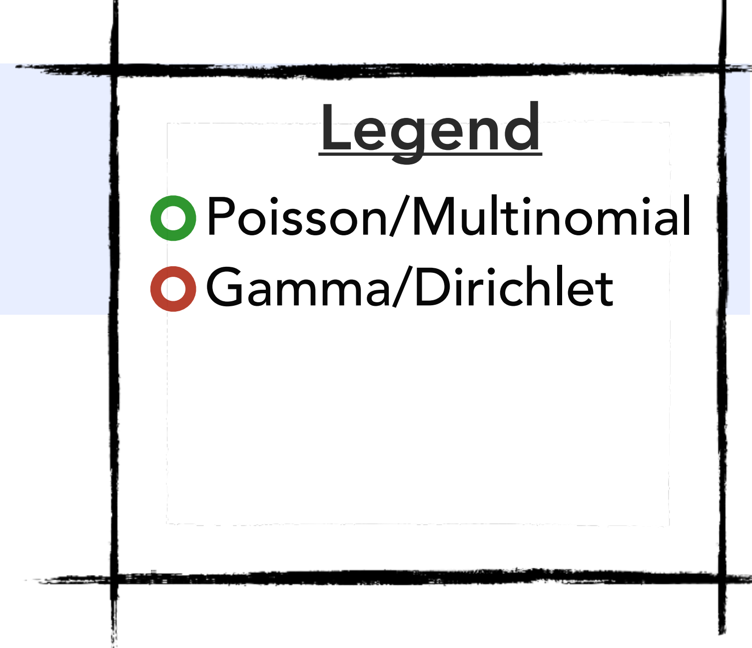
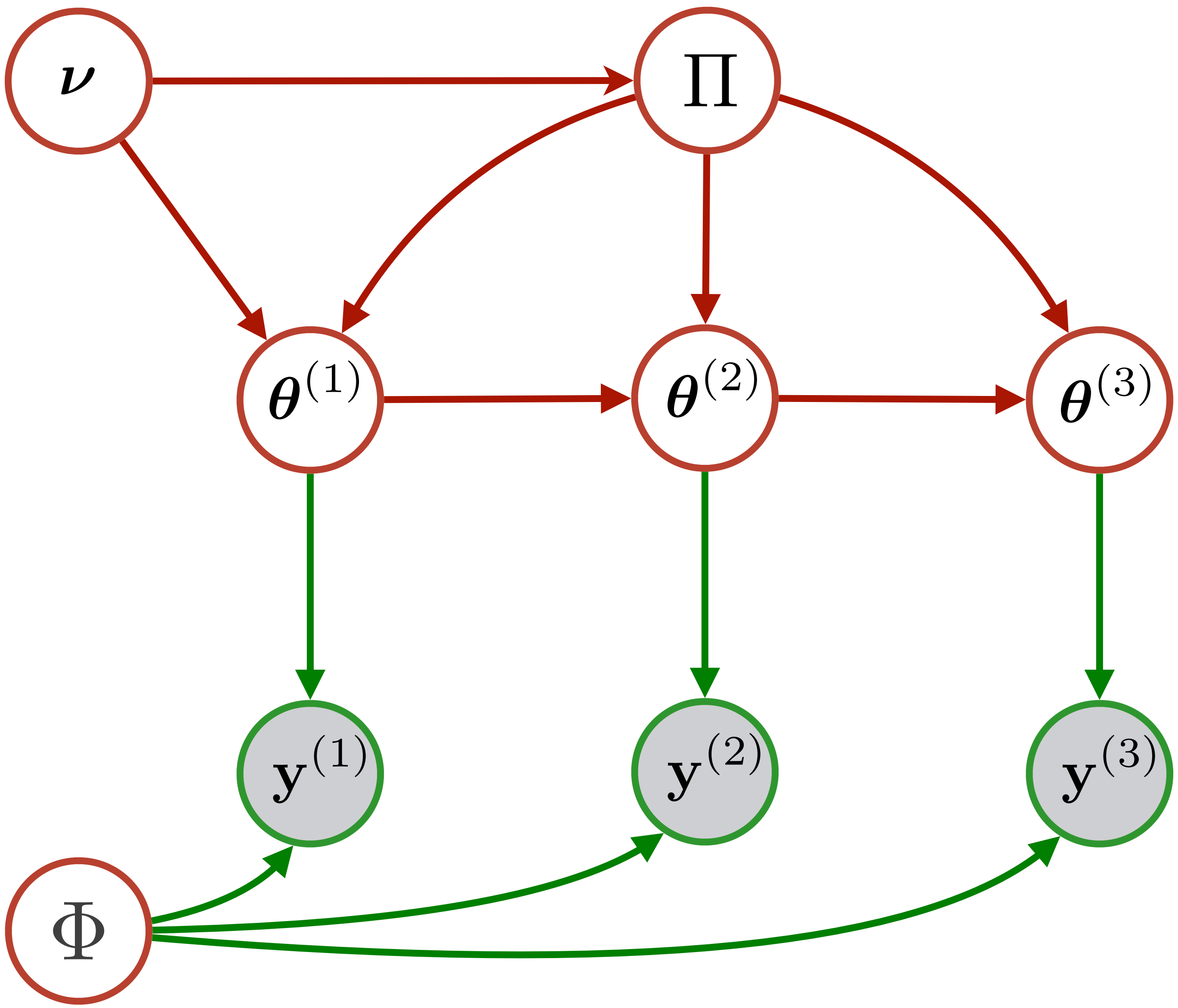




$$\Pi \sim P(\Pi \mid Y, \Theta, \boldsymbol{\nu})$$ ✗

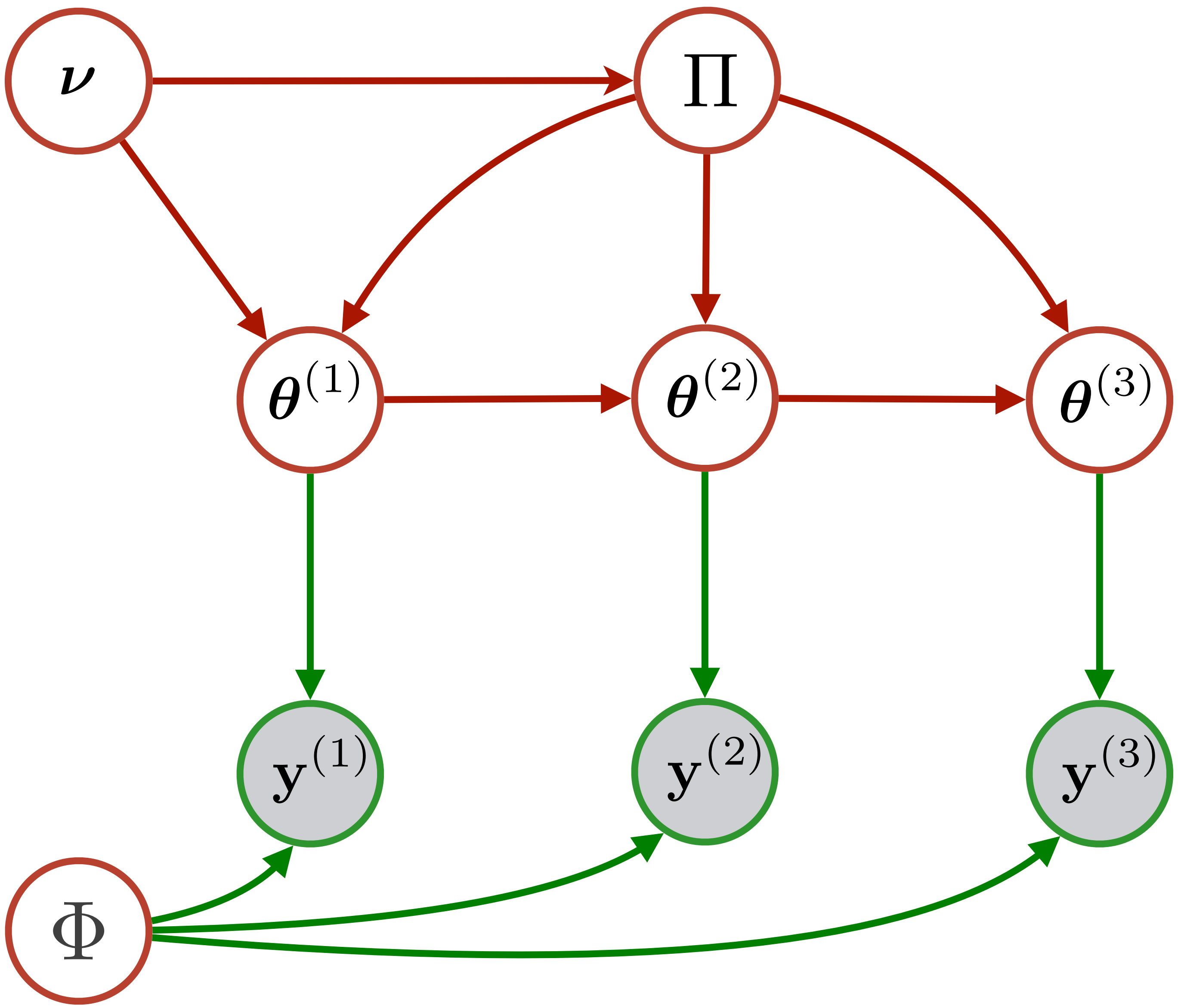
$$\Phi \sim P(\Phi \mid Y)$$ ✓

(conditional posterior has closed form)

Technical challenge

Legend

- Poisson/Multinomial
- Gamma/Dirichlet



$$\Pi \sim P(\Pi \,|\, Y, \Theta, \boldsymbol{\nu})$$ ✗

$$\Theta \sim P(\Theta \,|\, Y, \Pi, \boldsymbol{\nu})$$ ✗

$$\Phi \sim P(\Phi \,|\, Y)$$ ✓

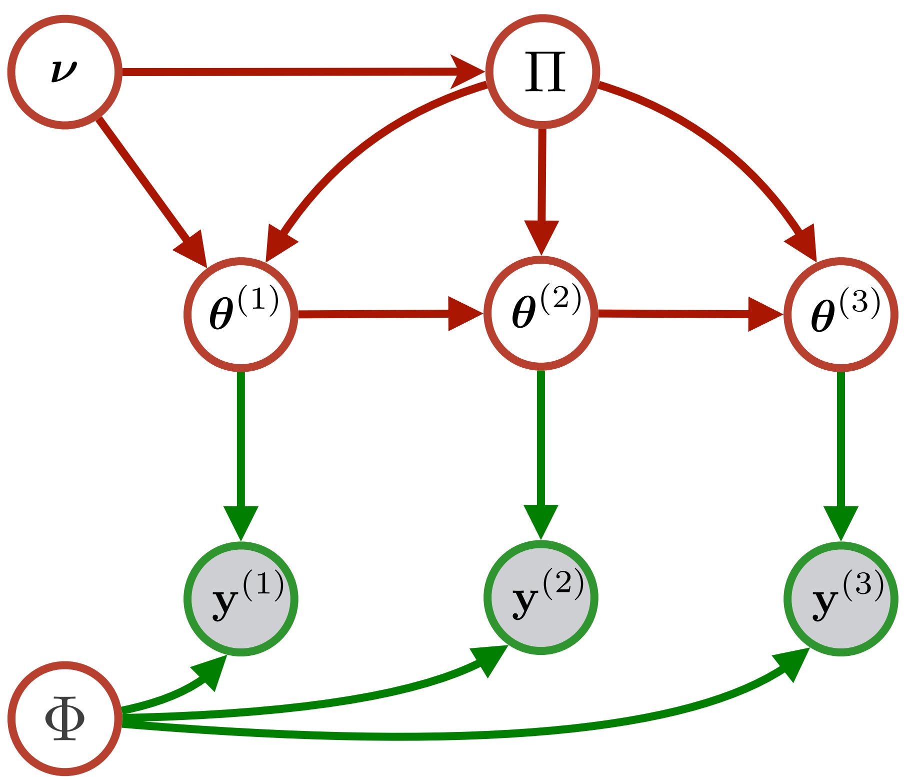
(conditional posterior has closed form)

Augment and Conquer

Legend

Poisson/Multinomial
Gamma/Dirichlet

Original model



Augment and Conquer

Legend

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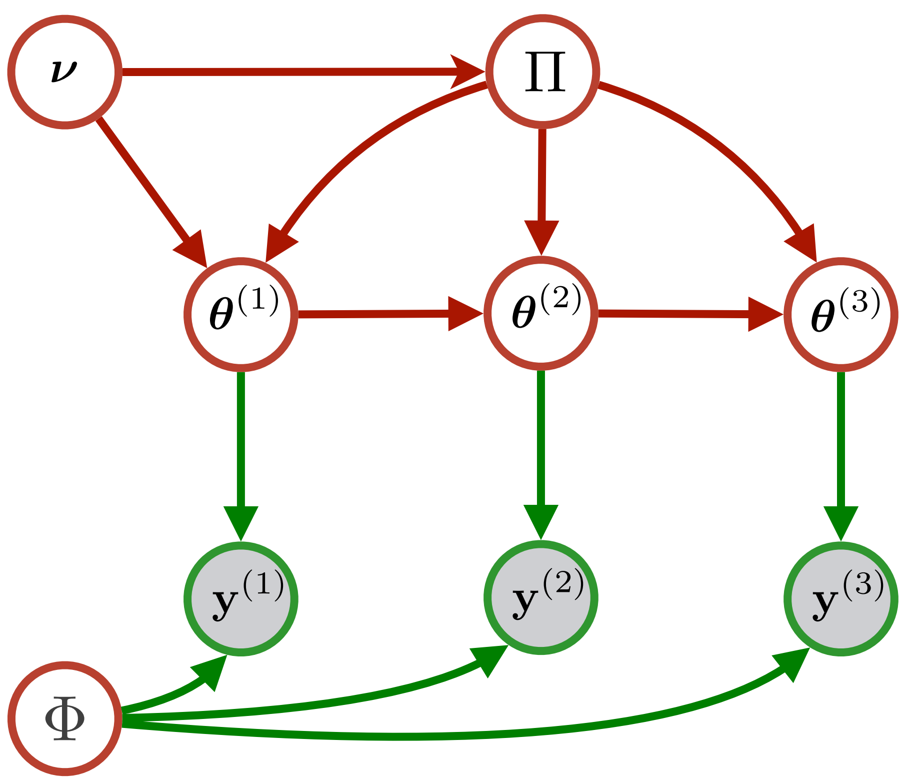
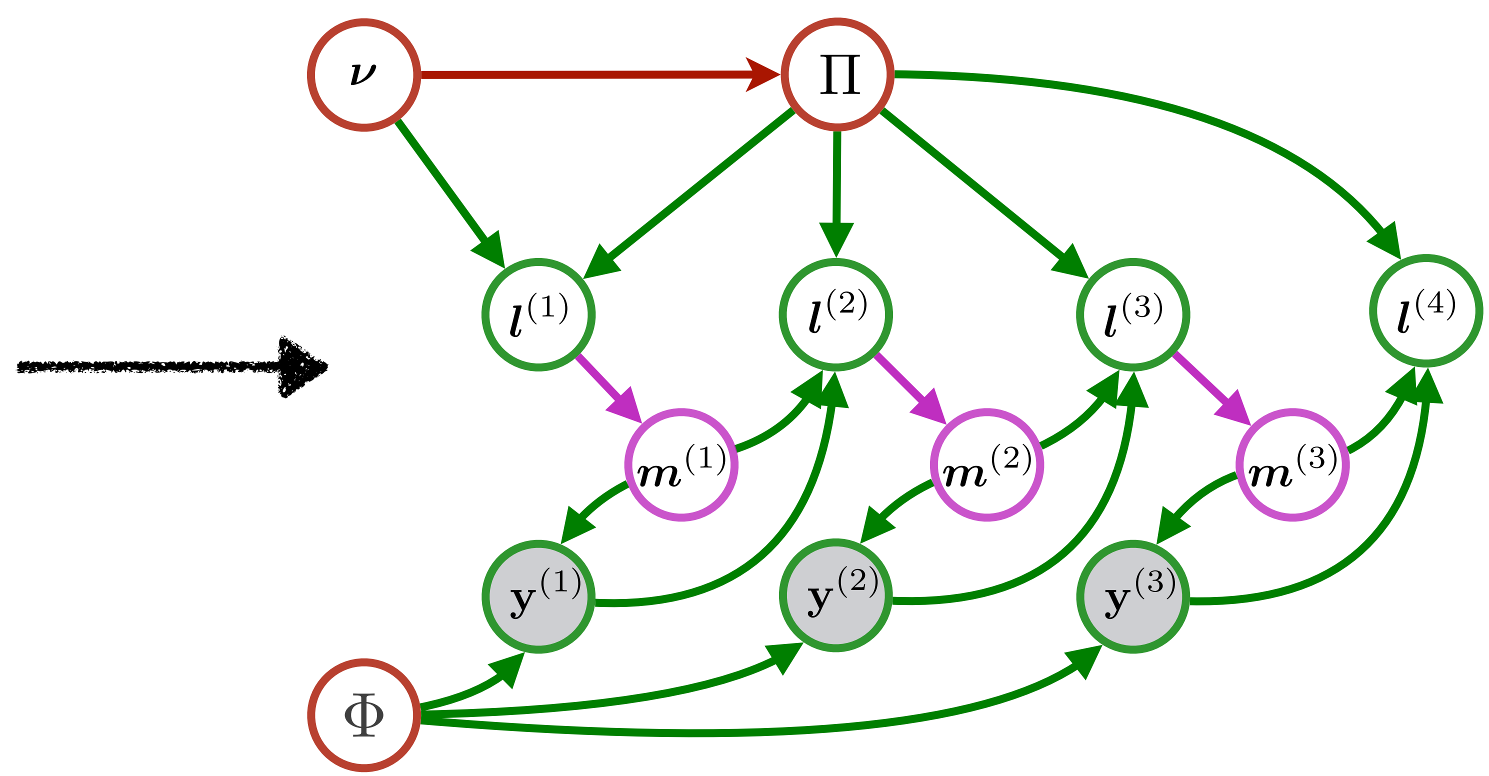


Alternative model



Augment and Conquer

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Poisson/Multinomial

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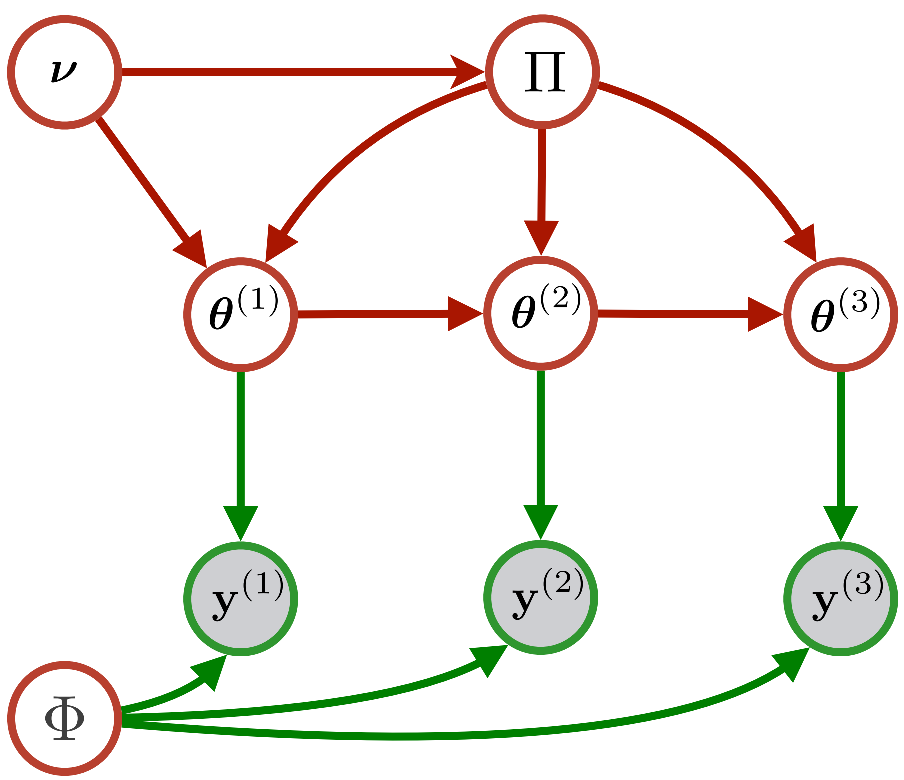

