

GR5206: lecture 3

Computational Statistics
And Introduction to Data Science

The two programming paradigms



Imperative:

- The programmer instructs the machine how to change its state.
- Two kinds:
 - **Procedural:** groups instructions into procedures.
 - Object-oriented: groups instructions together with the part of the state they operate on.

Declarative:

- ► The programmer declares properties of the desired result, but not how to compute it.
- Three kinds:
 - Functional: the output results of a series of function applications.
 - Logic: the output is the answer to a question about a system of facts and rules.
 - Mathematical: the output is the solution of an optimization problem.

What about R?



- A bit of everything:
 - Powerful but complex.
- Imperative:
 - Procedural: functions loaded with source().
 - Object-oriented: the S3 class system (and others).
- Declarative:
 - Mathematical: optimization with optim and specialized packages.
 - Functional: the hearth of R.

Functional programming languages



- What makes a programming language functional?
 - Many definitions but two common threads:
 - First-class functions.
 - Pure functions.

Functional style:

- Hard to describe exactly, but essentially:
 - Decompose a problem into small pieces, then solve each piece with a (combination of) function(s).
 - Each function is simple and straightforward to understand.
 - Complexity is handled by composing functions.

First-class functions



- Functions behave like any other data structure.
- In R, means that you can:
 - Assign them to variables.
 - Store them in lists.
 - Pass them as arguments to other functions.
 - Create them inside functions.
 - And even return them as the result of a function.

```
f1 <- function(x) x
l1 <- list(
    mean,
    sd,
    median
)
y <- rnorm(le1)
sapply(l1, function(f) f(y))
#> [1] -0.441 1.118 -0.246
```

Pure functions



- Two main properties:
 - ► The output only depends on the inputs:
 - Call it again with the same inputs, get the same outputs.
 - Excludes functions like runif() or read.csv() (why?).
 - No side-effects:
 - E.g., no changing the value of a global variable, writing to disk, or displaying to the screen.
 - Excludes functions like print(), write.csv() and <-.
- Two remarks:
 - Much easier to reason about, but some downsides:
 - How to do data analysis without generate random numbers or read files from disk?
 - Strictly speaking:
 - R isn't a functional language (why?).
 - While you don't have to write pure functions, you often should.

Functional style



- Three techniques:
 - Functionals:
 - Replace many loops.
 - E.g., lapply(), sapply().
 - The most important, used all the time in data analysis.
 - Function factories:
 - Functions that create functions.
 - Partition work between different parts of your code.
 - Function operators:
 - Functions that take/return functions as inputs/output.
 - Typically modify the operation of a function.

Called higher-order functions

In Out		Vector	Function	
Ve	ctor	Regular function	Function factory	
Function		Functional	Function operator	

Outline



- 1 Functionals
- 2 Map
- 3 Reduce
- 4 Predicate functionals
- 5 Base functionals
- 6 Function factories
- 7 Function operators

Functionals



To become significantly more reliable, code must become more transparent. In particular, nested conditions and loops must be viewed with great suspicion. Complicated control flows confuse programmers. Messy code often hides bugs.

— Bjarne Stroustrup

Functional:

- Takes/returns a function/vector as an input/output.
- lapply(), apply(), tapply(), purrr's map(), integrate()
 or optim().

```
randomise <- function(f) f(runif(1e3))
randomise(mean)
#> [1] 0.509
randomise(mean)
#> [1] 0.498
randomise(sum)
#> [1] 492
```

Outline



- purrr::map():
 - ► Combine multiple simple functionals to solve larger problems.
 - ► The 18 (!!) important variants of purrr::map().
- purrr::reduce().
- Predicates and the functionals using them.
- Some functionals in base R not members of those families.
- Focus on the purrr package:
 - Consistent interface that makes it easier to use/understand.
 - We'll compare to base R functions equivalents.

library(purrr)

Outline



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- The most fundamental functional:
 - Takes a vector and a function.
 - Calls the function once for each element of the vector
 - Returns the results in a list
- \blacksquare map(1:3, f) is equivalent to list(f(1), f(2), f(3)).
- The R base equivalent: lapply().

```
triple <- function(x) x * 3
map(1:3, triple)

#> [[1]]

#> [1] 3

#>

#> [[2]]

#> [1] 6

#>

#> [[3]]

#> [1] 9
```

How does that work?



