Image filters

Computer Vision Crash Course

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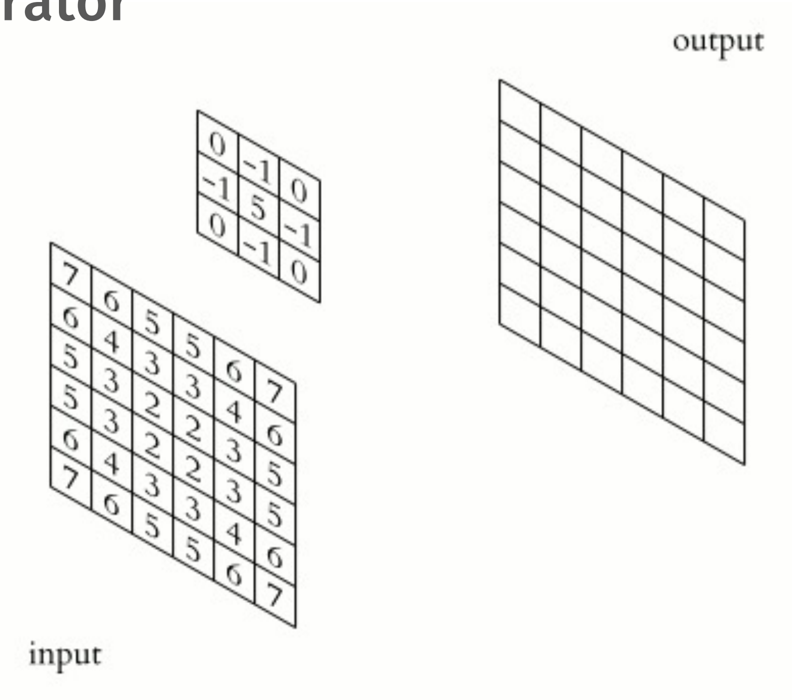
Contents

Image filters

Filters are a basic tool in image processing and computer vision used for a variety of tasks, included noise reduction and signal enhancement

Linear filtering: convolution

In linear cases, filtering corresponds to applying to a signal f a convolution operator



Linear filtering: convolution

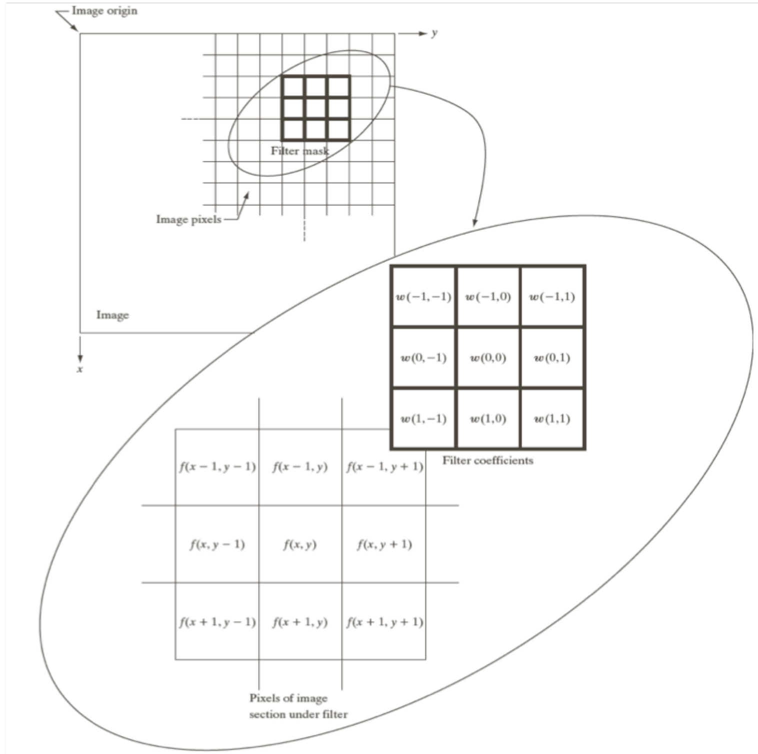
In linear cases, filtering corresponds to applying to a signal f a convolution operator

$$g = f * h$$

filtered signal

signal

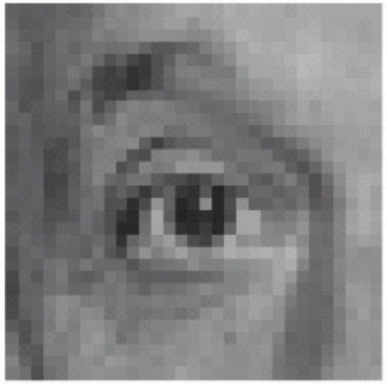
kernel



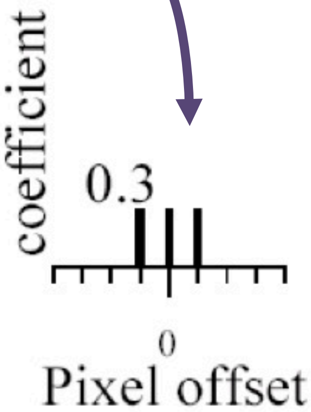
$$g(i,j) = \sum_{k,l} f(i-k, j-l)h(k,l) = \sum_{k,l} f(k,l)h(i-k, j-l)$$

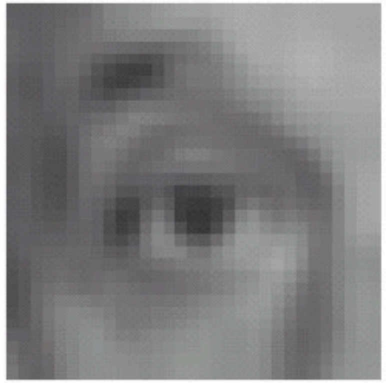
Linear filtering: low-pass / smoothing examples

Simple 1D average filter: it produces a smoothing effect along 1 direction



original





Blurred (filter applied in both dimensions).

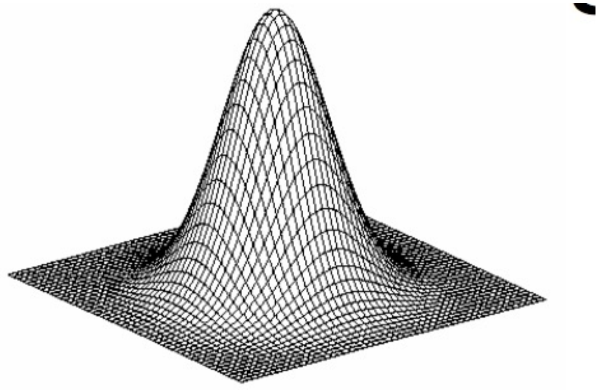
Weighted version

A similar effect can be obtained by a 2D average filter!

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

A classical smoothing filter: the Gaussian

It is computed as a discretization of a 2D Gaussian function

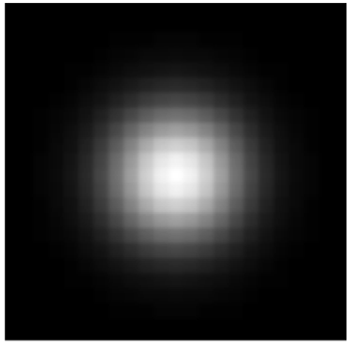


Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

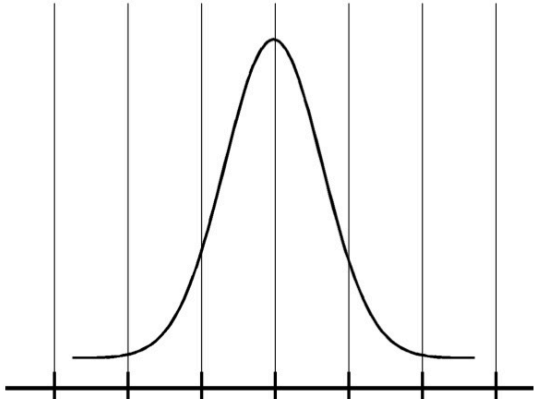
It has several good mathematical properties