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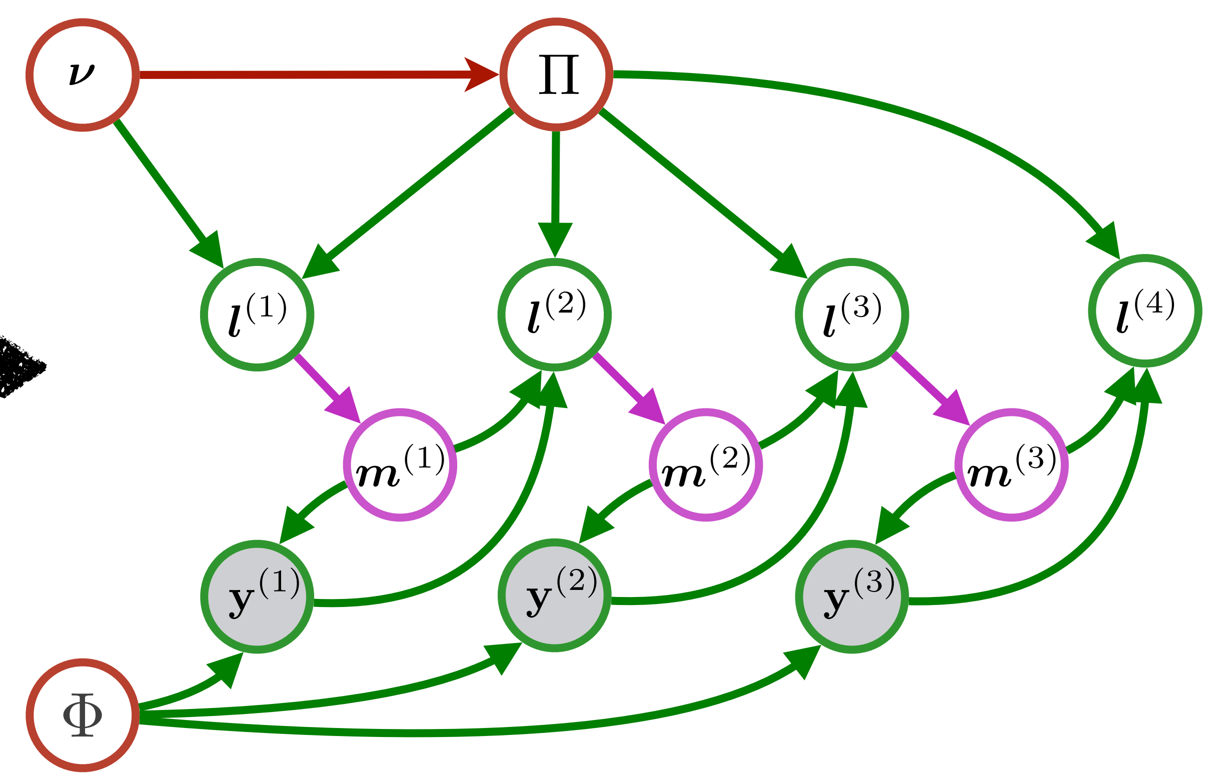

$$\Pi \sim P\left(\Pi \,|\, \mathcal{A}, Y, \boldsymbol{\nu}\right) \checkmark$$

Poisson—gamma dynamical systems

[NeurIPS ’16]

Analogue to linear dynamical systems for high-dimensional **discrete time-series**

with APF, good things happen…

- **robust** to sparsity, overdispersion, burstiness
- model-fitting **scales with number of non-zeros**

	T	$\hat{B}$	Task	Mean Relative Error (MRE) PGDS	GP-DPFA	LDS
GDELT	365	1.27	S	**2.335** ±0.19	2.951 ±0.32	3.493 ±0.53
			F	**2.173** ±0.41	2.207 ±0.42	2.397 ±0.29
ICEWS	365	1.10	S	**0.808** ±0.11	0.877 ±0.12	1.023 ±0.15
			F	**0.743** ±0.17	0.792 ±0.17	0.937 ±0.31
SOTU	225	1.45	S	**0.233** ±0.01	0.238 ±0.01	0.260 ±0.01
			F	**0.171** ±0.00	0.173 ±0.00	0.225 ±0.01
DBLP	14	1.64	S	0.417 ±0.03	0.422 ±0.05	**0.405** ±0.05
			F	**0.322** ±0.00	0.323 ±0.00	0.369 ±0.06
NIPS	17	0.33	S	0.415 ±0.07	**0.392** ±0.07	1.609 ±0.43
			F	0.343 ±0.01	**0.312** ±0.00	0.642 ±0.14

Poisson-randomized gamma dynamical systems

[NeurIPS '19]

Another analogue to linear dynamical systems
for high-dimensional discrete time-series

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- **robust** to sparsity, overdispersion, burstiness
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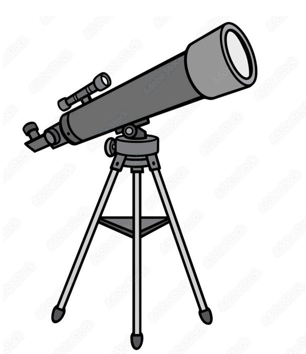
Based on a **sparse discrete** representation…



…which lets it ``see'' things other models can't.

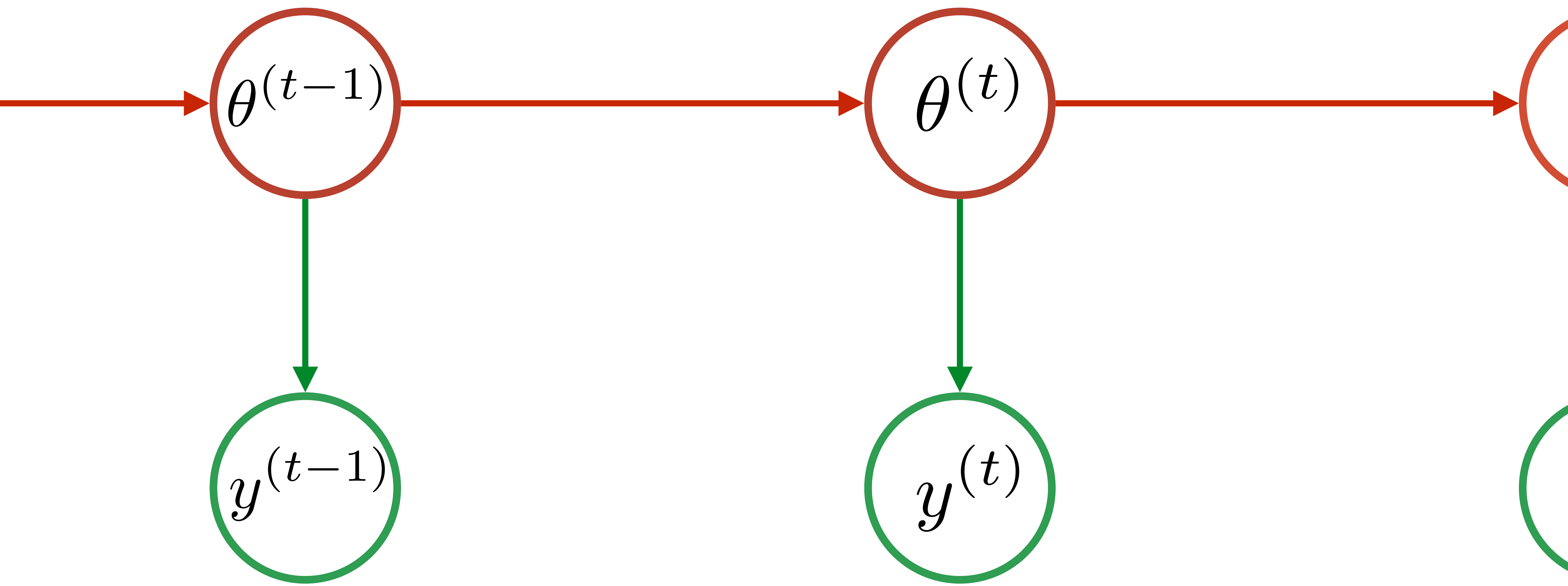
The gamma—Poisson—gamma chain

Legend

Poisson/Multinomial

Gamma/Dirichlet

$$\theta^{(t)} \sim \mathrm{Gam}(\tau\,\theta^{(t-1)},\ \tau)$$



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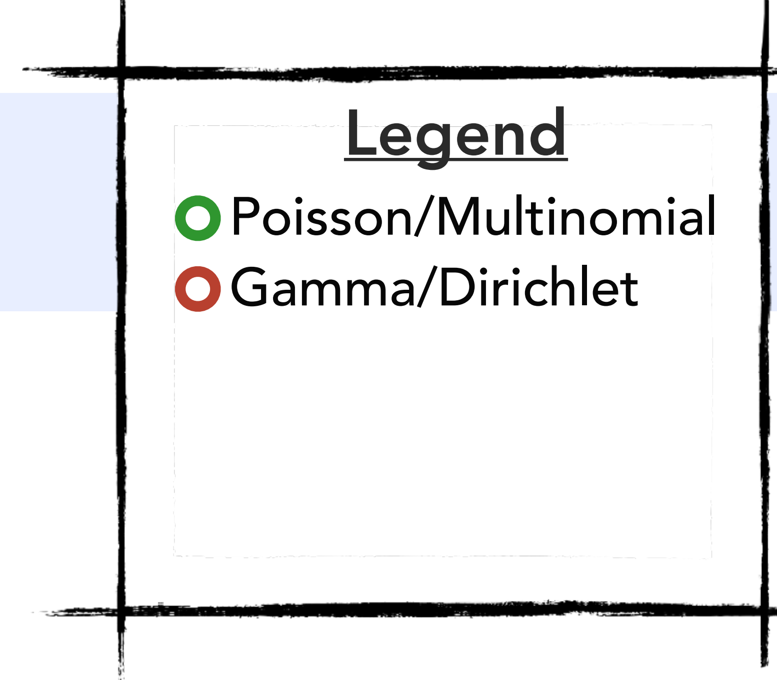

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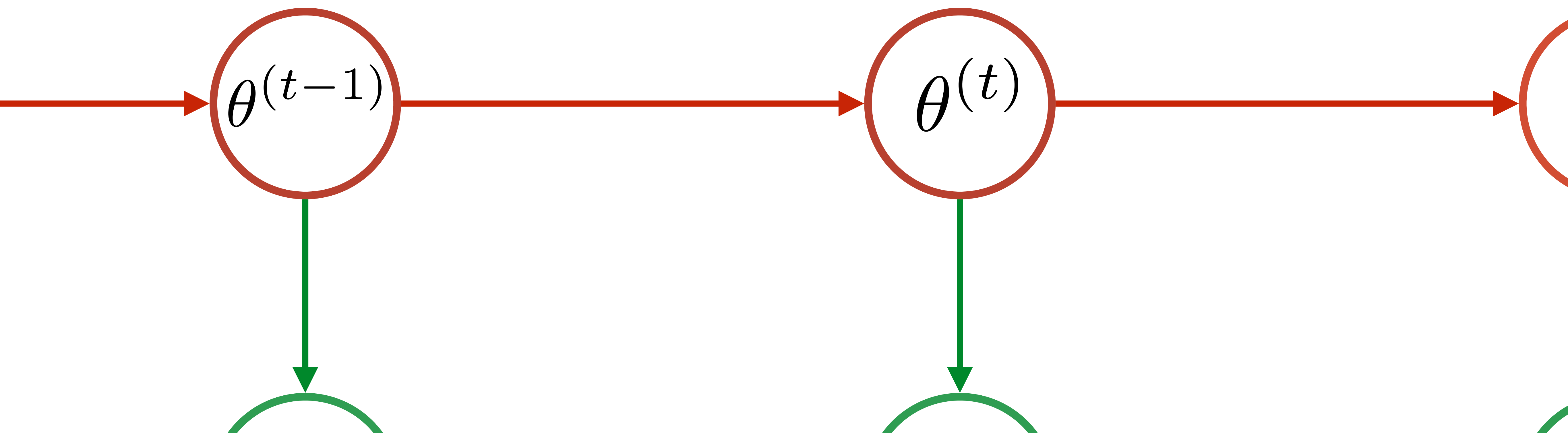
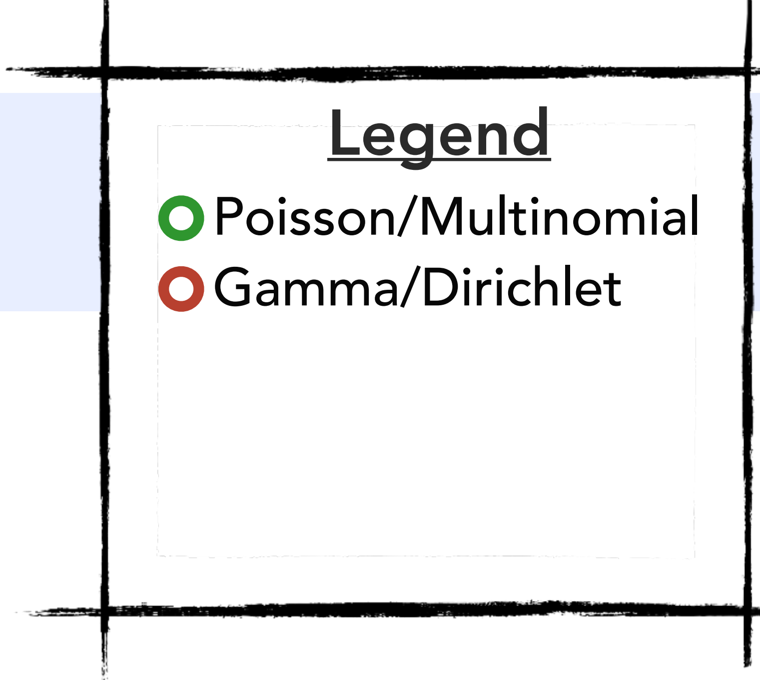
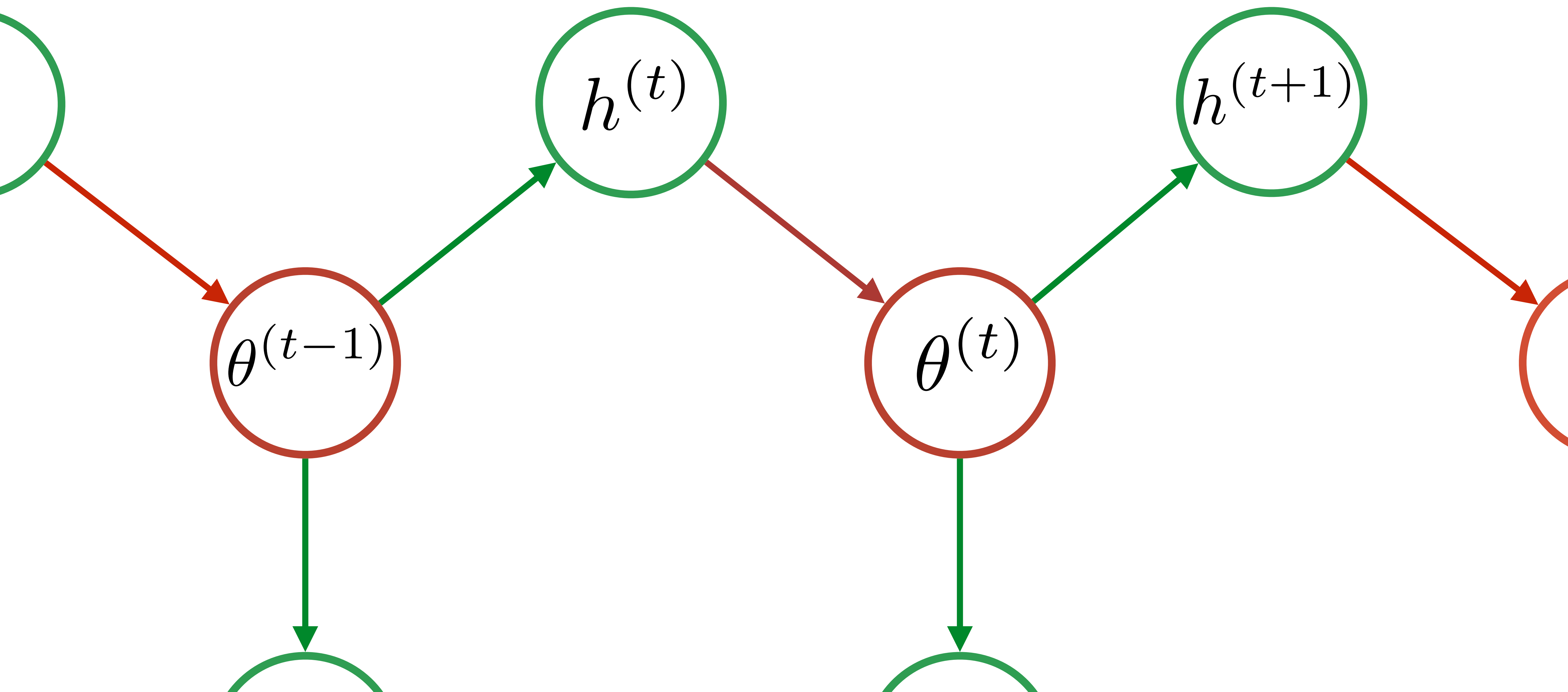


The gamma—Poisson—gamma chain

$$h^{(t)} \sim \mathrm{Pois}(\tau\theta^{(t-1)})$$ **discrete** latent state

$$\theta^{(t)} \sim \mathrm{Gam}(h^{(t)}, \tau)$$ **continuous** latent state





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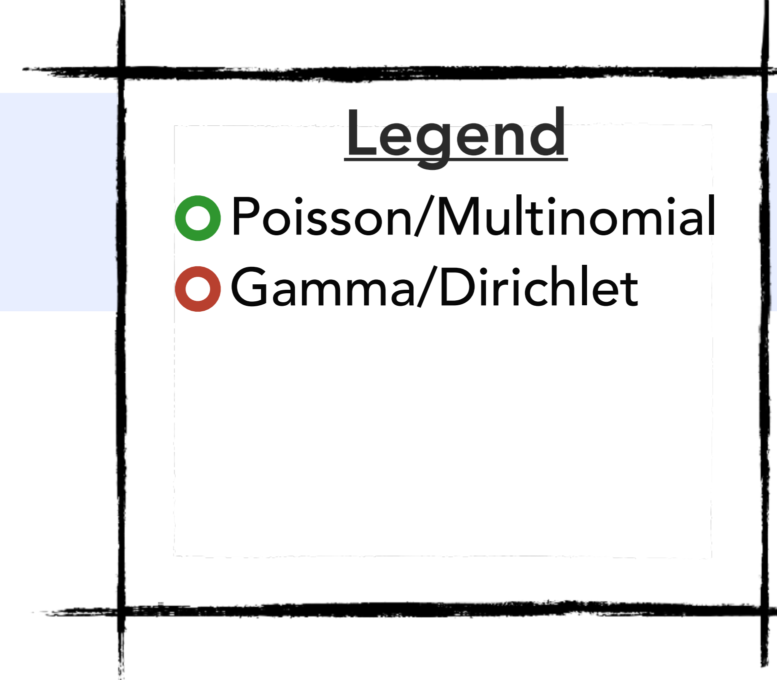

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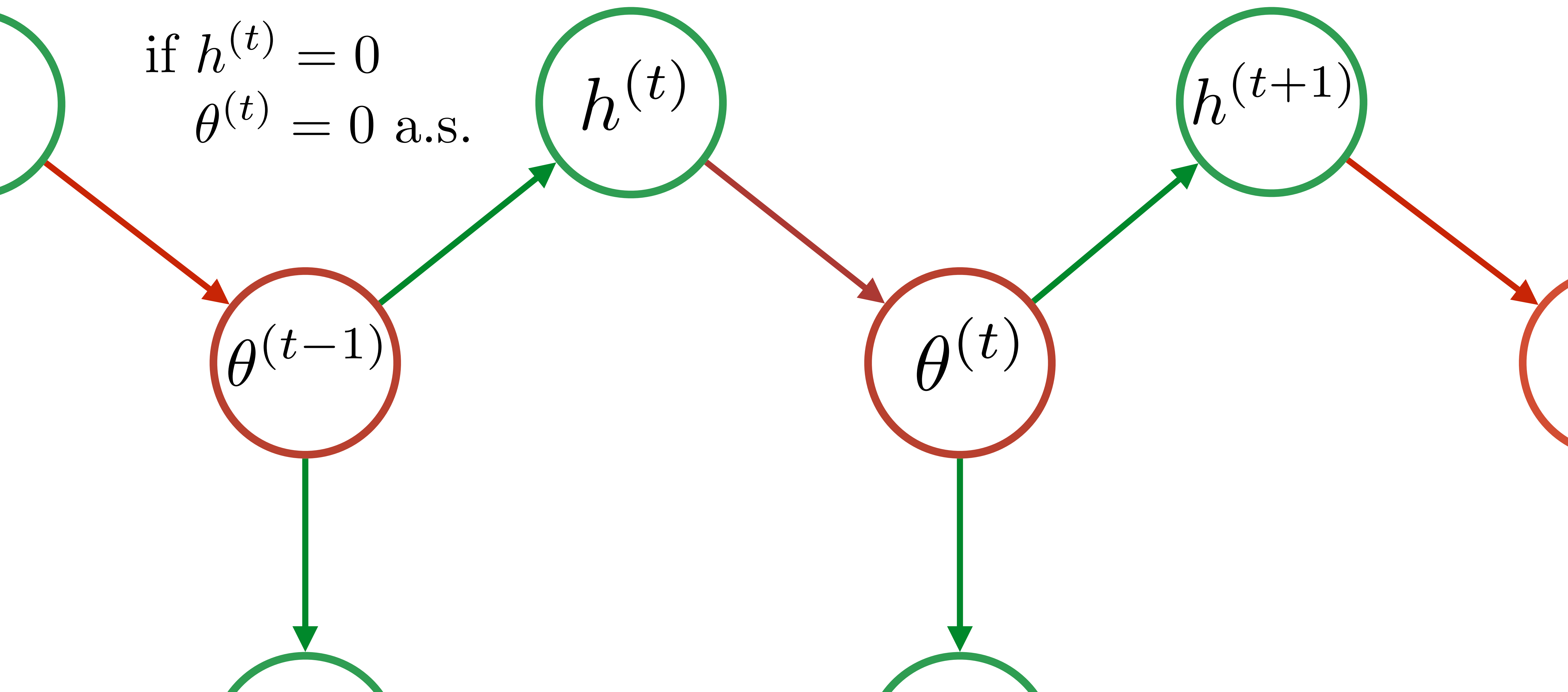
sparse representation

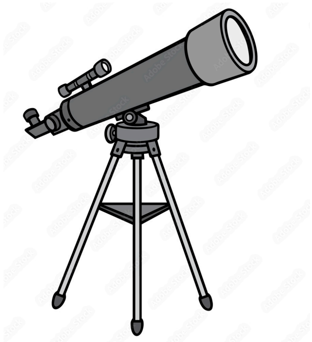
$$\text{if } h^{(t)} = 0$$
$$\theta^{(t)} = 0 \text{ a.s.}$$



Poisson-randomized gamma dynamical systems

[NeurIPS '19]

Sparse discrete representation…



…lets it ``see'' things other models can't.