- Simple implementation:
 - Allocate a list the same length as the input.
 - Fill in the list with a for loop.

```
simple_map <- function(x, f, ...) {
  out <- vector("list", length(x))
  for (i in seq_along(x)) {
    out[[i]] <- f(x[[i]], ...)
  }
  out
}</pre>
```

- A few differences for the real implementation:
 - Written in C for performance.
 - Preserves names
 - Supports a few shortcuts.

Producing atomic vectors



- map() returns a list
- 4 more specific variants:
 - map_dbl(), map_chr(), map_int() and map_lgl().
- map_dbl() always returns a double vector.

map_chr() always returns a character vector

Producing atomic vectors con'd

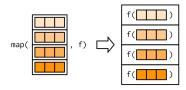


map_int() always returns an integer vector.

```
map_int(mtcars, function(x) length(unique(x)))
#> mpg cyl disp hp drat wt qsec vs am gear carb
#> 25 3 27 22 22 29 30 2 2 3 6
```

■ map_lgl() always returns a logical vector.

- Remarks:
 - Suffixes refer to the output.
 - But map_*() can take any type of vector as input.
- Examples rely on two facts:
 - mtcars is a data frame.
 - data frames are lists containing vectors of the same length.



Producing atomic vectors con'd



■ Each call to the function must return a single value.

```
map_dbl(1:2, function(x) c(x, x))
#> Result 1 must be a single double, not an integer vector of length 2
```

And obviously return the correct type.

```
map_dbl(1:2, as.character)
#> Error: Can't coerce element 1 from a character to a double
```

- In either case, use map() to see the problematic output!
- In base R:
 - sapply().
 - Tries to simplify the result,.
 - Can return a list, a vector, or a matrix.
 - Difficult to program with, avoid in non-interactive settings.
 - vapply().
 - FUN. VALUE to describe the output shape.
 - Verbosity: vapply(x, mean, na.rm = TRUE, FUN.VALUE = double(1)) for map_dbl(x, mean, na.rm = TRUE).

Anonymous functions and shortcuts



map can use anonymous functions.

```
map_dbl(mtcars, function(x) length(unique(x)))
#> mpg cyl disp hp drat wt qsec vs am gear carb
#> 25 3 27 22 22 29 30 2 2 3 6
```

Less verbose shortcut.

```
map_dbl(mtcars, ~ length(unique(.x)))
#> mpg cyl disp hp drat wt qsec vs am gear carb
#> 25 3 27 22 22 29 30 2 2 3 6
```

Useful for generating random data.

```
x <- map(1:3, ~ runif(2))
str(x)

#> List of 3

#> $ : num [1:2] 0.307 0.254

#> $ : num [1:2] 0.656 0.839

#> $ : num [1:2] 0.834 0.425
```

■ Rule of thumb: a function spans lines/uses {}, give it a name.

Extracting elements from a vector



```
x \leftarrow list(
 list(-1, x = 1, y = c(2), z = "a"),
  list(-2, x = 4, y = c(5, 6), z = "b"),
  list(-3, x = 8, y = c(9, 10, 11))
# Select by name
map_dbl(x, "x")
#> [1] 1 4 8
# Or by position
map_dbl(x, 1)
#> [1] -1 -2 -3
# Or by both
map_dbl(x, list("y", 1))
#> [1] 2 5 9
# You'll get an error if a component doesn't exist:
map_chr(x, "z")
#> Result 3 must be a single string, not NULL of length 0
```

Passing arguments with ...



 To pass along additional arguments, use an anonymous function.

```
x <- list(1:5, c(1:10, NA))
map_dbl(x, ~ mean(.x, na.rm = TRUE))
#> [1] 3.0 5.5
```

Or the simpler form.

```
map_dbl(x, mean, na.rm = TRUE)
#> [1] 3.0 5.5
```

A subtle difference.

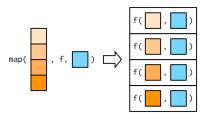
```
plus <- function(x, y) x + y
x <- c(0, 0, 0, 0)

map_dbl(x, plus, runif(1))

#> [1] 0.281 0.281 0.281 0.281

map_dbl(x, ~ plus(.x, runif(1)))

#> [1] 0.358 0.946 0.570 0.589
```



Map variants



- 23 primary variants of map():
 - map(), map_dbl(), map_chr(), map_int(), map_lgl()
 - 18 (!!) more to learn.
 - Five new ideas:
 - Output same type as input with modify()
 - Iterate over two inputs with map2().
 - Iterate with an index using imap()
 - Return nothing with walk().
 - Iterate over any number of inputs with pmap().