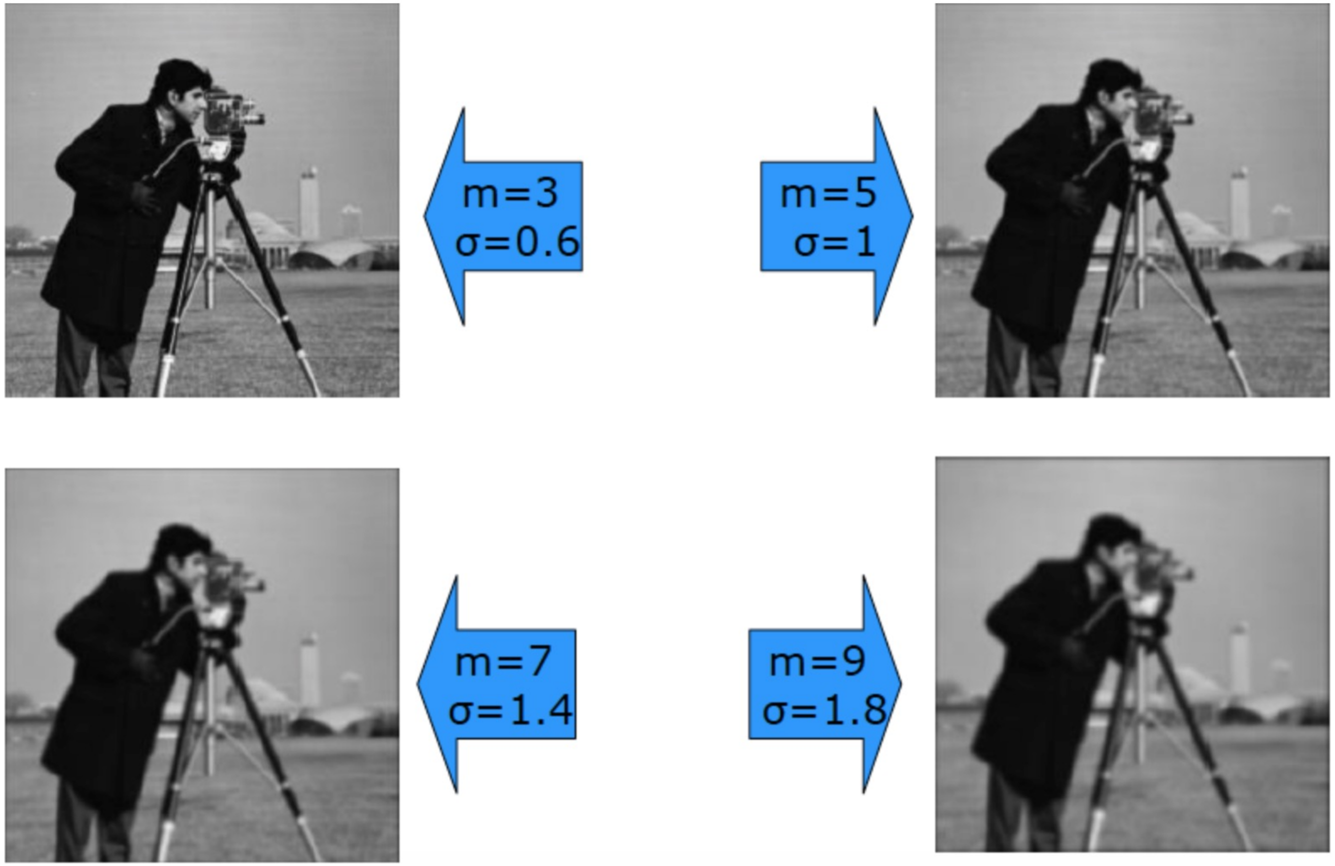
from a computational view-point can be seen as a way to weigh pixels



Smoothing effect and parameter choice

If you apply an effective discretization of the Gaussian you get a very tight relationship between size of the kernel and size of the "bell"

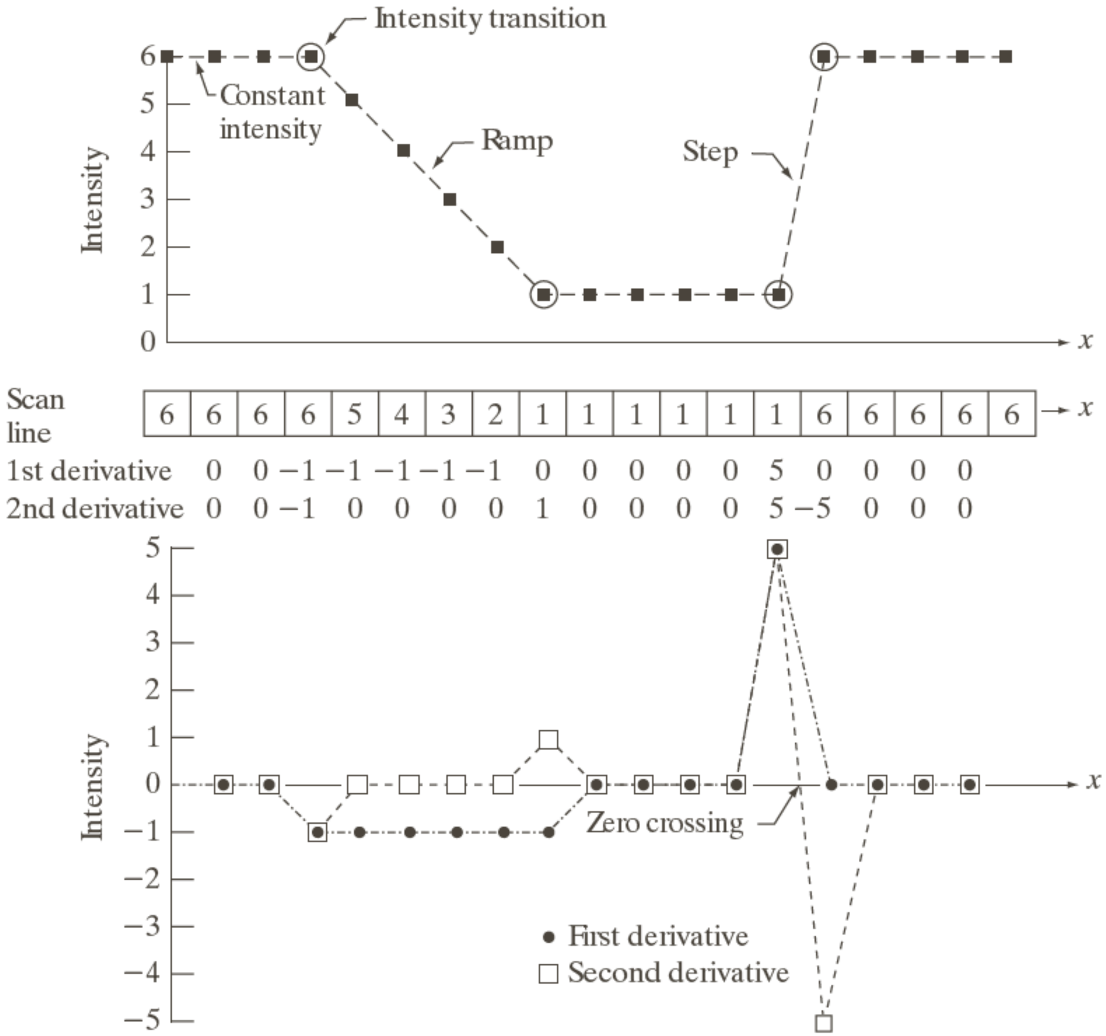




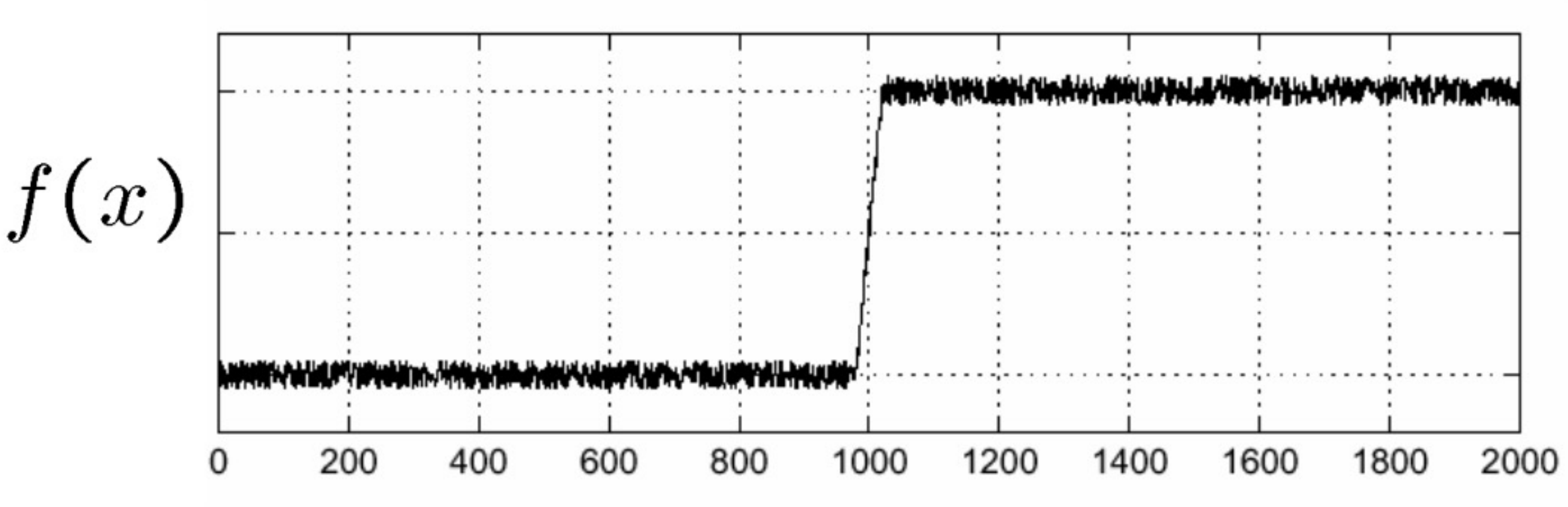
Linear filtering: high-pass / enhancement