Poisson-randomized gamma dynamical systems

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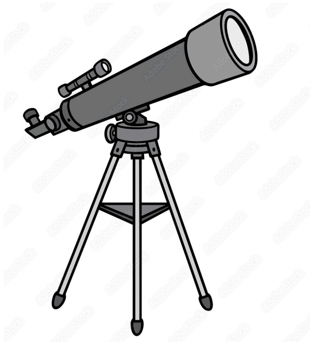

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Qualitative study: PGDS [NeurIPS '16] **vs.** PRGDS [NeurIPS '19]

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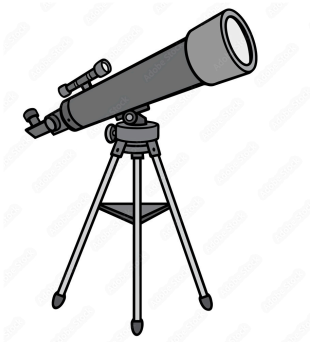

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Qualitative study: PGDS [NeurIPS '16] **vs.** PRGDS [NeurIPS '19]

1) Fit them both to the same IR data.

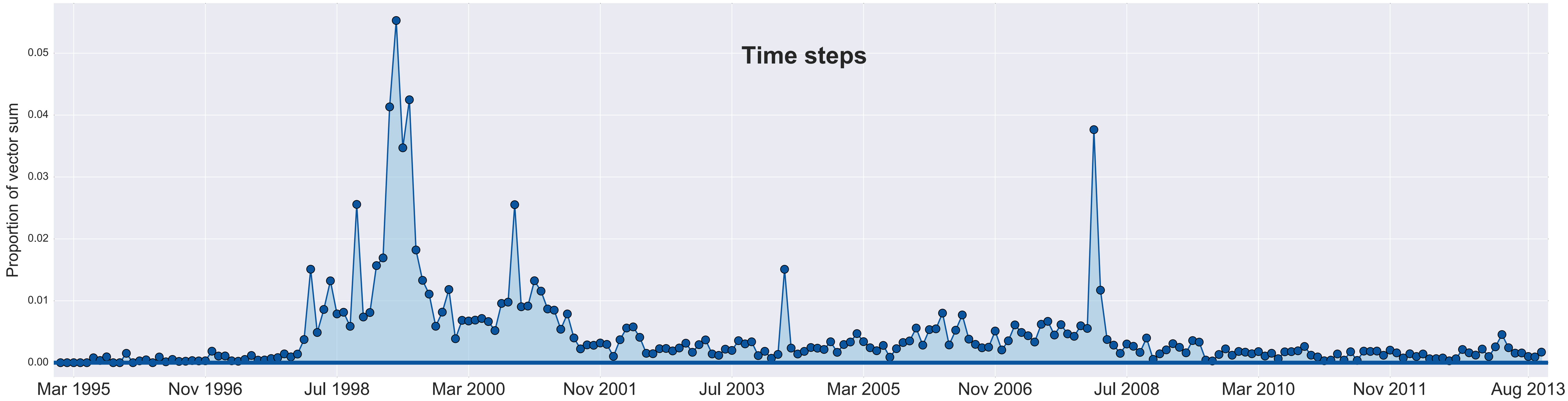
2) **Align** their latent components.

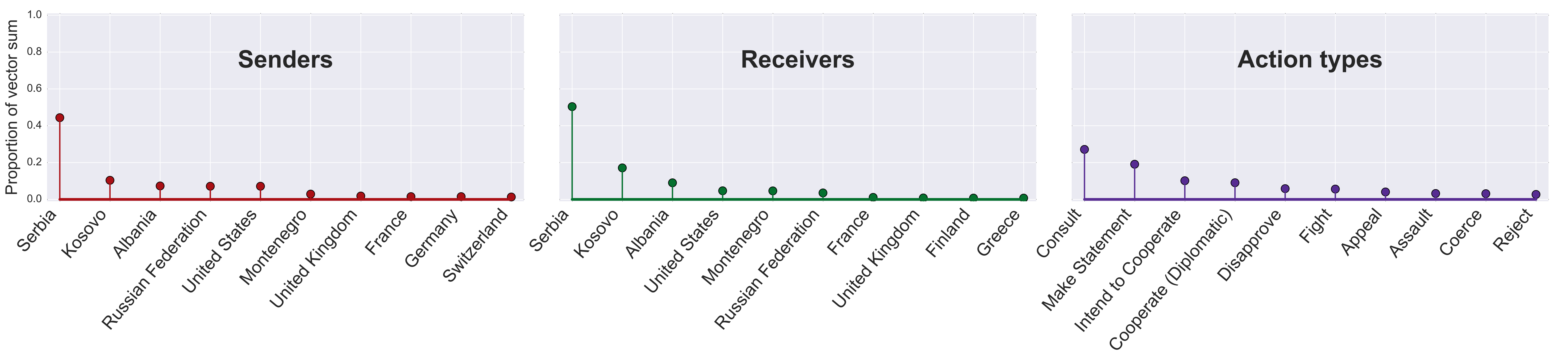
3) Check for systematic discrepancy.

Aligned component: **PGDS**

[NeurIPS '16]

Both models ``see'' this component…

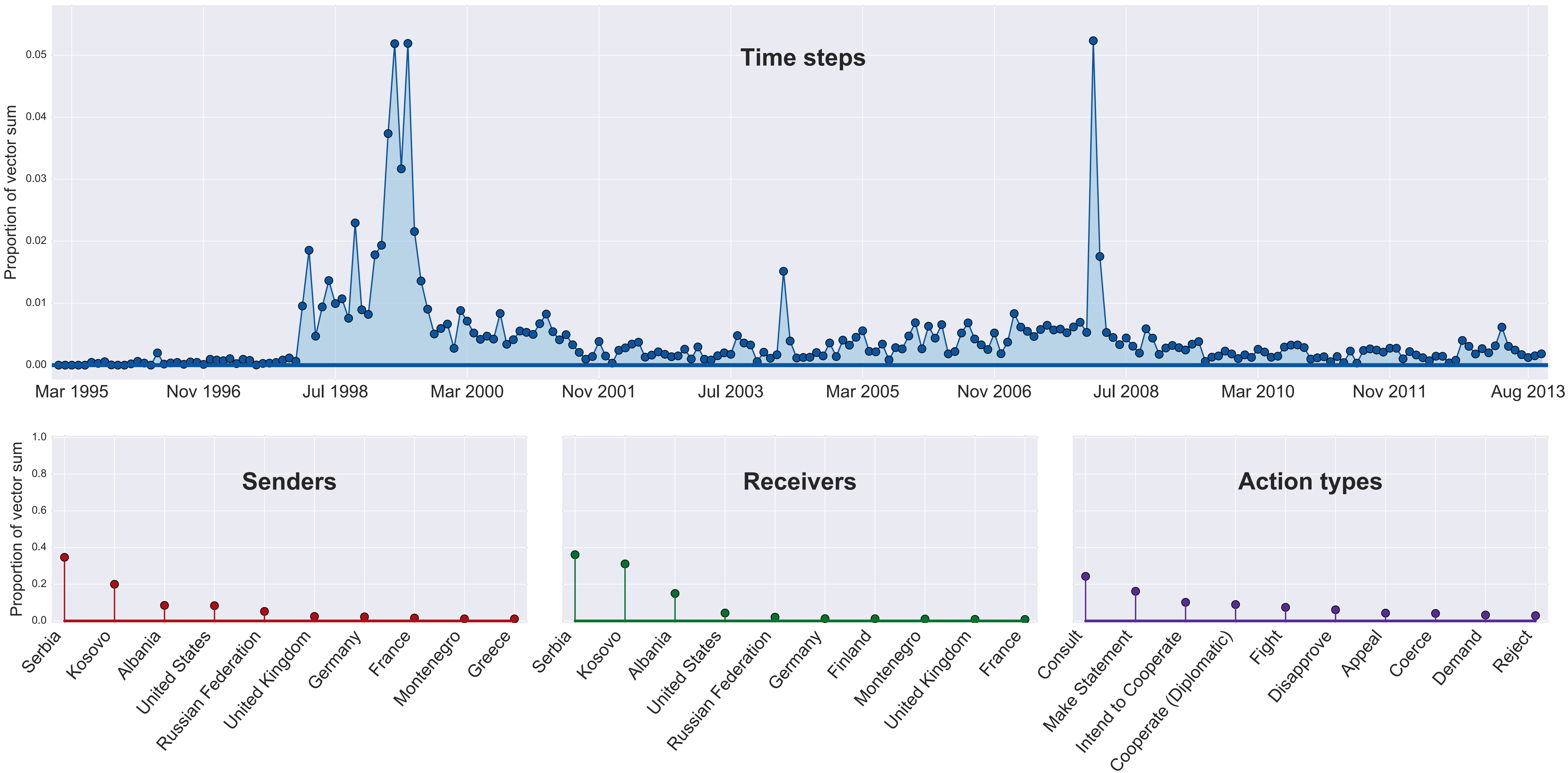




Aligned component: **PRGDS**

[NeurIPS '19]

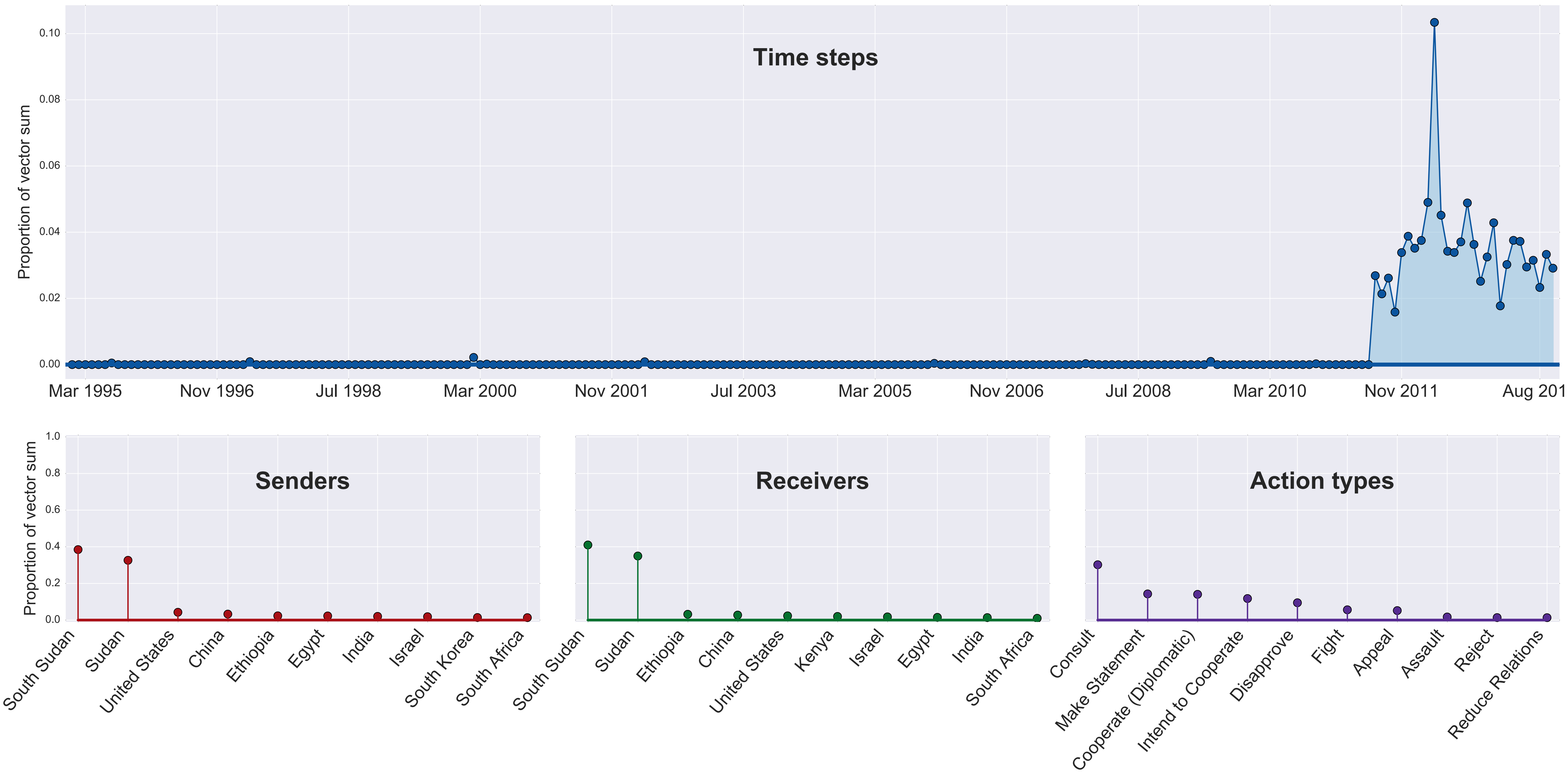
Both models "see" this component…



Discrepant component: **PRGDS**

[NeurIPS '19]

Only PRGDS ``sees'' this one.



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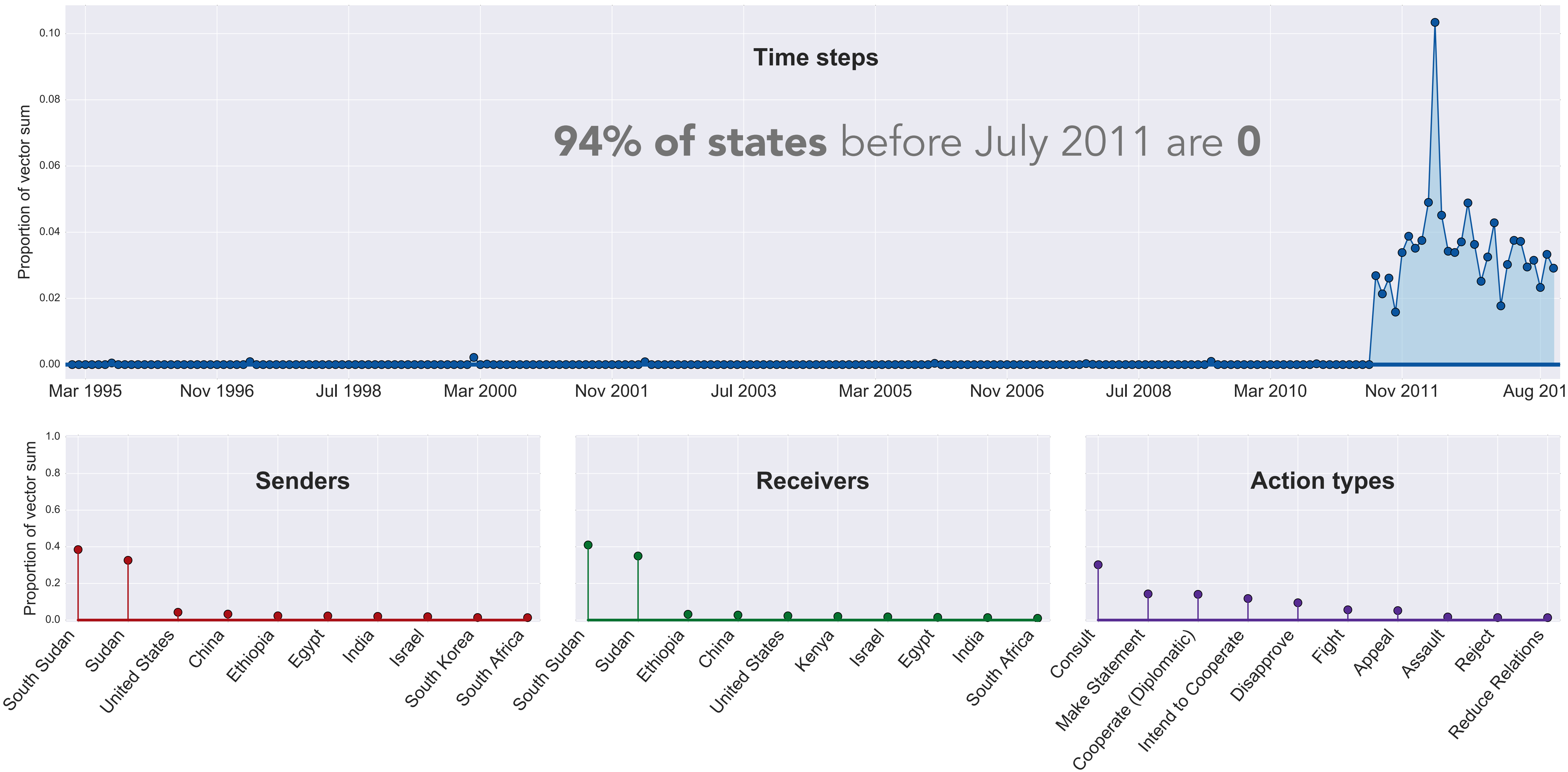
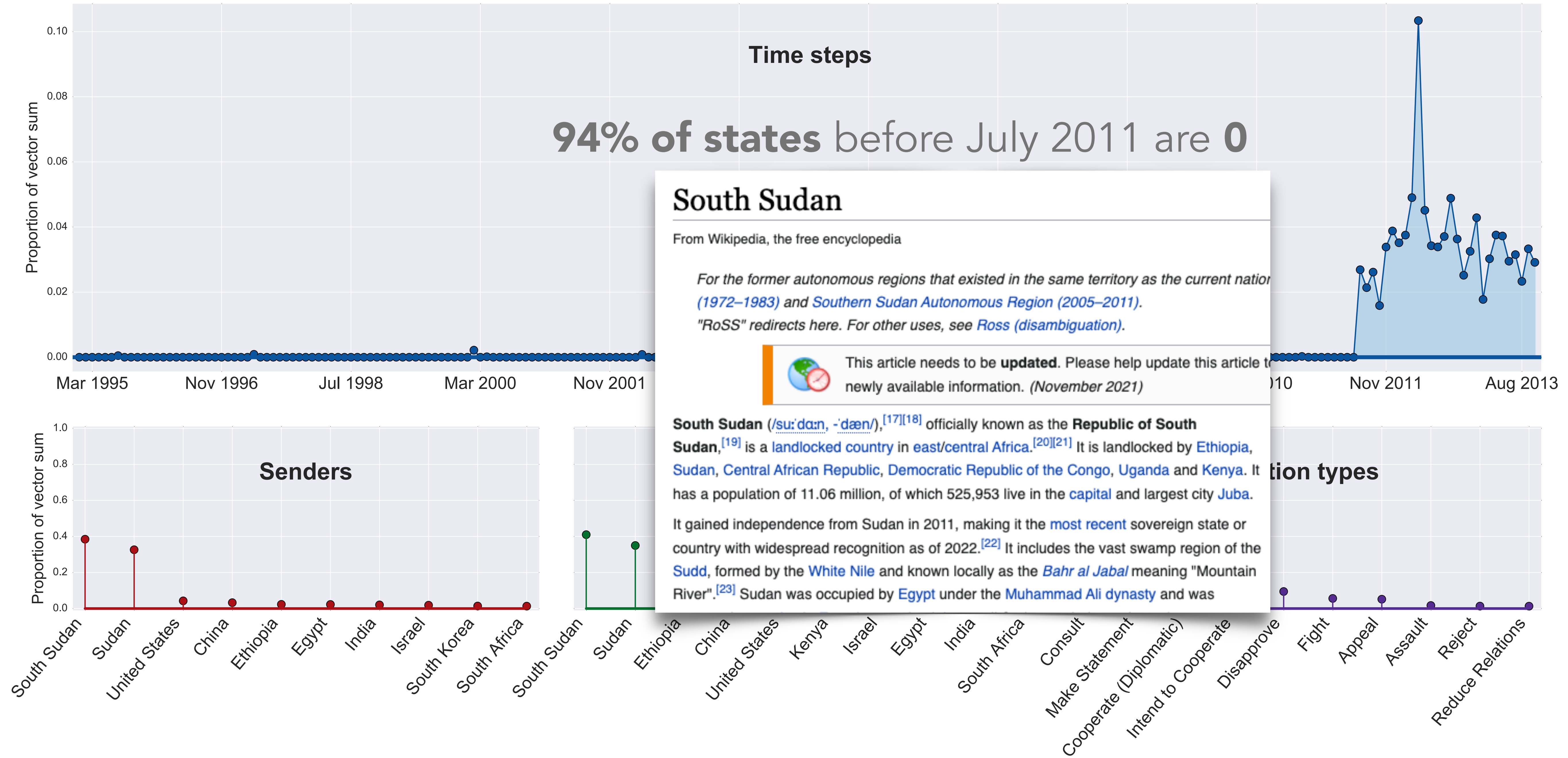


Discrepant component: **PRGDS**

[NeurIPS '19]

Only PRGDS ``sees'' this one.