	List	Atomic	Same type	Nothing
One argument Two arguments	map() map2()	map_lgl(), map2_lgl(),	<pre>modify() modify2()</pre>	walk() walk2()
One argument + index N arguments	<pre>imap() pmap()</pre>	<pre>imap_lgl(), pmap_lgl(),</pre>	imodify()	iwalk() pwalk()

```
df <- data.frame(x = 1:3, y = 6:4)
map(df, ~ .x * 2)
#> $x
#> [1] 2 4 6
#>
#> $y
#> [1] 12 10 8
modify(df, ~ .x * 2)
#> x y
#> 1 2 12
#> 2 4 10
#> 3 6 8
```

A simple implementation.

```
simple_modify <- function(x, f, ...) {
  for (i in seq_along(x)) {
    x[[i]] <- f(x[[i]], ...)
  }
  x
}</pre>
```

Two inputs: map2() and friends



■ How do we find the vector of weighted means?

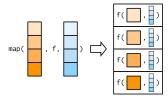
```
xs <- map(1:8, ~ runif(10))
xs[[1]][[1]] <- NA
ws <- map(1:8, ~ rpois(10, 5) + 1)</pre>
```

■ Use map_dbl() to compute the unweighted means.

```
map_dbl(xs, mean)
#> [1] NA 0.520 0.510 0.634 0.475 0.560 0.486 0.495
```

Passing ws as an additional argument doesn't work.

```
map_dbl(xs, weighted.mean, w = ws)
#> Error in weighted.mean.default(.x[[i]], ...): 'x' and 'w' must have the same
```





Both arguments are varied in each call.

```
map2_dbl(xs, ws, weighted.mean)
#> [1] NA 0.467 0.470 0.638 0.516 0.550 0.471 0.524
```

Additional arguments still go afterwards.

```
map2_dbl(xs, ws, weighted.mean, na.rm = TRUE)
#> [1] 0.458 0.467 0.470 0.638 0.516 0.550 0.471 0.524
```



No outputs: walk() and friends



```
welcome <- function(x) {</pre>
  cat("Welcome ", x, "!\n", sep = "")
}
names <- c("Hadley", "Jenny")</pre>
# As well as generate the welcomes, it also shows
# the return value of cat()
map(names, welcome)
#> Welcome Hadley!
#> Welcome Jenny!
#> [[1]]
#> NULL
#>
#> [[2]]
#> NTIT.T.
## Avoid this with walk
walk(names, welcome)
#> Welcome Hadley!
#> Welcome Jenny!
```

Iterating over values and indices



- Three basic ways to loop over a vector with for:
 - Over the elements: for (x in xs) f(xs)
 - Over the names: for (nm in names(xs)) f(nm)
 - Over the indices: for (i in seq_along(xs)) f(i)
- First kind: similar to map(xs, f).
- The other two: imap(xs, f).
 - ► Same as map2(xs, names(xs), f) if xs as names.
 - Same as map2(xs, seq_along(xs), f) otherwise.

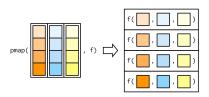
```
imap_chr(iris, ~ paste0("The first value of ", .y, " is ", .x[[1]]))
#>
                                Sepal.Lenath
   "The first value of Sepal.Length is 5.1"
#>
                                 Sepal. Width
    "The first value of Sepal. Width is 3.5"
#>
#>
                                Petal.Length
   "The first value of Petal.Length is 1.4"
#>
                                 Petal. Width
#>
    "The first value of Petal. Width is 0.2"
#>
                                     Species
#>
     "The first value of Species is setosa"
```

Any number of inputs: pmap()



- map() and map2()... map3(), map4(), map5()?
- Instead, there is pmap():
 - Supply it a single list, which contains any number of arguments.
 - In most cases, a list of equal-length vectors (e.g., a data frame).

```
params <- tibble::tribble(
  ~ n, ~ min, ~ max,
  1L. 0. 1.
  2L, 10, 100,
  3L, 100, 1000
pmap(params, runif)
#> [[1]]
#> [1] 0.15
#>
#> [[2]]
#> [1] 17.3 74.8
#>
#> [[3]]
#> [1] 933 994 402
```



pmap() cont'd



pmap(list(x, y), f, na.rm = TRUE) is the same as map2(x, y, f, na.rm = TRUE).

```
pmap_dbl(list(xs, ws), weighted.mean)
#> [1] NA 0.467 0.470 0.638 0.516 0.550 0.471 0.524
pmap dbl(list(xs, ws), weighted.mean, na.rm = TRUE)
#> [1] 0.458 0.467 0.470 0.638 0.516 0.550 0.471 0.524
    In base R:
```

- Map():
 - Vectorises over all arguments
 - Cannot supply arguments that do not vary.
 - mapply():
 - Multidimensional version of sapply().