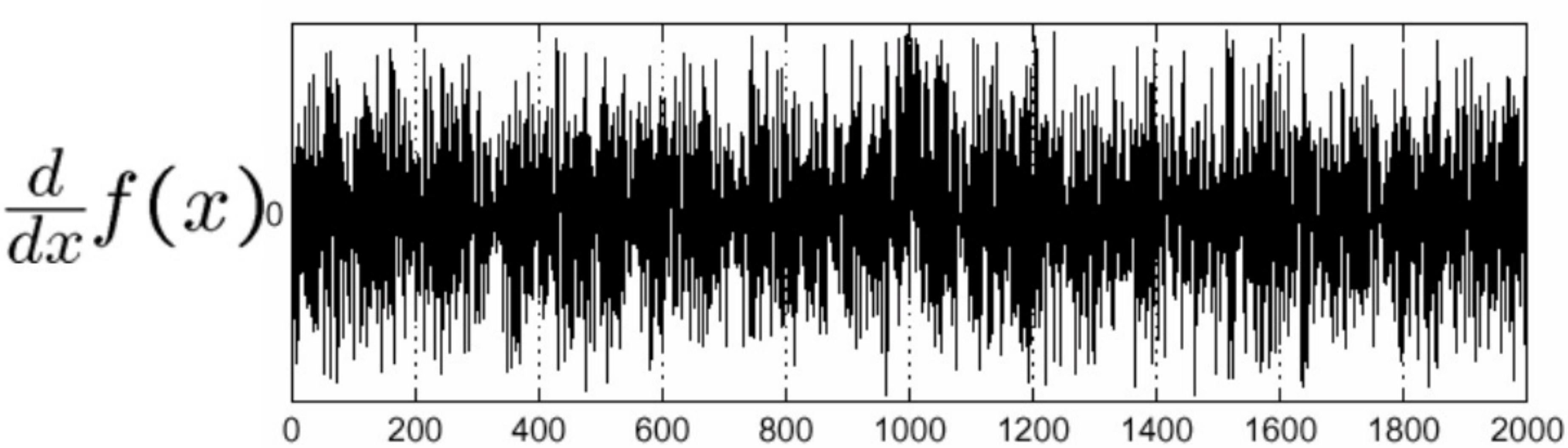
Digital differentiation

Goal : detecting points of variations in a signal



Why smoothing? The effect of noise





based on S. Seitz slides

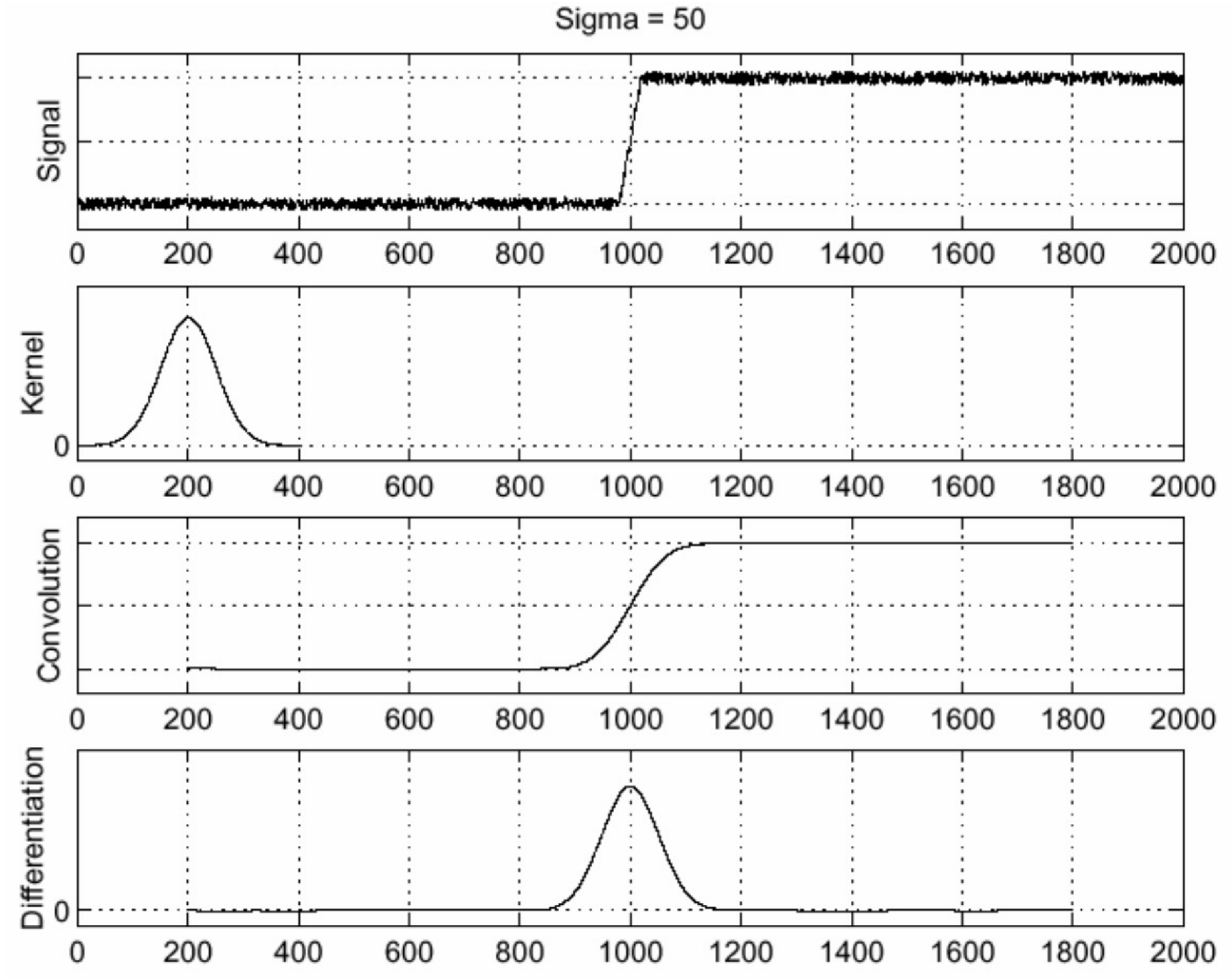
Solution: smooth first

$$f$$

$$h$$

$$h \star f$$

$$\frac{\partial}{\partial x}(h \star f)$$



based on S. Seitz slides

Solution: convolution properties

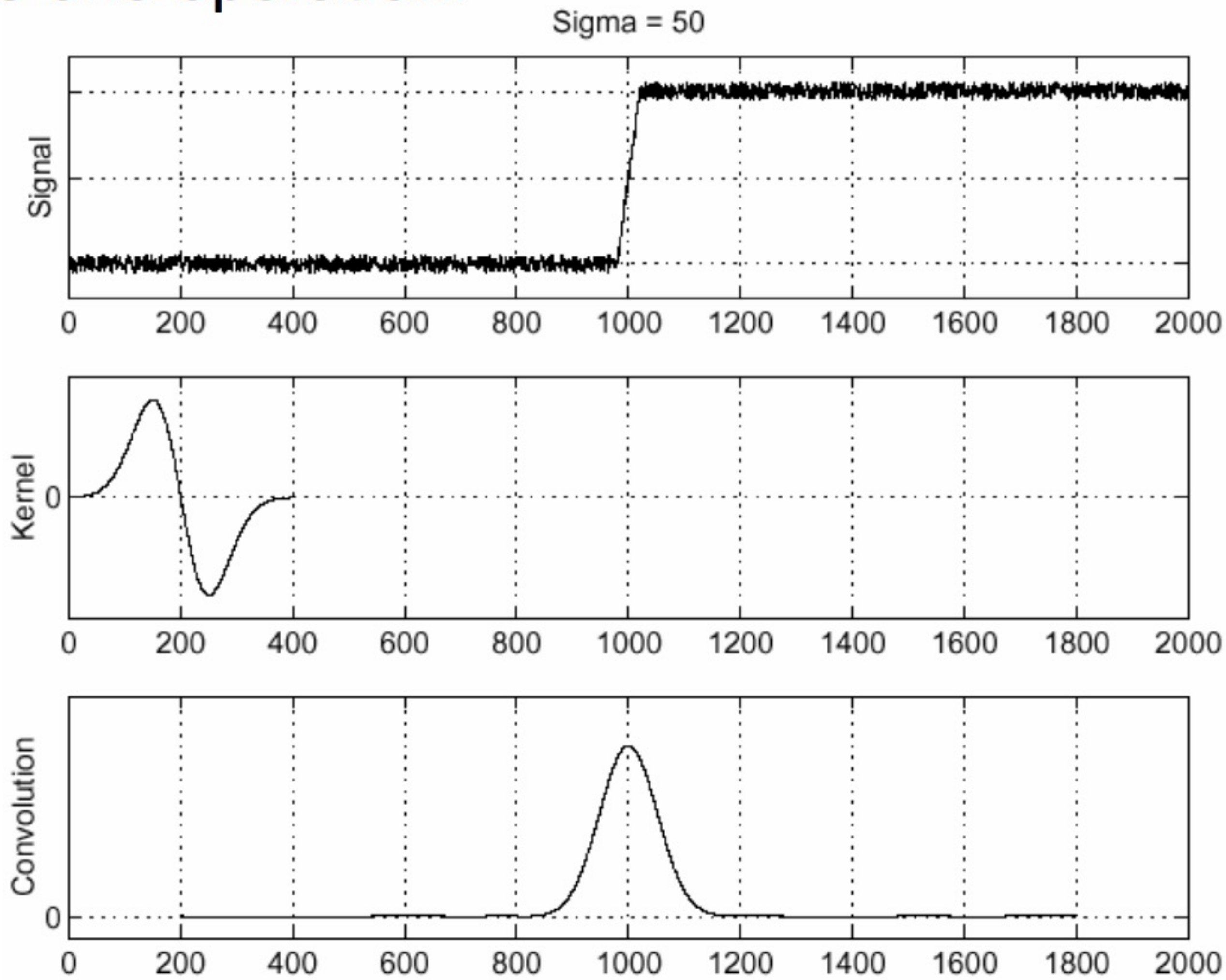
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

