Table of Contents

1. Prelude

In [1]:
```
import matplotlib.pyplot as plt
import pulp
import math
import random
import pandas as pd
import numpy as np
import time
import simpy
```

2. Utilities (as before)

2.1. Points and Distances

In [2]:
```
def dist(p1, p2):
    (x1, y1) = p1
    (x2, y2) = p2
    return int(math.sqrt((x1-x2)**2+(y1-y2)**2))
```

2.2 PlotMap

In [3]:
```
def plotMap(G, T=[], P=[], W=None,
            style='r-o', lw=1, ms=3,
            styleT='go', msT=5,
            styleP='b-o', lwP=3, msP=1,
            stylePT='go', msPT=7,
            styleW='bo', msW=9,
            text=None, grid=False):
    fig = plt.gcf()
    fig.set_size_inches(6, 6)
    V, E = G

    if not grid:
        plt.axis('off')
    plt.plot( [ p[0] for p in V ], [ p[1] for p in V ], 'ro', lw=lw, ms=ms)
    for (p, q) in E:
        plt.plot( [ p[0], q[0] ], [ p[1], q[1] ], 'r-o', lw=lw, ms=ms)
    for t in T:
        plt.plot( [ t[0] ], [ t[1] ],
                  styleT, ms=msT)
    plt.plot( [ p[0] for p in P ],
              [ p[1] for p in P ],
              styleP, lw=lwP, ms=msP)
    for p in P:
        if p in T:
            plt.plot( [ p[0] ], [ p[1] ],
                      stylePT, ms=msPT)
    if W is not None:
        plt.plot( [ W[0] ], [ W[1] ],
                  styleW, ms=msW)
    if text is not None:
        maxX = max([p[0] for p in V])
        plt.text(0.8*maxX, 0, text)
    if grid:
        plt.grid()
    plt.show()
```

2.3. Add Targets

In [4]:
```
def addTargets(M, T):
    V, E = M
    E = E.copy()
    V = V.copy()
    for t in T:
        minD = math.inf
        minE = None
        for e in E:
            P, Q = e
            distT = dist(P, t)+dist(t, Q)-dist(P, Q)
            if distT < minD:
                minD = distT
                minE = e
        P, Q = minE
        E.remove( (P, Q) )
        E.append( (P, t) )
```

```
        E.append( (t, Q) )
        V.append(t)
    return V, E
```

2.4. Generate Warehouse Location

In [5]:
```
def generateWarehouseLocation(M):
    V, _ = M
    W = random.sample(V, k=1)[0]
    return W
```

2.5.Time Handling

Convention: In this project we measure time in seconds. The simulation will start at 0:00. Time related methods will be added as they are needed.

`timestamp(t)` generates a timestamp string in the form `[dd] hh:mm:ss.d`

In [6]:
```
def timestamp(t):
        day = int(t)//(24*3600)
        t = t - day*24*3600
        hour = int(t)//3600
        t = t - hour*3600
        mins = int(t)//60
        t = t - mins*60
        secs = int(math.floor(t))
        t = int(math.floor((t - secs)*10))
        return f"[{day:2d}] {hour:02d}:{mins:02d}:{secs:02d}.{t:1d}"
```

In [7]:
```
timestamp(24*3600*3+17*3600+615.1)
```

Out[7]: '[3] 17:10:15.0'

In [8]:
```
def nextHour(env, hour):
    beginningOfDay = int(env.now//(24*3600))*24*3600
    timeOfDay = env.now-beginningOfDay
    if hour*3600 > timeOfDay:
        return hour*3600 - timeOfDay
    else:
        return hour*3600 + 24*3600 - timeOfDay
```

In [9]:
```
def day(now):
    return int(now//(24*3600))
```

2.6. Plotting Routines

In [10]:
```
import scipy.stats as stats

def histplot(data, title="", xlabel="",
```

```
             width=None, height=None):

    minx = min(data)
    maxx = max(data)
    μ = np.mean(data)
    σ = np.std(data)

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 4)
    fig.set_figheight(height if height is not None else 2.5)
    ax = fig.gca()

    hist=plt.hist(data, density=True)
    plt.xlabel(xlabel)
    plt.ylabel('Density')
    plt.title(title)

    x = np.linspace(minx, maxx, 100)
    y = [ stats.norm(loc=μ, scale=σ).pdf(p) for p in x]
    ax.plot(x, y, lw=1, color='red')
    ax.axvline(x=μ, color='red')
    maxy = max(max(y), max(hist[0]))
    ax.text(maxx, maxy,
            f'μ={μ:2.2f}\nσ={σ:2.2f}',
            ha='right', va='top',
            color='red', fontsize=12)
    ax.grid(True)
    plt.show()
```

In [11]:
```
def dailyPlot(data,
              title="", ylabel="",
              width=None, height=None):

    days = len(data)

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 6)
    fig.set_figheight(height if height is not None else 2)

    ax = fig.gca()
    diff = (max(data)-min(data))*0.1
    ymin = int(math.floor(min(data)-diff))
    ymax = int(math.ceil(max(data)+diff))
    ax.set_xlim(-1, days)
    ax.set_ylim(ymin, ymax)
    ax.grid(True)

    ms = 2 if len(data)>100 else 5
    lw = 0.5 if len(data)>100 else 1

    x = np.arange(0, len(data))
    y = np.array([ y for y in data ])
    b, m = np.polynomial.polynomial.polyfit(x, y, 1)

    plt.plot(x, y, 'bo-', linewidth=lw, markersize=ms)
    plt.plot(x, m*x+b, 'r-')
```

```
    plt.xlabel('Day')
    plt.ylabel(ylabel)
    plt.title(title)
    plt.show()
```

3. Finding Shortest Path (as before)

In [12]:
```
def dist(p1, p2):
    (x1, y1) = p1
    (x2, y2) = p2
    return int(math.sqrt((x1-x2)**2+(y1-y2)**2))
```

In [13]:
```
def pathLength(P):
    return 0 if len(P)<=1 else \
            dist(P[0], P[1])+pathLength(P[1:])
```

In [14]:
```
def shortestPath(M, A, B):

    def h(p):
        return pathLength(p)+dist(p[-1],B)

    # candidates C are pairs of the path so far and
    # the heuristic function of that path,
    # sorted by the heuristic function, as maintained by
    # insert function
    def insert(C, p):
        hp = h(p)
        c = (p, hp)
        for i in range(len(C)):
            if C[i][1]>hp:
                return C[:i]+[c]+C[i:]
        return C+[c]

    V, E = M
    assert(A in V and B in V)
    C = insert([], [A])

    while len(C)>0:
        # take the first candidate out of the list of candidates
        path, _ = C[0]
        C = C[1:]
        if path[-1]==B:
            return path
        else:
            for (x, y) in E:
                if path[-1]==x and y not in path:
                    C = insert(C, path+[y])
                elif path[-1]==y and x not in path:
                    C = insert(C, path+[x])
    return None
```

4. Finding Shortest Delivery Route (as before)

4.1. Iterative Integer Programming

In [15]:
```
def createTables(M, T):

    def reverse(P):
        return [ P[-i] for i in range(1,len(P)+1) ]

    def index(x, L):
        for i in range(len(L)):
            if x==L[i]:
                return i
        return None

    n = len(T)
    d = [ [ math.inf for t in T ] for t in T ]
    p = [ [ None for t in T ] for t in T ]
    for i in range(n):
        d[i][i] = 0
        p[i][i] = [ T[i] ]
    for i in range(n):
        for j in range(n):
            if p[i][j] is None:
                s = shortestPath(M, T[i], T[j])
                d[i][j] = d[j][i] = pathLength(s)
                p[i][j] = s
                p[j][i] = reverse(s)
                for m in range(len(s)-1):
                    smi = index(s[m], T)
                    if smi is None:
                        continue
                    for l in range(m+1, len(s)):
                        sli = index(s[l], T)
                        if sli is None:
                            continue
                        sub = s[m:l+1]
                        if p[smi][sli] is None:
                            p[smi][sli] = sub
                            p[sli][smi] = reverse(sub)
                            d[smi][sli] = d[sli][smi] = pathLength(sub)
    return d,p
```

In [16]:
```
def roundtrips(x, n):

    def isElem(x, l):
        for i in range(len(l)):
            if l[i]==x:
                return True
        return False
```

```
    def startpoint(trips):
        for i in range(n):
            for t in trips:
                if isElem(i, t):
                    break
            else:
                return i

    def totalLength(trips):
        s=0
        for i in range(0, len(trips)):
            s += len(trips[i])-1
        return s

    trips = []
    while totalLength(trips)<n:
        start = startpoint(trips)
        trip = [ start ]
        i = start
        while len(trip) < n-totalLength(trips):
            for j in range(0, n):
                if pulp.value(x[i][j])==1:
                    trip.append(j)
                    i=j
                    break
            if pulp.value(x[trip[-1]][start])==1:
                trip.append(start)
                break
        trips.append(trip)
    return sorted(trips, key=lambda t: len(t), reverse=True)
```

In [17]:
```
import time

def createLoop(M, T, timing=False):

    if timing:
        start_time = time.time()
        last_time = time.time()

    D, P = createTables(M, T)   # These are the distances between customers and war

    if timing:
        print(f"createTables:   {time.time()-start_time:6.2f}s")
        last_time = time.time()

    n = len(T)

    if n==1:
        return T

    # create variables
    x = pulp.LpVariable.dicts("x", ( range(n), range(n) ),
                              lowBound=0, upBound=1, cat=pulp.LpInteger)
    # create problem
    prob = pulp.LpProblem("Loop",pulp.LpMinimize)
```

```
    # add objective function
    prob += pulp.lpSum([ D[i][j]*x[i][j]
                              for i in range(n) for j in range(n) ])
    # add constraints
    constraints=0
    for j in range(n):
        prob += pulp.lpSum([ x[i][j] for i in range(n) if i!=j ]) ==1
    constraints += n
    for i in range(n):
        prob += pulp.lpSum([ x[i][j] for j in range(n) if i!=j ]) ==1
    constraints += n
    for i in range(n):
        for j in range(n):
            if i!=j:
                prob += x[i][j]+x[j][i] <= 1
                constraints += 1
    # initialise solver
    solvers = pulp.listSolvers(onlyAvailable=True)
    solver = pulp.getSolver(solvers[0], msg=0)
    prob.solve(solver)

    if timing:
        print(f"Solver:         {time.time()-last_time:6.2f}s {constraints:6,d} Con
        last_time = time.time()

    trips = roundtrips(x, n)
    while len(trips)>1:
        longest = max([ len(t) for t in trips ])
        for t in trips:
            if len(t)<longest:
                prob += pulp.lpSum([ x[t[i]][t[i+1]] + x[t[i+1]][t[i]]
                                        for i in range(0,len(t)-1) ]) <= len(t)-2
                constraints += 1
            else:
                longest = math.inf
        prob.solve(solver)

        if timing:
            print(f"Solver:         {time.time()-last_time:6.2f}s {constraints:6,d}
            last_time = time.time()

        trips = roundtrips(x, n)

    trip = trips[0]
    loop = []
    for k in range(len(trip)-1):
        sub = P[trip[k]][trip[k+1]]
        loop += sub if len(loop)==0 else sub[1:]

    if timing:
        print(f"createLoop:     {time.time()-start_time:6.2f}s")

    return loop
```

4.2. Heuristic Algorithm

In [18]:
```
def FW(M):

    V, E = M

    n = len(V)
    d = [ [ math.inf for j in range(n) ] for i in range(n) ]
    p = [ [ None for j in range(n) ] for i in range(n) ]

    for (A, B) in E:
        a = V.index(A)
        b = V.index(B)
        d[a][b] = d[b][a] = dist(A, B)
        p[a][b] = [A, B]
        p[b][a] = [B, A]

    for i in range(n):
        d[i][i] = 0
        p[i][i] = [V[i]]

    for k in range(n):
        for i in range(n):
            for j in range(n):
                dk = d[i][k] + d[k][j]
                if d[i][j] > dk:
                    d[i][j] = dk
                    p[i][j] = p[i][k][:-1] + p[k][j]

    return d, p
```

In [19]:
```
def createLoopH(M, T, timing=False):

    def makeLoop(L):
        loop = []
        for i in range(len(L)-1):
            A = L[i]
            B = L[i+1]
            a = V.index(A)
            b = V.index(B)
            sub = P[a][b]
            loop += sub if len(loop)==0 else sub[1:]
        return loop

    if timing:
        start_time = time.time()
        last_time = time.time()

    V, E = M
    D, P = FW(M)   # note these are the distances between all vertices in M (and T)

    if timing:
        print(f"createTables:   {time.time()-start_time:6.2f}s")
        last_time = time.time()

    W = T[0]
```

```
    customers = T[1:]
    if len(T)==1:
        L = T
    elif len(T)<=3:
        L = T + [T[0]]
    else:
        L = T[:3]+[T[0]]
        T = T[3:]
        while len(T)>0:
            minExt = math.inf
            minInd = None
            selInd = None
            for k in range(len(T)):
                C = T[k]
                c = V.index(C)
                for i in range(0, len(L)-1):
                    A = L[i]
                    B = L[i+1]
                    a = V.index(A)
                    b = V.index(B)
                    ext = D[a][c] + D[c][b] - D[a][b]
                    if ext<minExt:
                        minExt, minInd, selInd = ext, i+1, k
            L = L[:minInd]+[T[selInd]]+L[minInd:]
            T = T[:selInd]+T[selInd+1:]

    if timing:
        print(f"createLoopH:    {time.time()-start_time:6.2f}s")

    return makeLoop(L)

def shortcut2(roundtrip):
    #Attempt to shorten the route by reversing segments of the route.
    n = len(roundtrip)
    best_route = roundtrip[:]
    for i in range(n - 1):
        for j in range(i + 2, n):  # ensure at least one node between i and j
            new_route = roundtrip[:i+1] + list(reversed(roundtrip[i+1:j+1])) + roun
            if calculate_total_distance(new_route) < calculate_total_distance(best_
                best_route = new_route
    return best_route

def shortcut3(roundtrip):
    #Attempt to improve the route by repositioning nodes.
    n = len(roundtrip)
    best_route = roundtrip[:]
    for i in range(1, n - 1):
        for j in range(n):
            if j != i and j != i + 1:  # Prevents index errors and unnecessary swap
                new_route = roundtrip[:i] + roundtrip[i+1:]
                new_route.insert(j, roundtrip[i])
                if calculate_total_distance(new_route) < calculate_total_distance(b
                    best_route = new_route
    return best_route

def calculate_total_distance(route):
```

```
    #Calculate the total distance of a route using the dist function.
    return sum(dist(route[i], route[i + 1]) for i in range(len(route) - 1))
```

5. Class Recorder

We willl use a class Recorder as a reference point for capturing data during the simulation. There will be only one recorder. It will be created at the beginning of every simulation run. Every entity will carry a reference to the Recorder.

