CHEM4502 - HW1 - Created:2022-06-09

Chapter 1

1.18: When a clean surface of silver is irradiated with light of wavelength 230 nm, the kinetic energy of the ejected electrons is found to be 0.805 eV. Calculate the work function and the threshold frequency of silver

$$eKE = hv - \phi = \frac{hc}{\lambda} - \phi \qquad \rightarrow \qquad 0.805(1.60 \times 10^{-19}) = \frac{6.626 \times 10^{-34}(2.997 \times 10^{8})}{(230 \times 10^{-9})} - \phi$$

$$\boxed{\phi = 7.346 \times 10^{-19} J}$$

$$\text{Threshold Frequency is when eKE} = 0 \qquad \rightarrow \qquad v = \frac{\phi}{h} = \frac{7.346 \times 10^{-19}}{6.626 \times 10^{-34}} = \boxed{1.110 \times 10^{15}\ s^{-1}}$$

1.21: A line in the Lyman series of hydrogen has a wavelength of 1.03 × 10-7 m. Find the original energy level of the electron.

$$\frac{1}{\lambda} = Z^2 R_H \left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right] \qquad \text{Note that for Lyman Series, } n_i = 1$$

$$\frac{1}{1.03 \times 10^{-7} m^{-1}} = 1^2(1.097 \times 10^{7}) \left[\frac{1}{1} - \frac{1}{n_f^2}\right] \qquad \text{Solve for } n_f$$

$$\boxed{n_f = 2.94 \approx 3}$$

1.24: Calculate the wavelength and the energy of a photon associated with the series limit of the Lyman series.

$$\frac{1}{\lambda} = Z^2 R_H \left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right] \qquad \text{Note that for Lyman Series, } n_i = 1$$

$$\text{Note that the series limit happens at } n_f = \infty$$

$$\frac{1}{\lambda} = 1^2(1.097 \times 10^{7} m^{-1}) \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] \qquad \text{Solve for } \lambda \text{ in m}$$

$$\boxed{\lambda = 9.12 \times 10^{-8} m}$$

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} Js)(2.9979 \times 10^{8} ms^{-1})}{9.12 \times 10^{-8} m} = \boxed{2.178 \times 10^{-18} J}$$

1.28: Calculate the energy and wavelength associated with an a particle that has fallen through a potential difference of 4.0 V. Take the mass of an a particle to be 6.64 × 10-27 kg.

$$\text{Kinetic Energy and Potential Formula: } KE = qV$$

$q =$ Charge of particle
$V =$ Potential

$$KE = qV = ((2)(1.60 \times 10^{-19}))(4.0V) = \boxed{1.28 \times 10^{-18} J}$$

$$KE = 0.5mv^2 \qquad \text{Solve for } v \ \rightarrow v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(1.28 \times 10^{-18})}{6.64 \times 10^{-27}}} = 19635.2\ ms^{-1}$$

$$\lambda = \frac{h}{mu} = \frac{6.626 \times 10^{-}34}{(6.64 \times 10^{-27}(19635.2))}$$

$$\boxed{\lambda = 5.08 \times 10^{-12}\ m = 5.08 \times 10^{-3} nm}$$

1.31: Derive the Bohr formula for v for a nucleus of atomic number Z.

$$E = hv, \text{ where } v = c\tilde{v} \qquad \therefore E = hc\tilde{v}$$

$$\text{Bohr's Formula for Energy} \qquad E = hc\tilde{v} = -\frac{Z^2 m_e e^4}{8\epsilon_o^2 h^2 n^2} = Z^2 R_H[\frac{1}{n_i^2} - \frac{1}{n_f^2}]$$

$$\text{Solve for } \tilde{v} \ \rightarrow \tilde{v} = -\frac{Z^2 m_e e^4}{8\epsilon_o^2 h^2 n^2 ch} = Z^2 R_H[\frac{1}{n_i^2} - \frac{1}{n_f^2}]$$

$$\text{Where } R_H = -\frac{Z^2 m_e e^4}{8\epsilon_o^2 h^2 n^2 ch} \qquad \rightarrow \boxed{\tilde{v} = Z^2 R_H[\frac{1}{n_i^2} - \frac{1}{n_f^2}]}$$