94% of states before July 2011 are **0**



South Sudan

From Wikipedia, the free encyclopedia

For the former autonomous regions that existed in the same territory as the current natior (1972–1983) and Southern Sudan Autonomous Region (2005–2011). "RoSS" redirects here. For other uses, see Ross (disambiguation).

This article needs to be **updated**. Please help update this article t newly available information. *(November 2021)*

South Sudan (/suːˈdɑːn, -ˈdæn/),[17][18] officially known as the **Republic of South Sudan**,[19] is a landlocked country in east/central Africa.[20][21] It is landlocked by Ethiopia, Sudan, Central African Republic, Democratic Republic of the Congo, Uganda and Kenya. It has a population of 11.06 million, of which 525,953 live in the capital and largest city Juba.

It gained independence from Sudan in 2011, making it the most recent sovereign state or country with widespread recognition as of 2022.[22] It includes the vast swamp region of the Sudd, formed by the White Nile and known locally as the *Bahr al Jabal* meaning "Mountain River".[23] Sudan was occupied by Egypt under the Muhammad Ali dynasty and was

Discrepant component: **PRGDS**

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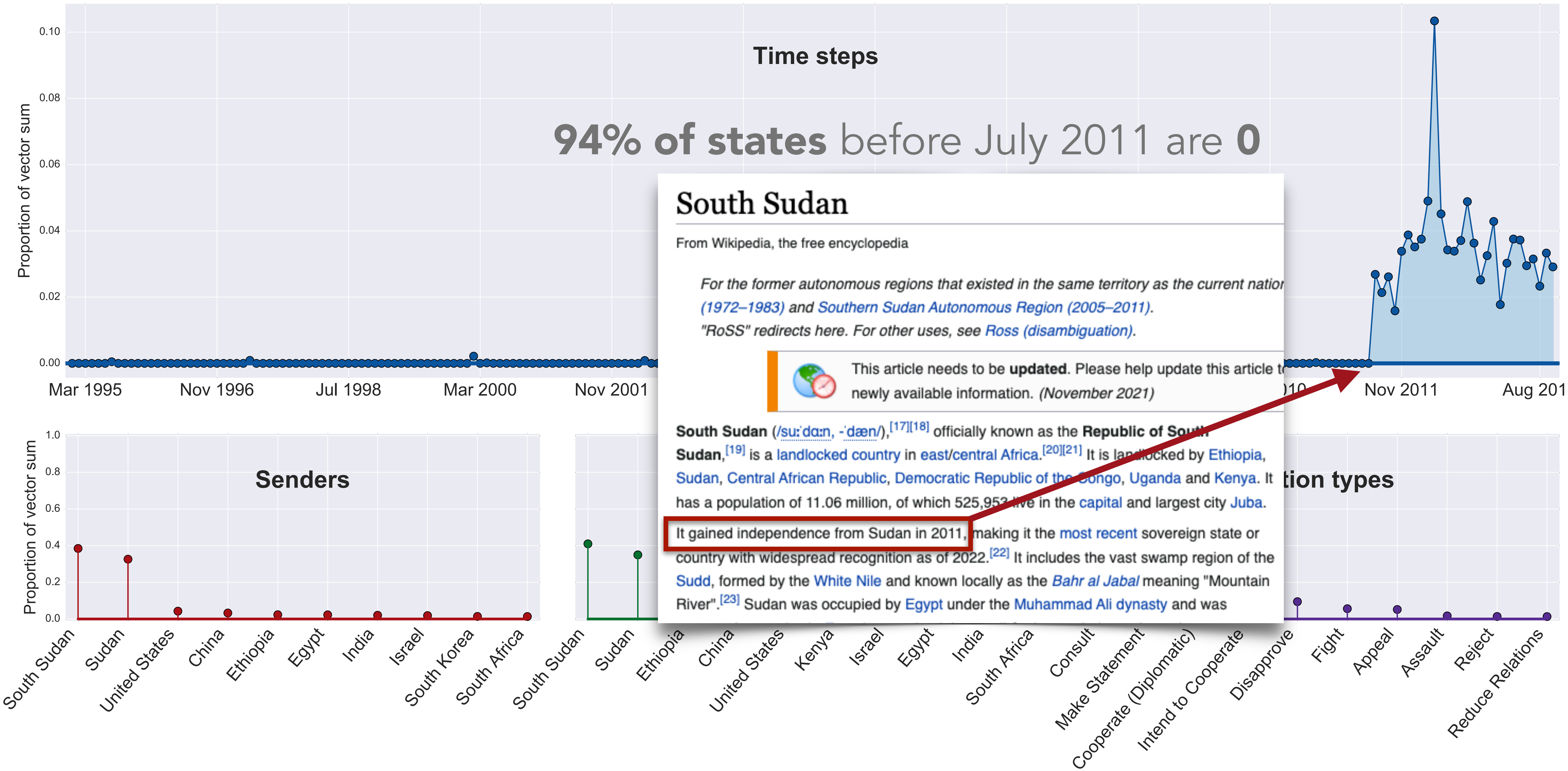

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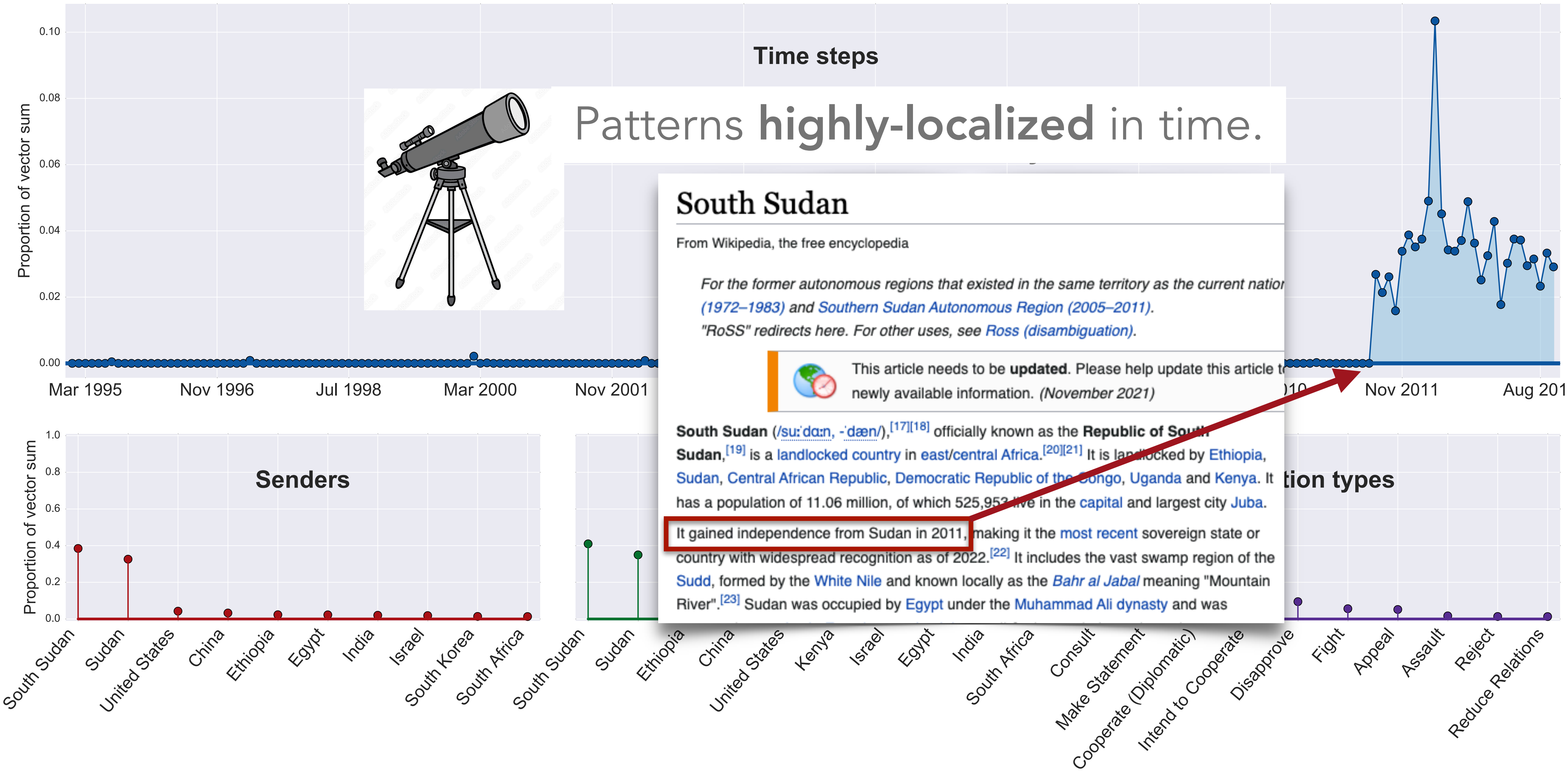


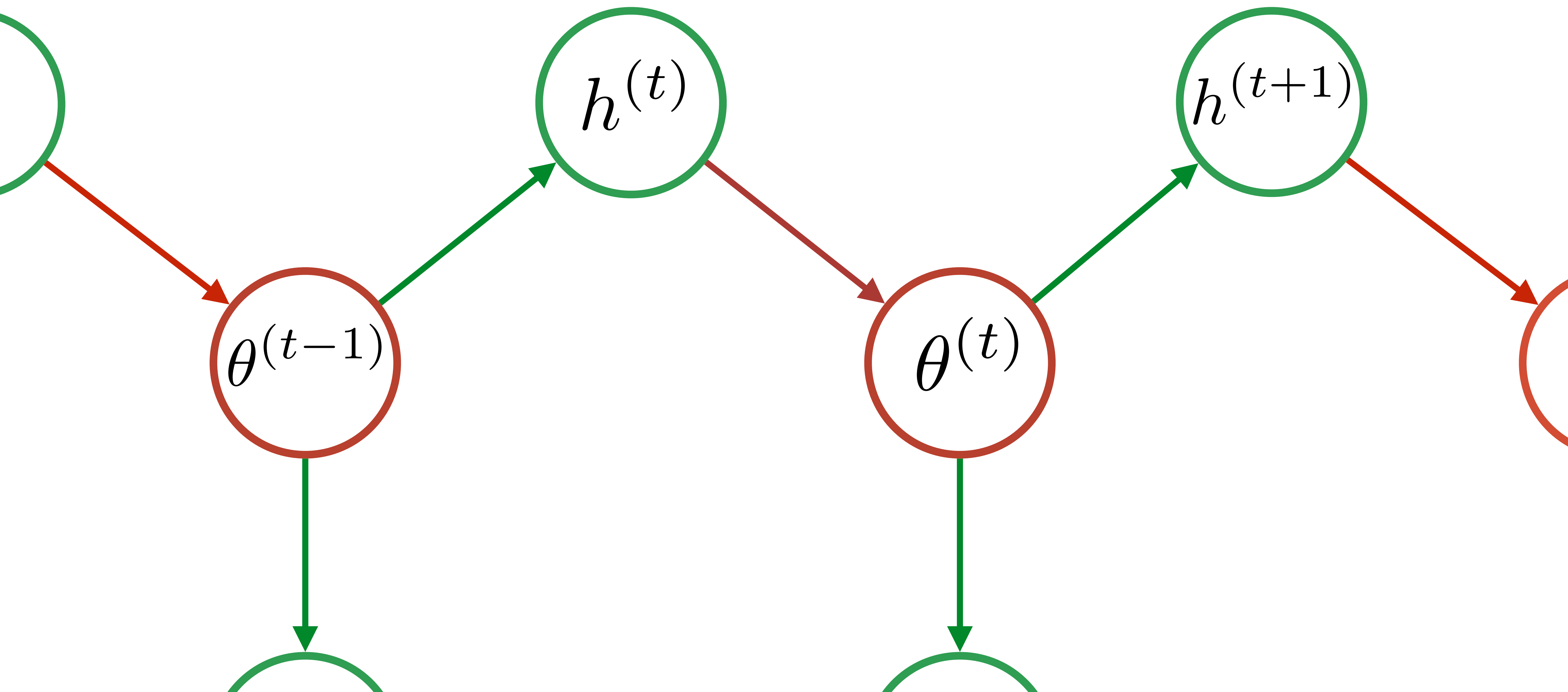
Approximate Bayesian inference

Legend

- Poisson/Multinomial
- Gamma/Dirichlet

Partially conditionally conjugate

$$\left(\theta^{(t)} \mid -\right) \sim P\left(\theta^{(t)} \mid h^{(t)}, h^{(t+1)}, -\right)$$



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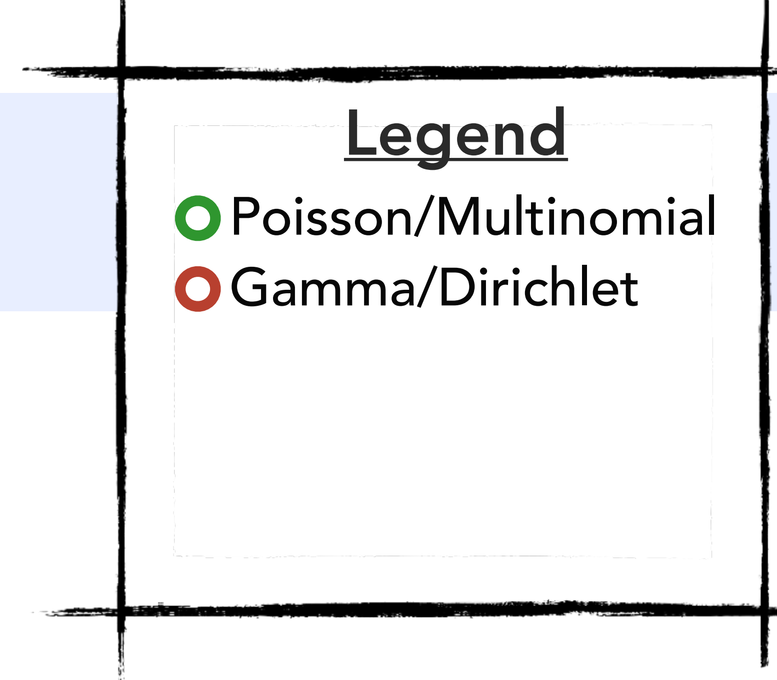
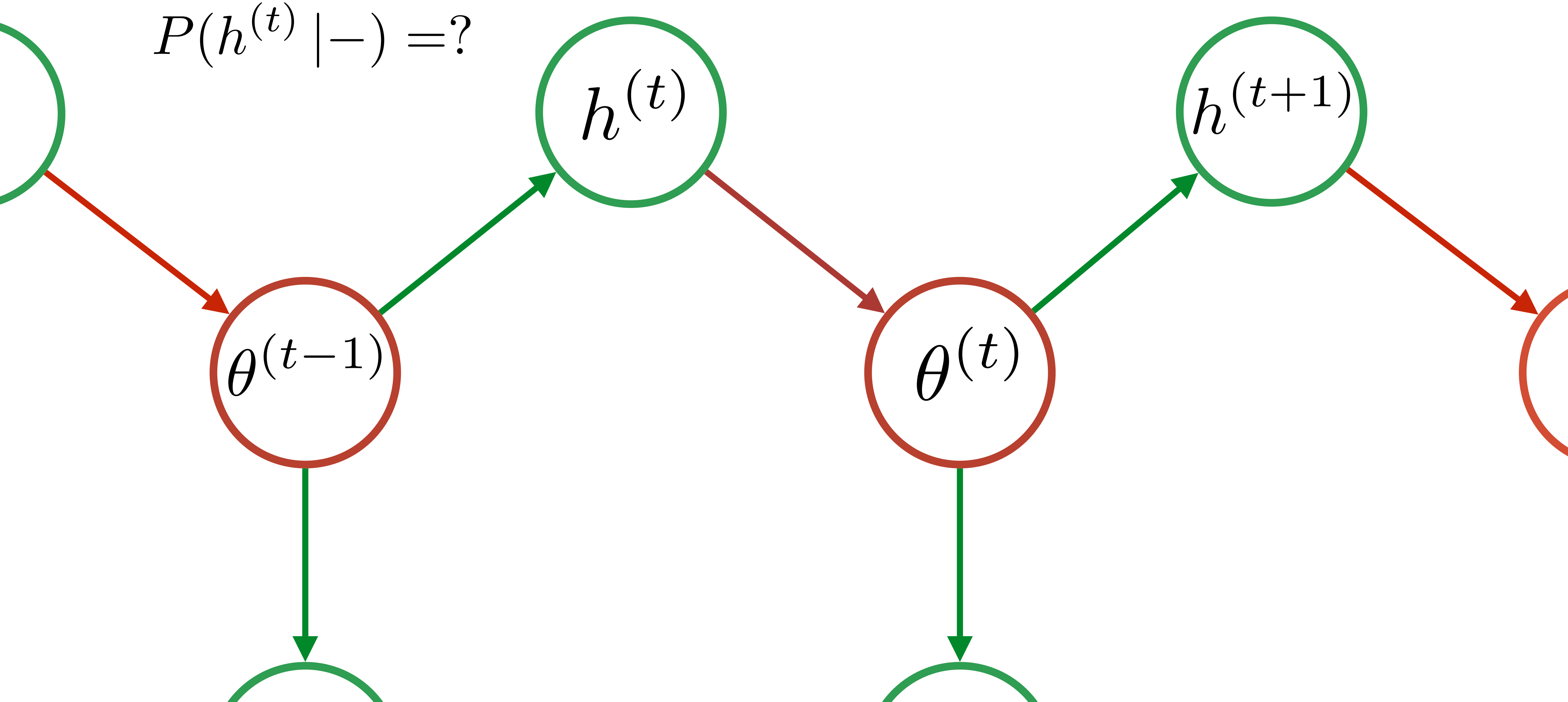


Partially conditionally conjugate

$$\left(\theta^{(t)} \mid -\right) \sim P\left(\theta^{(t)} \mid h^{(t)}, h^{(t+1)}, -\right) \checkmark$$

Partially not

$$P(h^{(t)} \mid -) = ?$$



Approximate Bayesian inference

Partially conditionally conjugate

$$\left(\theta^{(t)} \mid -\right) \sim P\left(\theta^{(t)} \mid h^{(t)}, h^{(t+1)}, -\right) \checkmark$$

Partially not

$$P(h^{(t)} \,|{-}) = ?$$

…but even without conjugacy…

closed-formedness!

Approximate Bayesian inference

Partially conditionally conjugate

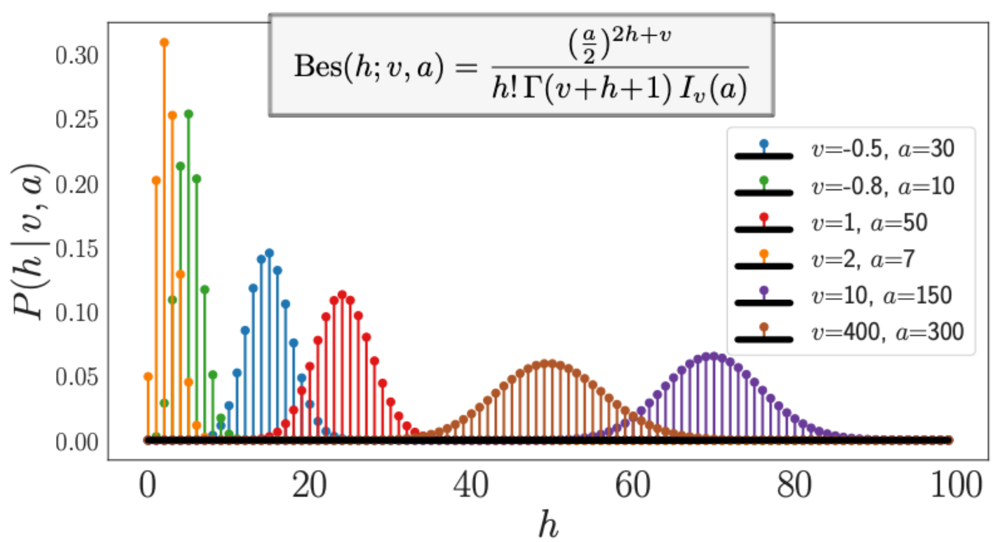
$$\left(\theta^{(t)} \mid -\right) \sim P\left(\theta^{(t)} \mid h^{(t)}, h^{(t+1)}, -\right) ✓$$

Partially not

$$P(h^{(t)} \mid -) = ✓$$ …but even without conjugacy…

closed-formedness!