In [20]:
```
class Recorder:

    def __init__(self, env, M, W, C, days,
                 log=False, plot=False, timing=False):
        self.env = env
        self.M = M
        self.W = W
        self.C = C
        self.days = days
        self.log = log
        self.plot = plot

        self.timing = timing
        self.start_time = time.time()
        self.last_time = self.start_time
        self.cum_timer = {}

        Customer.REGISTER = []
        Parcel.REGISTER = []

        # create a data frame for records per working day
        self.daily = pd.DataFrame()
        self.daily['begin work at'] = [None]*days
        self.daily['end work at'] = [None]*days
        self.daily['dist'] = [None]*days
        self.daily['left'] = [None]*days

    def timer(self, s):
        t = time.time()
        Δt = t-self.last_time
        if self.timing:
            print(f"==== t: {t-self.start_time:6.2f}s "
                  f"Δt: {Δt:6.2f}s [{s:s}]")
        if s in self.cum_timer:
            self.cum_timer[s] += Δt
        else:
            self.cum_timer[s] = Δt
        self.last_time = t

    def reportTimer(self):
        print(f"==== t: {self.total_time:6.2f}s Total")
        for k in sorted(self.cum_timer, key=lambda x: self.cum_timer[x], reverse=Tr
            print(f"==== ΣΔt: {self.cum_timer[k]:6.2f}s "+ k)
```

```
    def trace(self, event):
        if self.log:
            print(timestamp(self.env.now), event)

    def recordDriverBeginsWork(self):
        self.trace("Driver arrives for work")
        self.daily.at[day(self.env.now), 'begin work at'] = int(round(self.env.now)

    def recordDriverEndsWork(self):
        self.trace("Driver goes home")
        self.daily.at[day(self.env.now), 'end work at'] = int(round(self.env.now))

    def recordTourLength(self, length):
        self.daily.at[day(self.env.now), 'dist'] = int(length)

    def recordParcelsLeftOver(self, numberOfParcels):
        self.trace(f"{numberOfParcels:d} left over for next day")
        self.daily.at[day(self.env.now), 'left'] = numberOfParcels

    def finish(self):
        # simulation is finished for good
        # by removing the simulation environment we can
        # pickle recorder
        self.env = None
        self.total_time = time.time()-self.start_time
        self.daily['working time'] = (self.daily['end work at']-self.daily['begin w

    def histWorkingTime(self):
        histplot(self.daily['working time'],
                 xlabel='Working Time [min]',
                 title='Daily Working Time')

    def plotWorkingTime(self):
        dailyPlot(self.daily['working time'],
                  ylabel='Working Time [min]',
                  title='Daily Working Time')

    def histTourLength(self):
        histplot(self.daily['dist'],
                 xlabel='Tour Length [m]',
                 title='Daily Tour Length')

    def plotTourLength(self):
        dailyPlot(self.daily['dist'],
                  ylabel='Tour Length [m]',
                  title='Daily Tour Length')

    def histLeftOver(self):
        histplot(self.daily['left'],
                 xlabel='Left-Over Parcels',
                 title='Daily Left-Over Parcels')

    def plotLeftOver(self):
        dailyPlot(self.daily['left'],
                  ylabel='Number of Parcels',
                  title='Daily Left-Over Parcels')
```

6. Class Parcel

No description has been provided for this image

Parcels follow through a sequence of states:

- processing
- in transit (from manufacture to distribution centre)
- arrived in distribution centre
- ready for delivery
- out for delivery
- customer not present
- returned to distribution centre
- delivered

In [21]:
```
class Parcel:

    REGISTER = []

    def __init__(self, rec, i, cust, custIndex):
        self.rec = rec
        self.i = i  # row index in data frames of input data
```

```
        self.dest = cust.location
        self.custIndex = custIndex
        self.status = [ 'processing' ] # status record and
        self.timing = [ self.rec.env.now ]      # timing
        assert(len(Parcel.REGISTER)==i)
        Parcel.REGISTER += [ self ]

    # factory method ensures that there is only
    # one Parcel per location
    def getParcel(rec, i, location, custIndex):
        for p in Parcel.REGISTER:
            if p.i == i:
                return p
        return Parcel(rec, i, location, custIndex)


    def __str__(self):
        return f"Parcel: {self.i:3d} ({self.custIndex:3d})"

    def index(self):
        return self.i

    def destination(self):
        return self.dest

    def __reg(self, state):
        self.status += [ state ]
        self.timing += [ self.rec.env.now ]
        self.rec.trace(str(self)+" "+state)

    def arrivedAtDeliveryCentre(self):
        self.__reg('arr at delivery centre')

    def outForDelivery(self):
        self.__reg('out for delivery')

    def returnFromDelivery(self):
        self.__reg('return from delivery')
```

7. Class Customer

No description has been provided for this image

In [22]:
```
class Customer:

    REGISTER = []

    def __init__(self, rec, location):
        self.rec = rec
        self.location = location
        self.i = len(Customer.REGISTER)
        Customer.REGISTER += [ self ]
        self.atHome = True
        self.answersDoor = False
        self.parcelsReceived = []
        rec.env.process(self.process())

    def __str__(self):
        return f"Customer: {self.i:2d} {str(self.location):s}"

    # factory method ensures that there is only
    # one customer per location
    def getCustomer(rec, location):
        for c in Customer.REGISTER:
            if c.location == location:
                return c
        return Customer(rec, location)
```

```
    def leaveHouse(self):
        assert(self.atHome and not self.answersDoor)
        # self.rec.trace(str(self)+" leaves house")
        self.atHome = False

    def returnHome(self):
        assert(not self.atHome)
        # self.rec.trace(str(self)+" returns home")
        self.atHome = True

    def answerDoor(self):
        if self.atHome:
            yield self.rec.env.timeout(random.expovariate(1/AVERAGE_TIME_ANSWER_DOO
            self.rec.trace(str(self)+" answers door")
            self.answersDoor = True
        else:
            yield self.rec.env.timeout(WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR)
            self.rec(str(self)+" not at home")

    def acceptParcel(self, parcel):
        assert(self.answersDoor)
        self.parcelsReceived += [parcel]
        self.rec.trace(str(self)+" accepts "+str(parcel))

    def signOff(self):
        assert(self.answersDoor)
        self.rec.trace(str(self)+" signs off")
        self.answersDoor = False

    def process(self):
        yield self.rec.env.timeout(nextHour(self.rec.env, 8))
        while day(self.rec.env.now)<self.rec.days:
            # in a refinement we may use random times
            self.leaveHouse()
            yield self.rec.env.timeout(nextHour(self.rec.env, 18))
            self.returnHome()
            yield self.rec.env.timeout(nextHour(self.rec.env, 8))
```

8. Class Driver

No description has been provided for this image

In [23]:
```
class Driver:

    def __init__(self, rec, DC):
        self.rec = rec
        self.DC = DC
        self.location = None
        self.parcels = None
        self.tour = None
        self.rec.env.process(self.process())

    # activity
    def __drive(self, target):
        assert(self.tour[0] == self.location)
        while self.location!=target:
            d = dist(self.location, self.tour[1])
            yield self.rec.env.timeout(d / AVERAGE_SPEED)
            self.location = self.tour[1]
            self.tour = self.tour[1:]
        assert(self.tour[0] == self.location == target)

    def arriveForWork(self):
```

```
        self.location = self.DC.W
        self.parcels = []
        self.returns = []
        self.tour = [ self.DC.W ]
        self.rec.recordDriverBeginsWork()

    def leaveForDelivery(self, tour, parcels):
        self.tour, self.parcels = tour, parcels
        self.rec.trace(f"Driver leaves for delivery " \
                       f"of {len(parcels):d} parcels")

    def process(self):
        yield self.rec.env.timeout(nextHour(self.rec.env, 18))
        while day(self.rec.env.now)<self.rec.days:
            self.arriveForWork()
            tour, parcels = self.DC.sendForDelivery()
            yield self.rec.env.timeout(PREP_TIME_PER_PARCEL*len(parcels))
            self.rec.recordTourLength(pathLength(tour))
            self.leaveForDelivery(tour, parcels)
            while len(self.parcels)>0:
                # drive to customer
                custLocation = self.parcels[0].dest
                cust = Customer.getCustomer(self.rec, custLocation)
                self.rec.trace("Driver drives to "+str(cust))
                yield from self.__drive(custLocation)
                self.rec.trace("Driver arrived at "+str(cust))
                # call at customer
                yield from cust.answerDoor()

                if cust.answersDoor:
                    while len(self.parcels)>0 and \
                            custLocation == self.parcels[0].dest:
                        cust.acceptParcel(self.parcels[0])
                        yield self.rec.env.timeout(random.expovariate(1/10))
                        self.parcels = self.parcels[1:]
                    cust.signOff()
                    yield self.rec.env.timeout(random.expovariate(1/10))
                else:
                    while len(self.parcels)>0 and  \
                            custLocation == self.parcels[0].dest:
                        self.returns += self.parcels[0]
                        self.parcels = self.parcels[1:]

            # return to delivery centre
            self.rec.trace("Driver returns to delivery centre")
            yield from self.__drive(self.DC.W)
            self.rec.trace("Driver arrived at delivery centre")

            for parcel in self.returns:
                self.DC.returnFromDelivery(parcel)
                yield self.rec.env.timeout(RETURN_TIME_PER_PARCEL)
            yield self.rec.env.timeout(600)

            self.rec.recordParcelsLeftOver(len(self.DC.parcels)+
                                           len(self.DC.leftOver))
```

```
            self.rec.recordDriverEndsWork()

            yield self.rec.env.timeout(nextHour(self.rec.env, 18))
```

9. Class Delivery Centre

No description has been provided for this image

In [24]:
```
class DeliveryCentre:

    def __init__(self, rec, M, W):
        self.rec = rec
        self.M = M
        self.W = W
        self.limit = 40000

        self.leftOver = []    # list of parcels
        self.parcels = []     # list of parcels scheduled for delivery
        self.dest = []        # list of unique customer destinations
        self.tour = None      # tour planned for delivery
```

```
    def __accept(self, parcel):
        custLoc = parcel.dest
        if custLoc not in self.dest:
            MT = addTargets(self.M, self.dest + [custLoc])
            self.rec.timer("addTarget")
            SH = createLoopH(MT, [self.W] + self.dest + [custLoc],
                             timing=self.rec.timing)
            self.rec.timer("createLoopH")
            if self.tour is None and pathLength(SH)<self.limit:
                self.parcels.append(parcel)
                self.dest += [custLoc]
            else:
                S = createLoop(MT, [self.W] + self.dest + [custLoc],
                               timing=self.rec.timing)
                self.rec.timer("createLoop")
                if pathLength(S)<self.limit:
                    self.parcels.append(parcel)
                    self.dest += [custLoc]
                    self.tour = S
                else:
                    self.leftOver.append(parcel)
        else:
            self.parcels.append(parcel)

    def acceptParcel(self, parcel):
        parcel.arrivedAtDeliveryCentre()
        self.__accept(parcel)

    def sendForDelivery(self):
        parcels = []
        if self.tour is None:
            MT = addTargets(self.M, self.dest)
            self.rec.timer("addTarget")
            self.tour = createLoop(MT, [self.W] + self.dest,
                                   timing=self.rec.timing)
            self.rec.timer("createLoop")
        tour = self.tour
        addresses = self.dest

        # pick parcels in sequence to be delivered
        for i in range(1, len(tour)-1):
            dest = tour[i]
            for p in self.parcels:
                if p.dest == dest and p not in parcels:
                    parcels += [p]
                    p.outForDelivery()

        # arrange the left overs
        L = self.leftOver
        self.tour = None
        self.parcels = []
        self.leftOver = []
        self.dest = []
        for p in L:
            self.__accept(p)
```

```
        if self.rec.plot:
            plotMap(self.rec.M, T=addresses, P=tour, W=tour[0],
                    text=f"Day {day(self.rec.env.now):2d}, {pathLength(tour):,d}m")

        return tour, parcels

    def returnFromDelivery(self, parcel):
        parcel.returnFromDelivery()
        self.__accept(parcel)

    def getInventory(self):
        return len(self.parcels)+len(self.leftOver)
```

10. Simulation

10.1 Parameters from Specification

The time required for driving is based on the distance between way points at an average speed of 15km/h.

In [25]:
```
AVERAGE_SPEED = 15/3.6
```

The **cumulative preparation time** (route planning and sorting of the parcels in the delivery order and packing the cargo-bike) is assumed to be 50 sec per parcel to be delivered.

In [26]:
```
PREP_TIME_PER_PARCEL = 50
```

Additional assumption: The time to **process returned parcels** in the delivery centre is 30 sec per parcel.

In [27]:
```
RETURN_TIME_PER_PARCEL = 30
```

The average time to answer the door.

