June 14, 2022 Problems

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Problems

3.1: Which of the following functions are eigenfunctions of the inversion operator ($\hat{l}$), which essentially transforms x into -x. For any functions which are eigenfunctions, give the associated eigenvalue. For any functions which are eigenfunctions, give the associated eigenvalue.

$$a) x^3 - kx \qquad b) \cos(kx) \qquad c) x^2 + 3x - 1$$

$$a)\ \hat{A}f(x) = af(x) = f(-x) = -x^3 + kx \qquad \text{It is an eigenfunction. The eigenvalue is } -1$$

$$b)\ \hat{A}f(x) = af(x) = f(-x) = \cos(-kx) \qquad \text{It is an eigenfunction. The eigenvalue is } 1$$

$$c)\ \hat{A}f(x) = af(x) = f(-x) = x^2 - 3x - 1 \qquad \text{It is not an eigenfunction.}$$

3.2: if $[\hat{A}, \hat{B}] = 0$, does $[\hat{A}^2, \hat{B}] = \hat{A}[\hat{A}, \hat{B}] + [\hat{A}, \hat{B}]\hat{A}$?

$$[\hat{A}^2, \hat{B}] = \hat{A}[\hat{A}, \hat{B}] + [\hat{A}, \hat{B}]\hat{A} \rightarrow \hat{A}^2\hat{B} - \hat{B}\hat{A}^2 = \hat{A}(\hat{A}\hat{B} - \hat{B}\hat{A}) + (\hat{A}\hat{B} - \hat{B}\hat{A})\hat{A}$$

$$\hat{A}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{A} = \hat{A}\hat{A}\hat{B} - \hat{A}\hat{B}\hat{A} + \hat{A}\hat{B}\hat{A} - \hat{B}\hat{A}\hat{A} = \hat{A}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{A}$$

$$\therefore \text{ Both sides are equal. Hence, it is proven that } [\hat{A}^2, \hat{B}] = \hat{A}[\hat{A}, \hat{B}] + [\hat{A}, \hat{B}]\hat{A}$$