Problem 1

a) 3d Orbital

$n = 3$, $l = 2$

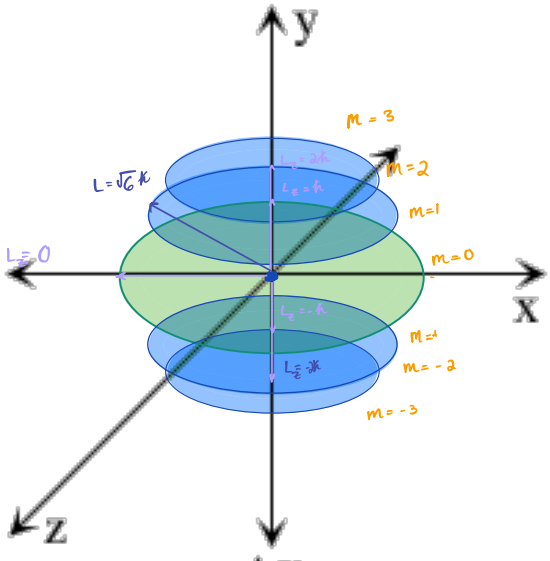
$m = 0, \pm 1, \pm 2$

Angular nodes $= 2$

Radial nodes $= n - l - 1$
$= 3 - 2 - 1 = 0$

$L = \sqrt{l(l+1)}\,\hbar = \sqrt{6}\,\hbar$

$L_z = 0, \pm\hbar, \pm 2\hbar$



b) 4f Orbital

$n = 4$, $l = 3$

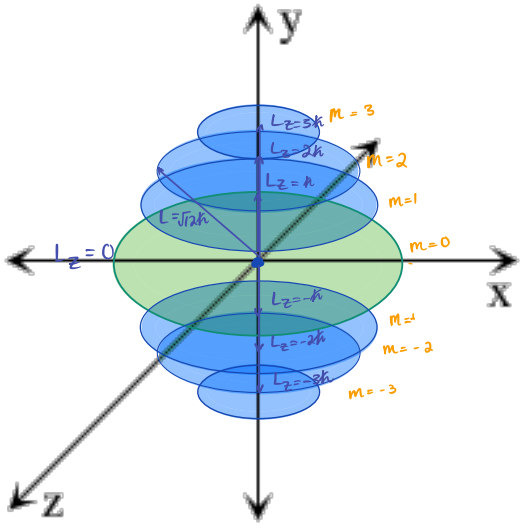
$m = 0, \pm 1, \pm 2, \pm 3$

Angular nodes $= l = 3$

Radial nodes $= n - l - 1$
$= 4 - 3 - 1 = 0$

$L = \sqrt{l(l+1)}\,\hbar = \sqrt{12}\,\hbar$

$L_z = 0, \pm\hbar, \pm 2\hbar, \pm 3\hbar$



c) 3s Orbital

$n = 3$, $l = 0$

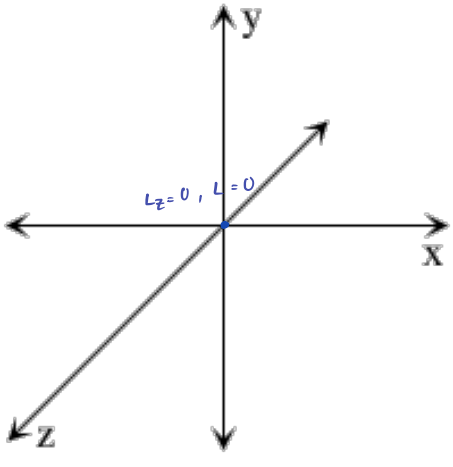
$m = 0$

Angular nodes $= l = 0$

Radial nodes $= n - l - 1$
$= 3 - 0 - 1 = 2$

$L = \sqrt{l(l+1)}\,\hbar = \sqrt{0}\,\hbar = 0$

$L_z = 0$



Problem 2:

Average value formula : $\langle x \rangle = \int_{\text{all space}} x \psi^* \psi \, dx$

$$\Rightarrow \langle r \rangle = \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \psi^* r \psi \, r^2 dr \sin(\theta) \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \left[\left(\frac{1}{4\sqrt{2\pi}} \right)^2 \left(\frac{Z}{a_0} \right)^3 \left(\frac{r}{a_0} \right)^2 e^{-\frac{r}{a_0}} \cos^3\theta \, r^3 \right] dr \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{32\pi} \cdot \frac{Z^3}{a_0^3} \cdot \frac{1}{a_0^2} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} e^{-\frac{r}{a_0}} \cos^3\theta \, r^5 \, dr \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \frac{1}{32\pi} \frac{1}{a_0^5} \int_0^{2\pi} d\phi \int_0^{\pi} \cos^3\theta \sin\theta \, d\theta \int_0^{\infty} r^5 e^{\frac{-r}{a_0}} dr$$

$$= \frac{1}{32\pi a_0^5} (2\pi) \left(-\frac{1}{3} (-1 - 1) \right) \left(120 a_0^6 \right) = 5a_0 < 6a_0$$