In [28]:
```
AVERAGE_TIME_ANSWER_DOOR = 40
```

In [29]:
```
WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR = 60
```

10.2. Generate Input Data

In [30]:
```
def generateDeliveryData(p, C, days, seed=0):
    ## p is the average number of parcels per day per customer
    ## C is the number of customers to be served
    ## days is the number of days for which data are to be generated.
    np.random.seed(seed)
    R = np.random.poisson(lam=len(C)*p, size=days)
```

```
    D = [ sorted(list(np.random.choice(range(len(C)), size=i))) for i in R ]
    return D
```

In [31]:
```
def generateInputData(D, log=False):

    R = [ len(d) for d in D ]
    N = sum(R)

    DAY_LENGTH = 24*3600    # measured in minutes
    DAY_START = 8*3600      # first delivery in the morning
    DAY_END = 17*3600     # last delivery during day time

    x = pd.DataFrame()

    x['iarr'] = [None]*N
    x['time'] = [None]*N
    x['day'] = [None]*N
    x['dest'] = [None]*N

    current_day = 0
    last_time = 0
    i = 0

    for d in D: # for each day

        if log:
            print("generating for day: ",current_day, D[current_day])

        time = current_day*DAY_LENGTH + DAY_START

        for c in d: # for each customer that should get a

            IARR = (DAY_END-DAY_START-2*3600) / len(d)  # estimated average IAT for

            iat = random.expovariate(1.0/IARR)
            new_time = time + iat

            x.at[i, 'iarr'] = round(new_time - last_time,1)
            x.at[i, 'time'] = round(new_time - current_day*DAY_LENGTH , 1)
            x.at[i, 'day'] = current_day
            x.at[i, 'dest'] = c

            i += 1
            last_time = time = new_time

        current_day += 1

    return x
```

10.3. Simulation Routine

In [32]:
```
def simulation(M, W, C, p=0.15, days=25, seed=5640, log=False, plot=False, timing=F

    if timing:
```

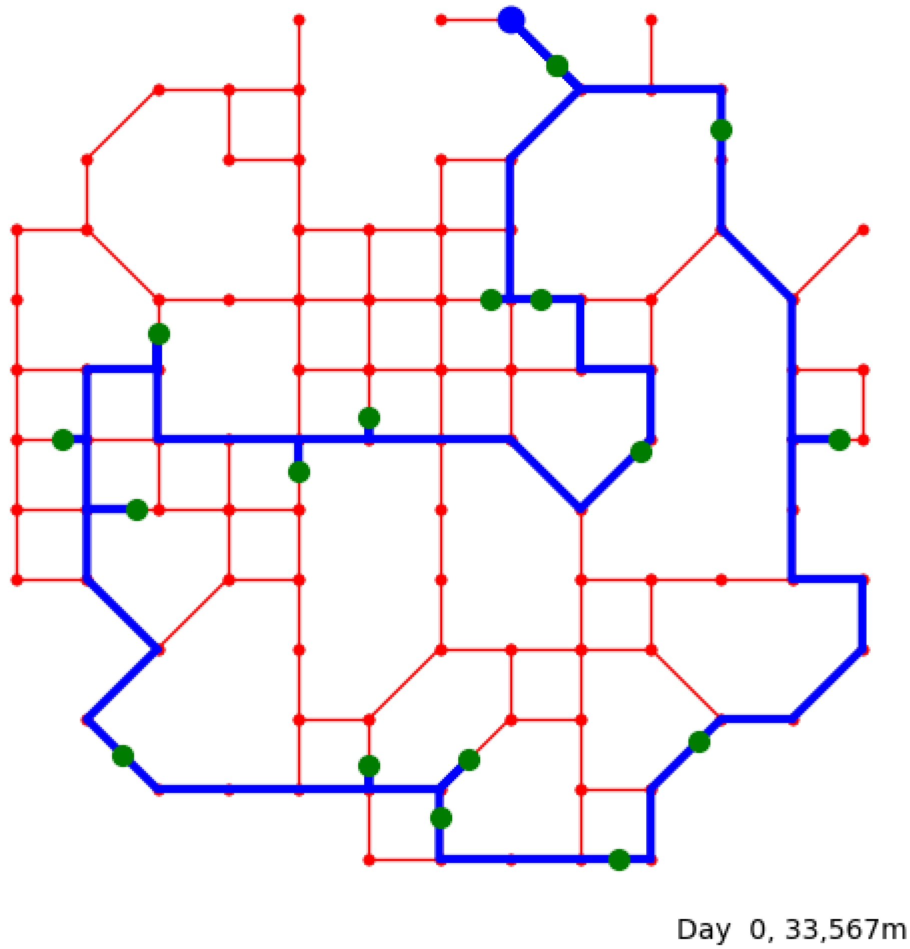
```
        start_time = time.time()

    random.seed(seed)
    D = generateDeliveryData(p, C, days, seed)
    X = generateInputData(D, log=log)

    env = simpy.Environment()
    rec = Recorder(env, M, W, C, days, log=log, plot=plot, timing=timing)

    print(f"Simulating delivery of {len(X):d} parcels "
          f"over {len(D):d} days to {len(C):d} customers")

    for c in C:
        Customer.getCustomer(rec, c)
    DC = DeliveryCentre(rec, M, W)
    D = Driver(rec, DC)

    def generatorProcess(env):
        # generate the parcels based on input data x
        for i in range(len(X)):
            yield env.timeout(X.at[i, 'iarr'])
            custIndex = X.at[i, 'dest']
            custLoc = C[custIndex]
            cust = Customer.getCustomer(rec, custLoc)
            p = Parcel.getParcel(rec, i, cust, custIndex)
            DC.acceptParcel(p)

    env.process(generatorProcess(env))
    env.run()

    rec.finish()

    if log:
        print(f"Delivery Centre Inventory: {DC.getInventory():d} parcels")

    return rec
```

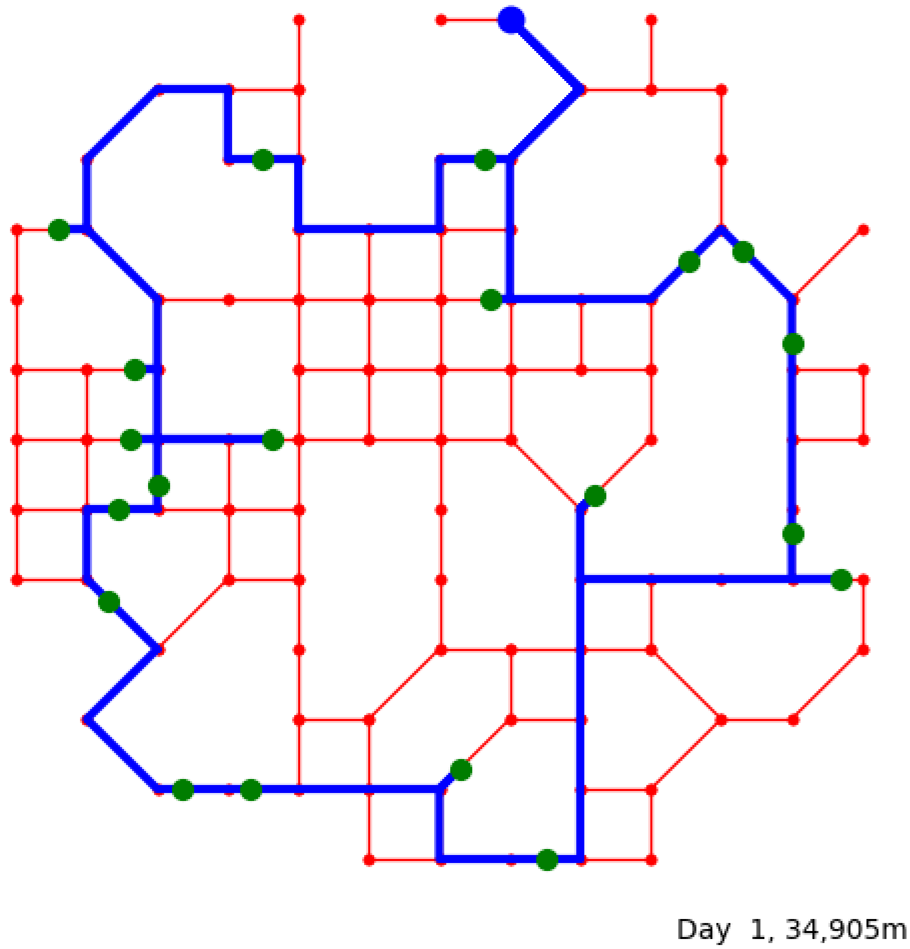
10.4. Small Simulation Run

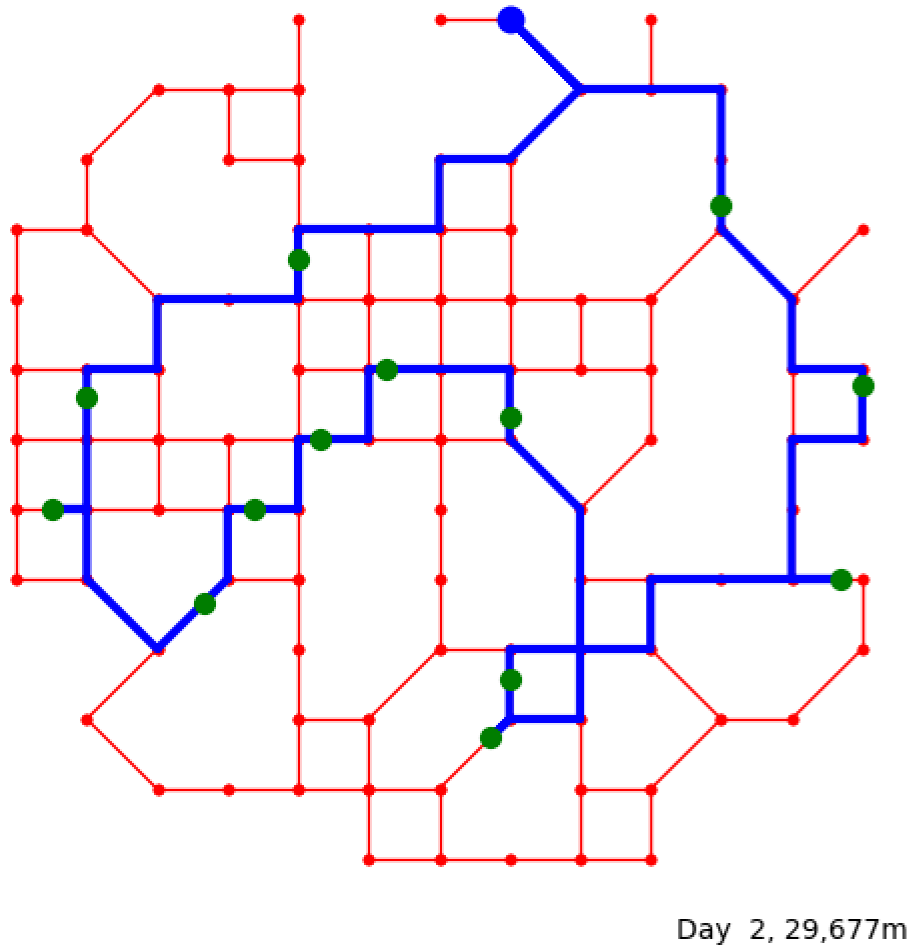
In [33]:
```
import pickle
with open('data.pickled', 'rb') as f:
    M, C = pickle.load(f)
```

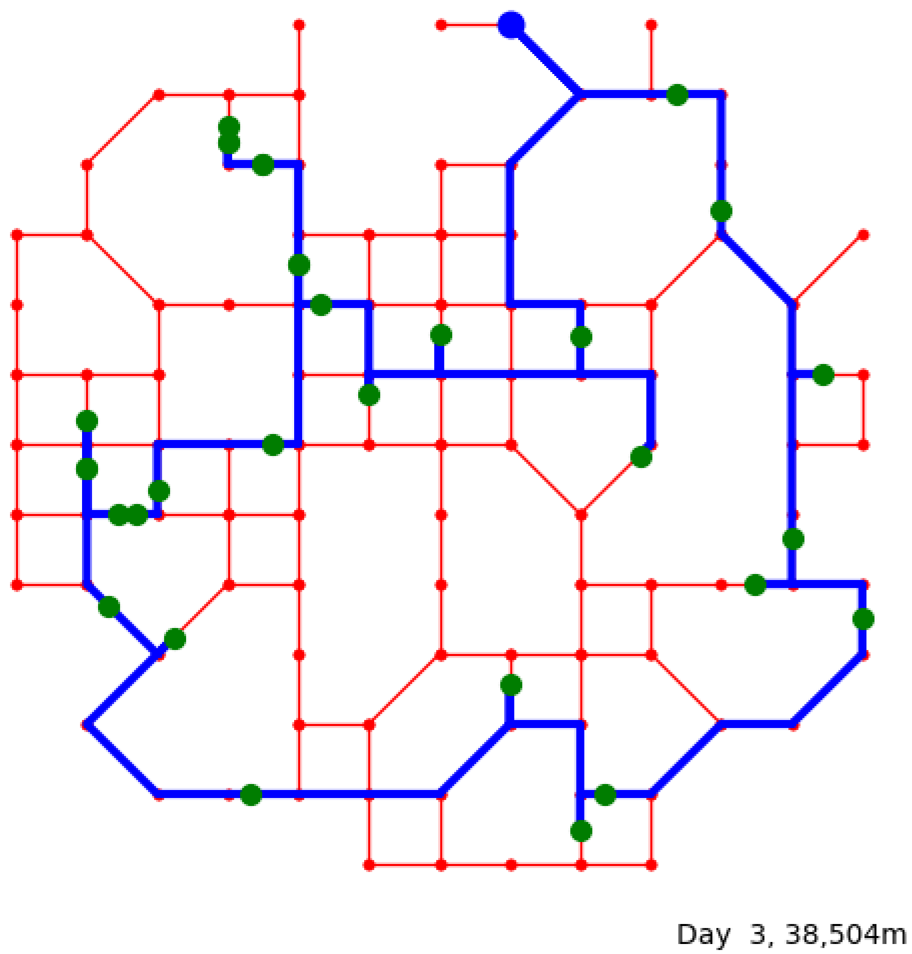
In [34]:
```
random.seed(5640)
W = generateWarehouseLocation(M)
rec = simulation(M, W, C, p=0.15, days=25, plot=True)
print(rec.daily.head())
```