This saves us one operation:

$$f$$

$$\frac{\partial}{\partial x}h$$

$$(\frac{\partial}{\partial x}h) \star f$$

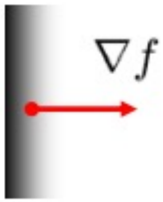


based on S. Seitz slides

Image gradient

The gradient points in the direction of most rapid change in intensity

$$\nabla f = grad(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$

$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

Gradient magnitude: $M(x,y) = \sqrt{g_x^2 + g_y^2}$

Gradient orientation: $\theta(x,y) = \arctan \frac{g_y}{g_x}$

Image gradient estimation with convolution

K

-0.5	0	0.5

Central difference

K

-1	1

Forward difference

$$g_x = K * f$$
$$g_y = K^\top * f$$

Rx

-1	0
0	1

Ry

0	-1
1	0

Roberts filter

$$g_x = R_x * f$$
$$g_y = R_y * f$$

Sx

-1	-2	-1
0	0	0
1	2	1

Sobel filter

$$g_x = S_x * f$$
$$g_y = S_y * f$$
$$S_y = S_x^\top$$

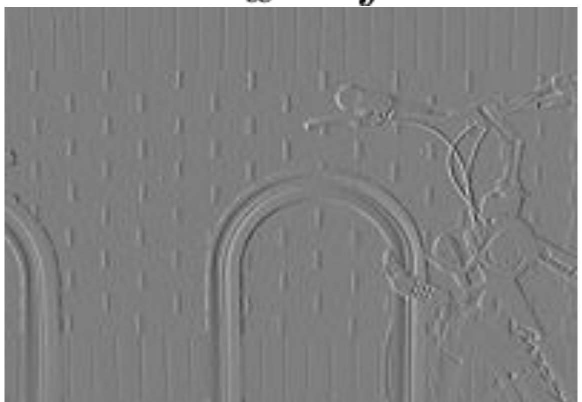
Image gradient estimation with convolution

Example – Sobel operator

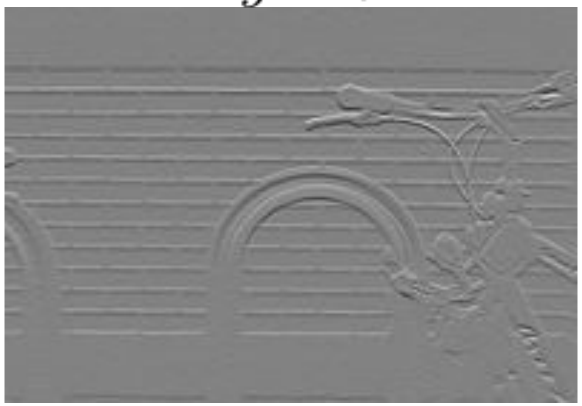
It can be decomposed as the product of an averaging and a differentiation kernel

$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$S_x * f$



$S_y * f$

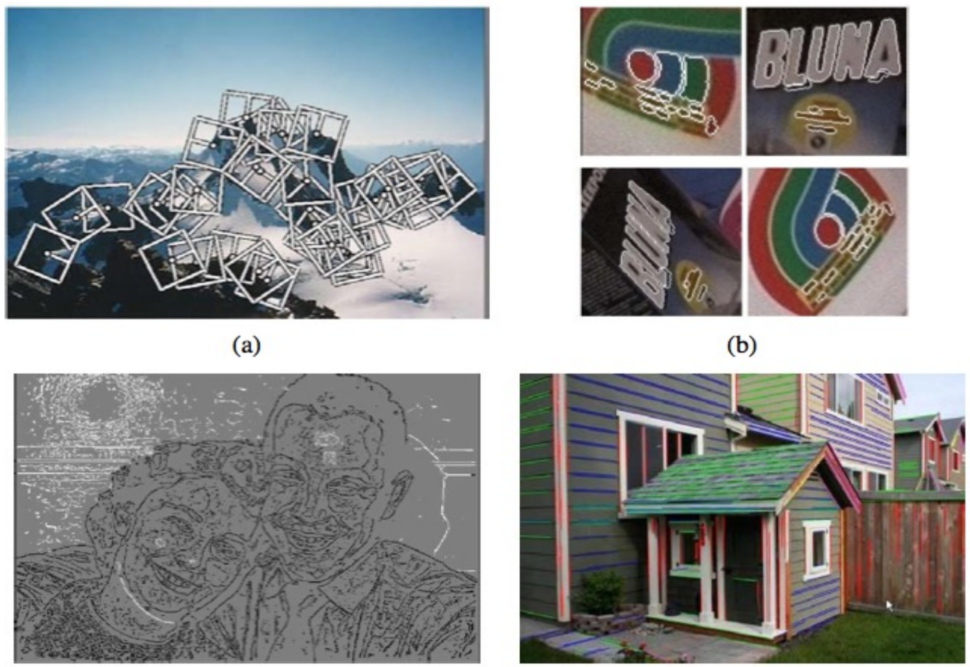


M

Feature detection

Enhancement is the starting point of feature detection algorithms

Image features are local, meaningful, detectable part of the image



(a) (b) (c) (d)

Edge points

Edge points (or edges) are pixels at or around which the image values undergo a sharp variation

Edge detection

Edges are pixels at which the image values undergo a *sharp variation*

Edge detection: given a image locate edges most likely to be generated by scene elements and not by noise

- Noise smoothing
- Edge enhancement
- Edge localization (thresholding)

Local features: edges

Noise smoothing
(gaussian filter)