1.32: The series in the He+ spectrum that corresponds to the set of transitions where the electron falls from a higher level into the n = 4 state is called the Pickering series, an important series in solar astronomy. Derive the formula for the wavelengths of the observed lines in this series. In what region of the spectrum does it occur? (See Problem 1-31.)

$$\text{Problem 1.31 Formula: } \tilde{v} = Z^2 R_H[\frac{1}{n_i^2} - \frac{1}{n_f^2}] = \frac{1}{\lambda}$$

☐Minimum: Z will be equal to 2 and $n_f = 5$

$$\tilde{v} = 2^2(1.097 * 10^7 m^{-1})[\frac{1}{4^2} - \frac{1}{5^2}] = \frac{1}{\lambda} \qquad \text{Solve for } \lambda$$

$$\boxed{\lambda = 1.013 \times 10^{-6} m}$$

☐Maximum: Z will be equal to 2 and $n_f = \infty$

$$\tilde{v} = 2^2(1.097 * 10^7 m^{-1})[\frac{1}{4^2} - \frac{1}{\infty^2}] = \frac{1}{\lambda} \qquad \text{Solve for } \lambda$$

$$\boxed{\lambda = 3.646 \times 10^{-7} m}$$

$\therefore$ This will be between the ultraviolet and the infrared regions (Refer to Figure 1.7)

1.36: What is the uncertainty of the momentum of an electron if we know its position is somewhere in a 10 pm interval? How does the value compare to momentum of an electron in the first Bohr orbit?

$$\Delta x \Delta p \geq \frac{\hbar}{2} \ \rightarrow \Delta p \geq \frac{\hbar}{(2)\Delta x} \rightarrow \Delta p \geq \frac{1.05 \times 10^{-34} Js}{(2)(10 \times 10^{-12} m)} \qquad \rightarrow \Delta p = 5.25 \times 10^{-24}$$

$$p = m_e v = m_e \left(\frac{e^2}{2nh\epsilon_o}\right) = 9.11 \times 10^{-31} \left(\frac{(1.60 \times 10^{-19})^2}{2(1)(6.626 \times 10^{-34})(8.85 \times 10^{-12})}\right)$$

$$\text{Solve for } p \rightarrow \boxed{p = 1.989 \times 10^{-24} kg\ m\ s^{-1}}$$

Chapter 2

2.8: Consider the linear second-order differential equation

$$\frac{d^2 y}{dx^2} + a_1(x)\frac{dy}{dx} + a_o(x)y(x) = 0$$

Note that this equation is linear because y(x) and its derivatives appear only to the first power and there are no cross terms. It does not have constant coefficients, however, and there is no general, simple method for solving it like there is if the coefficients were constants. In fact, each equation of this type must be treated more or less individually. Nevertheless, because it is linear, we must have that if y1 (x) and y2(x) are any two solutions, then a linear combination,

$$y(x) = c_1 y_1(x) + c_2 y_2(x)$$

where c_1 and c_2 are constants, is also a solution. Prove that y(x) is a solution.