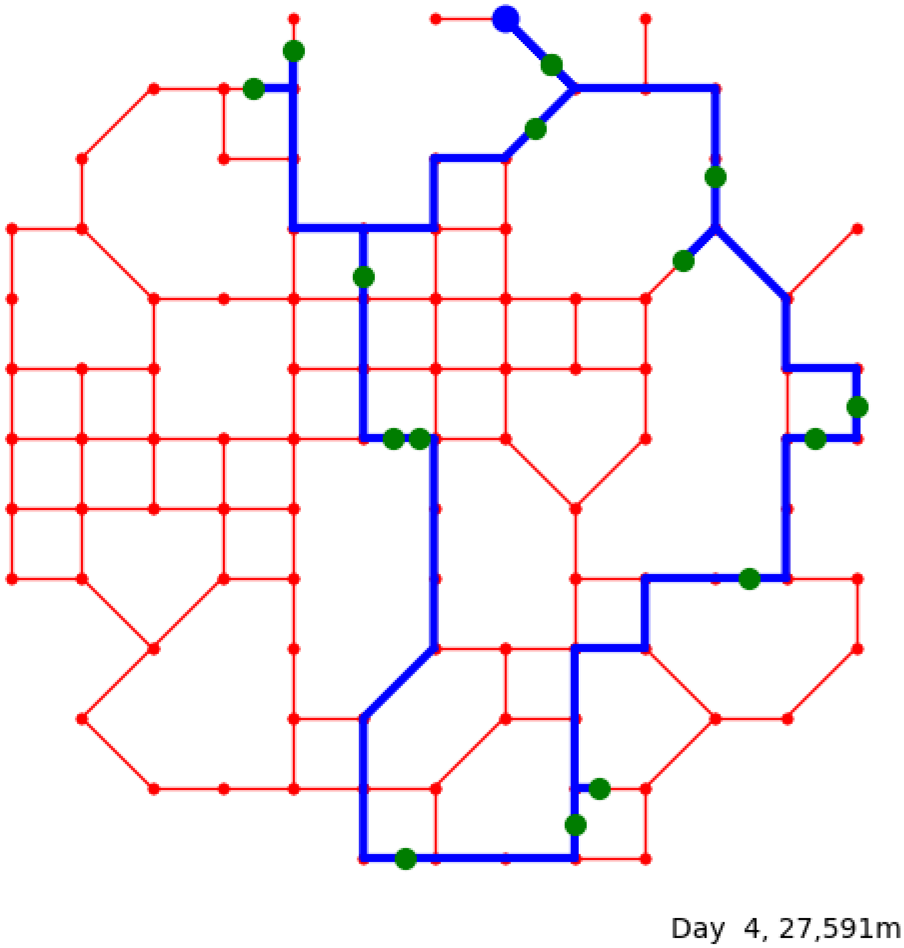
```
Simulating delivery of 578 parcels over 25 days to 150 customers
```