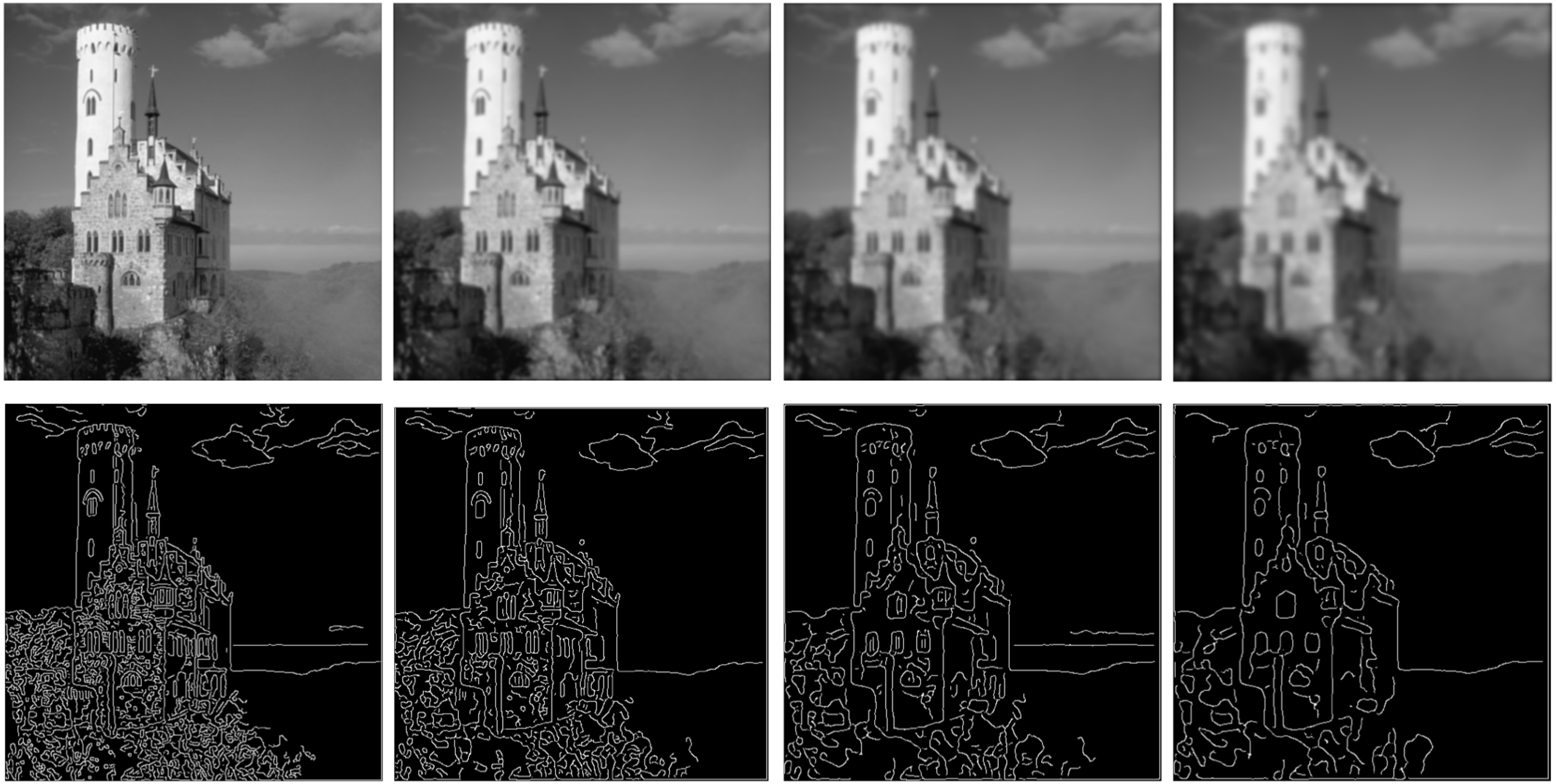
Edge enhancement
(gradient magnitude)

Edge localization
(non-maxima suppression
+ thresholding)

Reference algorithm: Canny edge detector

TRADE-OFF Localization - Detection

on the selection of appropriate parameters



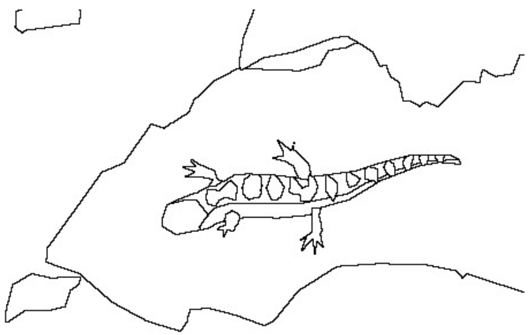
Edges or objects boundaries?

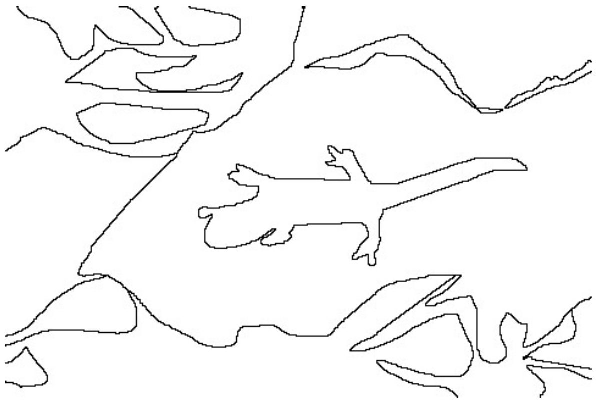
As a side note: if we are interested in image understanding why are we looking for edges?

edge detection —> boundary detection

Supervised approach: ask humans to label boundaries and learn them from examples

(see the Berkeley Segmentation Dataset BSDS500)





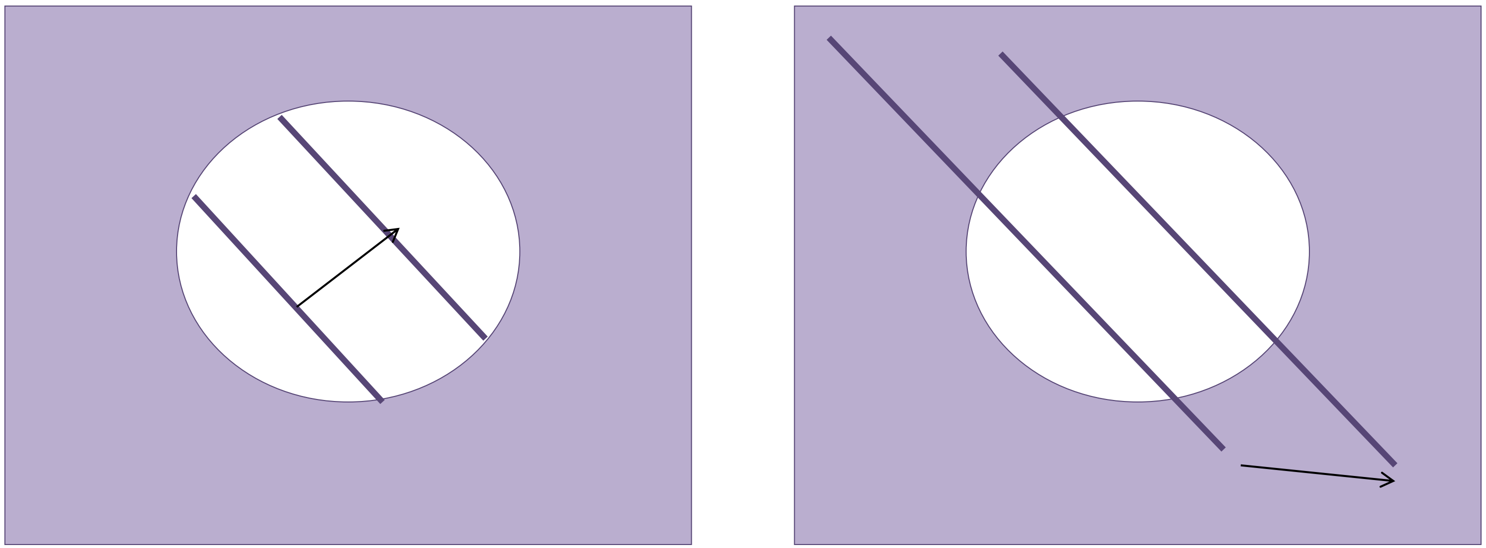
Good features to match

- A main task in CV is image matching
- A way for addressing it is to associate pixels or features in different images of the same scene by similarity

- Edges are interesting features on the image but they are not stable and they are not easy to match

Good features to match

- Edges are interesting points on the image but they are not stable and they are not easy to match



Good features to match

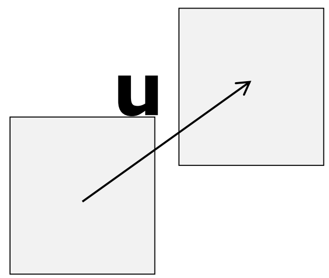
Corners

We observe how keypoints with gradients varying in at least two (significantly) different orientations are more stable

This can be formalized by analysing a simple matching criterium (Summed Square Difference)

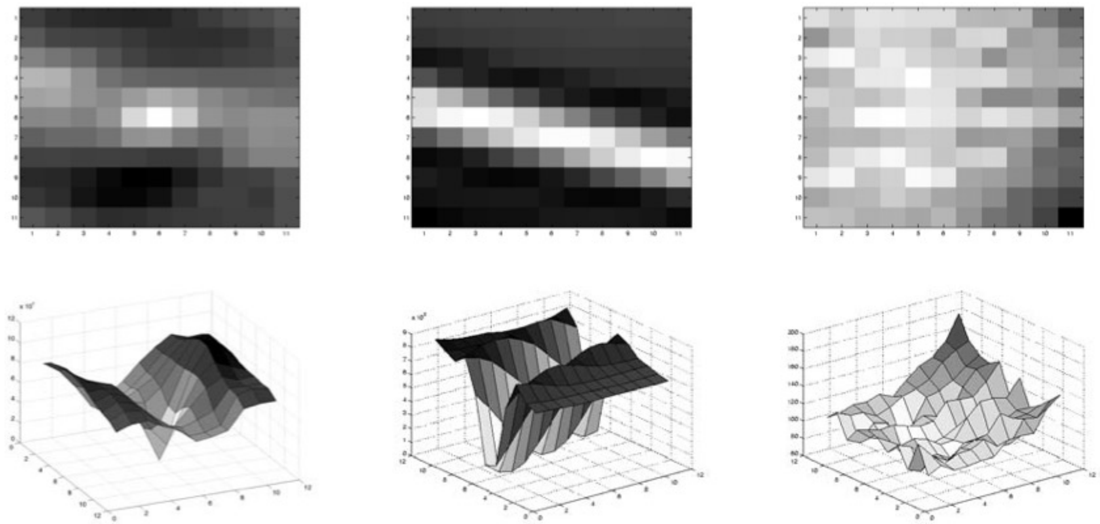
In particular we use it to check how stable the patch is wrt small variations in position **u** (SSD **autocorrelation function**):

$$E_{AC}(\mathbf{u}) = \sum_i [I(\mathbf{x}_i + \mathbf{u}) - I(\mathbf{x}_i)]^2$$