Outline

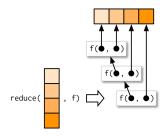


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Reduce family



- The next most important (family of) functionals.
 - Much smaller (two main variants).
 - Powers the map-reduce framework.
- purrr::reduce():
 - Takes a vector of length n.
 - Produces a vector of length 1 by calling a function with a pair of values at a time.
 - reduce(1:4, f) is equivalent to f(f(f(1, 2), 3), 4).





- Useful to generalize a function that works with two inputs to work with any number of inputs.
- Problem: find the values that occur in every element.

```
1 <- map(1:4, ~ sample(1:10, 15, replace = T))
str(1)
#> List of 4
#> $: int [1:15] 75729528105...
#> $: int [1:15] 9214713554...
#> $: int [1:15] 87516610921...
#> $: int [1:15] 10362756984...
```

Two solutions:

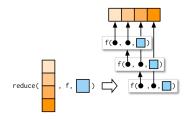
```
out <- 1[[1]]
out <- intersect(out, 1[[2]])
out <- intersect(out, 1[[3]])
out <- intersect(out, 1[[4]])
out
#> [1] 7 5 2 9 10 4 3
```

Reduce family cont'd



- Can also pass additional arguments.
- Simple implementation.

```
simple_reduce <- function(x, f, ...) {
  out <- x[[1]]
  for (i in seq(2, length(x))) {
    out <- f(out, x[[i]], ...)
  }
  out
}</pre>
```



- In base R:
 - Reduce().
 - The function comes first, followed by the vector.
 - No way to supply additional arguments.

Accumulate



```
accumulate(1, intersect)
#> [[1]]
#> [1] 7 5 7 2 9 5 2 8 10 5 4 2 3 6 5
#>
#> [[2]]
#> [1] 7 5 2 9 10 4 3
#>
#> [[3]]
#> [1] 7 5 2 9 10 4 3
#>
#> [[4]]
#> [1] 7 5 2 9 10 4 3
x < -c(4, 3, 10)
reduce(x, `+`)
#> [1] 17
reduce(x, +) == sum(x)
#> [1] TRUE
accumulate(x, `+`)
#> [1] 4 7 17
accumulate(x, `+`) == cumsum(x)
#> [1] TRUE TRUE TRUE
```

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A predicate:

- Function that returns a single TRUE or FALSE.
- ► E.g., is.character(), is.null(), or all().
- A predicate matches a vector if it returns TRUE.

A predicate functional:

- Applies a predicate to each element of a vector.
- 6 functions in 3 pairs.
- some(.x, .p)/every(.x, .p).
 - Returns TRUE if any/all element matches.
 - Similar to any(map_lgl(.x, .p))/all(map_lgl(.x, .p)).
 - But terminate early.
- detect(.x, .p)/detect_index(.x, .p).
 - Returns the *value/location* of the first match.
- keep(.x, .p)/discard(.x, .p).
 - Keeps/drops all matching elements.

Predicate functionals cont'd



```
df <- data.frame(x = 1:3, y = c("a", "b", "c"))
detect(df, is.factor)
#> [1] a b c
#> Levels: a b c
detect_index(df, is.factor)
#> [1] 2

str(keep(df, is.factor))
#> 'data.frame': 3 obs. of 1 variable:
#> $ y: Factor w/ 3 levels "a", "b", "c": 1 2 3
str(discard(df, is.factor))
#> 'data.frame': 3 obs. of 1 variable:
#> $ x: int 1 2 3
```