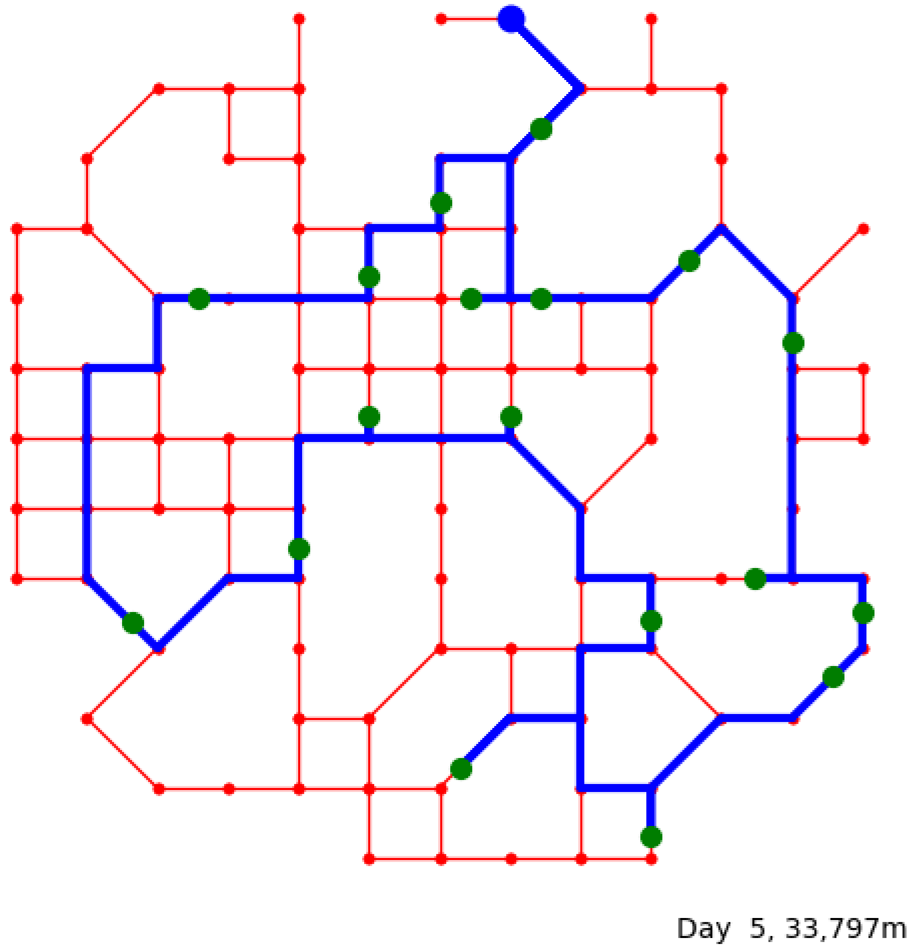

Day 0, 33,567m

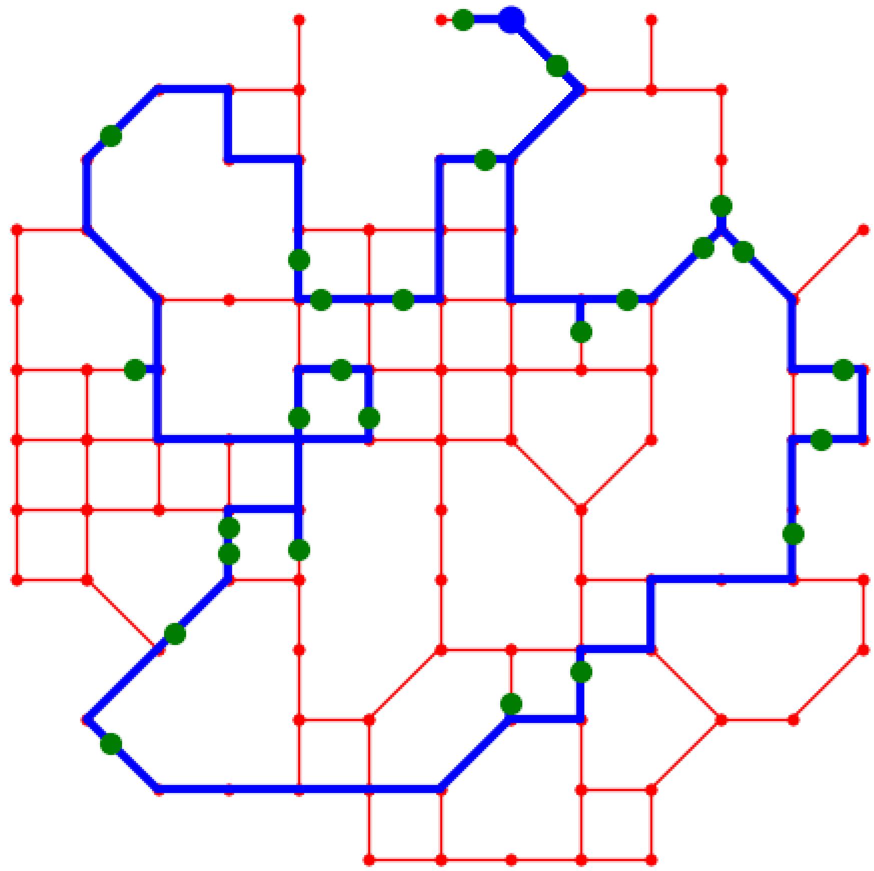

Day 1, 34,905m


Day 2, 29,677m

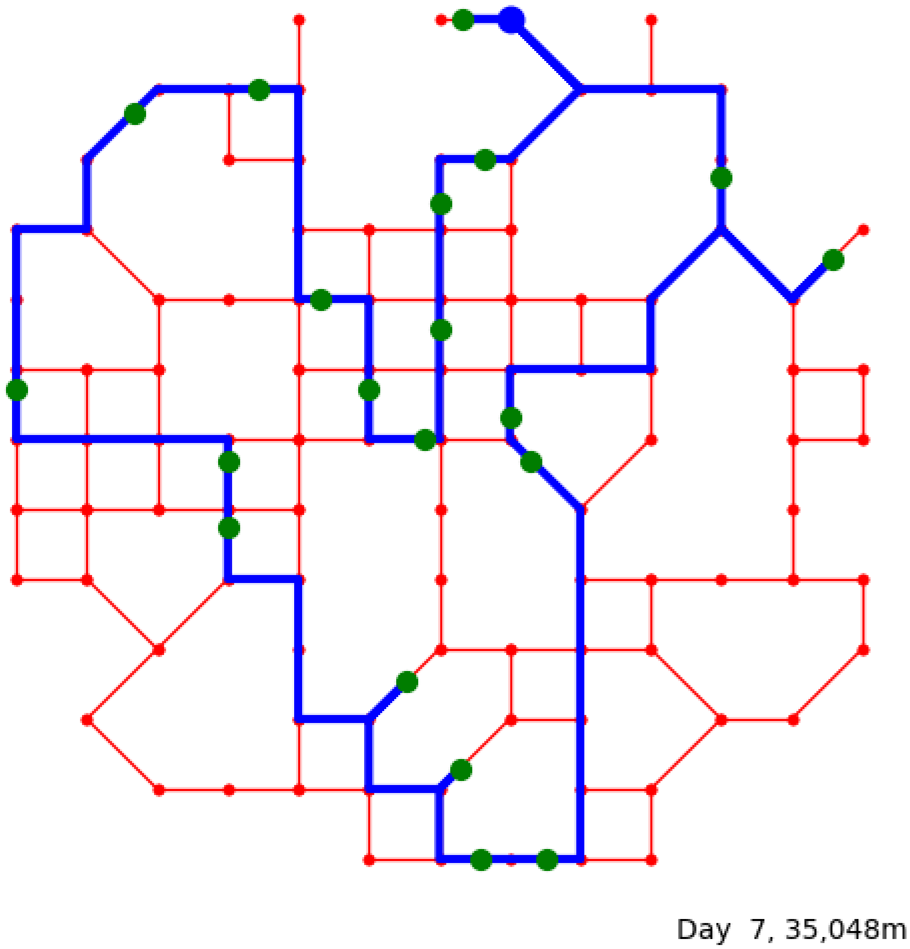

Day 3, 38,504m

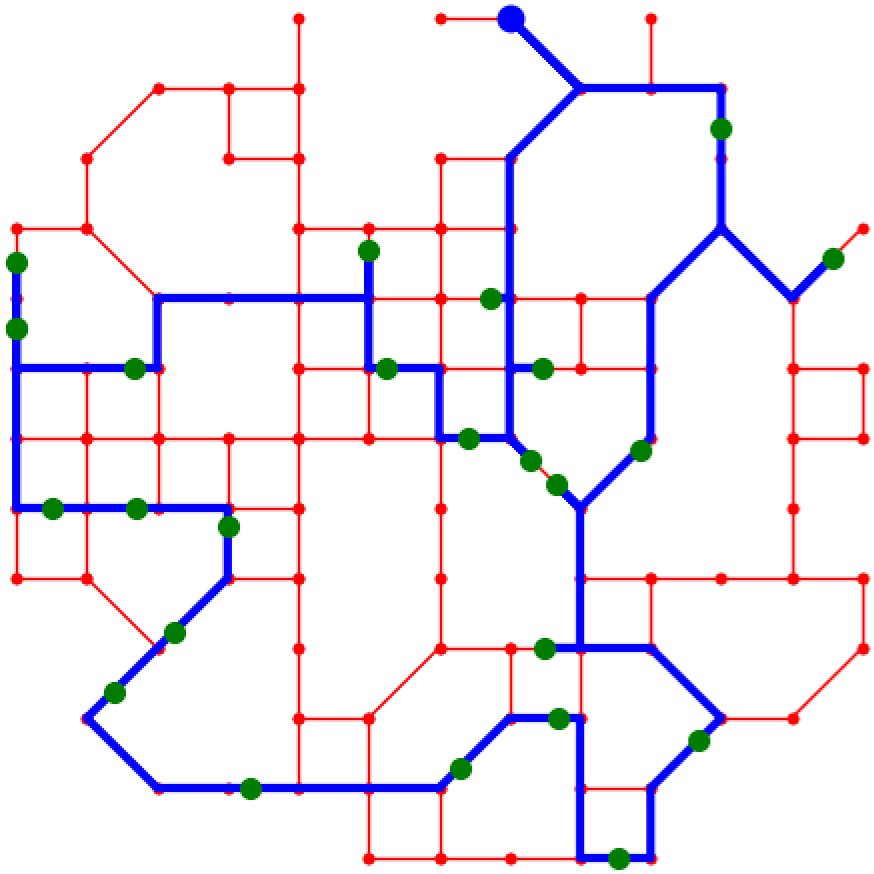

Day 4, 27,591m


Day 5, 33,797m

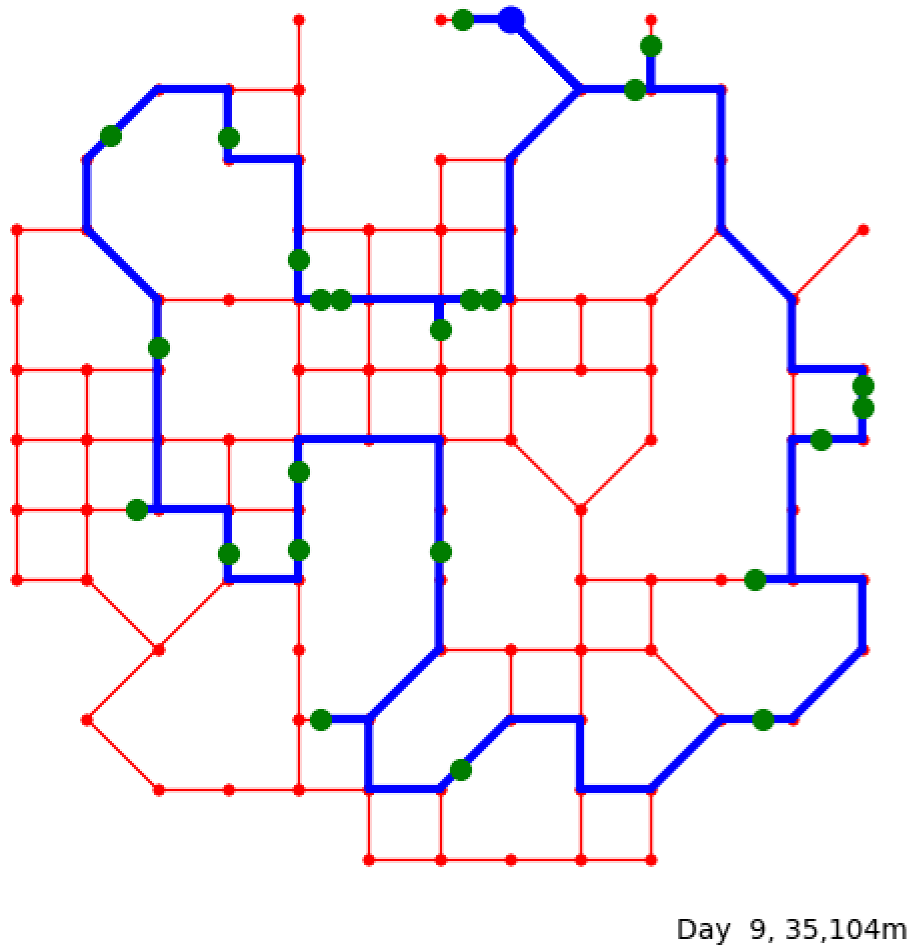


Day 6, 35,932m

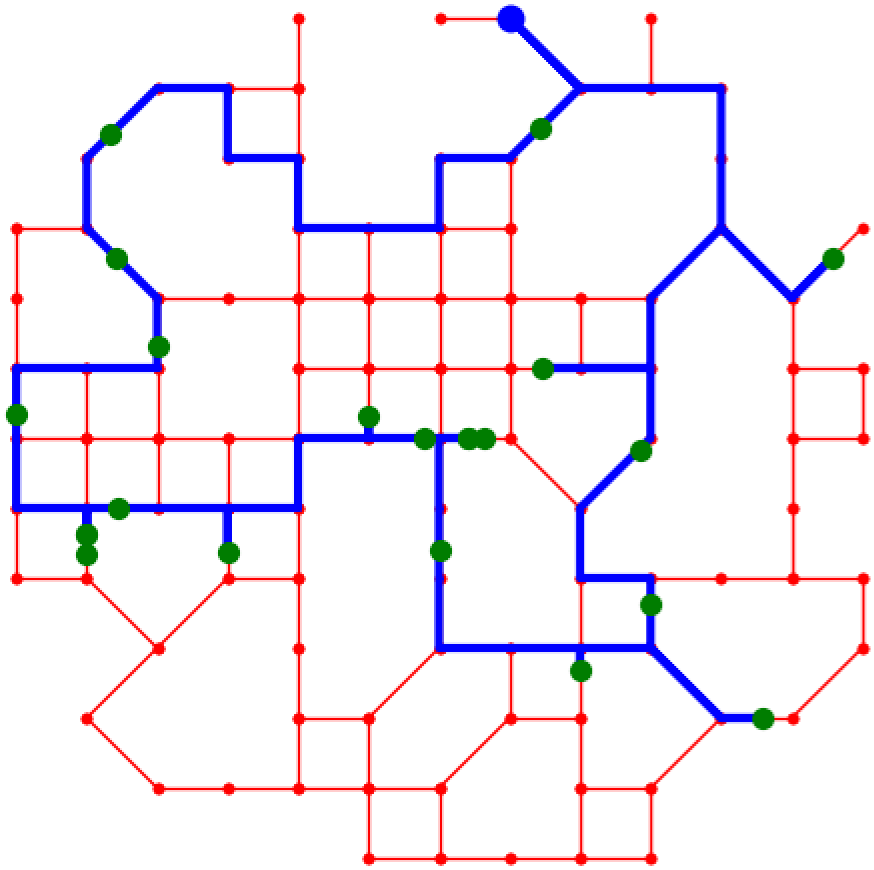

Day 7, 35,048m



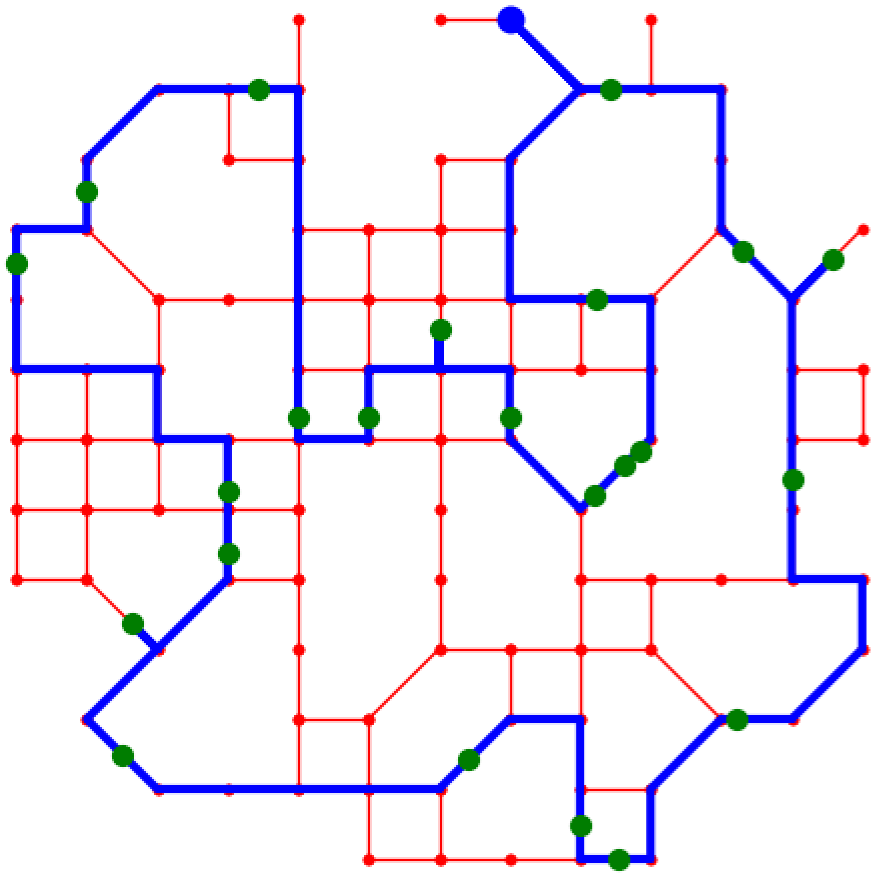
Day 8, 37,363m



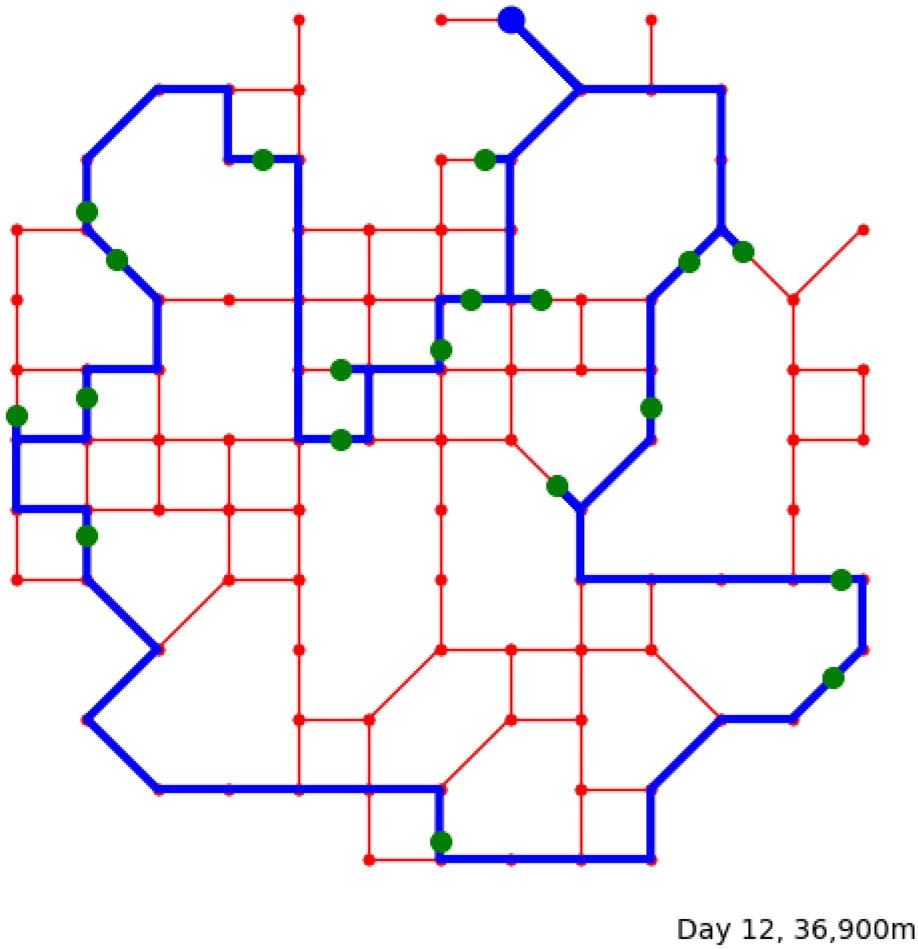
Day 9, 35,104m

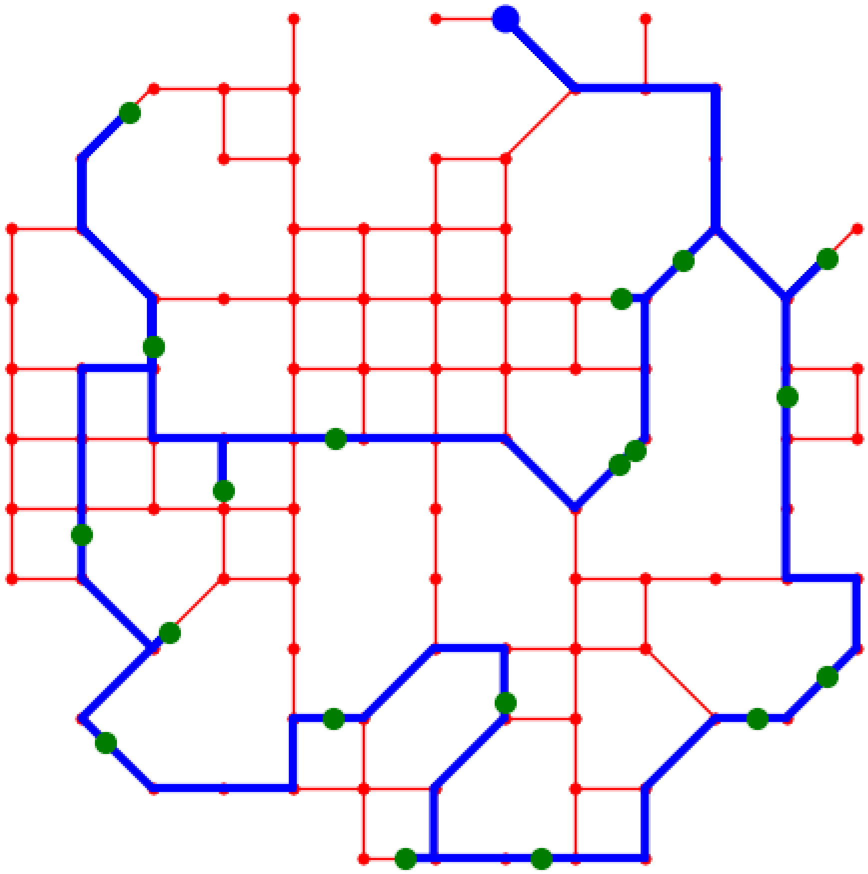


Day 10, 34,999m

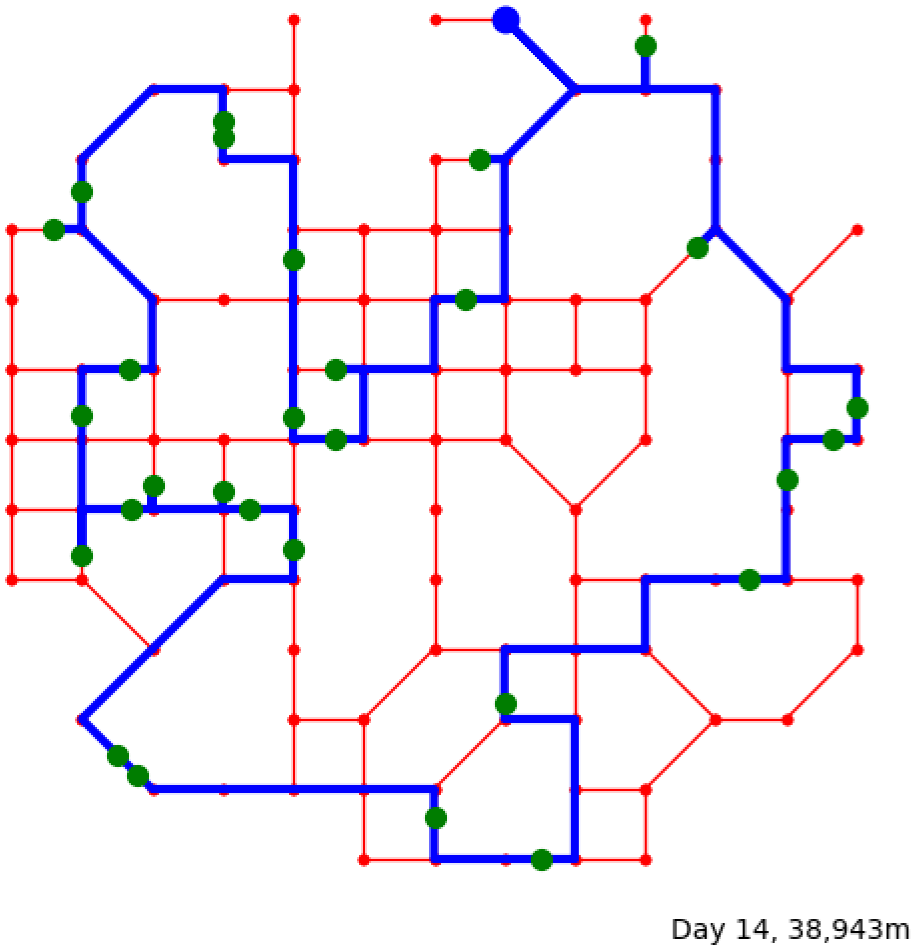


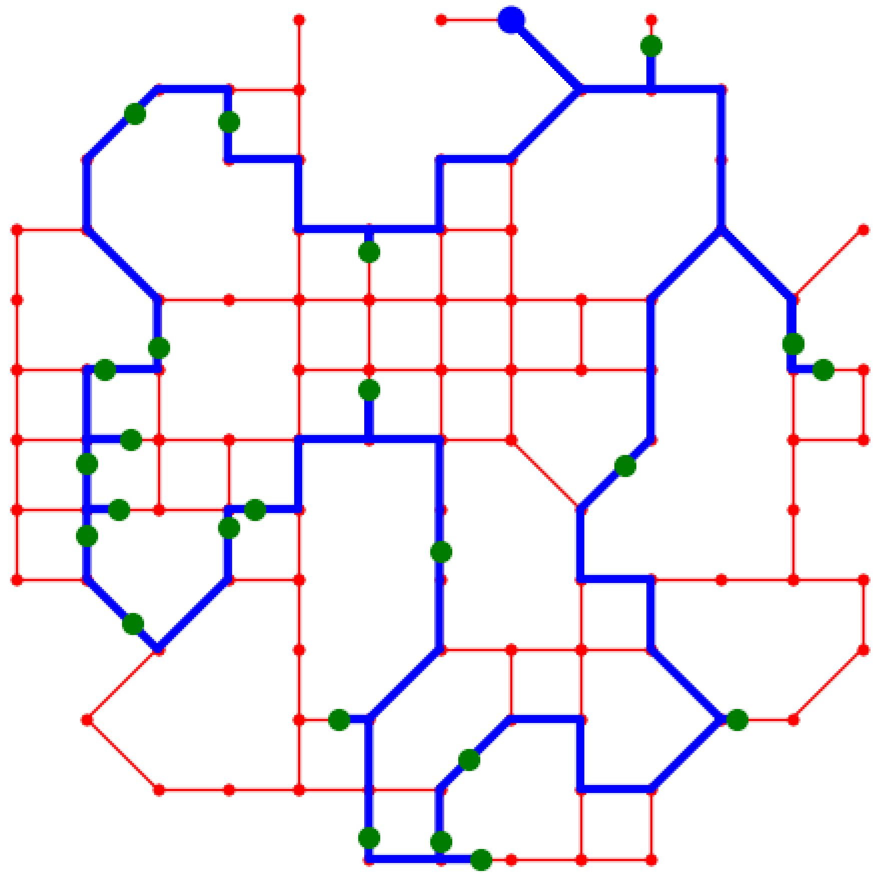
Day 11, 39,280m


Day 12, 36,900m

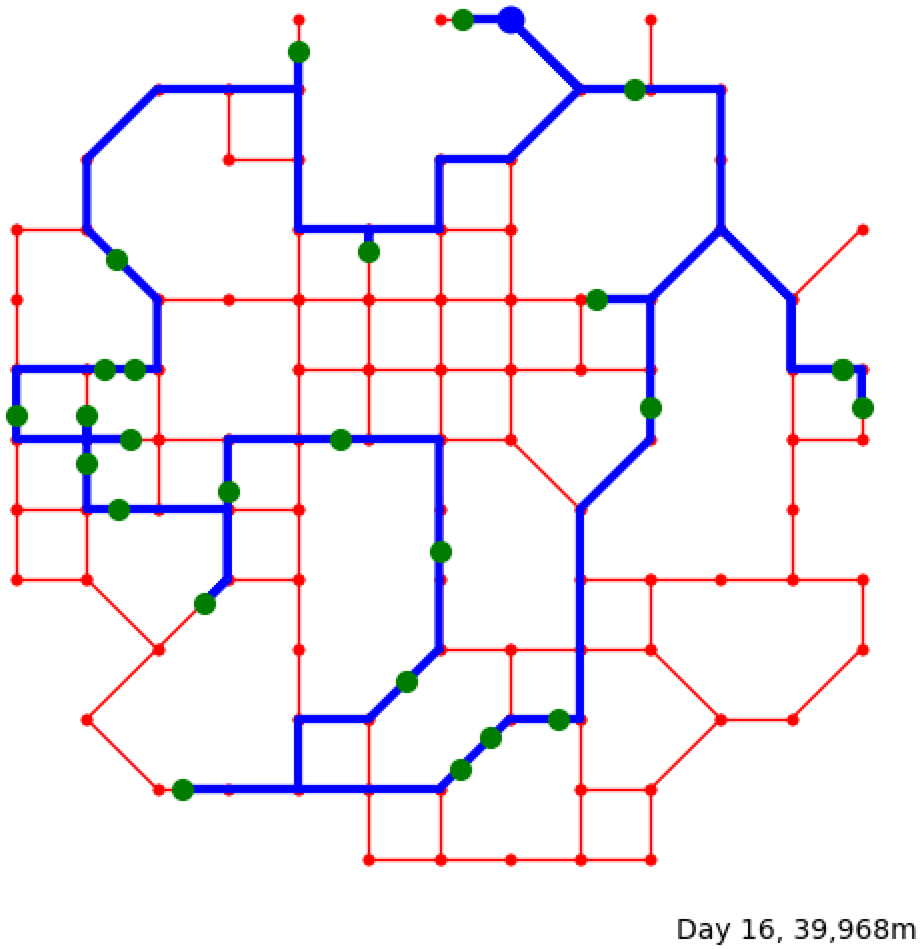


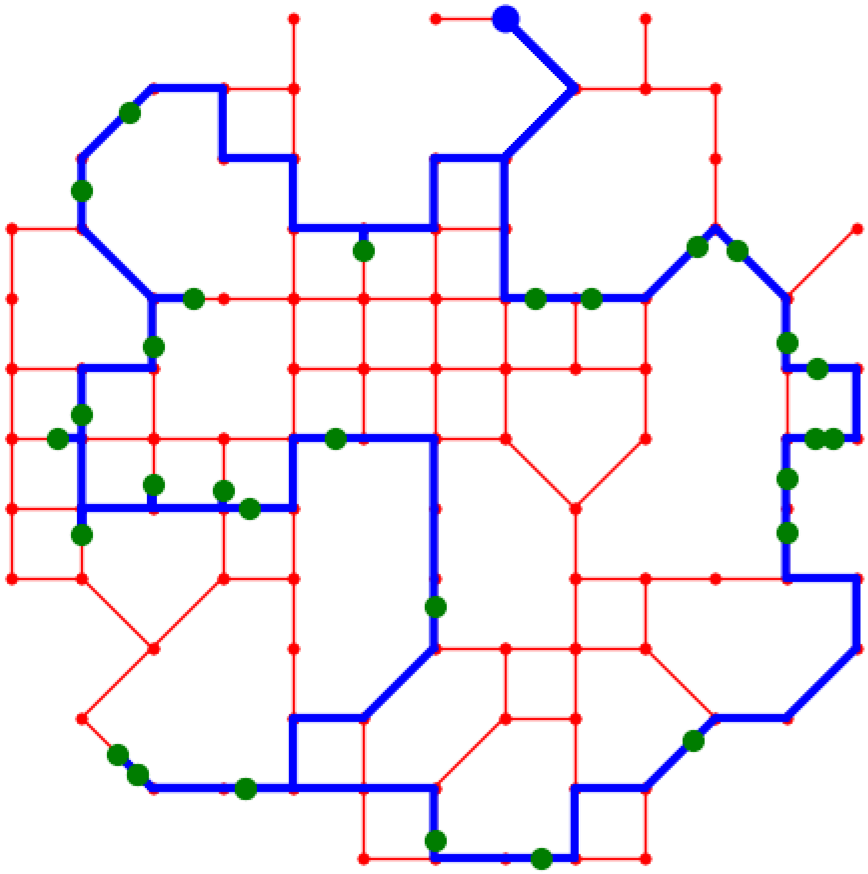
Day 13, 39,413m


Day 14, 38,943m

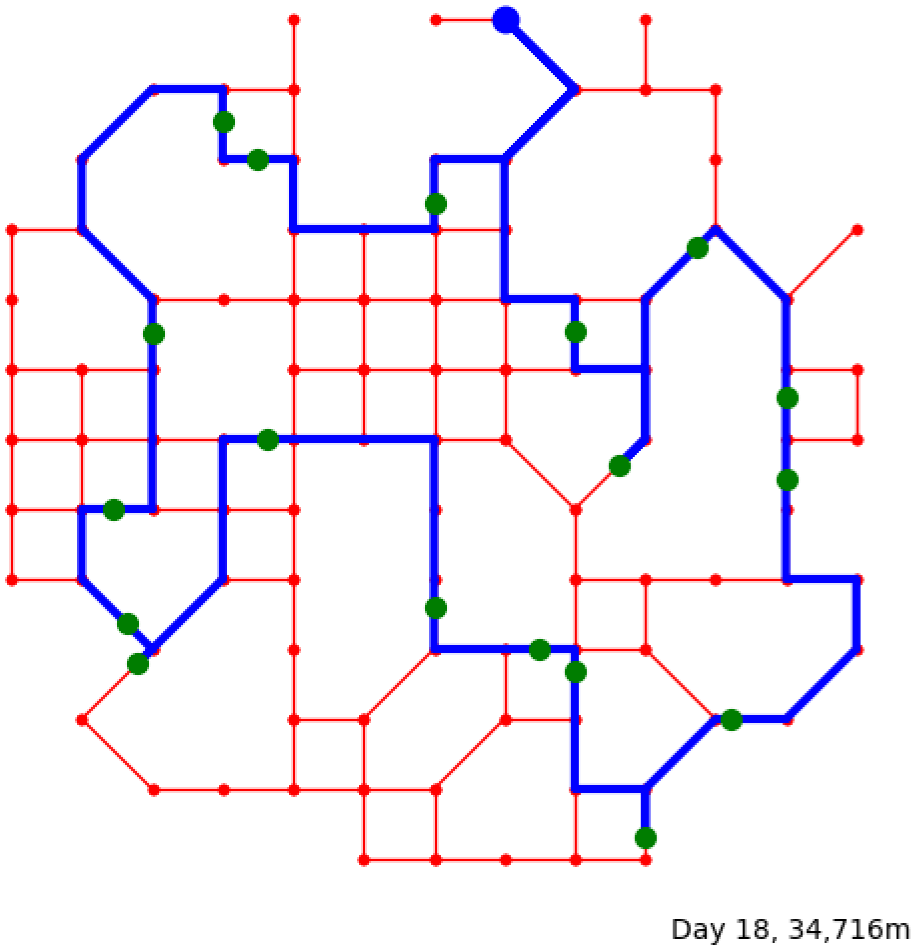