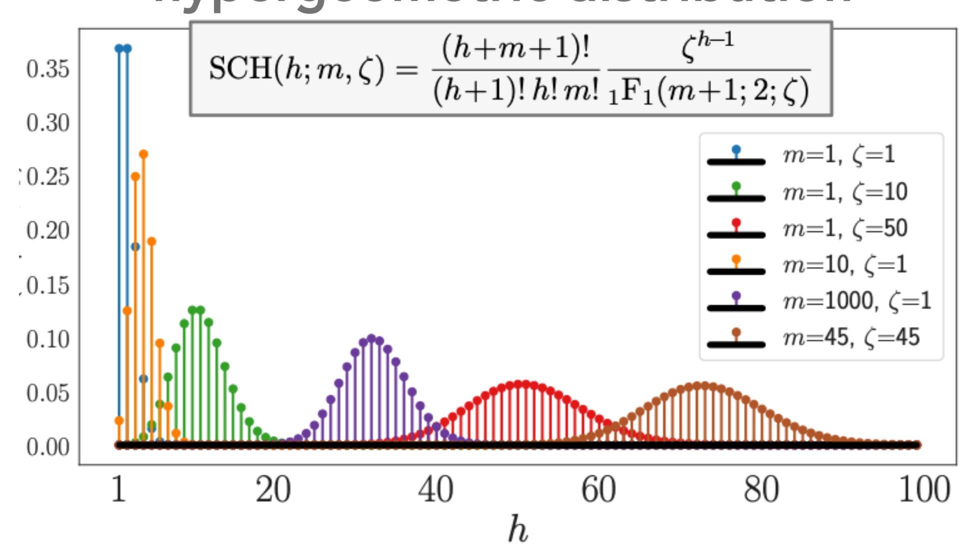
Bessel distribution



(Yuan and Kalbfleisch, 2000)

(Devroye 2002)

Size-biased confluent hypergeometric distribution



OUTLINE

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  - **CP** extracts "multilateral" structure [Schein et al., 2015]
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Escalation in international relations

Escalation is a central **concept** in IR…

Table 2
WEIS Coding of 1990 Iraq-Kuwait Crisis

Date	Source	Target	Type of Action
900717	IRQ	KUW	CHARGE
900725	IRQ	EGY	ASSURE
900727	IRQ	KUW	WARN
900731	IRQ	KUW	MOBILIZATION
900801	KUW	IRQ	REFUSE
900802	IRQ	KUW	MILITARY FORCE

…but **not observed** directly

AN ESCALATION LADDER
A Generalized (or Abstract) Scenario

AFTERMATHS

CIVILIAN CENTRAL WARS
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DISAGREEMENT—COLD WAR

Stoehr et al. [AISTATS 2023]

The Ordered Matrix Dirichlet for State-Space Models

Niklas Stoehr[ʕ] **Benjamin J. Radford**[ŋ] **Ryan Cotterell**[ʕ] **Aaron Schein**[ɘ]
[ʕ]ETH Zurich [ŋ]UNC Charlotte [ɘ]The University of Chicago
niklas.stoehr@inf.ethz.ch bradfor7@uncc.edu ryan.cotterell@inf.ethz.ch schein@uchicago.edu

State-space models to **measure (de-)escalation** in dyadic event data

Niklas Werner Stöhr

Lead author:

Address

ETH Zürich
Doctorate at D-INFK
Niklas Werner Stöhr
Professur für Informatik
OAT W 14
Andreasstrasse 5
8092 Zürich
Switzerland

Ordinal action types

action type *a*	action name	Goldstein value
0	provide aid	7.0
1	engage material cooperation	6.0
2	yield	5.0
3	express intent cooperate	4.0
4	engage diplomatic cooperation	3.5
5	appeal	3.0
6	consult	1.0
7	make public statement	0.0
9	investigate	-2.0
10	disapprove	-2.0
11	reject	-4.0
12	reduce relations	-4.0
13	demand	-5.0
14	threaten	-6.0
15	protest	-6.5
16	coerce	-7.0
17	exhibit force posture	-7.2
18	assault	-9.0
19	fight	-10.0
20	unconventional mass violence	-10.0

State-space model

$$\mathbb{E}\left[y_{i \xrightarrow{a} j}^{(t)}\right] = \sum_{k=1}^{K} \lambda_{i \xrightarrow{k} j}^{(t)} \phi_{ka}$$

State-space model

how much **dyad** $i \rightarrow j$
is in **state** k at **time** t

how much **state** k
emits **action** a

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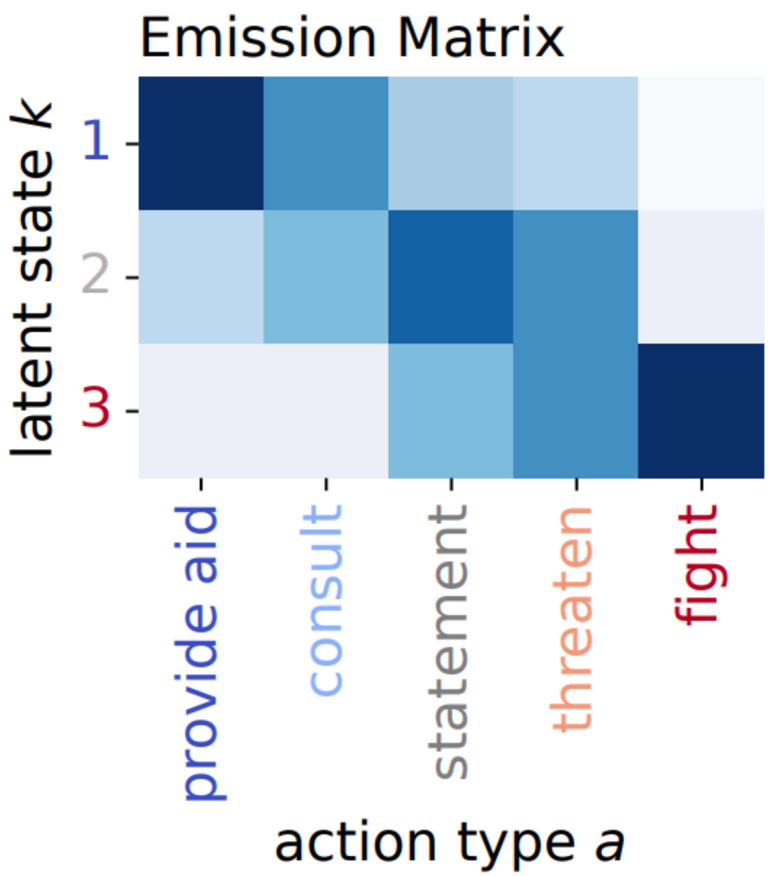
(k^{th} row of the emission matrix Φ)



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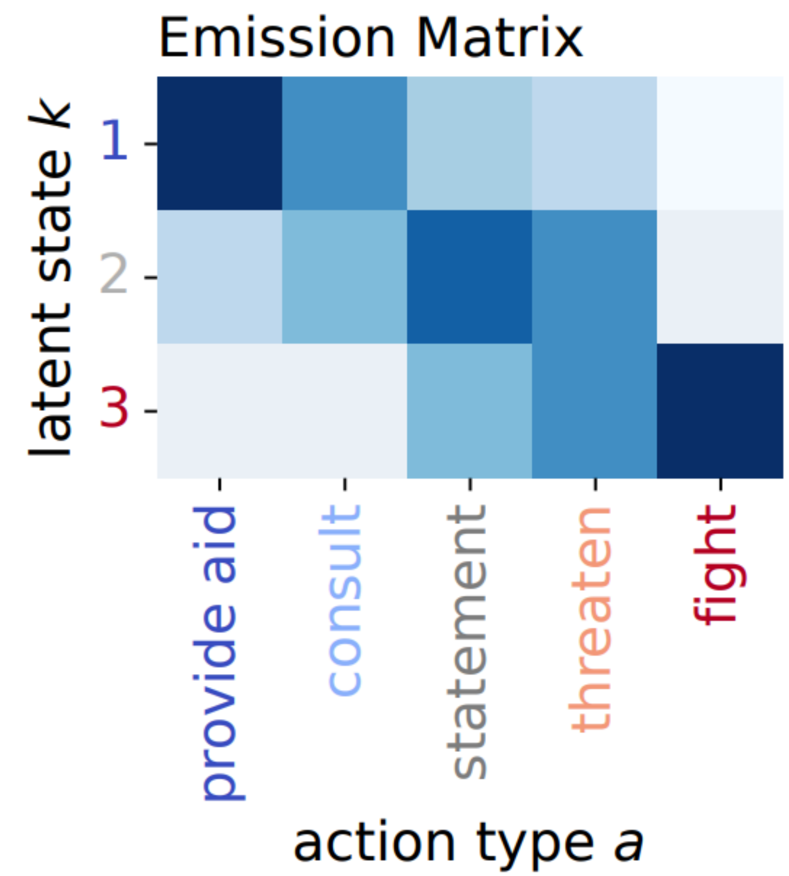
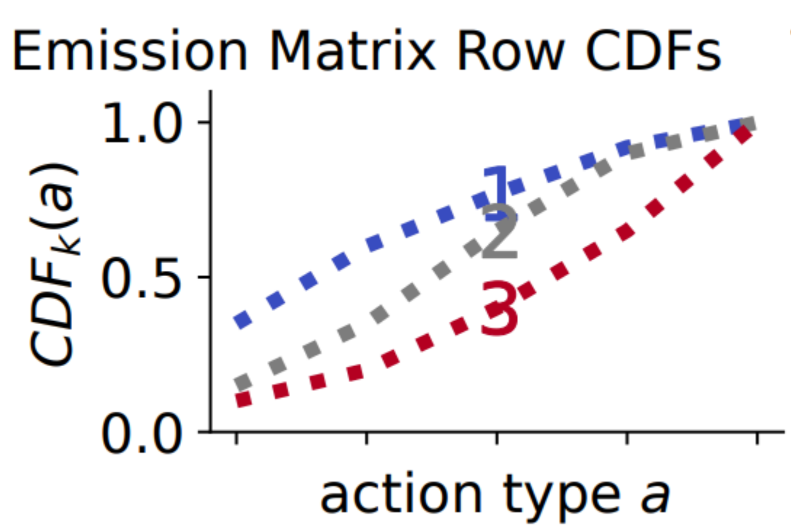
k^{th} row *stochastically dominates* $(k-1)^{\text{th}}$

$$\sum_{a' \le a} \phi_{ka'} \ge \sum_{a' \le a} \phi_{k'a'} \quad \text{for any } a \text{ and } k < k'$$





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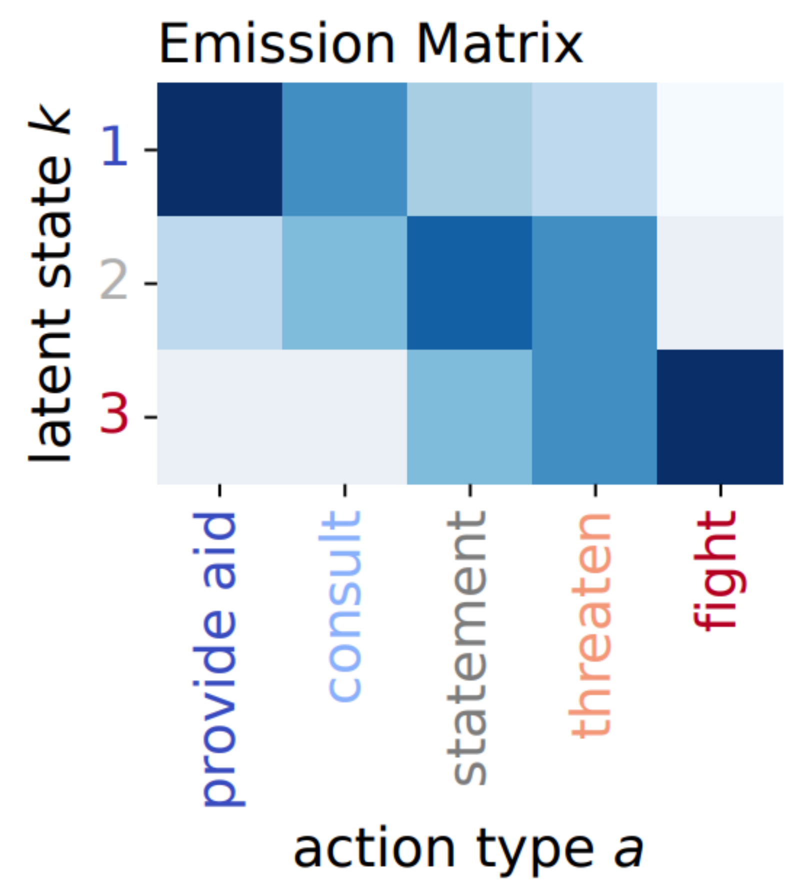
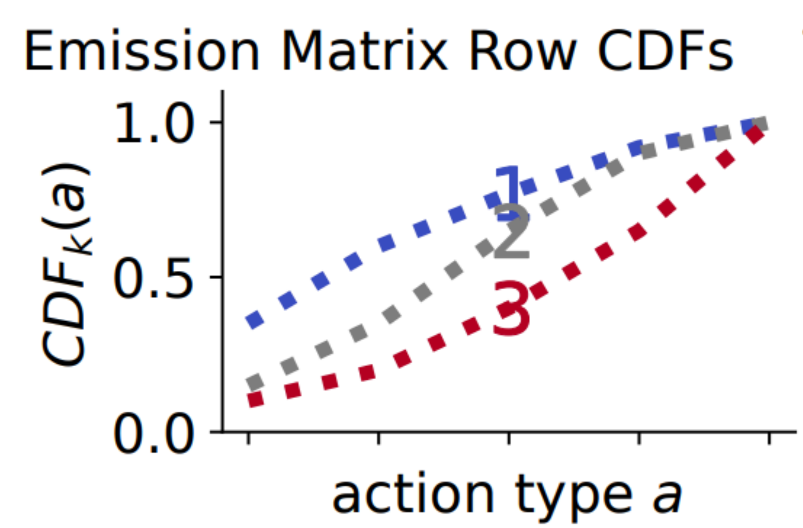
``well-ordered stochastic matrix"
(probability mass shifts down and right)



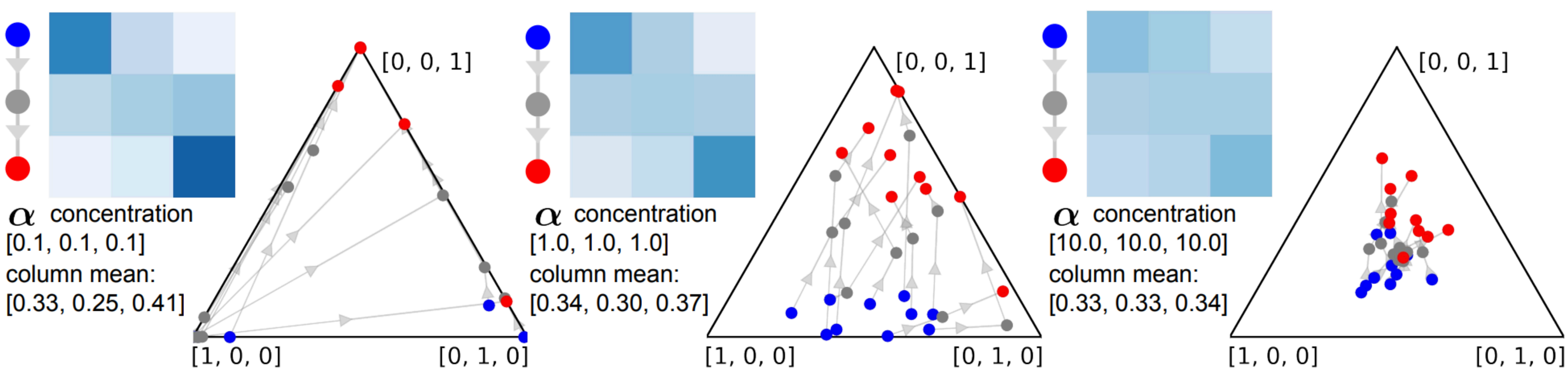


Ordered Matrix Dirichlet

A distribution over ``well-ordered" row-stochastic matrices

$$\Phi \sim \mathrm{OMD}(\alpha)$$

Defined implicitly using a **stick-breaking construction**



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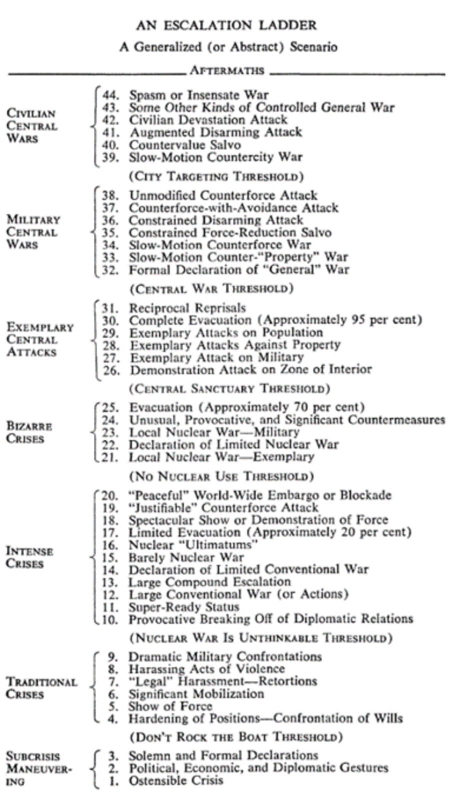
Ladder of escalation: *Allies do not become enemies overnight*



AN ESCALATION LADDER
A Generalized (or Abstract) Scenario

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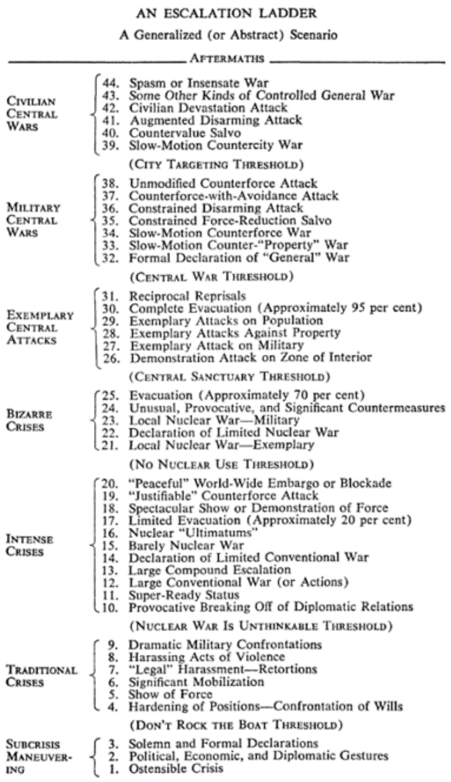

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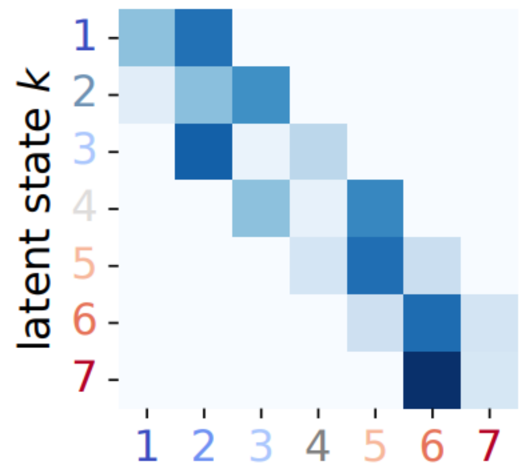
Traditional way to operationalize this:

Banded transition matrix



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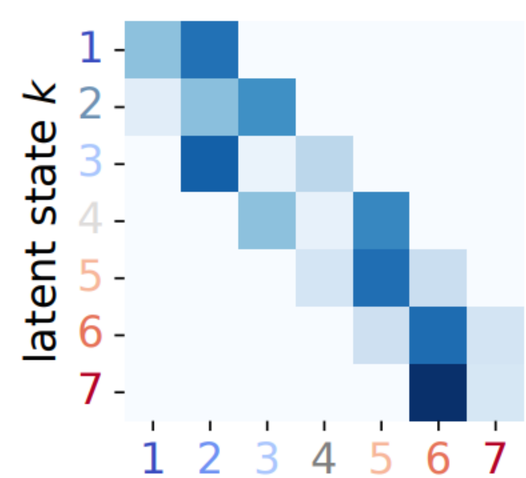
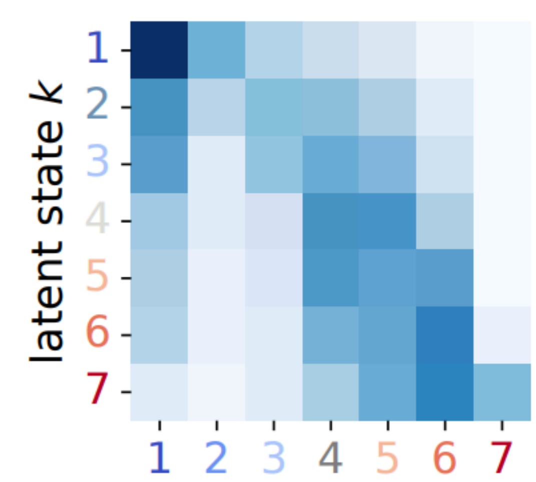
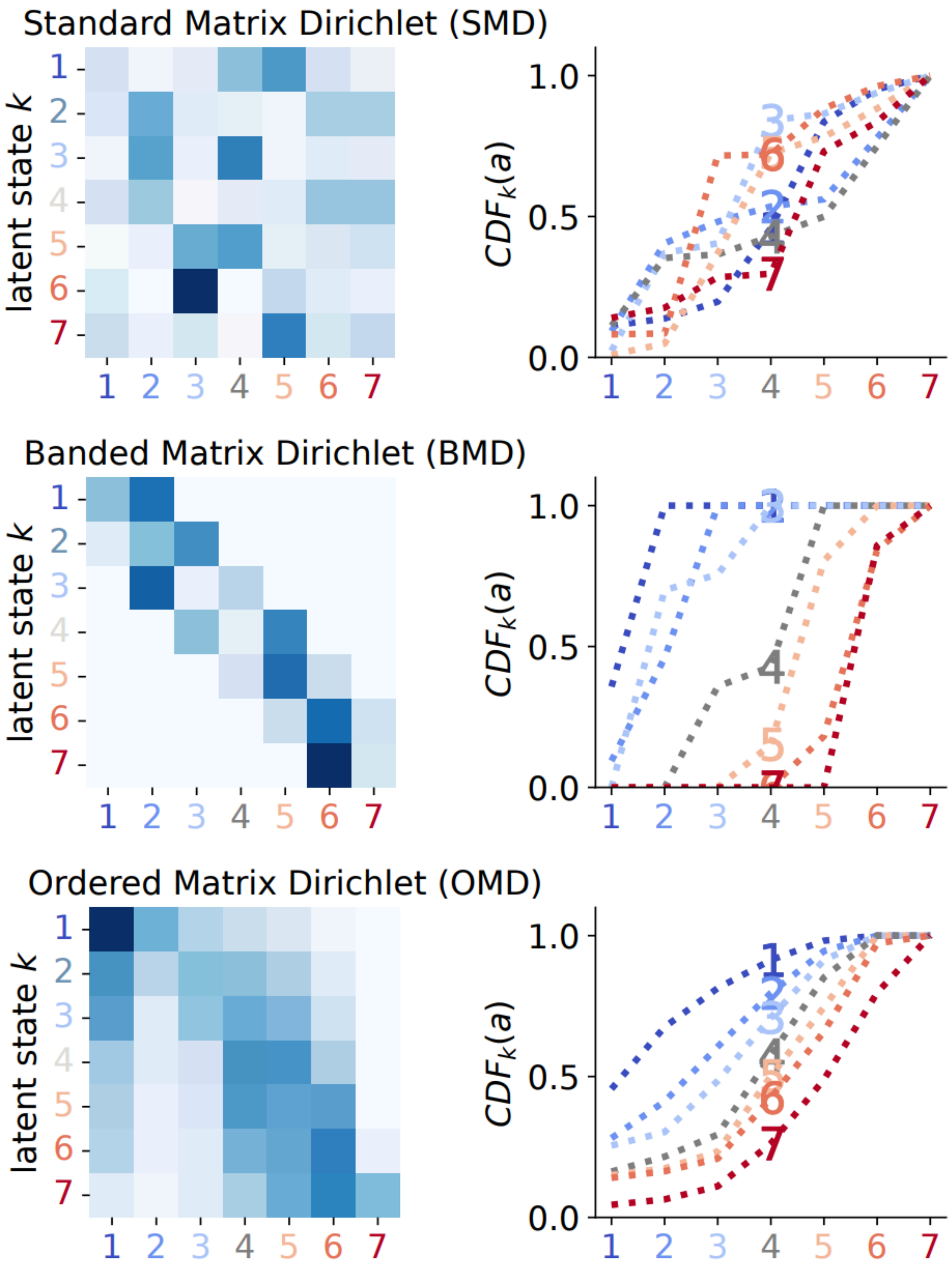


New way: $\Pi \sim \mathrm{OMD}(\boldsymbol{\alpha})$

OMD transition matrix



Ordered transitions



Putting it all together: **full model**

well-orderd **emission** matrix

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$$\lambda^{(t)}_{i \xrightarrow{k} j} \equiv \sum_{c=1}^{C} \sum_{d=1}^{C} \psi_{ic}\, \psi_{jd}\, \lambda^{(t)}_{c \xrightarrow{k} d}$$

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$$\mathbb{E}\left[\lambda^{(t)}_{c\overset{k}{\to}d}\right] = \sum_{k'=1}^{K} \pi_{k'k}\,\lambda^{(t-1)}_{c\overset{k'}{\to}d}$$

One extra piece…

$$\lambda^{(t)}_{i\overset{k}{\to}j} \equiv \sum_{c=1}^{C}\sum_{d=1}^{C} \psi_{ic}\,\psi_{jd}\,\lambda^{(t)}_{c\overset{k}{\to}d}$$

how much **countries** i and j
participate in **communities** c and d

how much **dyad** $c \to d$
is in **state** k at **time** t

Putting it all together: **full model**

well-ordered **emission** matrix

$$\Phi \sim \text{OMD}(\boldsymbol{\alpha})$$

$$\mathbb{E}\left[y^{(t)}_{i \xrightarrow{a} j}\right] = \sum_{k=1}^{K} \sum_{c=1}^{C} \sum_{d=1}^{C} \psi_{ic}\, \psi_{jd}\, \lambda^{(t)}_{c \xrightarrow{k} d}\, \phi_{ka}$$

Tucker decomposition

well-ordered **transition** matrix

$$\Pi \sim \text{OMD}(\boldsymbol{\alpha})$$

$$\mathbb{E}\left[\lambda^{(t)}_{c \xrightarrow{k} d}\right] = \sum_{k'=1}^{K} \pi_{k'k}\, \lambda^{(t-1)}_{c \xrightarrow{k'} d}$$

state-space model over the **core tensor**

Putting it all together: **full model**

well-ordered **emission** matrix

$$\Phi \sim \text{OMD}(\boldsymbol{\alpha})$$

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Tucker decomposition

$$y^{(t)}_{i \xrightarrow{a} j} \sim \text{Pois}\left(\sum_{k=1}^{K}\sum_{c=1}^{C}\sum_{d=1}^{C} \psi_{ic}\,\psi_{jd}\,\lambda^{(t)}_{c \xrightarrow{k} d}\,\phi_{ka}\right)$$

Poisson Tucker decomposition [ICML '16]

well-ordered **transition** matrix

$$\Pi \sim \text{OMD}(\boldsymbol{\alpha})$$

$$\mathbb{E}\left[\lambda^{(t)}_{c \xrightarrow{k} d}\right] = \sum_{k'=1}^{K} \pi_{k'k}\,\lambda^{(t-1)}_{c \xrightarrow{k'} d}$$

state-space model over the **core tensor**

$$\lambda^{(t)}_{c \xrightarrow{k} d} \sim \text{Gamma}\left(\tau_0 \sum_{k'=1}^{K} \pi_{k'k}\,\lambda^{(t-1)}_{c \xrightarrow{k'} d},\ \tau_0\right)$$

Poisson-Gamma dynamical system [NeurIPS '16]

Putting it all together: **full model**

well-ordered **emission** matrix

$$\Phi \sim \text{OMD}(\boldsymbol{\alpha})$$

$$\mathbb{E}\left[y^{(t)}_{i \xrightarrow{a} j}\right] = \sum_{k=1}^{K} \sum_{c=1}^{C} \sum_{d=1}^{C} \psi_{ic}\, \psi_{jd}\, \lambda^{(t)}_{c \xrightarrow{k} d}\, \phi_{ka}$$

Tucker decomposition

$$y^{(t)}_{i \xrightarrow{a} j} \sim \text{Pois}\left(\sum_{k=1}^{K} \sum_{c=1}^{C} \sum_{d=1}^{C} \psi_{ic}\, \psi_{jd}\, \lambda^{(t)}_{c \xrightarrow{k} d}\, \phi_{ka} \right)$$

Poisson Tucker decomposition
[ICML '16]

well-ordered **transition** matrix

$$\Pi \sim \text{OMD}(\boldsymbol{\alpha})$$

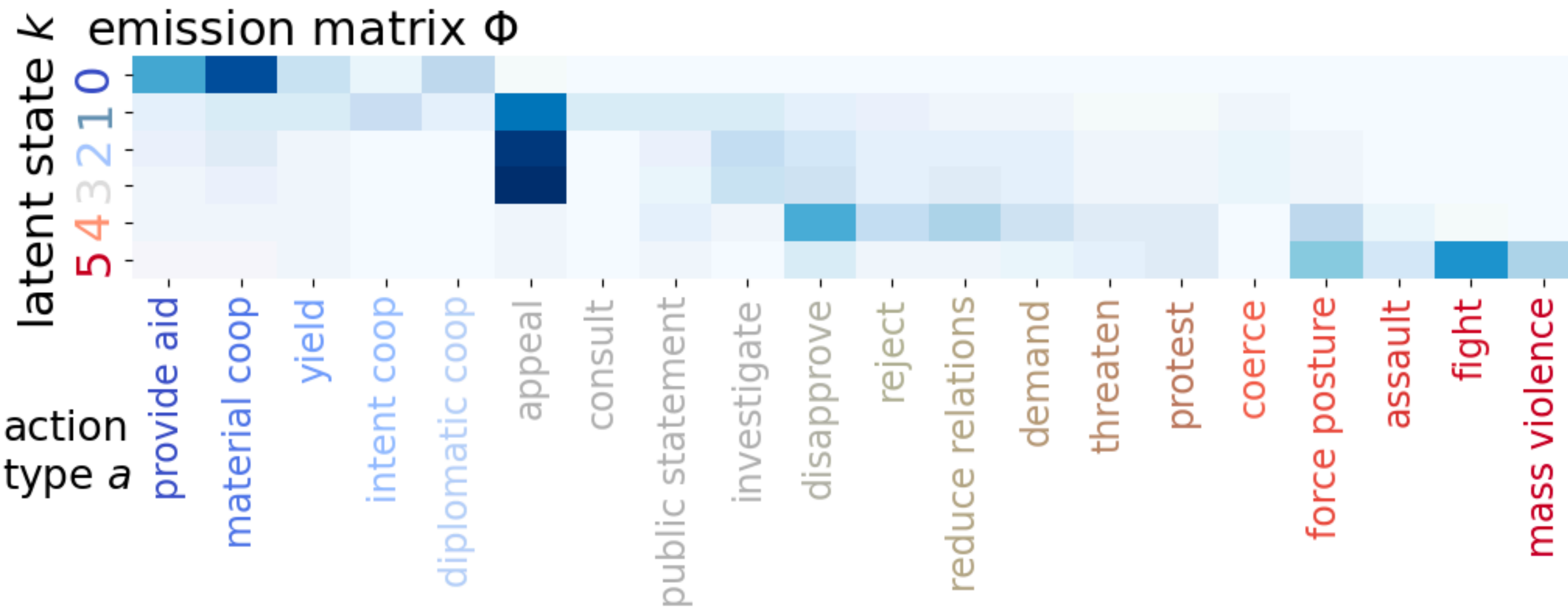
$$\mathbb{E}\left[\lambda^{(t)}_{c \xrightarrow{k} d}\right] = \sum_{k'=1}^{K} \pi_{k'k}\, \lambda^{(t-1)}_{c \xrightarrow{k'} d}$$

state-space model over
the **core tensor**

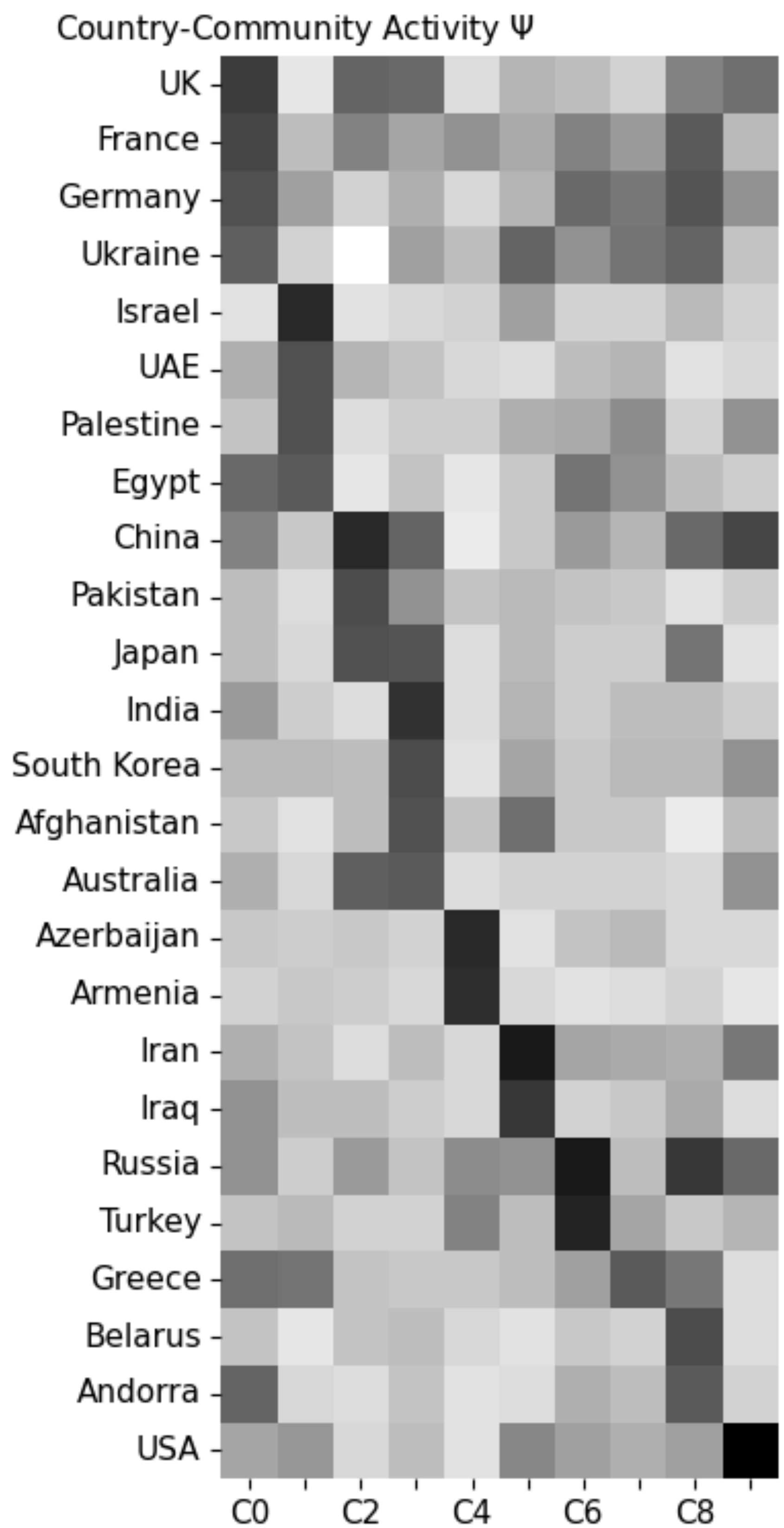
$$\lambda^{(t)}_{c \xrightarrow{k} d} \sim \text{Gamma}\left(\tau_0 \sum_{k'=1}^{K} \pi_{k'k}\, \lambda^{(t-1)}_{c \xrightarrow{k'} d},\ \tau_0 \right)$$

Poisson-Gamma dynamical system
[NeurIPS '16]