Analysing local variations with autocorrelation

(a)



Analysing local variations with autocorrelation

– Using a Taylor series expansion

$$I(\mathbf{x}_i + \mathbf{u}) = I(\mathbf{x}_i) + \nabla I(\mathbf{x}_i) \cdot \mathbf{u} + \mathcal{O}(\mathbf{x}_i^2)$$

– we obtain an auto-correlation function as follows

$$\begin{aligned}
E_{AC}(\mathbf{u}) &= \sum_i [I(\mathbf{x}_i + \mathbf{u}) - I(\mathbf{x}_i)]^2 \\
&\sim \sum_i [I(\mathbf{x}_i) + \nabla I(\mathbf{x}_i) \cdot \mathbf{u} - I(\mathbf{x}_i)]^2 \\
&= \sum_i [\nabla I(\mathbf{x}_i) \cdot \mathbf{u}]^2 \\
&= \mathbf{u}^\top A \mathbf{u}
\end{aligned}$$

$$A = \begin{bmatrix} \sum I_x^2 & \sum_p I_x I_y \\ \sum I_x I_y & \sum_p I_y^2 \end{bmatrix}$$

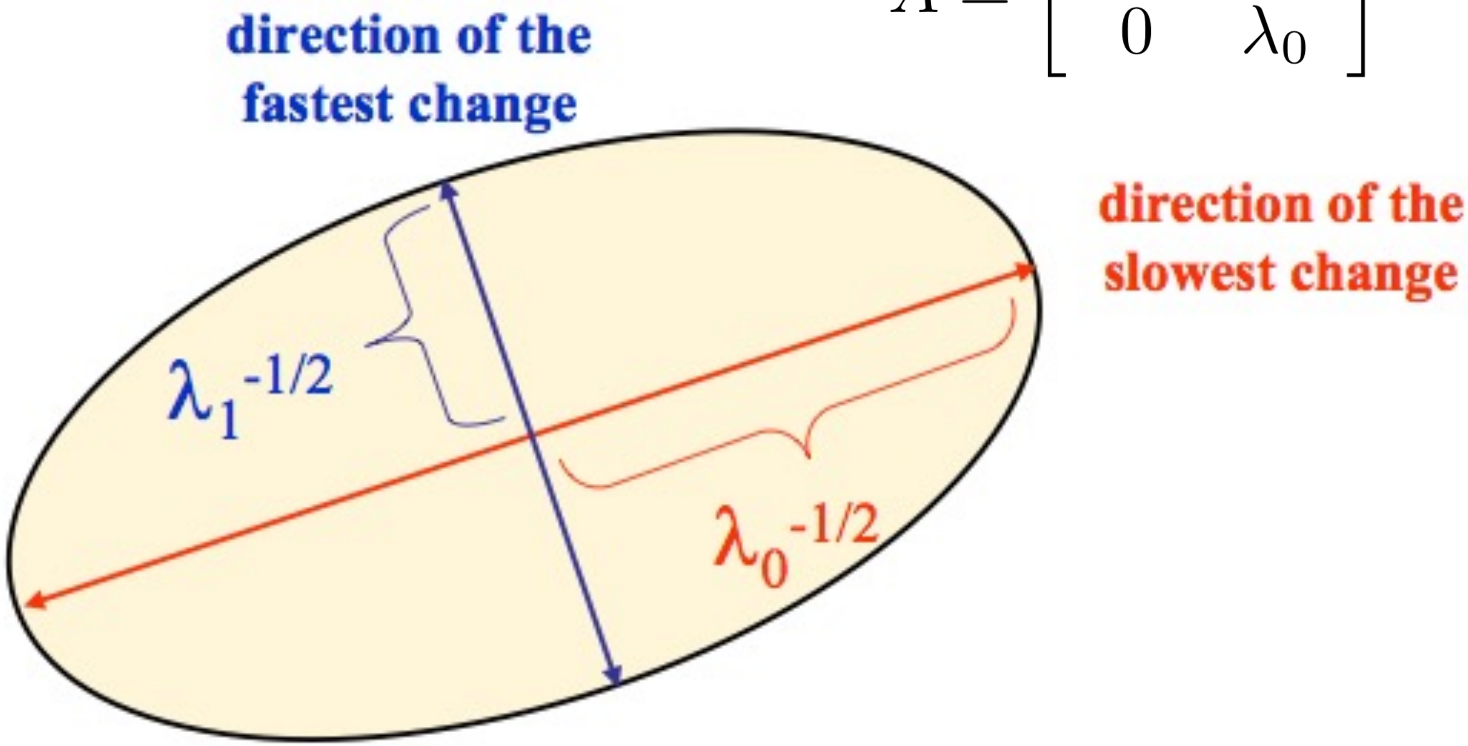
Analysing local variations with autocorrelation

– Autocorrelation matrix and its eigenvalues

$$A = \begin{bmatrix} \sum I_x^2 & \sum_p I_x I_y \\ \sum I_x I_y & \sum_p I_y^2 \end{bmatrix}$$

$$\mathrm{eig}(A) = [\lambda_1, \lambda_0] \quad \lambda_1 > \lambda_0$$

$$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_0 \end{bmatrix}$$



Corners detection (Shi-Tomasi algorithm)

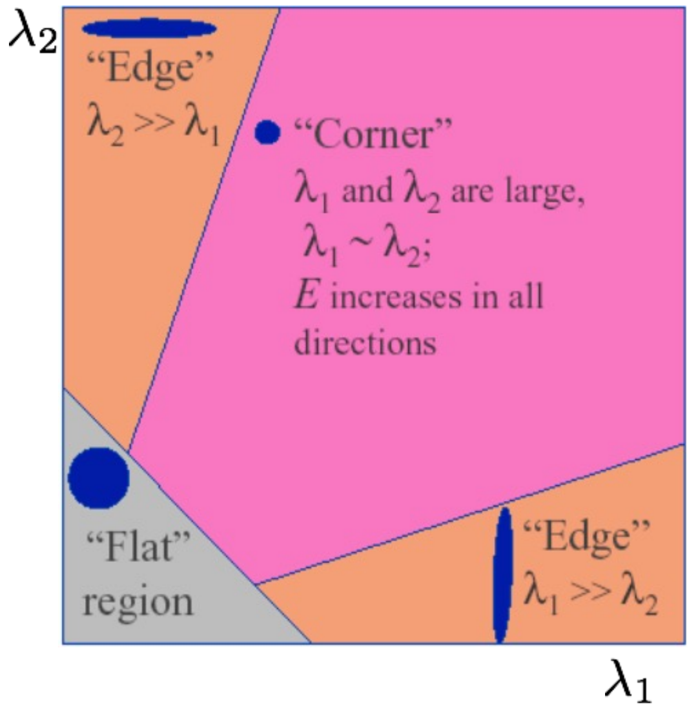
Corners correspond to points where the image gradient varies in at least two directions.

they can be detected by computing and analyzing the autocorrelation matrix A of a patch around each point

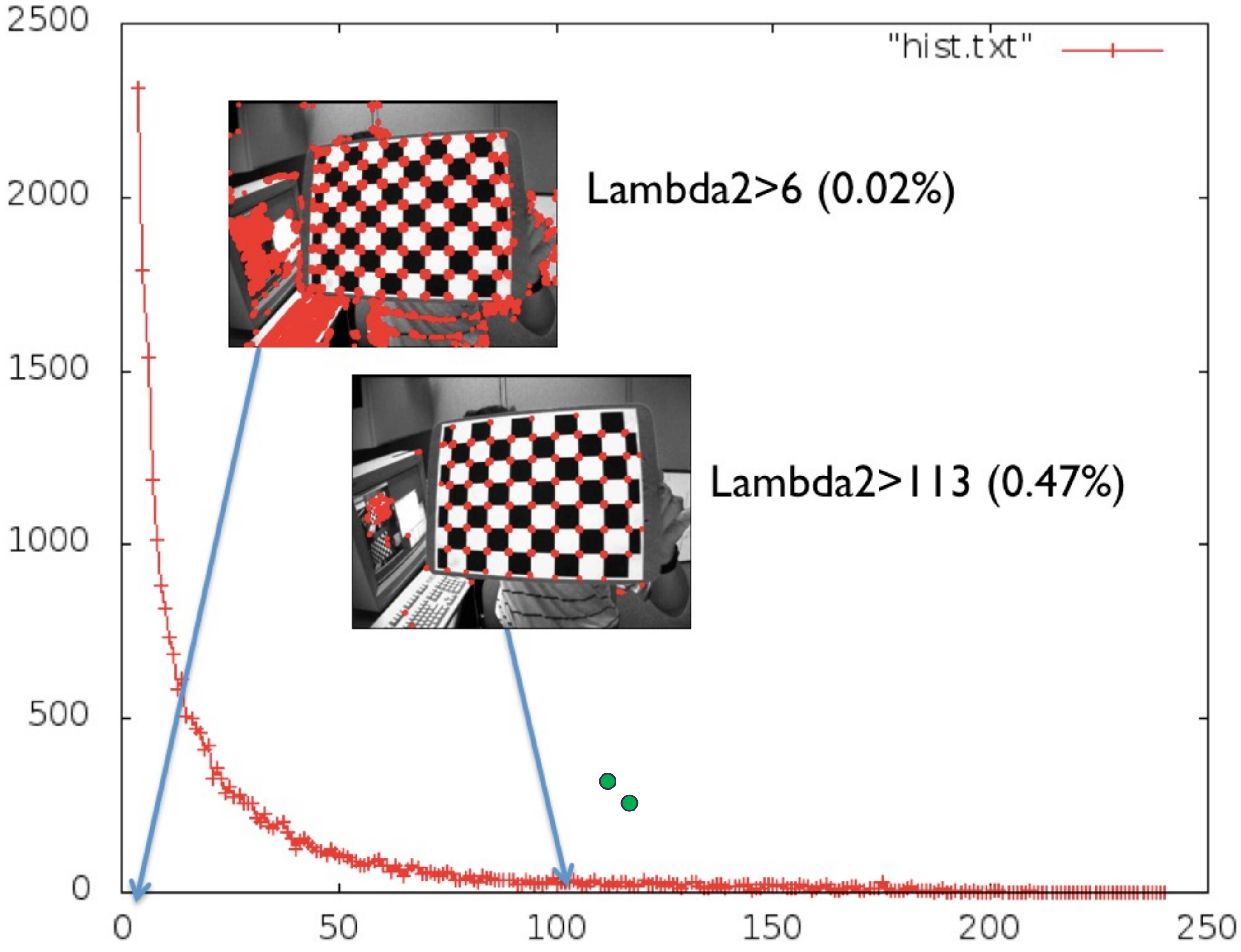
$$\nabla I = [I_x, I_y]^\top$$

$$A = \left[\begin{array}{cc} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{array} \right]$$

$$eig(A) = [\lambda_2, \lambda_1] \lambda_2 > \lambda_1$$



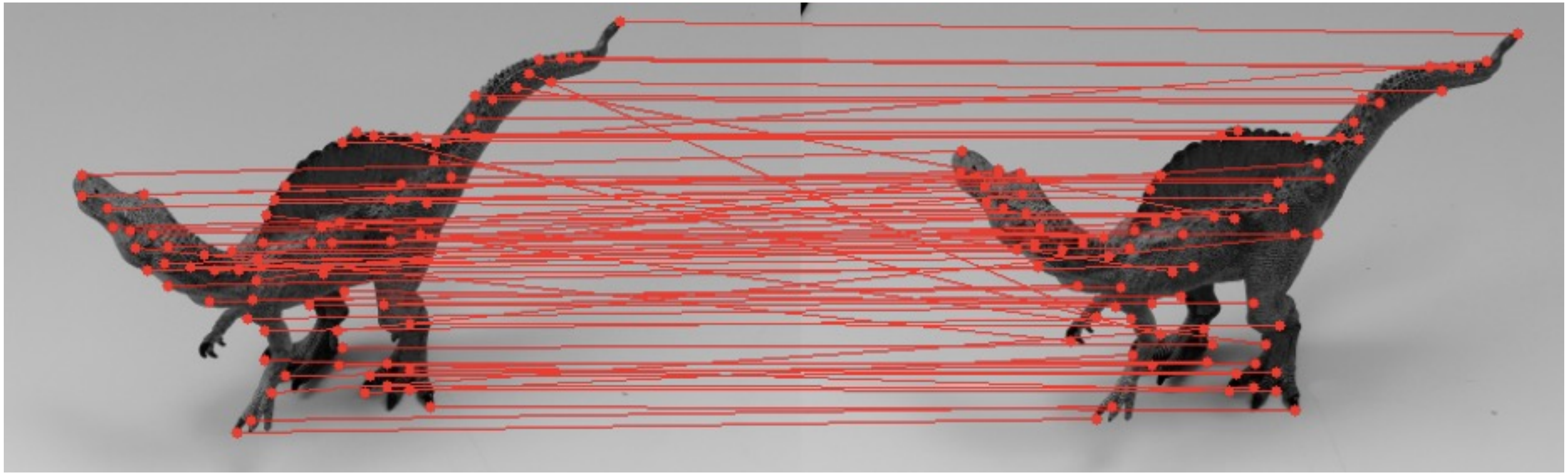
Corners detection (Shi-Tomasi algorithm)



Histogram of the Smallest eigenvalue (on all points of an image)

on the selection of appropriate parameters

Features and image matching



Pipeline

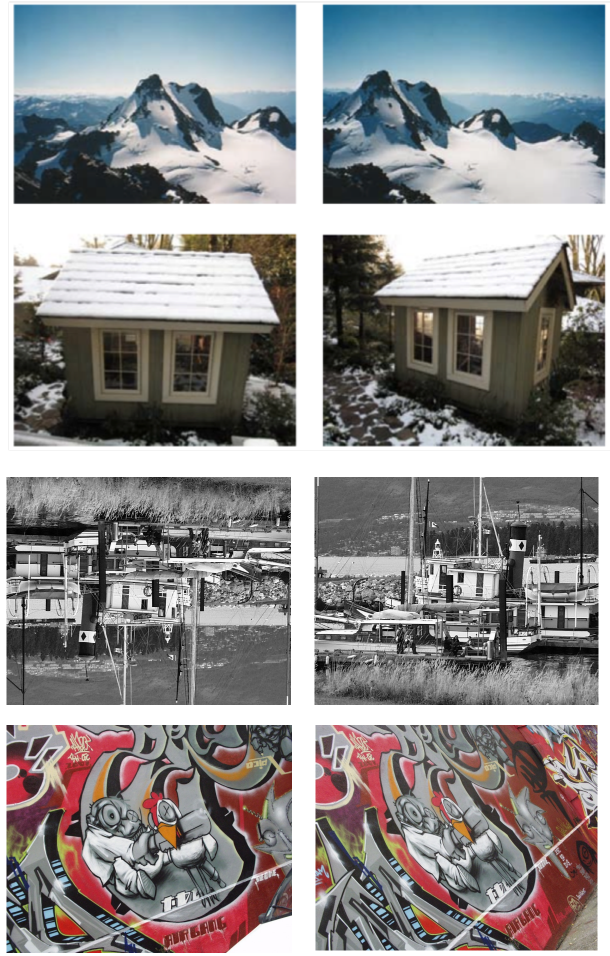
1. **Feature detection:** Find stable interest points on each image (corners)
2. **Feature description:** Compute a vector description for each point
3. **Feature matching** Compute the similarity between different feature descriptors

Local feature description

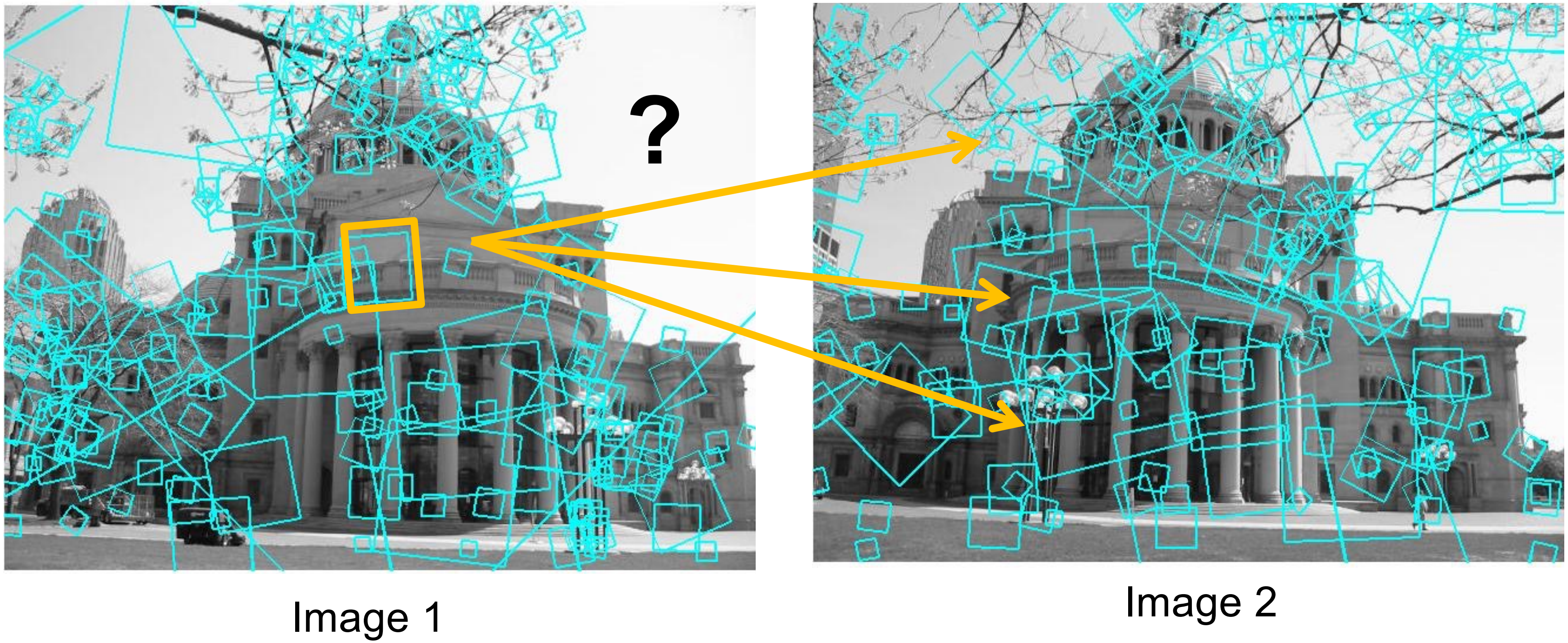
- If image pairs which are "similar enough" , then local features undergo a (quasi) translation transformation

- ***pixel neighbourhoods or larger NxN patches*** are an appropriate feature description

- In the case of scale, rotation, and more severe view-point changes we may need ***scale invariant*** interest points and better feature descriptors
  eg SIFT, SURF, ...



Matching strategy



To generate ***candidate matches***, find features with the most similar appearance

Brute force approach: compare them all, take the closest (or closest k, or within a thresholded distance)

Image credit: Kristen Grauman

Few more words on image similarity

Feature matching can be used, for instance, stitch images and compute panoramas

It can also be a way to assess (indirectly) image similarity -> how many features find a good match?

Other ways to use local features if we are interested in finding global similarities ... how about.... histograms of local features?

Bag of words

The inspiration comes from text analysis

Michael Schumacher (German pronunciation: [ˈmɪçaʔeːl ˈʃuːmaxɐ] (listen); born 3 January 1969) is a retired German racing driver. Schumacher is a seven-time Formula One World Champion and is widely regarded as one of the greatest F1 drivers of all time.[1][2][3][4] He holds many of Formula One's driver records, including most championships, race victories, fastest laps, pole positions, points scored and most races won in a single season – 13 in 2004. In 2002 he became the only driver in Formula One history to finish in the top three in every race of a season and then also broke the record for most consecutive podium finishes. According to the official Formula One website he is "statistically the greatest driver the sport has ever seen".[5]

After beginning with karting, Schumacher won German drivers' championships in Formula König and Formula Three before joining Mercedes in the World Sportscar Championship. After one Mercedes-funded race for the Jordan Formula One team, Schumacher signed as a driver for the Benetton Formula One team in 1991. After winning consecutive championships with Benetton in 1994/5, Schumacher moved to Ferrari in 1996 and won another five consecutive drivers' titles with them from 2000 to 2004. Schumacher retired from Formula One driving in 2006 staying with Ferrari as an advisor.[6] Schumacher agreed to return for Ferrari part-way through 2009, as cover for the badly injured Felipe Massa, but was prevented by a neck injury. He later signed a three-year contract to drive for the new Mercedes GP team starting in 2010.[7][8][9]

His career has not been without controversy, including being twice involved in collisions in the final race of a season that determined the outcome of the world championship, with Damon Hill in 1994 in Adelaide, and with Jacques Villeneuve in 1997 in Jerez.[10] Off the track Schumacher is an ambassador for UNESCO and a spokesman for driver safety. He has been involved in numerous humanitarian efforts throughout his life and donated tens of millions of dollars to charity.[11] Michael and his younger brother Ralf Schumacher are the only brothers to win races in Formula One, and they were the first brothers to finish 1st and 2nd in the same race, in Montreal in 2001. The two brothers repeated this achievement in four more races (the 2001 French Grand Prix, the 2002 Brazilian Grand Prix, the 2003 Canadian Grand Prix and the 2004 Japanese Grand Prix).

Term frequency

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Bag of keypoints

Michael Schumacher (German pronunciation: [ˈmɪçaʔeːl ˈʃuːmaxɐ] (listen); born 3 January 1969) is a retired German racing driver. Schumacher is a seven-time Formula One World Champion and is widely regarded as one of the greatest F1 drivers of all time.[1][2][3][4] He holds many of Formula One's driver records, including most championships, race victories, fastest laps, pole positions, points scored and most races won in a single season – 13 in 2004. In 2002 he became the only dri Formula One history to finish in the top three in every race of a season and then also broke the record for most co podium finishes. According to the official Formula One website he is "statistically the greatest driver the sport has seen".[5]

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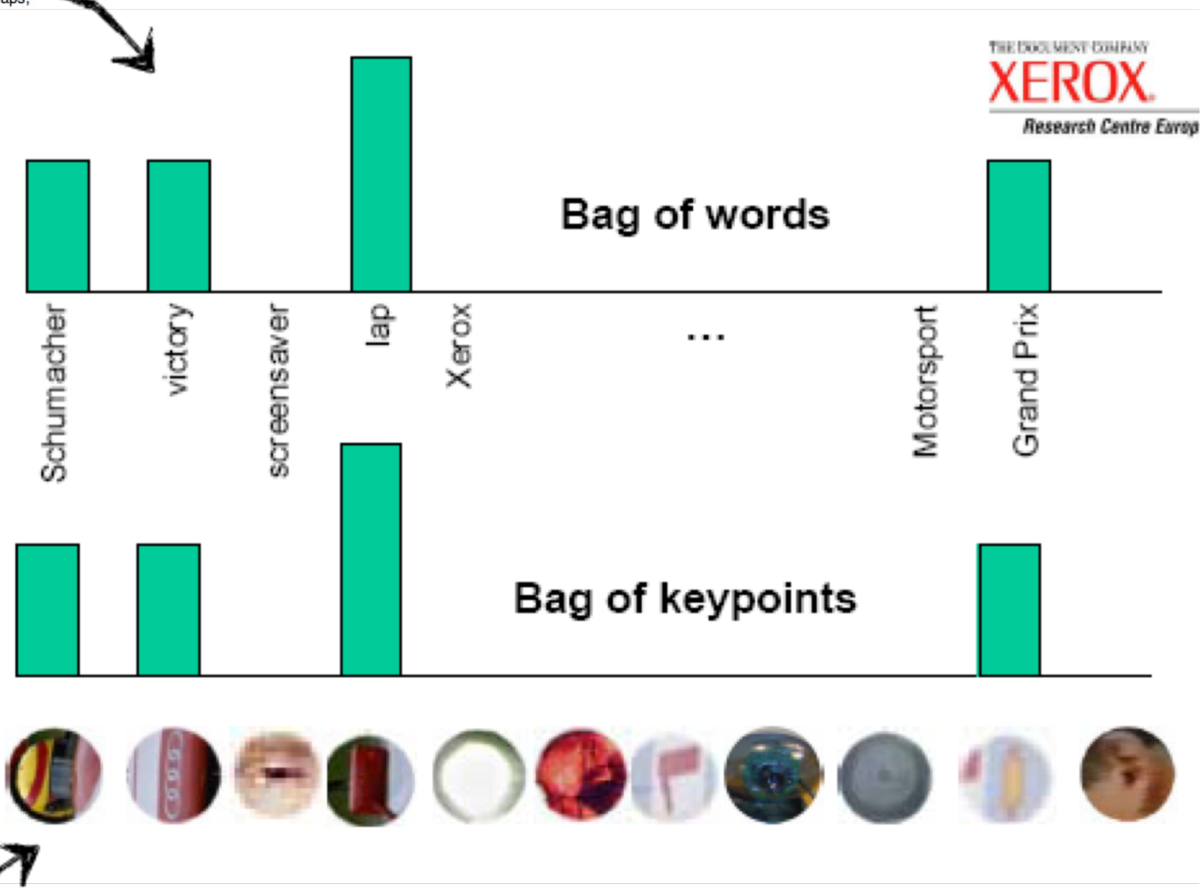

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