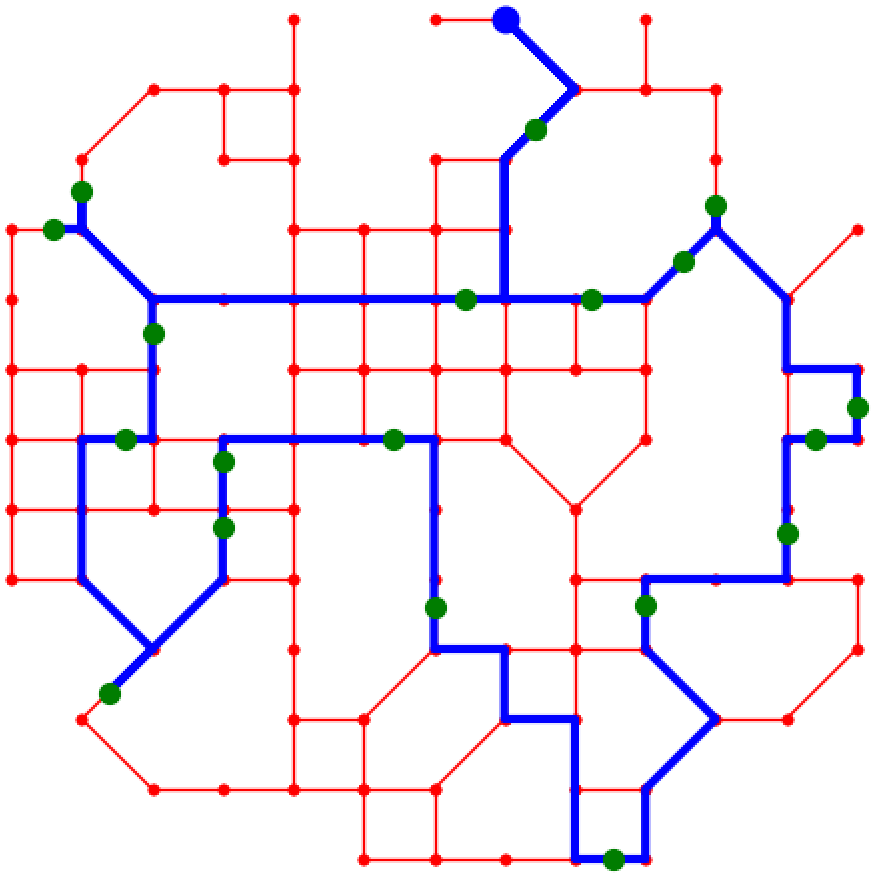
Day 15, 39,199m


Day 16, 39,968m

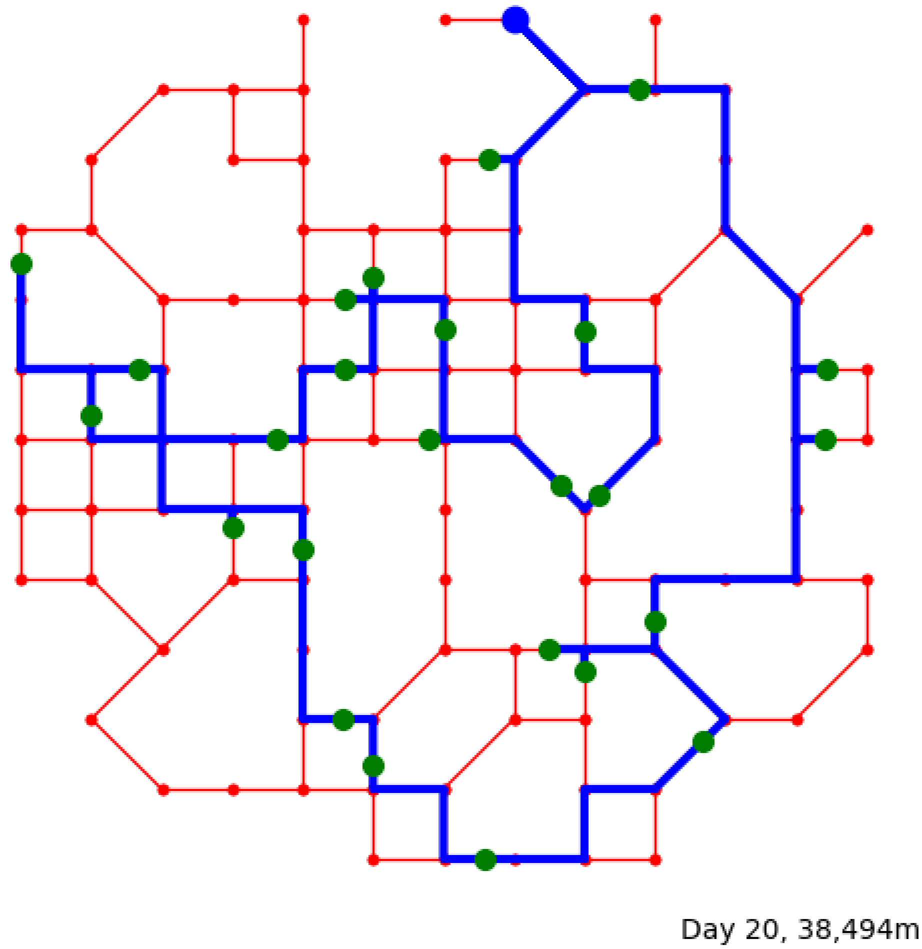


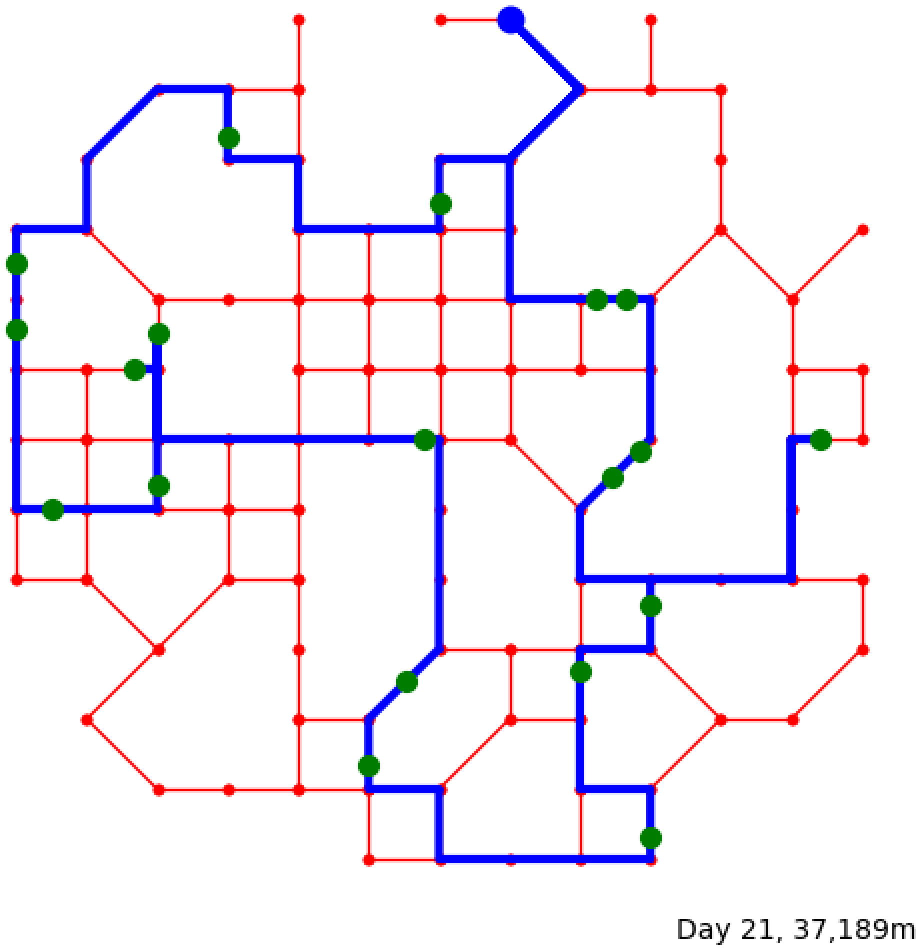
Day 17, 38,827m

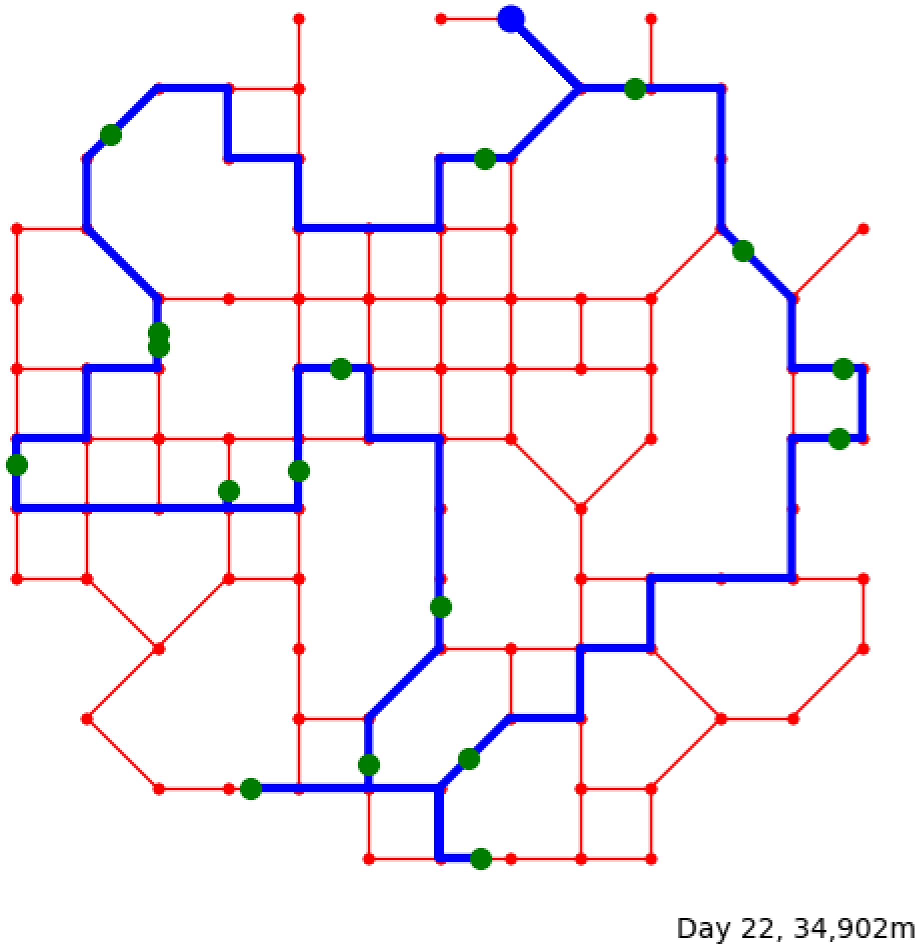

Day 18, 34,716m

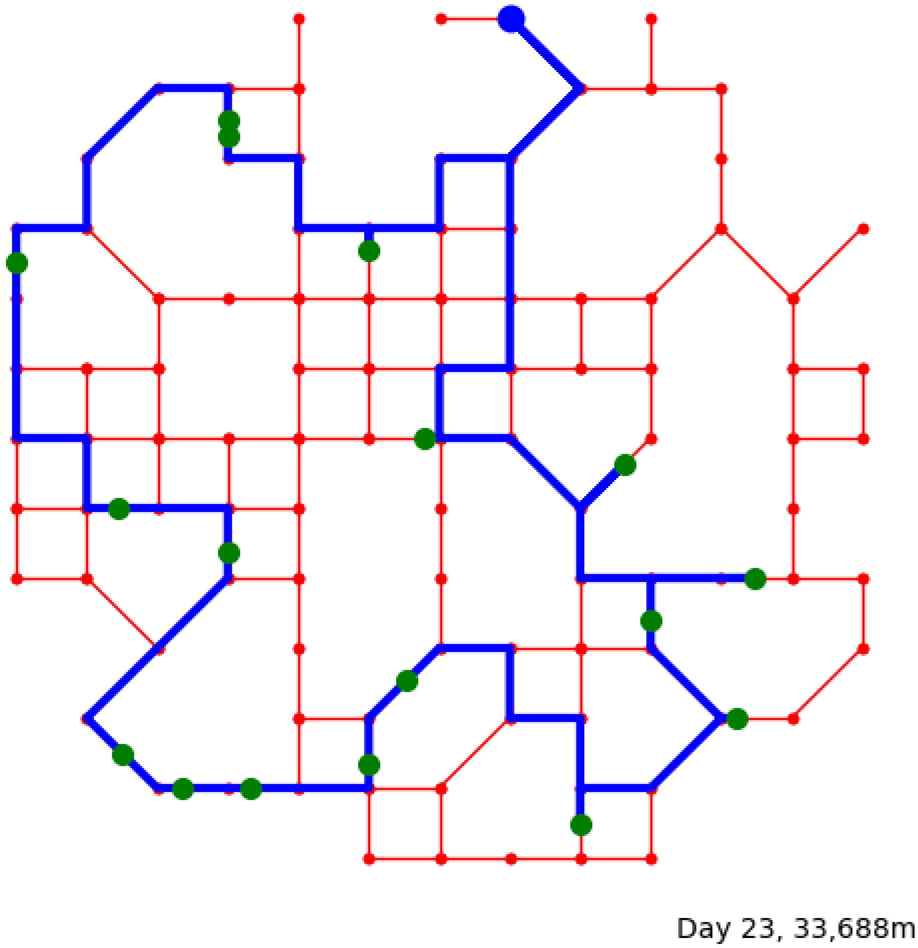


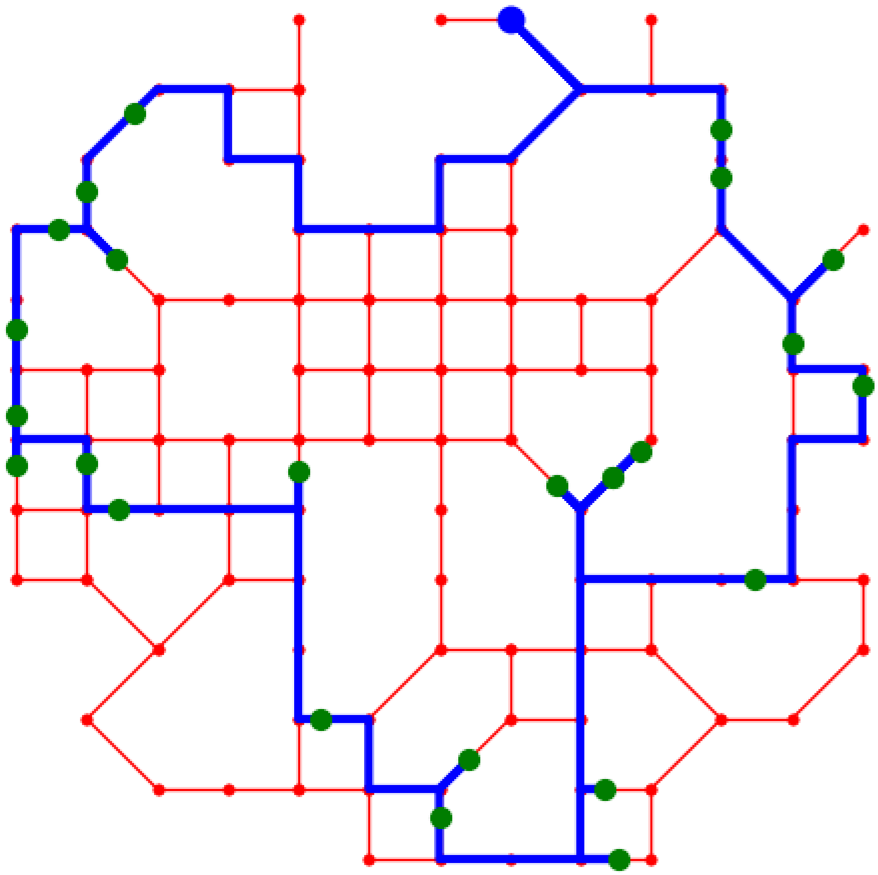
Day 19, 34,294m


Day 20, 38,494m


Day 21, 37,189m


Day 22, 34,902m


Day 23, 33,688m




```
   begin work at  end work at   dist  left  working time
0          64800        75030  33567     0           170
1         151200       162024  34905     0           180
2         237600       246940  29677     0           155
3         324000       337011  38504     0           216
4         410400       419447  27591     0           150
```

10.5 Calculating Operational Cost and Driver's Pay

In [35]:

```
def convert_distance_to_km(meters):
    "Convert distance from meters to kilometers."
    return meters / 1000

def convert_time_to_hours(seconds):
    "Convert time from seconds to hours."
    return seconds / 3600

def convert_time_to_hours_and_minutes(seconds):
    "Convert time from seconds to hours and minutes."
    hours = seconds // 3600
    minutes = (seconds % 3600) // 60
    return hours, minutes
```

In [36]:
```
def calculate_operational_costs(distance_km):
    "Calculate operational costs based on distance."
    cost_per_km = 0.08  # Example: 0.08 euros per km
    return distance_km * cost_per_km

def calculate_driver_pay(hours):
    "Calculate driver pay based on hours worked."
    pay_rate_per_hour = 30.00  # Example: 30 euros per hour
    minimum_daily_pay = 60.00  # Example: Minimum daily pay
    pay = hours * pay_rate_per_hour
    return max(pay, minimum_daily_pay)
```