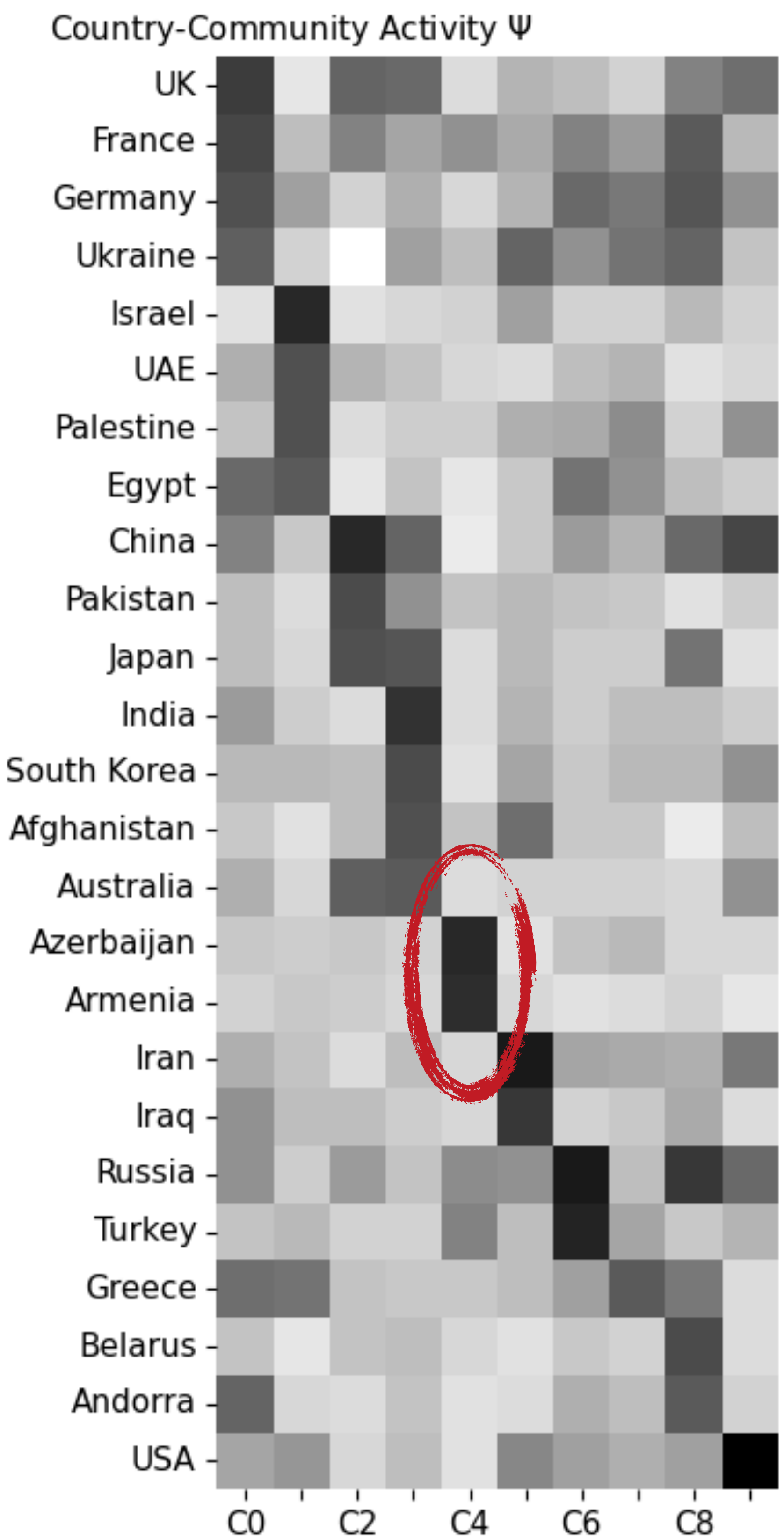
Results: ICEWS 2020 data

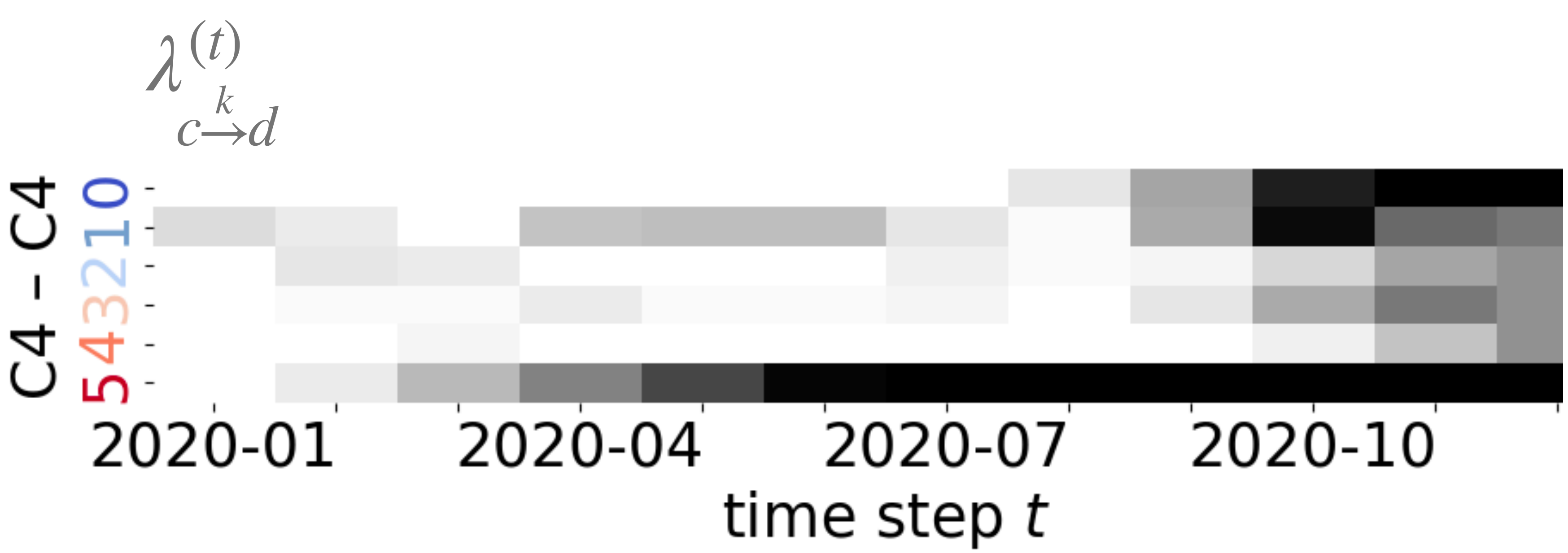


Results: ICEWS 2020 data



Results: ICEWS 2020 data





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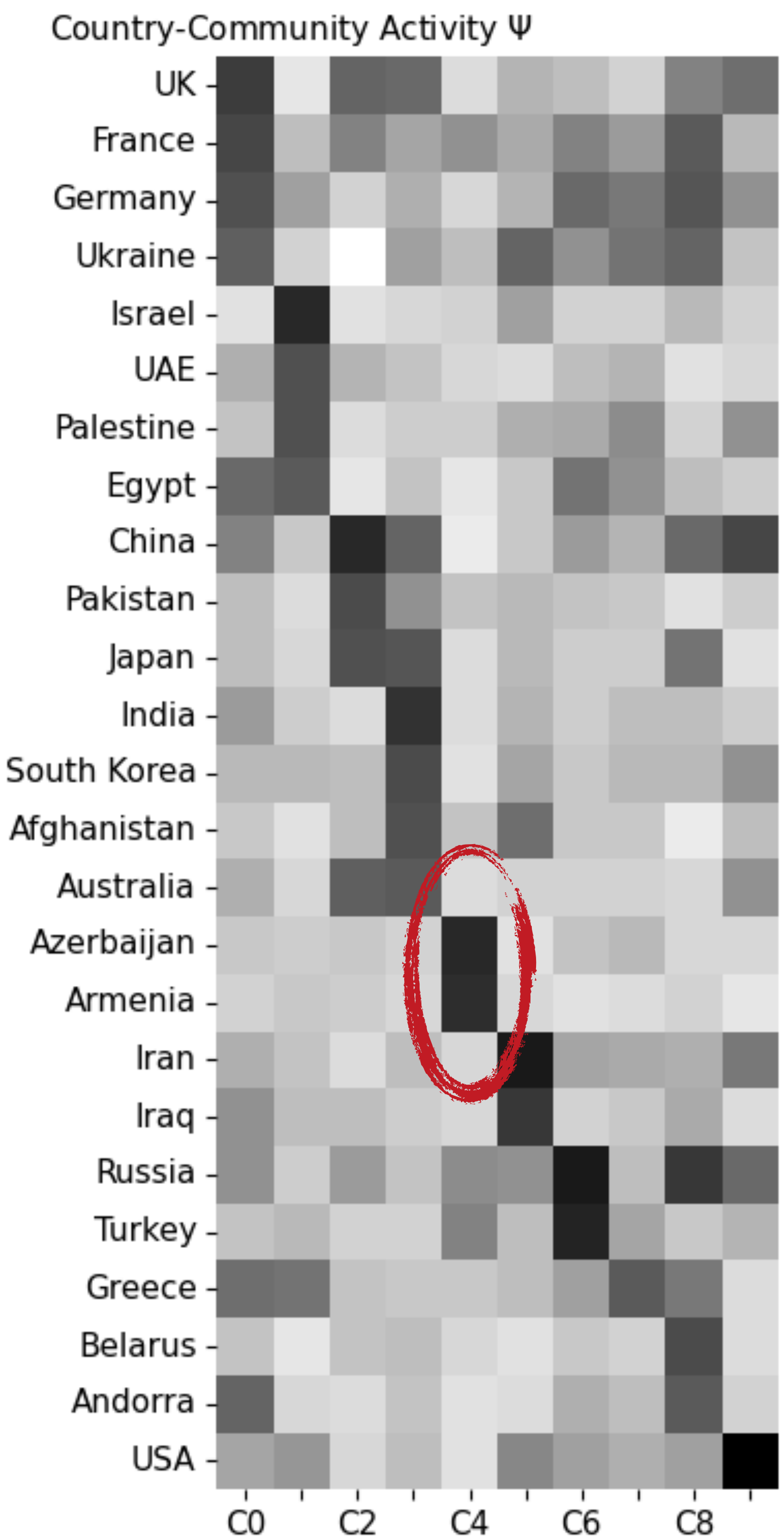

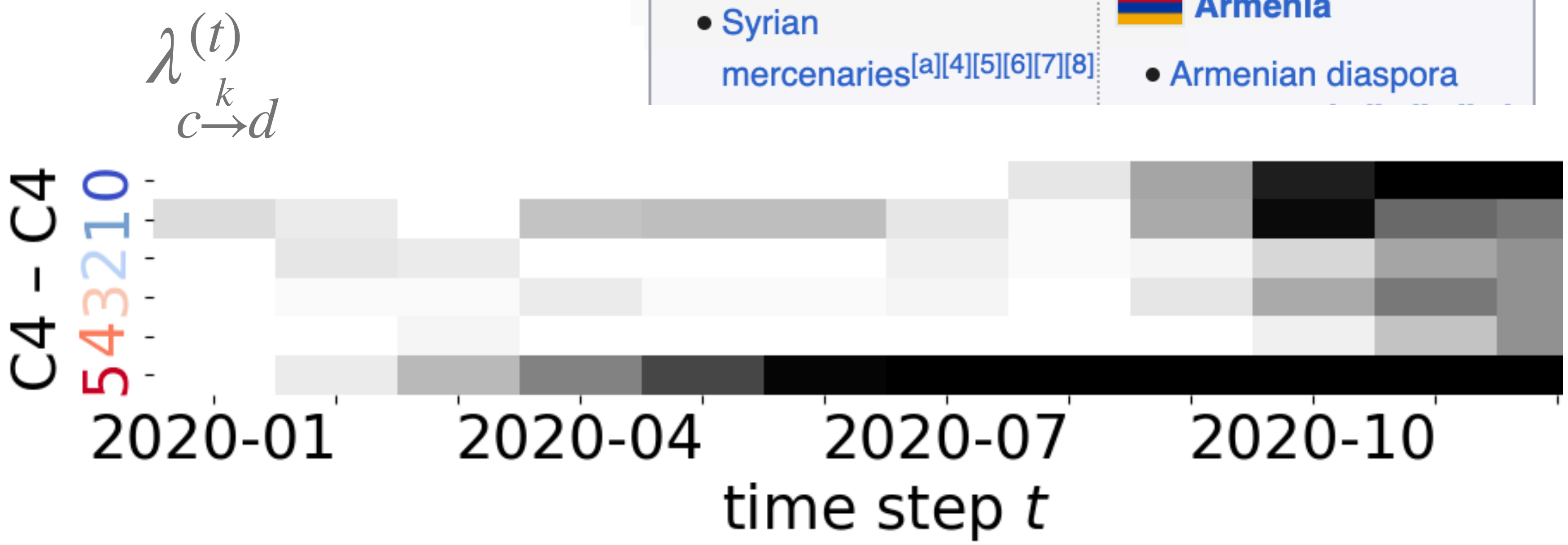

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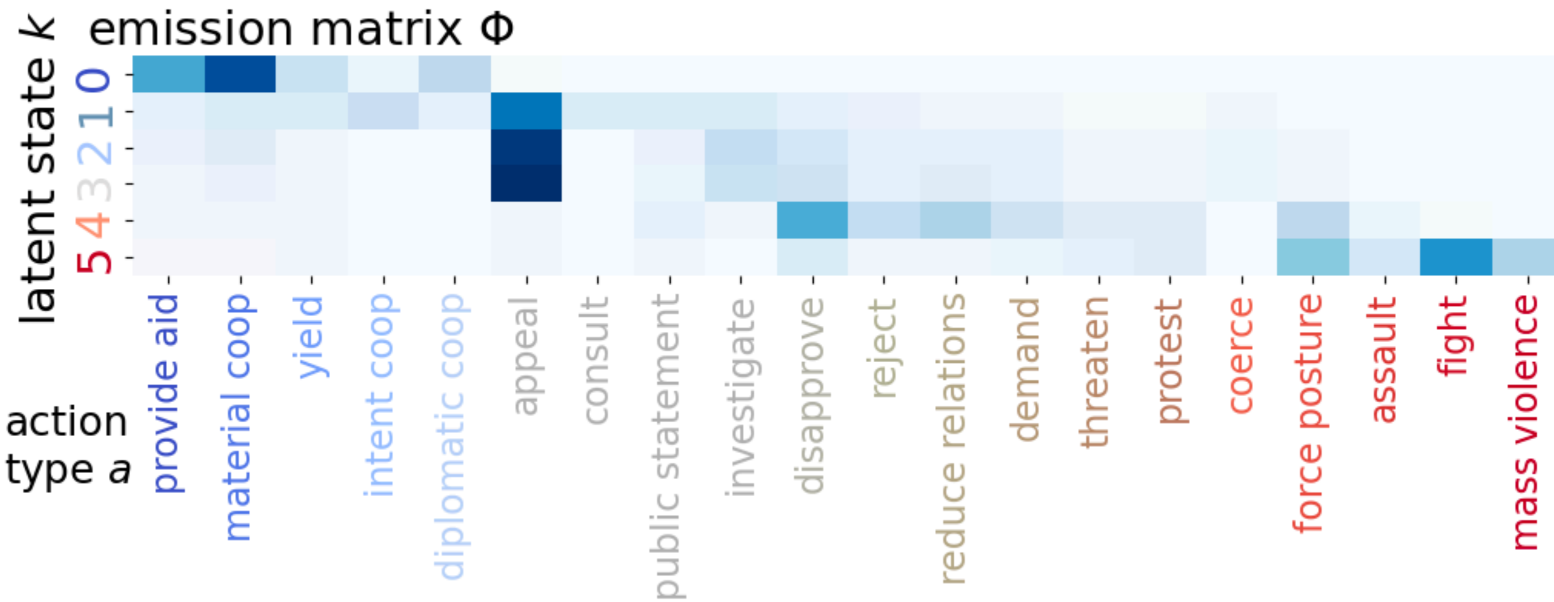

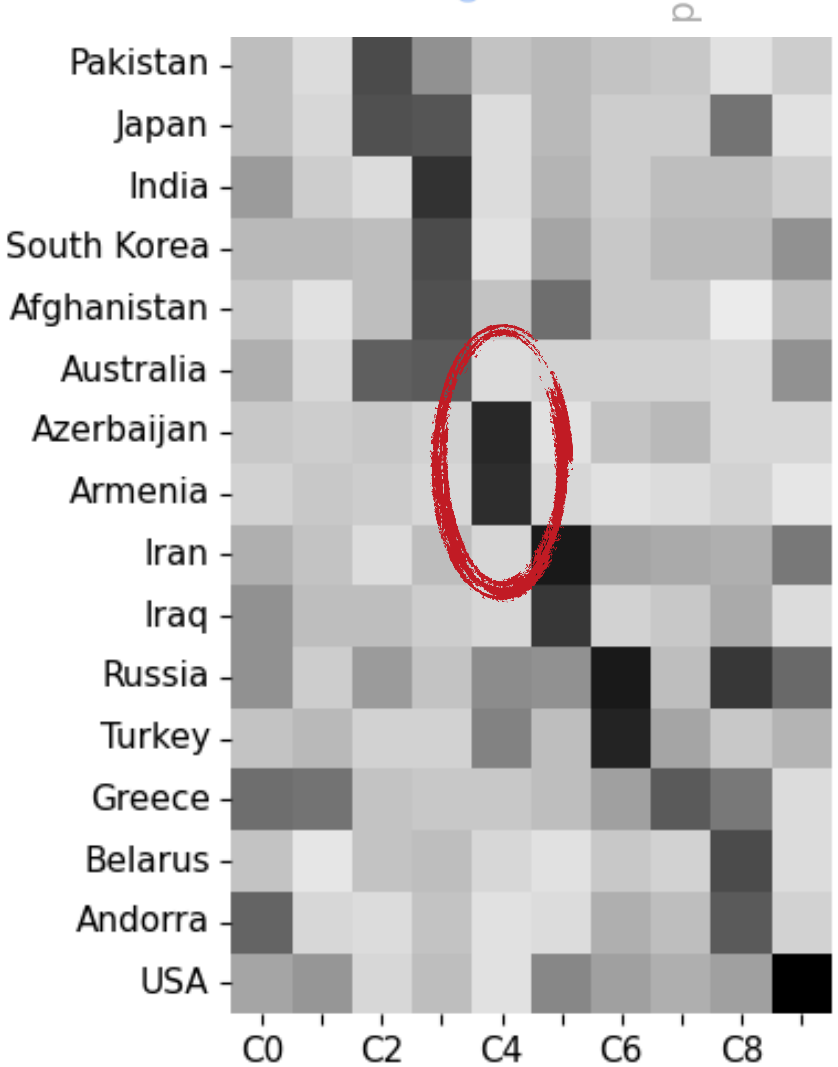

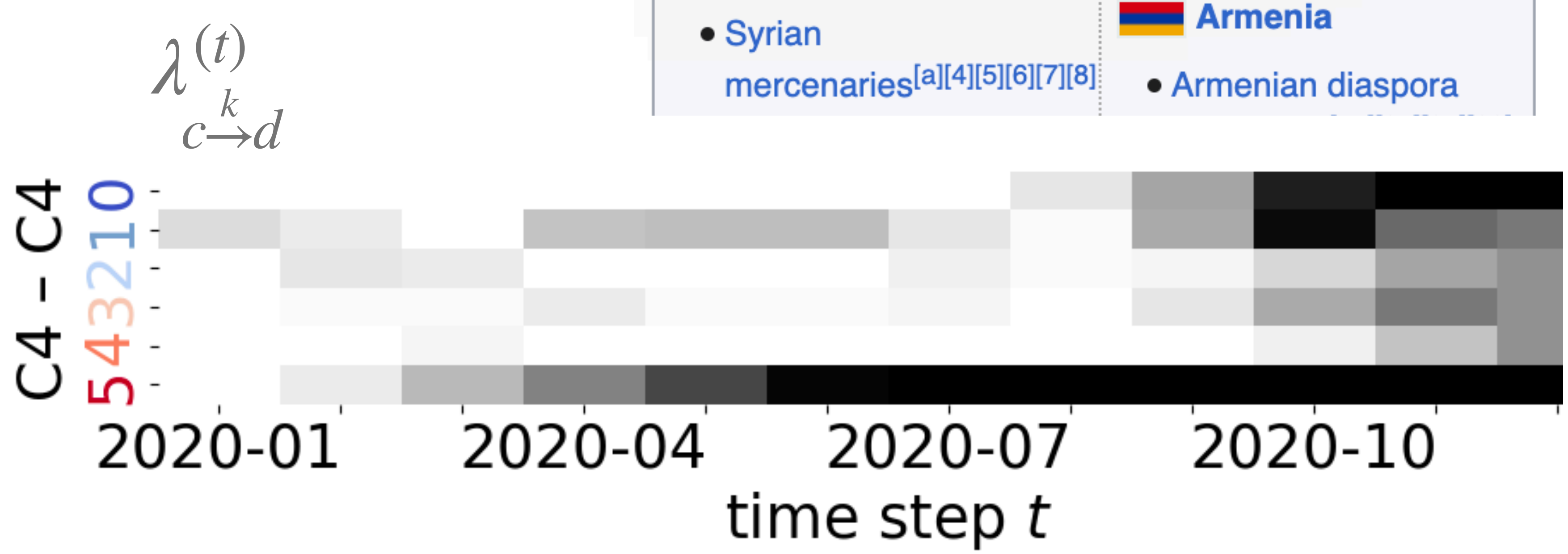

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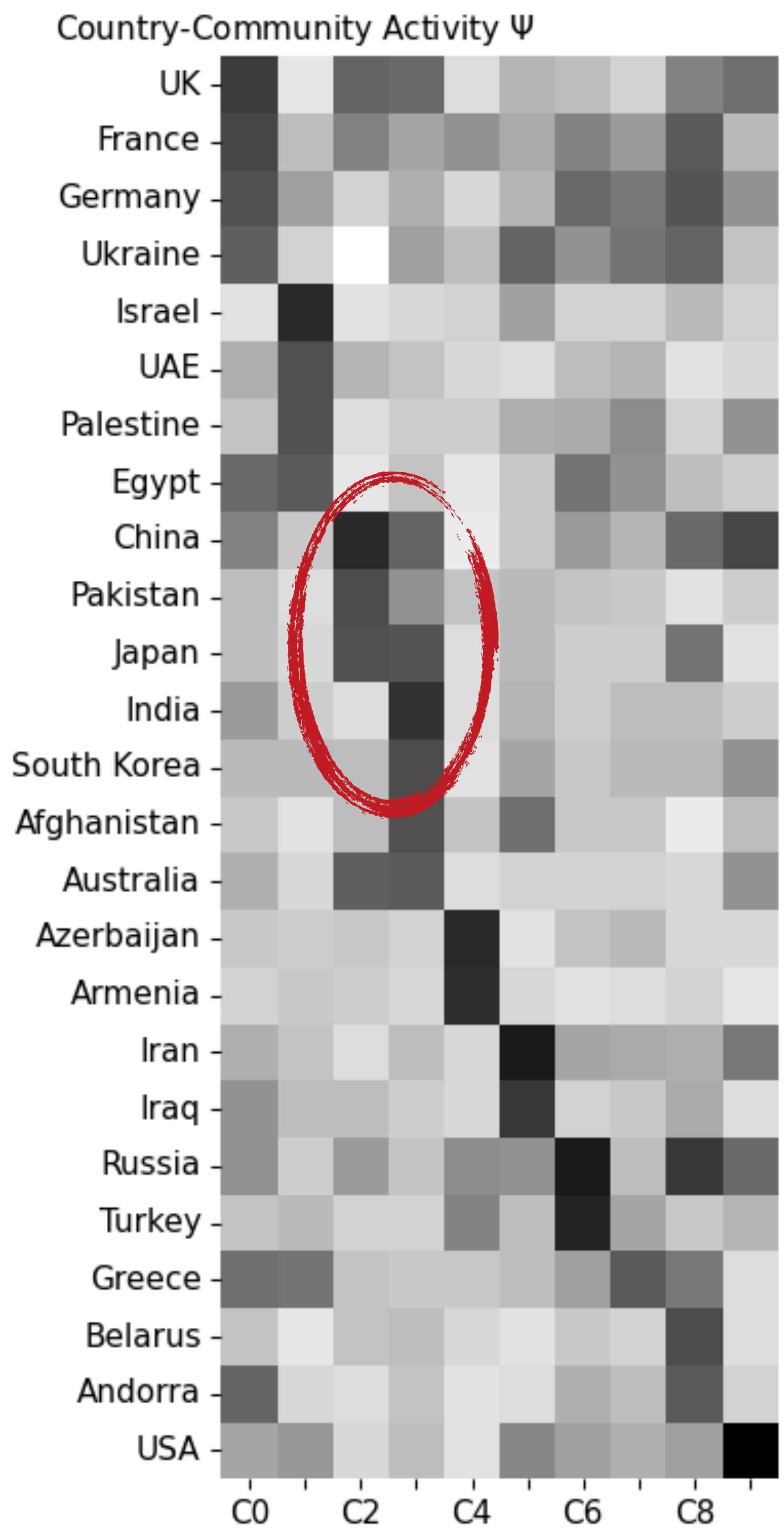