Take the first and second derivatives of $y(x)$:

$$\frac{dy}{dx} = c_1\frac{dy_1}{dx} + c_2\frac{dy_2}{dx}$$

$$\frac{d^2y}{dx^2} = c_1\frac{d^2y_1}{dx^2} + c_2\frac{d^2y_2}{dx^2}$$

$$\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_o(x)y(x) = c_1\frac{d^2y_1}{dx^2} + c_2\frac{d^2y_2}{dx^2} + a_1(c_1\frac{dy_1}{dx} + c_2\frac{dy_2}{dx}) + a_0(x)(c_1y_1(x) + c_2y_2(x))$$

$$= c_1\left(\frac{d^2y_1}{dx^2} + a_1\frac{dy_1}{dx} + a_0(x)y_1(x)\right) + c_2\left(\frac{d^2y_2}{dx^2} + a_1\frac{dy_2}{dx} + a_0(x)y_2(x)\right)$$

From the equation provided in the statement of the problem:

$$\boxed{= c_1(0) + c_2(0) = 0 + 0 = 0}$$

Math Chapter B

B.1: Consider a particle to be constrained to lie along a one-dimensional segment 0 to a. We will learn in the next chapter that the probability that the particle is found to lie between x and x + dx is given by:

$$p(x)dx = \frac{2}{a}sin^2(\frac{n\pi x}{a})dx$$

where n = 1, 2, 3, ... First show that p(x) is normalized. Now calculate the average position of the particle along the line segment.

$$P = \int_0^a p(x)dx = \int_0^a \frac{2}{a}sin^2(\frac{n\pi x}{a})dx = \frac{2}{a}\left[\frac{x}{2} - \frac{sin\left(2(\frac{n\pi}{a})x\right)}{4(\frac{n\pi}{a})}\right]_0^a$$

$$= \frac{2}{a}(\frac{a}{2} - \frac{sin(2n\pi)}{\frac{4n\pi}{a}}) - \frac{2}{a}(0 - \frac{sin(0)}{\frac{4n\pi}{a}}) = \frac{2}{a}(\frac{a}{2}) + \frac{2}{a}(0) = 1$$

$$\boxed{\therefore \text{Based on the value of the integral shown above, it is inferred that } p(x) \text{ is normalized.}}$$

Thus, a new variable will be introduced into the integral (x in this case) :

$$P = \int_0^a xp(x)dx = \int_0^a x\frac{2}{a}sin^2(\frac{n\pi x}{a})dx = \frac{2}{a}\left[\frac{x^2}{4} - \frac{x\sin(\frac{2n\pi}{a}x)}{\frac{4n\pi}{a}} - \frac{\cos(\frac{2n\pi}{a}x)}{8(\frac{n\pi}{a})^2}\right]_0^a$$

$$= \frac{2}{a}\left(\frac{a^2}{4} - \frac{a\sin(2n\pi)}{\frac{4n\pi}{a}} - \frac{\cos(2n\pi)}{8(\frac{n\pi}{a})^2}\right) - \frac{2}{a}\left(\frac{0^2}{4} - \frac{a\sin(0)}{\frac{4n\pi}{a}} - \frac{\cos(0)}{8\left(\frac{n\pi}{a}\right)^2}\right)$$

$$= \frac{2}{a}(\frac{a^2}{4} - \frac{1}{8(\frac{n\pi}{a})^2}) - \frac{2}{a}(0 - \frac{1}{8(\frac{n\pi}{a})^2}) = \frac{2}{a}(\frac{a^2}{4}) = \boxed{\frac{a}{2}}$$

B.3: Using the probability distribution given in Problem B-1, calculate the probability that the particle will be found between 0 and aj2. The necessary integral is given in Problem B-1.

$$P = \int_0^{\frac{a}{2}} p(x)dx = \int_0^{\frac{a}{2}} \frac{2}{a}sin^2(\frac{n\pi x}{a})dx = \frac{2}{a}\left[\frac{x}{2} - \frac{sin\left(2(\frac{n\pi}{a})x\right)}{4(\frac{n\pi}{a})}\right]_0^{\frac{a}{2}}$$

$$= \frac{2}{a}(\frac{a}{4} - \frac{sin(2n\pi)}{\frac{8n\pi}{a}}) - \frac{2}{a}(0 - \frac{sin(0)}{\frac{4n\pi}{a}}) = \frac{2}{a}(\frac{a}{4}) + \frac{2}{a}(0) = \boxed{\frac{1}{2}}$$