Map variants



```
df <- data.frame(</pre>
 num1 = c(0, 10, 20),
 num2 = c(5, 6, 7),
 chr1 = c("a", "b", "c"),
 stringsAsFactors = FALSE
str(map_if(df, is.numeric, mean))
#> List of 3
#> $ num1: num 10
#> $ num2: num 6
#> $ chr1: chr [1:3] "a" "b" "c"
str(modify_if(df, is.numeric, mean))
#> 'data.frame': 3 obs. of 3 variables:
#> $ num1: num 10 10 10
#> $ num2: num 6 6 6
#> $ chr1: chr "a" "b" "c"
str(map(keep(df, is.numeric), mean))
#> List of 2
#> $ num1: num 10
#> $ num2: num 6
```

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Base functionals



- Some base R functionals have no purrr equivalent:
 - Working with two-dimensional and higher vectors:
 - base::apply(): summarizes by collapsing rows/columns to a single value.
 - Mathematical tools:
 - integrate(): the area under the curve defined by f()
 - uniroot(): where f() hits zero
 - optimise(): the location of the lowest (or highest) value of f()



Summarizes by collapsing rows/columns to a single value.

```
a2d <- matrix(1:20, nrow = 5)

apply(a2d, 1, mean)

#> [1] 8.5 9.5 10.5 11.5 12.5

apply(a2d, 2, mean)

#> [1] 3 8 13 18
```

- Two caveats:
 - No control over the output type.
 - Doesn't work (well) with data frames.

```
df <- data.frame(x = 1:3, y = c("a", "b", "c"))
apply(df, 2, mean)
#> Warning in mean.default(newX[, i], ...): argument is not numeric or
#> logical: returning NA

#> Warning in mean.default(newX[, i], ...): argument is not numeric or
#> logical: returning NA
#> x y
#> NA NA
```

Mathematical tools



```
integrate(sin, 0, pi)
#> 2 with absolute error < 2.2e-14
str(uniroot(sin, pi * c(1 / 2, 3 / 2)))
#> List of 5
#> $ root : num 3.14
#> $ f.root : num 1.22e-16
#> $ iter : int 2
#> $ init.it : int NA
#> $ estim.prec: num 6.1e-05
str(optimise(sin, c(0, 2 * pi)))
#> List of 2
#> $ minimum : num 4.71
#> $ objective: num -1
str(optimise(sin, c(0, pi), maximum = TRUE))
#> List of 2
#> $ maximum : num 1.57
#> $ objective: num 1
```

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Function factories



- A **function factory** is a function that makes functions.
- Problem: replace all the missing values with NAs.

A first approach.

Function factories cont'd



What about this dataset?

```
df[1:2, 1] <- -999
df

#> X1 X2 X3 X4

#> 1 -999 6 6 5

#> 2 -999 -99 9 3

#> 3 10 5 10 -99

#> 4 2 4 -99 9
```

■ Naive approach.

```
fix_missing_99 <- function(x) {
    x[x == -99] <- NA
    x
}
fix_missing_999 <- function(x) {
    x[x == -999] <- NA
    x
}</pre>
```

Or using a function factory.

```
missing_fixer <- function(na_value) {
  function(x) {
    x[x == na_value] <- NA
    x
  }
}
fix_missing_99 <- missing_fixer(-99)
fix_missing_999 <- missing_fixer(-999)</pre>
```

More compelling uses for the concept in 2 weeks!



Another example.

```
power1 <- function(exp) {
   function(x) {
      x ^ exp
   }
}
square <- power1(2)
square(3)
#> [1] 9
cube <- power1(3)
cube(3)
#> [1] 27
```

■ What will the following code return?

```
x <- 2
square <- power1(x)
x <- 3
square(2)</pre>
```



Remember that R is lazy.

```
power1 <- function(exp) {</pre>
                                         power2 <- function(exp) {</pre>
  function(x) {
                                            force(exp)
                                            function(x) {
    x ^ exp
                                              x ^ exp
x < -2
                                         x < -2
square <- power1(x)
                                          square <- power2(x)
x < -3
                                         x < -3
square(2)
                                         square(2)
#> [1] 8
                                          #> [1] 4
```

■ Don't forget to force the evaluation with force()!

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Function operators



Functions that takes one (or more) functions as input and returns a function as output.

```
chatty <- function(f) {
  function(x, ...) {
    cat("Processing ", x, "\n", sep = "")
    f(x, ...)
  }
}

f <- function(x) x ^ 2
map_dbl(c(3, 2, 1), chatty(f))
#> Processing 3
#> Processing 2
#> Processing 1
#> [1] 9 4 1
```

- Closely related to function factories
 - ► They're just a function factory that takes a function as input.
 - Nothing you can't do without, powerful to factor out complexity.
- Typically paired with functionals.
- For Python users: decorators is just another name!

Dealing with failure using safely()



- The modified function always returns a list with two elements:
 - 1. result is the original result.
 - 2. error is an error object.

```
safe_log <- safely(log)
str(safe_log(10))
#> List of 2
#> $ result: num 2.3
#> $ error : NULL
str(safe_log("a"))
#> List of 2
#> $ result: NULL
#> $ error :List of 2
#> ..$ message: chr "non-numeric argument to mathematical function"
#> ..$ call : language .Primitive("log")(x, base)
#> ..- attr(*, "class")= chr [1:3] "simpleError" "error" "condition"
```



```
x <- list(1, 10, "a")
y <- map(x, safely(log))
str(y)
#> List of 3
#> $ :List of 2
#> ..$ result: num 0
#> ..$ error : NULL
#> $ :List of 2
#> ...$ result: num 2.3
#> ..$ error : NULL
#> $ :List of 2
#> ..$ result: NULL
#> ..$ error :List of 2
    .... $ message: chr "non-numeric argument to mathematical function"
#>
#>
    ....$ call : language .Primitive("log")(x, base)
    ... - attr(*, "class")= chr [1:3] "simpleError" "error" "condit"...
#>
```

transpose()



```
y <- transpose(y)</pre>
str(y)
#> List of 2
#> $ result:List of 3
#> ..$ : num 0
#> ..$ : num 2.3
#> ..$ : NULL
#> $ error :List of 3
#> ..$ : NULL
#> ..$ : NULL
    ..$ :List of 2
#>
    .... $ message: chr "non-numeric argument to mathematical function"
    ....$ call : language .Primitive("log")(x, base)
#>
#>
     ....- attr(*, "class")= chr [1:3] "simpleError" "error" "condit"..
```







Typical use



```
is_ok <- map_lgl(y$error, is_null)
x[!is_ok]
#> [[1]]
#> [1] "a"
flatten_dbl(y$result[is_ok])
#> [1] 0.0 2.3
```



possibly(): "simpler" than safely(), because you give it a default value to return when there is an error.

```
map_dbl(x, possibly(log, NA_real_))
#> [1] 0.0 2.3 NA
```

quietly(): instead of capturing errors, it captures printed output, messages, and warnings.

```
map(list(1, -1), quietly(log)) %>% str()
#> List of 2
#> $:List of 4
#> ..$ result : num 0
#> ..$ output : chr ""
#> ..$ warnings: chr(0)
#> ..$ messages: chr(0)
#> $:List of 4
#> ..$ result : num NaN
#> ..$ output : chr ""
#> ..$ warnings: chr "NaNs produced"
#> ..$ messages: chr(0)
```

Caching computations with memoise() © COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK

- Provided by the memoise package.
- Memoises a function
 - The function remembers previous inputs/returns cached results.
 - Classic CS tradeoff of memory versus speed:
 - A memoised function is faster, but uses more memory.

```
library(memoise)
fast_fct <- memoise(slow_fct)

system.time(print(fast_fct(1)))
#> [1] 9.94
#> user system elapsed
#> 0.001 0.000 1.002
system.time(print(fast_fct(1)))
#> [1] 9.94
#> user system elapsed
#> 0.043 0.000 0.044
```

Fibonacci series



- Defined recursively:
 - f(0) = 0, f(1) = 1,
 - And then f(n) = f(n-1) + f(n-2).

```
fib <- function(n) {
                                      fib2 <- memoise(function(n) {
 if (n < 2) return(1)
                                        if (n < 2) return(1)
 fib(n-2) + fib(n-1)
                                        fib2(n - 2) + fib2(n - 1)
                                      })
system.time(fib(23))
                                      system.time(fib2(23))
     user system elapsed
                                      #> user system elapsed
                                      #> 0.073 0.000 0.073
    0.089 0.020 0.109
system.time(fib(24))
                                      system.time(fib2(24))
#>
   user system elapsed
                                      #> user system elapsed
#>
   0.159 0.000
                   0.159
                                          0.002 0.000 0.002
```

- An example of **dynamic programming**:
 - Complex problem broken down into overlapping subproblems.
 - ▶ Remembering the results of a subproblem considerably improves performance.