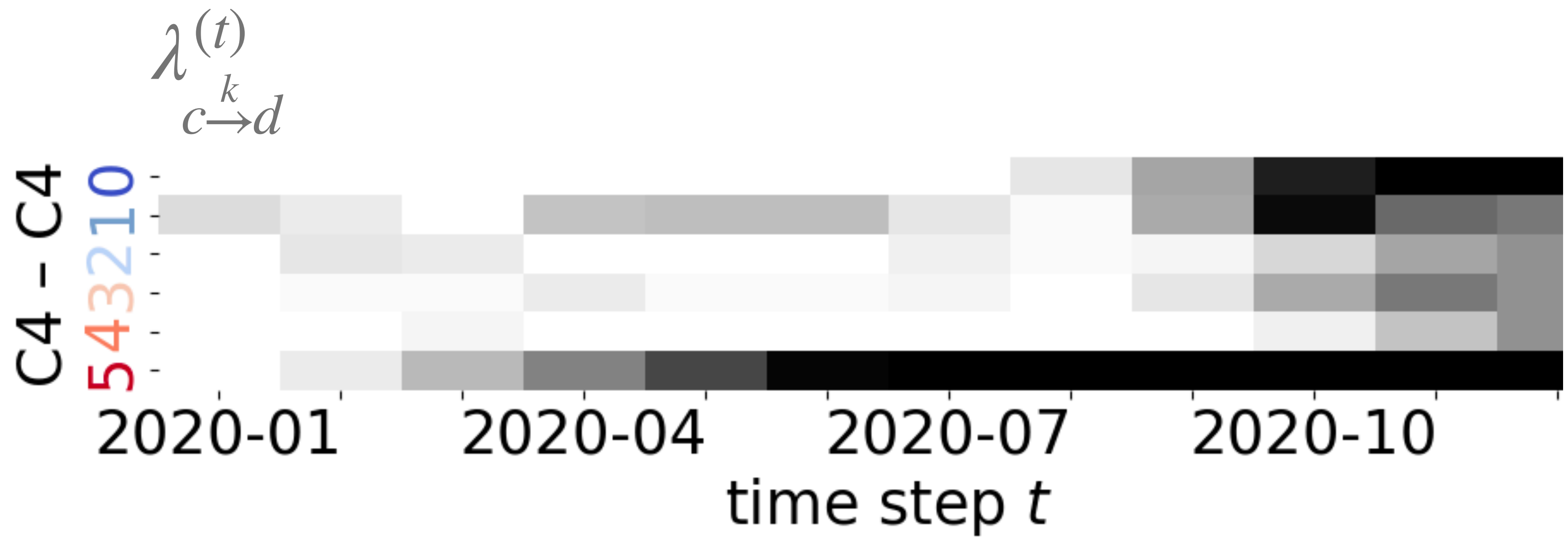

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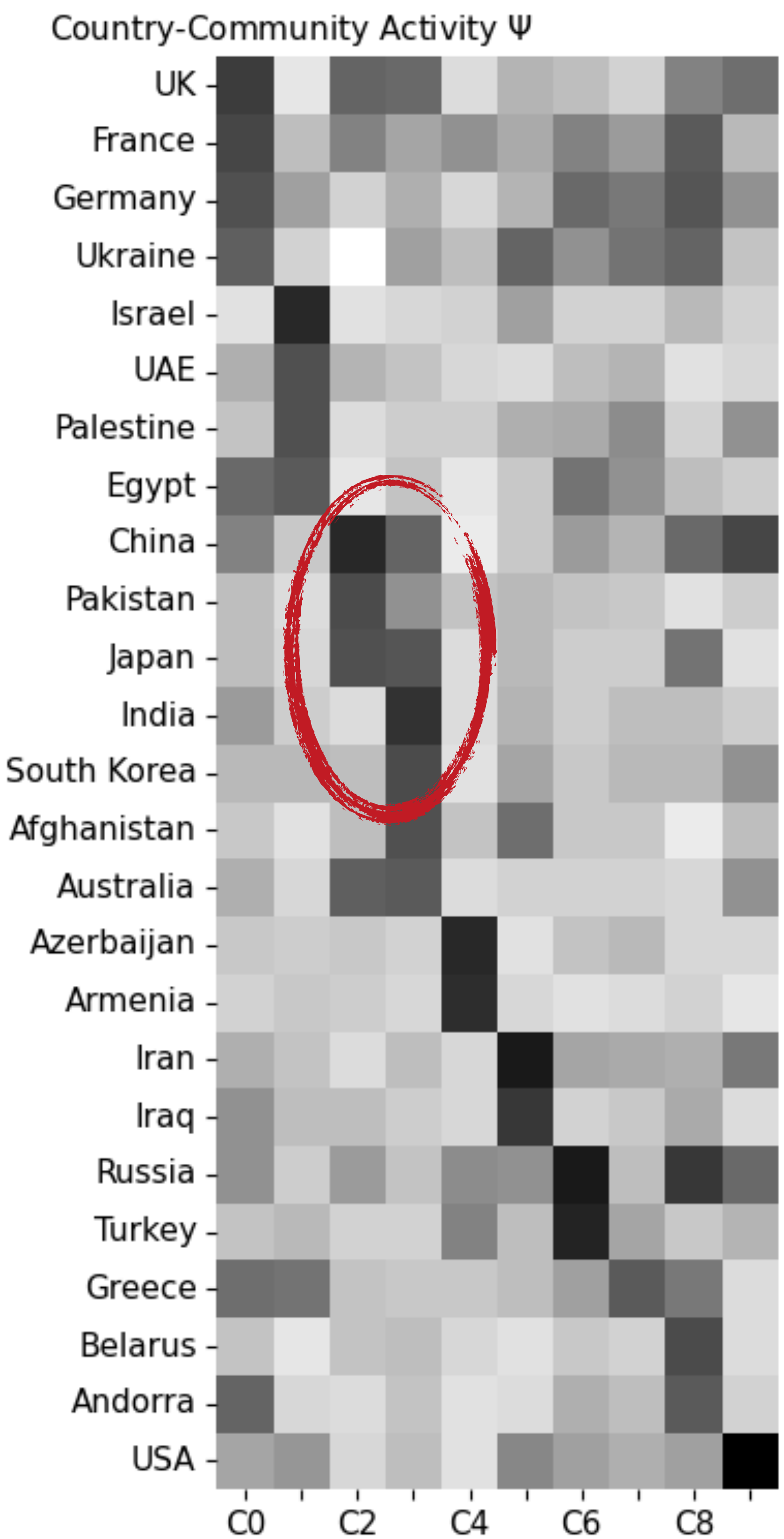
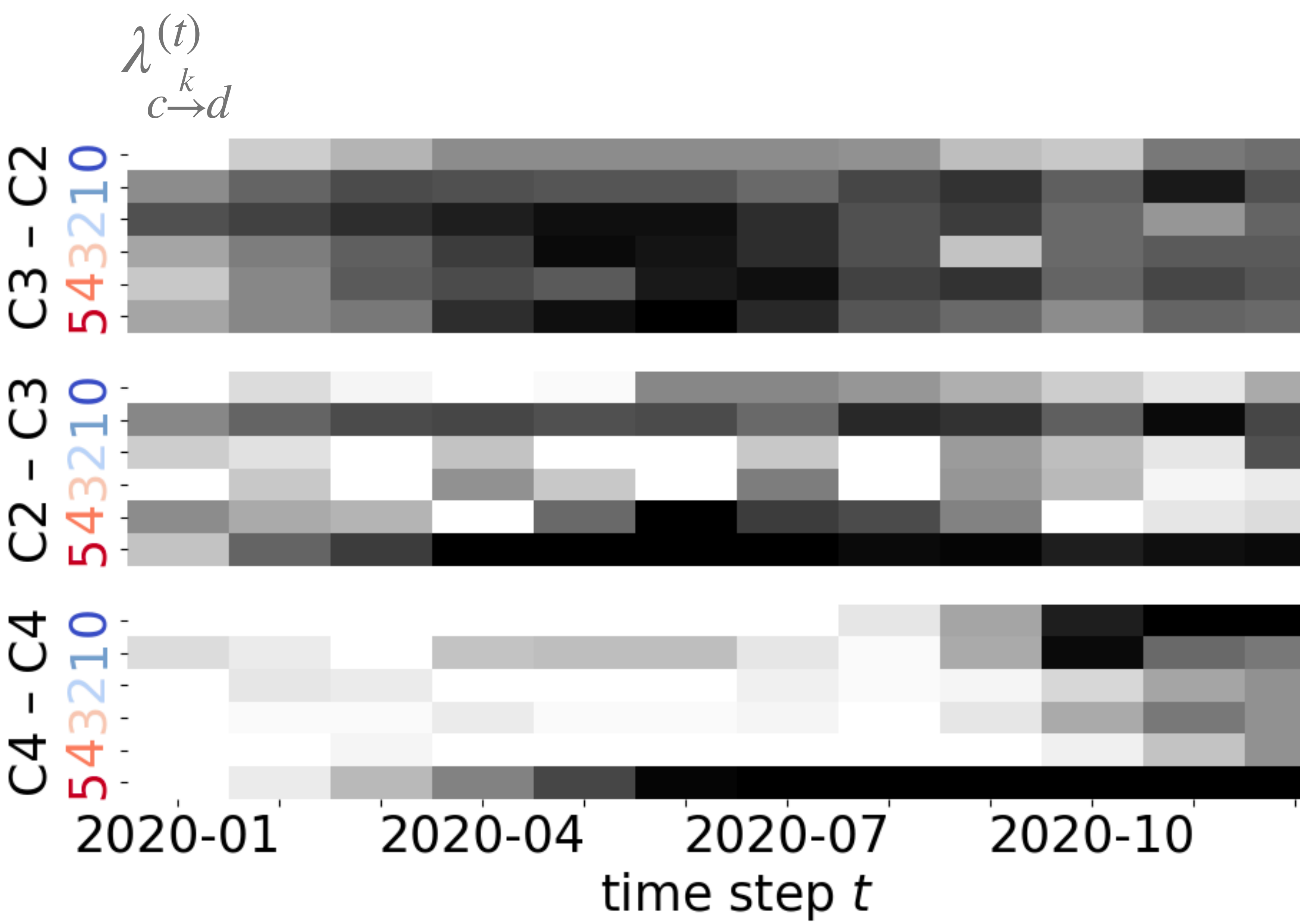
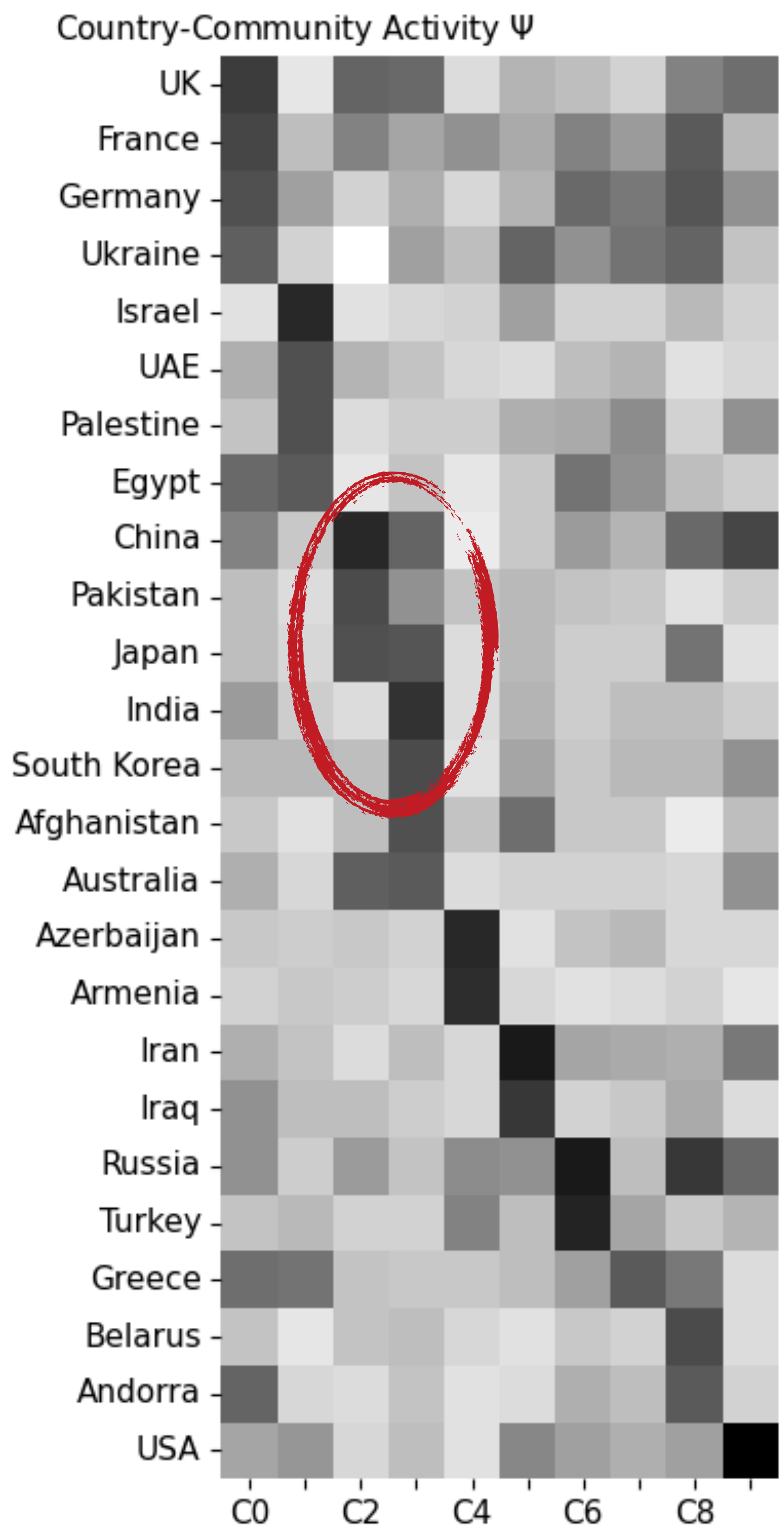




Results: ICEWS



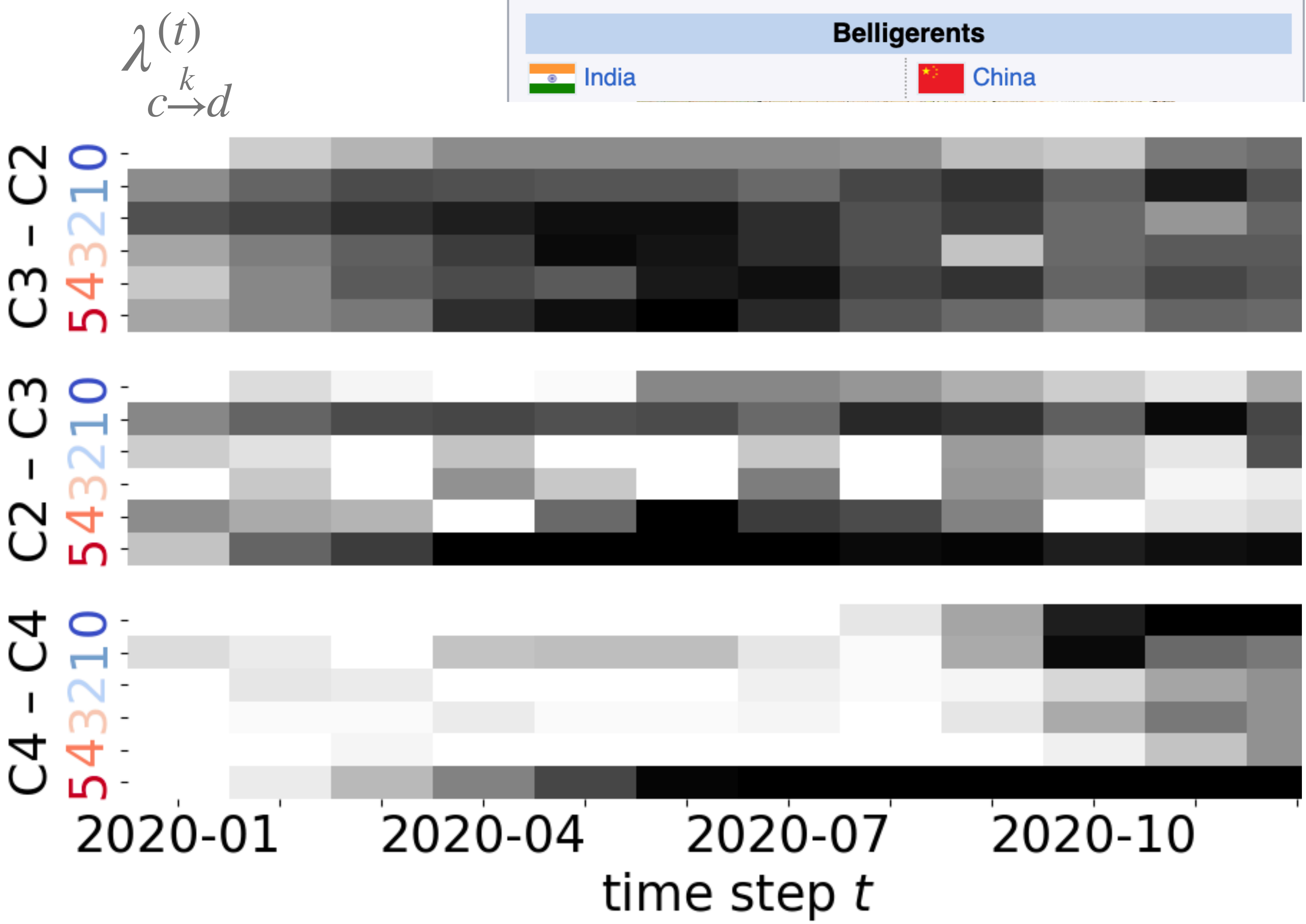
2020–2021 China–India skirmishes

Part of the Sino-Indian border dispute

Date	5 May 2020 – 20 January 2021 (8 months, 2 weeks and 1 day)
Location	Line of Actual Control (LAC), Sino-Indian border
Status	• Change in status quo of ground positions[1][2][3][4][5][6] • Disengagement and de-escalation in progress[7][8][9]

Belligerents

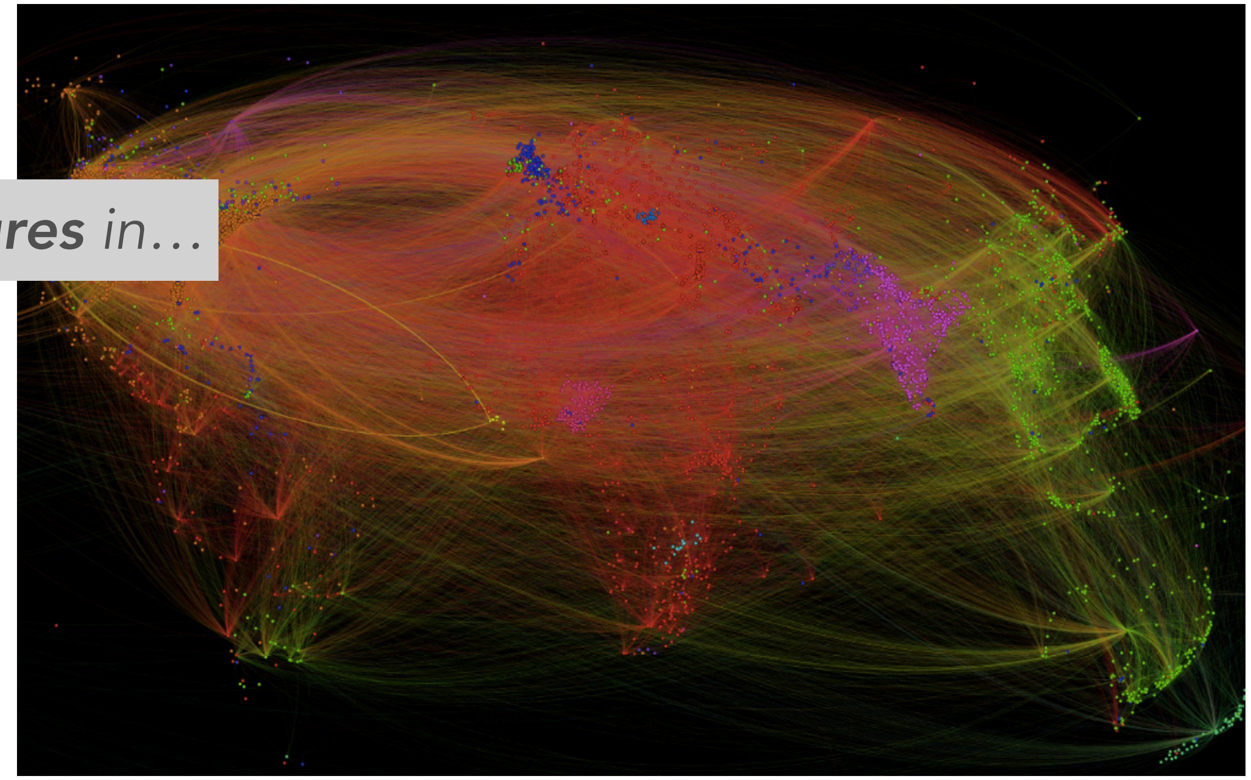
India	China



Summary and conclusion

Mission:

*Models to **measure complex dependence structures** in…*



…**high-dimensional discrete data**.

(sparse, bursty, overdispersed)

Requires:

- ***Blending** methodological frameworks:*
  - *Contingency table analysis*
  - *Tensor decomposition*
  - *Bayesian (ad)mixture / block modeling*
- ***New modeling motifs** for discrete and non-negative distributions*
- ***New techniques, theorems, tricks** for inference*

Data like this is **everywhere** e.g.,

- Genomics
- Neuroscience
- Demography

Broader data science goal:

*Develop the **toolkit** for **measuring scientifically-interesting complex structure** in ^^*