In [37]:
```
rec.finish()
```

In [38]:
```
for index, row in rec.daily.iterrows():
    if row['dist'] is not None and row['begin work at'] is not None and row['end wo
        distance_km = convert_distance_to_km(row['dist'])
        working_hours = convert_time_to_hours(row['end work at'] - row['begin work 
        working_hours, working_minutes = convert_time_to_hours_and_minutes(row['end

        # Now use these values in your cost and pay calculations
        operational_cost = calculate_operational_costs(distance_km)
        driver_pay = calculate_driver_pay(working_hours)

        # Store or output results
        print(f"Day {index+1}: Distance = {distance_km:.2f} km, Working Time = {wor
        print(f"Operational Cost = {operational_cost:.2f}, Driver Pay = {driver_pay
```

```
Day 1: Distance = 33.57 km, Working Time = 2.00 hours (2h 50m)
Operational Cost = 2.69, Driver Pay = 60.00
Day 2: Distance = 34.91 km, Working Time = 3.00 hours (3h 0m)
Operational Cost = 2.79, Driver Pay = 90.00
Day 3: Distance = 29.68 km, Working Time = 2.00 hours (2h 35m)
Operational Cost = 2.37, Driver Pay = 60.00
Day 4: Distance = 38.50 km, Working Time = 3.00 hours (3h 36m)
Operational Cost = 3.08, Driver Pay = 90.00
Day 5: Distance = 27.59 km, Working Time = 2.00 hours (2h 30m)
Operational Cost = 2.21, Driver Pay = 60.00
Day 6: Distance = 33.80 km, Working Time = 2.00 hours (2h 57m)
Operational Cost = 2.70, Driver Pay = 60.00
Day 7: Distance = 35.93 km, Working Time = 3.00 hours (3h 21m)
Operational Cost = 2.87, Driver Pay = 90.00
Day 8: Distance = 35.05 km, Working Time = 3.00 hours (3h 9m)
Operational Cost = 2.80, Driver Pay = 90.00
Day 9: Distance = 37.36 km, Working Time = 3.00 hours (3h 31m)
Operational Cost = 2.99, Driver Pay = 90.00
Day 10: Distance = 35.10 km, Working Time = 3.00 hours (3h 16m)
Operational Cost = 2.81, Driver Pay = 90.00
Day 11: Distance = 35.00 km, Working Time = 3.00 hours (3h 8m)
Operational Cost = 2.80, Driver Pay = 90.00
Day 12: Distance = 39.28 km, Working Time = 3.00 hours (3h 27m)
Operational Cost = 3.14, Driver Pay = 90.00
Day 13: Distance = 36.90 km, Working Time = 3.00 hours (3h 12m)
Operational Cost = 2.95, Driver Pay = 90.00
Day 14: Distance = 39.41 km, Working Time = 3.00 hours (3h 29m)
Operational Cost = 3.15, Driver Pay = 90.00
Day 15: Distance = 38.94 km, Working Time = 3.00 hours (3h 38m)
Operational Cost = 3.12, Driver Pay = 90.00
Day 16: Distance = 39.20 km, Working Time = 3.00 hours (3h 30m)
Operational Cost = 3.14, Driver Pay = 90.00
Day 17: Distance = 39.97 km, Working Time = 3.00 hours (3h 36m)
Operational Cost = 3.20, Driver Pay = 90.00
Day 18: Distance = 38.83 km, Working Time = 3.00 hours (3h 48m)
Operational Cost = 3.11, Driver Pay = 90.00
Day 19: Distance = 34.72 km, Working Time = 3.00 hours (3h 2m)
Operational Cost = 2.78, Driver Pay = 90.00
Day 20: Distance = 34.29 km, Working Time = 3.00 hours (3h 3m)
Operational Cost = 2.74, Driver Pay = 90.00
Day 21: Distance = 38.49 km, Working Time = 3.00 hours (3h 35m)
Operational Cost = 3.08, Driver Pay = 90.00
Day 22: Distance = 37.19 km, Working Time = 3.00 hours (3h 10m)
Operational Cost = 2.98, Driver Pay = 90.00
Day 23: Distance = 34.90 km, Working Time = 3.00 hours (3h 7m)
Operational Cost = 2.79, Driver Pay = 90.00
Day 24: Distance = 33.69 km, Working Time = 2.00 hours (2h 54m)
Operational Cost = 2.70, Driver Pay = 60.00
Day 25: Distance = 35.91 km, Working Time = 3.00 hours (3h 22m)
Operational Cost = 2.87, Driver Pay = 90.00
```

10.6. Finding Statistical Significance

In [39]:
```
import pandas as pd
import matplotlib.pyplot as plt
```

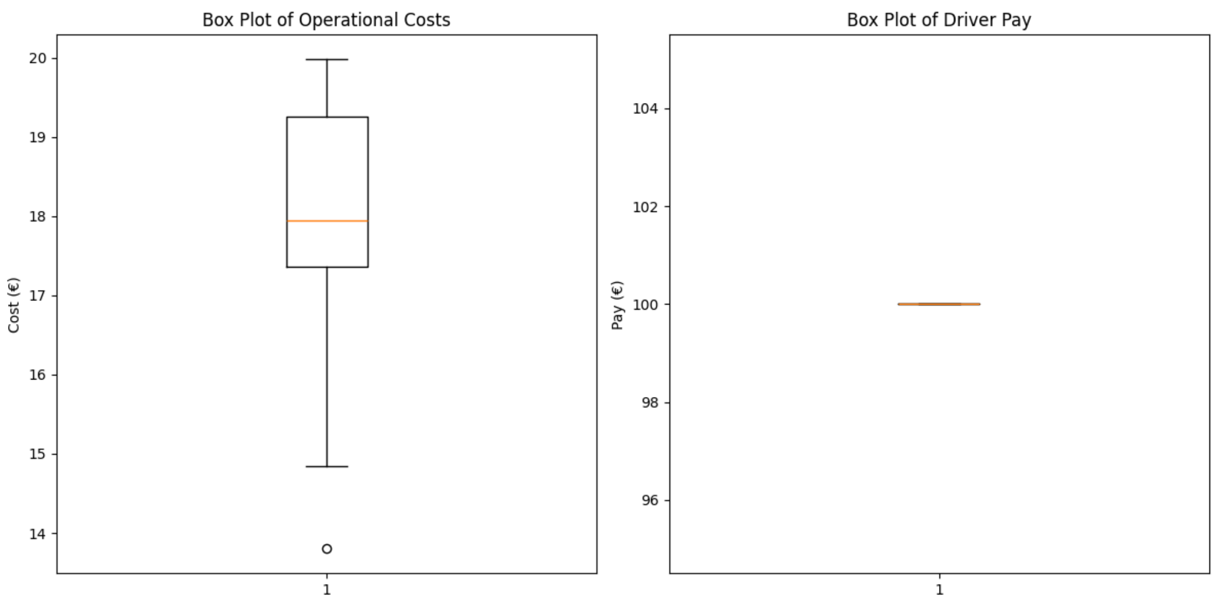
```
# Define the data
data = {
    'Day': range(1, 26),
    'Distance (km)': [33.57, 34.91, 29.68, 38.50, 27.59, 33.80, 35.93, 35.05, 37.36
                      35.00, 39.28, 36.90, 39.41, 38.94, 39.20, 39.97, 38.83, 34.72
                      38.49, 37.19, 34.90, 33.69, 35.91],
    'Working Time (h)': [2.00, 3.00, 2.00, 3.00, 2.00, 2.00, 3.00, 3.00, 3.00, 3.00
                         3.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00
                         3.00, 3.00, 3.00, 2.00, 3.00],
    'Operational Cost (€)': [16.78, 17.45, 14.84, 19.25, 13.80, 16.90, 17.97, 17.52
                             17.50, 19.64, 18.45, 19.71, 19.47, 19.60, 19.98, 19.41
                             19.25, 18.59, 17.45, 16.84, 17.95],
    'Driver Pay (€)': [100.00] * 25  # Since Driver's Pay is always 100 Euros
}

df = pd.DataFrame(data)
print(df)
df = pd.DataFrame(data)
```

```
    Day  Distance (km)  Working Time (h)  Operational Cost (€)  Driver Pay (€)
0     1          33.57               2.0                 16.78           100.0
1     2          34.91               3.0                 17.45           100.0
2     3          29.68               2.0                 14.84           100.0
3     4          38.50               3.0                 19.25           100.0
4     5          27.59               2.0                 13.80           100.0
5     6          33.80               2.0                 16.90           100.0
6     7          35.93               3.0                 17.97           100.0
7     8          35.05               3.0                 17.52           100.0
8     9          37.36               3.0                 18.68           100.0
9    10          35.10               3.0                 17.55           100.0
10   11          35.00               3.0                 17.50           100.0
11   12          39.28               3.0                 19.64           100.0
12   13          36.90               3.0                 18.45           100.0
13   14          39.41               3.0                 19.71           100.0
14   15          38.94               3.0                 19.47           100.0
15   16          39.20               3.0                 19.60           100.0
16   17          39.97               3.0                 19.98           100.0
17   18          38.83               3.0                 19.41           100.0
18   19          34.72               3.0                 17.36           100.0
19   20          34.29               3.0                 17.15           100.0
20   21          38.49               3.0                 19.25           100.0
21   22          37.19               3.0                 18.59           100.0
22   23          34.90               3.0                 17.45           100.0
23   24          33.69               2.0                 16.84           100.0
24   25          35.91               3.0                 17.95           100.0
```

In [40]:
```
def calculate_costs(df, cost_per_km=0.08, pay_per_hour=30.00, minimum_pay=100.00):
    # Assuming the cost per km and pay per hour are given, and minimum pay is enfor
    df['Calculated Operational Cost (€)'] = df['Distance (km)'] * cost_per_km
    df['Calculated Driver Pay (€)'] = df['Working Time (h)'] * pay_per_hour
    df['Calculated Driver Pay (€)'] = df['Calculated Driver Pay (€)'].apply(lambda
    return df
```

In [41]:
```
def plot_costs(df):
    plt.figure(figsize=(12, 6))
```

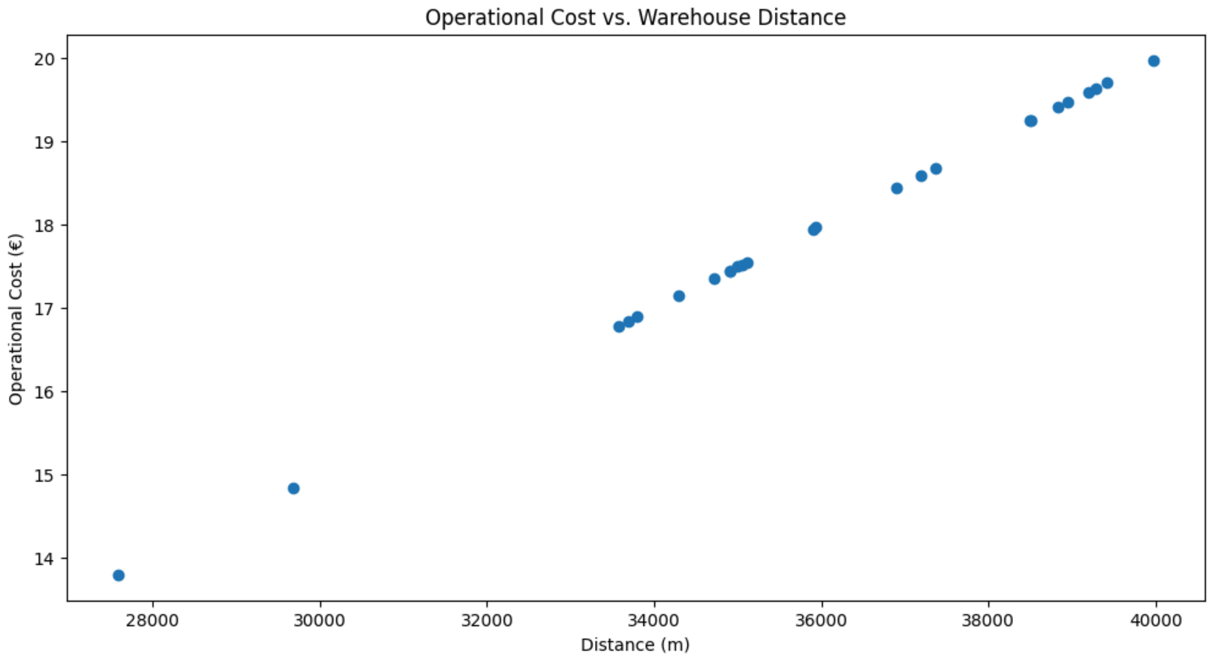
```
    # Plot Operational Costs
    plt.subplot(1, 2, 1)
    plt.boxplot(df['Operational Cost (€)'])
    plt.title('Box Plot of Operational Costs')
    plt.ylabel('Cost (€)')

    # Plot Driver Pay
    plt.subplot(1, 2, 2)
    plt.boxplot(df['Driver Pay (€)'])
    plt.title('Box Plot of Driver Pay')
    plt.ylabel('Pay (€)')

    plt.tight_layout()
    plt.show()

plot_costs(df)
```



In [42]:
```
import pandas as pd
import matplotlib.pyplot as plt
from scipy import stats

def analyze_distance_and_costs(df_distance, df_costs):
    # Extract 'Distance (m)' and 'Operational Cost (€)' columns for analysis
    distance_data = df_distance['Distance (m)']
    cost_data = df_costs['Operational Cost (€)']

    # Check if there are enough unique values for ANOVA analysis
    if distance_data.nunique() < 2:
        print("There is not enough variability in the data for ANOVA analysis.")
        return

    # Perform ANOVA
    try:
        f_value, p_value = stats.f_oneway(cost_data, distance_data)  # Perform ANOV
        print(f"F-Statistic: {f_value}, P-Value: {p_value}")
```

```
        # Check if the differences are statistically significant
        if p_value < 0.05:
            print("Statistically significant differences found between warehouse lo
        else:
            print("No statistically significant differences found. Consider increas

        # Generate scatter plot for visual inspection
        plt.figure(figsize=(12, 6))
        plt.scatter(distance_data, cost_data)
        plt.title('Operational Cost vs. Warehouse Distance')
        plt.xlabel('Distance (m)')
        plt.ylabel('Operational Cost (€)')
        plt.show()
    except Exception as e:
        print(f"An error occurred: {e}")

# Warehouse distance data
warehouse_distance = {
    'Day': ['Day 0', 'Day 1', 'Day 2', 'Day 3', 'Day 4', 'Day 5', 'Day 6', 'Day 7',
            'Day 10', 'Day 11', 'Day 12', 'Day 13', 'Day 14', 'Day 15', 'Day 16', '
            'Day 20', 'Day 21', 'Day 22', 'Day 23', 'Day 24'],
    'Distance (m)': [33567, 34905, 29677, 38504, 27591, 33797, 35932, 35048, 37363,
                     34999, 39280, 36900, 39413, 38943, 39199, 39968, 38827, 34716,
                     38494, 37189, 34902, 33688, 35907]
}

# Operational cost data
operational_costs = {
    'Day': ['Day 1', 'Day 2', 'Day 3', 'Day 4', 'Day 5', 'Day 6', 'Day 7', 'Day 8',
            'Day 11', 'Day 12', 'Day 13', 'Day 14', 'Day 15', 'Day 16', 'Day 17', '
            'Day 21', 'Day 22', 'Day 23', 'Day 24', 'Day 25'],
    'Operational Cost (€)': [16.78, 17.45, 14.84, 19.25, 13.80, 16.90, 17.97, 17.52
                             17.50, 19.64, 18.45, 19.71, 19.47, 19.60, 19.98, 19.4
                             19.25, 18.59, 17.45, 16.84, 17.95]
}

# Convert data to DataFrames
df_warehouse_distance = pd.DataFrame(warehouse_distance)
df_operational_costs = pd.DataFrame(operational_costs)

# Call the function
analyze_distance_and_costs(df_warehouse_distance, df_operational_costs)
```

```
F-Statistic: 3549.5555643068683, P-Value: 1.1680635943351185e-46
Statistically significant differences found between warehouse locations.
```


Operational Cost vs. Warehouse Distance
Operational Cost (€)
20
19
18
17
16
15
14
28000
30000
32000
34000
36000
38000
40000
Distance (m)

In [ ]: