s 与 t 的符号意义

s：时域微分 $\quad sF(s) \to \frac{d}{dt}f(t)$

$\frac{1}{s}$：时域积分 $\quad \frac{1}{s}F(s) \to \int_0^t f(x)\,dx$

t：频域微分 $\quad tf(t) \to -\frac{d}{ds}F(s) \qquad t^n f(t) \to (-1)^n \frac{d^n}{ds^n}F(s)$

$\frac{1}{t}$：频域积分 $\quad \frac{1}{t}f(t) \to \int_s^{+\infty} F(s)\,ds$

(i) For linear factor $as+b$ of $Q(s)$, exists a partial fraction of the form

$$\frac{A}{as+b}, \qquad A \text{ constant}$$

(ii) For repeated linear factor $(as+b)^n$ of $Q(s)$, exists a partial fraction of the form

$$\frac{A_1}{as+b}+\frac{A_2}{(as+b)^2}+\cdots+\frac{A_n}{(as+b)^n}, \qquad A_1, A_2, \ldots, A_n \text{ constants}$$

(iii) For quadratic factor as^2+bs+c of $Q(s)$, exists a partial fraction of the form

$$\frac{As+B}{as^2+bs+c}, \qquad A, B \text{ constants}$$

(iv) For repeated quadratic factor $(as^2+bs+c)^n$ of $Q(s)$, exists a partial fraction of the form

$$\frac{A_1s+B_1}{as^2+bs+c}+\frac{A_2s+B_2}{(as^2+bs+c)^2}+\cdots+\frac{A_ns+B_n}{(as^2+bs+c)^n}, \qquad A_1, \ldots, A_n, B_1, \ldots, B_n \text{ constants}$$

关于 s 的分式多项式进行 Laplace 反变换

求系数的通用方法：

① 系数所在分母为该式的最高次项，则同乘该项，令 s 取值使其余系数所在项为 0.

② 不为   则乘最高次项，并求导

Example:

$$\mathcal{L}^{-1}\left(\frac{s+1}{s^2(s-1)}\right) \xrightarrow{\text{Partial Fraction}} F(s)=\frac{s+1}{s^2(s-1)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s-1}$$

求系数的方法：乘分母，消去其余项

$$C=(s-1)F(s)|_{s=1}=2, \quad B=s^2F(s)\big|_{s=0}=-1, \quad A=\boxed{\frac{d}{ds}}\left(s^2F(s)\right)\big|_{s=0}=-2$$

存在比该项分母更高阶的分母项时用微分

Case：对微分方程进行拉普拉斯变换后仍为微分方程（ $aF(s)+bF'(s)+C=0$ ）

$$y''+ty'-2y=4 \qquad y(0)=-1 \qquad y'(0)=0$$

$$\boxed{\begin{aligned} &L(t\,y'(t)) = -\frac{d}{ds}\left(sY(s)\right) = -\left(Y(s)+sY'(s)\right) \\ &L(y'(t)) = sY(s) \end{aligned}}$$

$$\left[s^2Y(s) - sy(0) - y'(0)\right] - \left[Y(s)+sY'(s)\right] - 2Y(s) = \frac{4}{s}$$

$$s^2Y(s)+s-3Y(s)-sY'(s) = \frac{4}{s} \qquad (s^2-3)Y(s)-sY'(s)+s=\frac{4}{s} \qquad \left(s-\frac{3}{s}\right)Y(s)-Y'(s)=\frac{4}{s^2}-1$$

$$\boxed{Y'(s)+\left(\frac{3}{s}-s\right)Y(s) = 1-\frac{4}{s^2}}$$

$\downarrow$ 一般求解微分方程的办法

Intergrating Factor $\mu(s) = e^{\int\left(\frac{3}{s}-s\right)ds} = s^3e^{-s^2/2}$

微分方程两边同乘积分因子 $\mu(s)$，使得左边变为某个函数的导数形式

$$\mu'(s) = 3s^2e^{-s^2/2} + s^3\cdot e^{-s^2/2}\cdot(-s) = 3s^2e^{-s^2/2} - s^4e^{-s^2/2} = \left(\frac{3}{s}-s\right)\cdot s^3e^{-s^2/2} = \left(\frac{3}{s}-s\right)\mu(s)$$

左边：

$$\mu(s)\cdot Y'(s) + \mu(s)\cdot\left(\frac{3}{s}-s\right)Y(s) = \mu(s)Y'(s)+\mu'(s)Y(s) = \left(\mu(s)Y(s)\right)'$$

右边：

$$\left(1-\frac{4}{s^2}\right)\cdot\mu(s) = \left(1-\frac{4}{s^2}\right)\cdot\left(s^3e^{-s^2/2}\right)$$

方程：

$$\left(Y(s)\cdot s^3e^{-s^2/2}\right)' = s^3e^{-s^2/2} - 4se^{-s^2/2}$$

两边积分：

$$Y(s)\cdot s^3e^{-s^2/2} = \int s^3e^{-s^2/2}\,ds - \int 4se^{-s^2/2}\,ds$$

$$\Downarrow$$

$$Y(s) = \frac{2!}{s^3} - \frac{1}{s} + \underline{\frac{C}{s^3}e^{s^2/2}}$$

$\swarrow$?

$$y(t) = L^{-1}(Y(s)) = t^2-1 + 0$$

Free Undamped Vibration

$$m\ddot{x} + kx = 0 \quad \Rightarrow \quad \ddot{x} + \omega^2 x = 0, \quad \omega = \sqrt{\frac{k}{m}}$$

Laplace Transform

esign

Free undamped vibration (continue):

无阻尼自由振动

$$s^2\mathcal{L}(x) - sx(0) - \dot{x}(0) + \omega^2\mathcal{L}(x) = 0$$

$$\mathcal{L}(x) = x(0)\frac{s}{s^2+\omega^2} + \frac{\dot{x}(0)}{\omega}\frac{\omega}{s^2+\omega^2}$$

Inverse Laplace transform

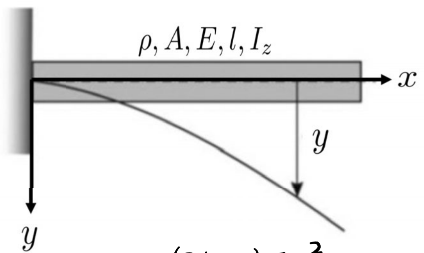
$$x(t) = A\cos\omega t + B\sin\omega t, \quad A = x(0), B = \frac{\dot{x}(0)}{\omega}$$

$$= X\sin(\omega t + \phi), \quad X = \sqrt{A^2+B^2}, \phi = \tan^{-1}\left(\frac{A}{B}\right)$$

$\omega = \sqrt{\frac{k}{m}}$

X is called the amplitude, ϕ is called the phase angle, ω is called the natural frequency.

Example 3: equivalent bending vibration (cantilever beam)



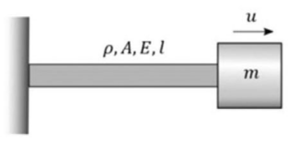
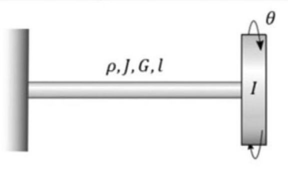
ρ: density
A: cross-section area
E: Young's modulus
l: length
I_z: area moment

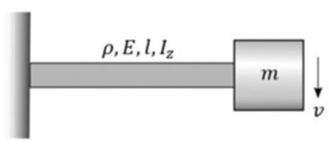
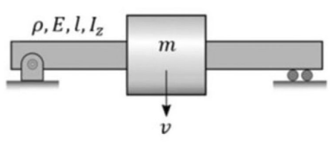
静态加载时: $y(x) = \dfrac{(3l-x)Fx^2}{6EI_z}$

amplitude at free end

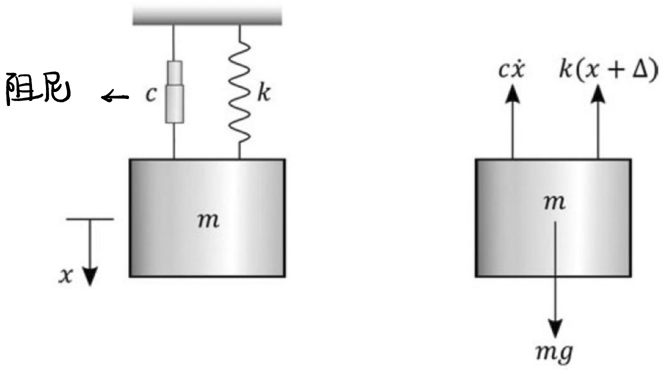
假设动态与静态加载变化一致: $y(x,t) = \phi(x)\cdot q(t)$

shape function

System	Shape Function	m_e	k_e
 Longitudinal vibration	$\phi(x) = \frac{x}{l}$	$m + \frac{\rho Al}{3}$	$\frac{EA}{l}$
 Torsional vibration	$\phi(x) = \frac{x}{l}$	$I + \frac{\rho Jl}{3}$	$\frac{GJ}{l}$

System	Shape Function	m_e	k_e
 Bending of a cantilever beam	$\phi(x) = \frac{3x^2l - x^3}{2l^3}$	$m + \frac{33\rho Al}{140}$	$\frac{3EI_z}{l^3}$
 Bending of a simply supported beam	$\phi(x) = \sin\left(\frac{\pi x}{l}\right)$	$m + \frac{\rho Al}{2}$	$\frac{\pi^4 EI_z}{2l^3}$

Free Damped Vibration



$$m\ddot{x} = mg - c\dot{x} - k(x+\Delta x)$$

$$mg = k\Delta x$$

$$\Downarrow$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

$\Downarrow$ Laplace Transform

$$m\cdot\left(s^2 L(x) - s x(0) - \dot{x}(0)\right) + c\left(s L(x) - x(0)\right) + k L(x) = 0$$

$$ms^2 L(x) - ms\, x_0 - m\dot{x}_0 + cs\, L(x) - c x_0 + k L(x) = 0$$

$$(ms^2 + cs + k) L(x) - (ms + c) x_0 - m\dot{x}_0 = 0$$

$$L(x) = \frac{(ms+c)x_0 + m\dot{x}_0}{ms^2 + cs + k} = \frac{(s+\frac{c}{m})x_0 + \dot{x}_0}{s^2 + \frac{c}{m}s + \frac{k}{m}} = \frac{x_0 s + \frac{c}{m}x_0 + \dot{x}_0}{(s+\frac{c}{2m})^2 + \frac{k}{m} - \frac{c^2}{4m^2}} = \frac{x_0 s + 2\xi\omega_n x_0 + \dot{x}_0}{(s+\xi\omega_n)^2 + (1-\xi^2)\omega_n^2}$$

damping ratio $\boxed{\xi = \frac{c}{2\sqrt{km}} \qquad \omega_n = \sqrt{\frac{k}{m}}}$ natural frequency

Case under-damped $\xi < 1$ 欠阻尼，系统仍会起振

$$L(x) = \frac{x_0 s + 2\xi\omega_n x_0 + \dot{x}_0}{(s+\xi\omega_n)^2 + (1-\xi^2)\omega_n^2}$$

$$= x_0 \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + (\sqrt{1-\xi^2}\,\omega_n)^2} - \frac{x_0\xi\omega_n + 2x_0\xi\omega_n + \dot{x}_0}{(s+\xi\omega_n)^2 + (\sqrt{1-\xi^2}\,\omega_n)^2}$$

$$= x_0 \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + (\sqrt{1-\xi^2}\,\omega_n)^2} - \frac{3x_0\xi\omega_n + \dot{x}_0}{(\sqrt{1-\xi^2}\,\omega_n)^2} \cdot \frac{(\sqrt{1-\xi^2}\,\omega_n)^2}{(s+\xi\omega_n)^2 + (\sqrt{1-\xi^2}\,\omega_n)^2}$$

$L^{-1}\downarrow$

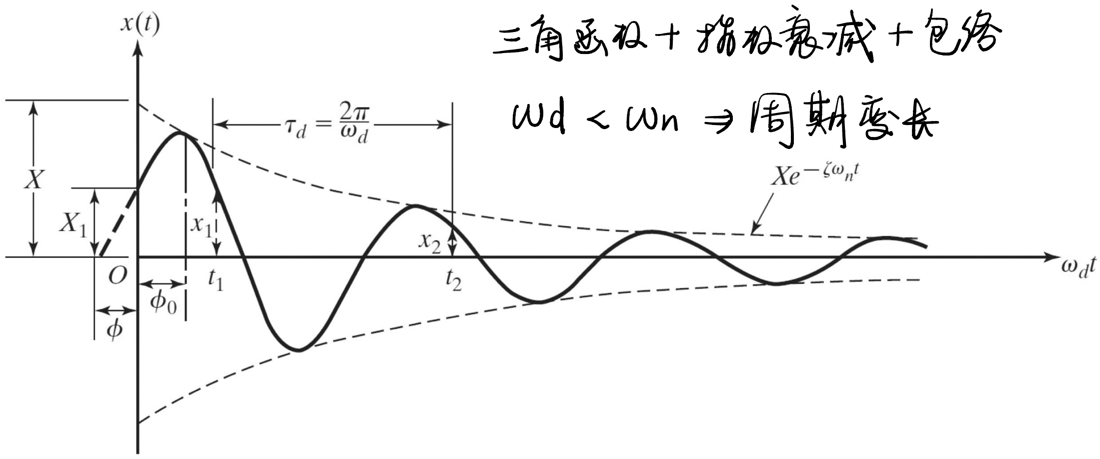
$$x(t) = x_0 \cdot e^{-\xi\omega_n t}\cos(\sqrt{1-\xi^2}\,\omega_n t) - \frac{3x_0\xi\omega_n + \dot{x}_0}{(\sqrt{1-\xi^2}\,\omega_n)^2} e^{-\xi\omega_n t}\sin(\sqrt{1-\xi^2}\,\omega_n t)$$

$$= e^{-\xi\omega t}\cdot\sqrt{x_0^2 + \frac{(3x_0\xi\omega_n + \dot{x}_0)^2}{(\sqrt{1-\xi^2}\,\omega_n)^4}}\cdot\cos(\sqrt{1-\xi^2}\,\omega_n t + \phi)$$

$\boxed{\omega_d = \sqrt{1-\xi^2}\,\omega_n}$ damped frequency

其中

$$\tan\phi = \frac{3x_0\xi\omega_n + \dot{x}_0}{x_0\cdot(\sqrt{1-\xi^2}\,\omega_n)^2}$$



三角函数 + 指数衰减 + 包络

$\omega_d < \omega_n \Rightarrow$ 周期变长

Case $\zeta = 1$ critical damped 系统不会起振 阶跃 + 斜坡 + 指数衰减

$$L(x) = \frac{x_0 s + 2\zeta\omega_n x_0 + \dot{x}_0}{(s+\zeta\omega_n)^2 + (1-\zeta^2)\omega_n^2} = \frac{x_0 s + 2\omega_n x_0 + \dot{x}_0}{(s+\omega_n)^2} = \frac{x_0(s+\omega_n) - x_0\omega_n}{(s+\omega_n)^2} + \frac{2\omega_n x_0 + \dot{x}_0}{(s+\omega_n)^2}$$

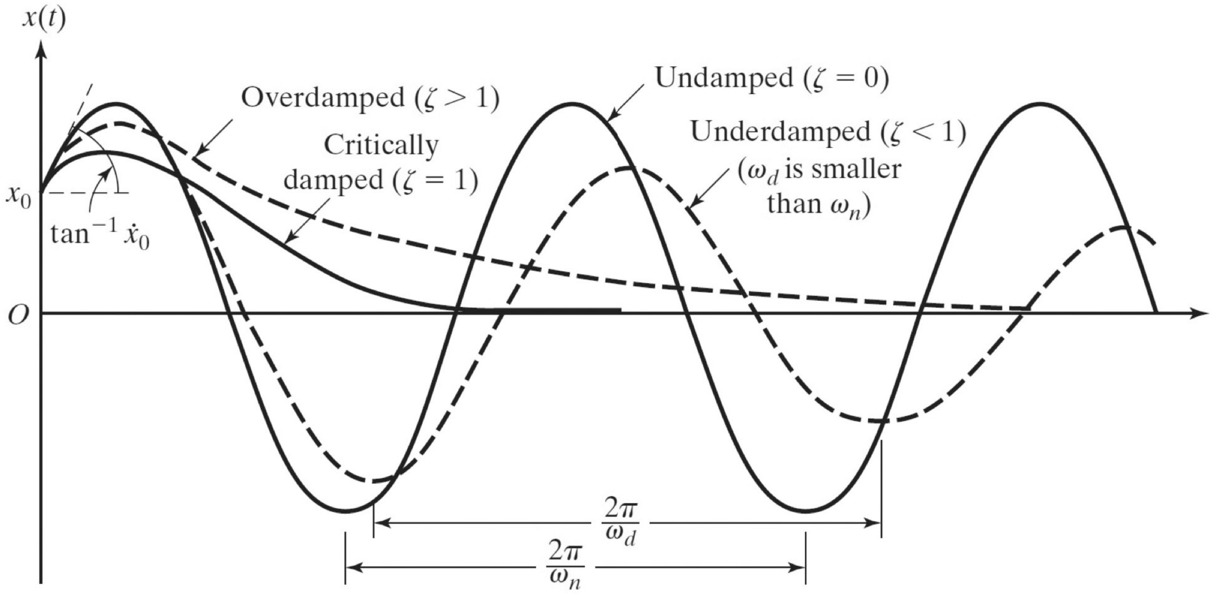
$$= x_0 \frac{(s+\omega_n)}{(s+\omega_n)^2} + (\omega_n x_0 + \dot{x}_0) \cdot \frac{1}{(s+\omega_n)^2}$$

$$x(t) = x_0 \cdot e^{-\omega_n t} \cdot u(t) + (\omega_n x_0 + \dot{x}_0) \cdot e^{-\omega_n t} \cdot t = [x_0 + (\omega_n x_0 + \dot{x}_0) t] e^{-\omega_n t}$$

Case $\zeta > 1$ over damped

$$L(x) = \frac{x_0 s + 2\zeta\omega_n x_0 + \dot{x}_0}{(s+\zeta\omega_n)^2 - (\zeta^2-1)\omega_n^2} = x_0 \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2 - (\sqrt{\zeta^2-1}\,\omega_n)^2} + \frac{-\zeta\omega_n x_0 + 2\zeta\omega_n x_0 + \dot{x}_0}{(s+\zeta\omega_n)^2 - (\sqrt{\zeta^2-1}\,\omega_n)^2}$$

$$x(t) = x_0 \cdot e^{-\zeta\omega_n t} \cdot \cosh(\sqrt{\zeta^2-1}\,\omega_n t) + \frac{\zeta\omega_n x_0 + \dot{x}_0}{\sqrt{\zeta^2-1}\,\omega_n} \sinh(\sqrt{\zeta^2-1}\,\omega_n t) \cdot e^{-\zeta\omega_n t}$$



$$L(x) = \frac{(ms+c)x_0 + m\dot{x}_0}{\boxed{ms^2+cs+k}} = \frac{x_0 s + 2\zeta\omega_n x_0 + \dot{x}_0}{(s+\zeta\omega_n)^2 + (1-\zeta^2)\omega_n^2} = \frac{x_0 s + 2\zeta\omega_n x_0 + \dot{x}_0}{\boxed{s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2}}$$

特征根(根轨迹法)

求解 $ms^2+cs+k=0$ / $s^2+2\zeta\omega_n s+\zeta^2\omega_n^2=0$ 的根

Case i) $\zeta = 0$ (harmonic oscillation)

$$s_1 = i\omega_n, \quad s_2 = -i\omega_n$$

Case ii) $\zeta < 1$ (underdamped)

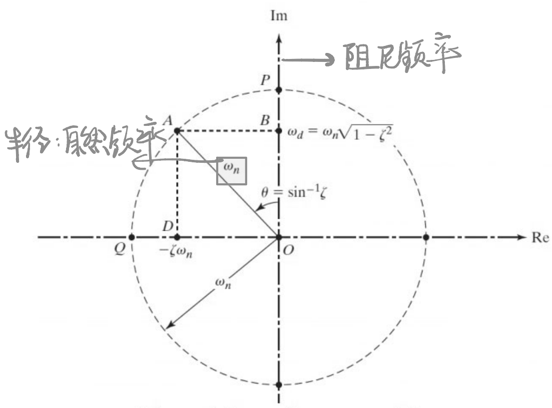
$$s_1 = (-\zeta + i\sqrt{1-\zeta^2})\omega_n, \quad s_2 = (-\zeta - i\sqrt{1-\zeta^2})\omega_n$$

Case iii) $\zeta = 1$ (critically damped)

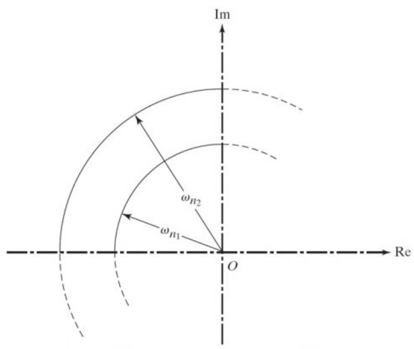
$$s_1 = s_2 = -\zeta\omega_n$$

Case iv) $\zeta > 1$ (overdamped)

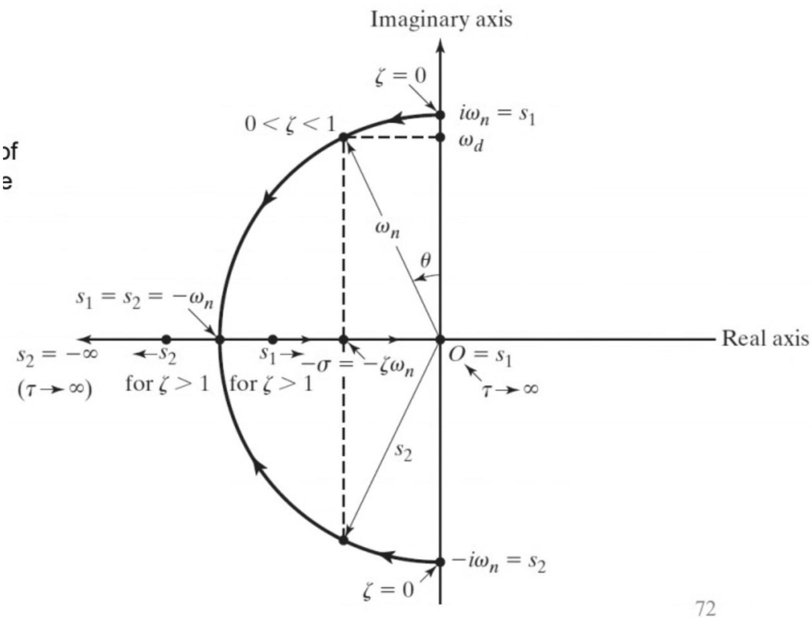
$$s_1 = (-\zeta + \sqrt{\zeta^2-1})\omega_n, \quad s_2 = (-\zeta - \sqrt{\zeta^2-1})\omega_n$$



Interpretations of ω_n, ω_d, and ξ.



Different concentric circles: different natural frequencies



有关 Hyperbolic Function

双曲函数（hyperbolic function）可借助指数函数定义 [1]

双曲正弦：$\sinh x = \dfrac{e^x - e^{-x}}{2}$

双曲余弦：$\cosh x = \dfrac{e^x + e^{-x}}{2}$

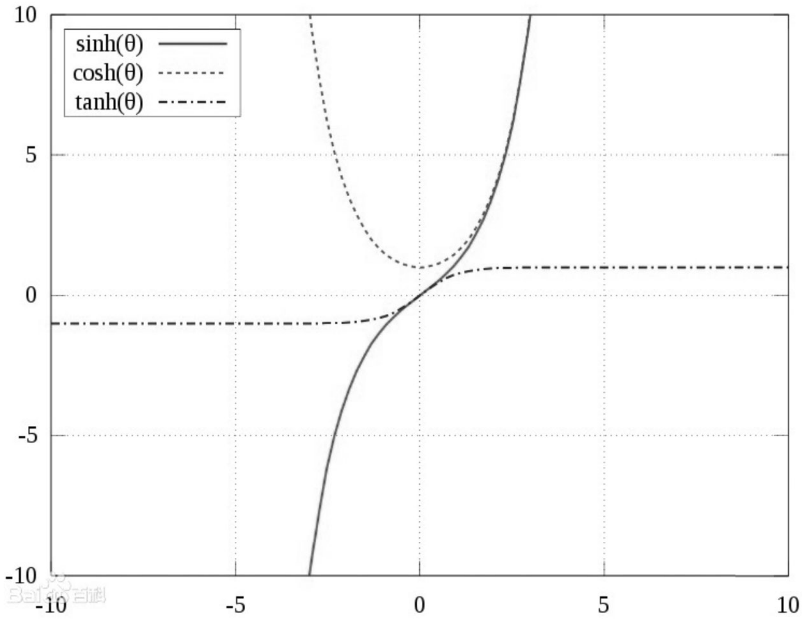
双曲正切：$\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

双曲余切：$\coth x = \dfrac{1}{\tanh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

双曲正割：$\mathrm{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

双曲余割：$\mathrm{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

$\cosh^2 x - \sinh^2 x = 1$



与三角函数关系

双曲函数与三角函数有如下的关系：

$$\sinh x = -i \sin ix$$

$$\cosh x = \cos ix$$

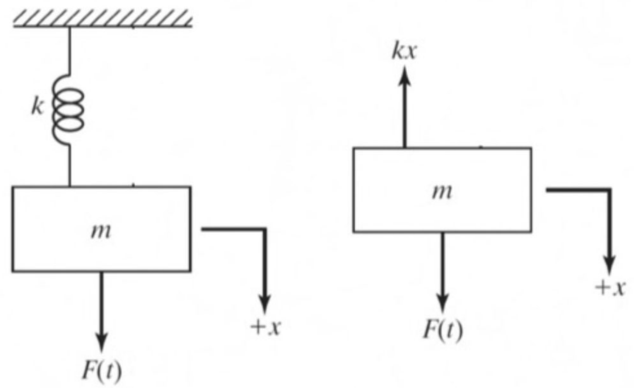
$$\tanh x = -i \tan ix$$

$$\coth x = i \cot ix$$

$$\mathrm{sech}\, x = \sec ix$$

$$\mathrm{csch}\, x = i \csc ix$$

Forced Vibration 单自由度受迫振动



(a) (b) Free-body diagram

equation of motion : $F(t)+mg-k(x+\Delta)=m\ddot{x}$

习惯上定义物体力平衡点为x原点

Laplace Transform:

$$\mathcal{L}(x)=\frac{F_0}{m}\frac{s}{(s^2+\omega^2)(s^2+\omega_n^2)}+\frac{x_0 s+\dot{x}_0}{s^2+\omega_n^2},\quad \omega_n=\sqrt{\frac{k}{m}}$$

$$\mathcal{L}(x)=\frac{F_0}{m(\omega_n^2-\omega^2)}\left(\frac{s}{s^2+\omega^2}-\frac{s}{s^2+\omega_n^2}\right)+x_0\frac{s}{s^2+\omega_n^2}+\frac{\dot{x}_0}{\omega_n}\frac{\omega_n}{s^2+\omega_n^2}$$

Define static deflection $\delta_{st}=F_0/k$

$$x(t)=\left(x_0-\frac{\delta_{st}}{m(\omega_n^2-\omega^2)/k}\right)\cos\omega_n t+\frac{\dot{x}_0}{\omega_n}\sin\omega_n t+\frac{\delta_{st}}{m(\omega_n^2-\omega^2)/k}\cos\omega t$$

$$=\underbrace{\boxed{\left(x_0-\frac{\delta_{st}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)\cos\omega_n t+\frac{\dot{x}_0}{\omega_n}\sin\omega_n t}}_{\text{Homogeneous solution } x_h \text{ (transient)}}+\underbrace{\boxed{\underbrace{\frac{\delta_{st}}{1-\left(\frac{\omega}{\omega_n}\right)^2}}_{X}\cos\omega t}}_{\text{Particular solution } x_p \text{ (steady-state)}}$$

plitude ratio X/δ_{st} and frequency ratio $r=\omega/\omega_n$

X 1

受迫振动中,{ 频率为自然频率 ω_n 的部分为瞬态 transient
(振动物体)
频率为施加力的频率 ω 的部分为稳态 steady state

振动图像为瞬态与稳态的叠加. (见Matlab Session)

存在阻尼时,瞬态部分会逐渐衰减,最终只剩下稳态部分

frequency ratio: $r = \omega/\omega_n$

amplitude ratio: $X/\delta_{st} = \dfrac{1-\left(\frac{\omega}{\omega_n}\right)^2}{F_0/k}$

$\downarrow$

Static deflection

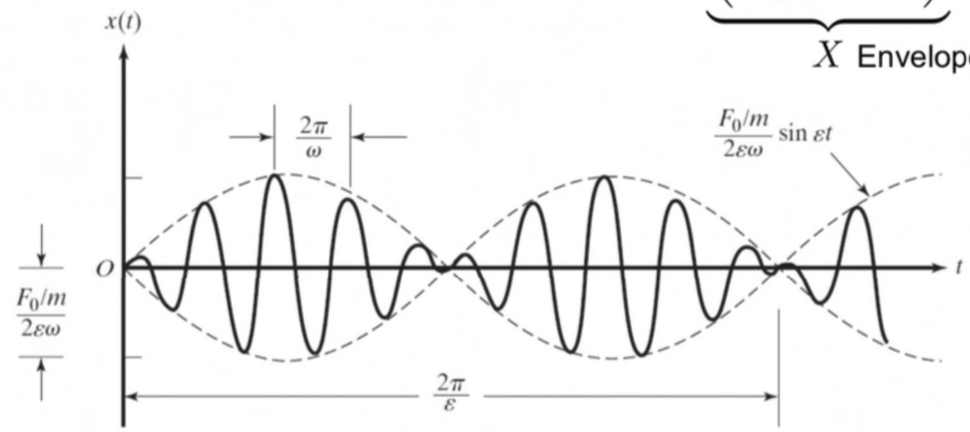
Beating $\omega = \omega_n$

Beating phenomenon (let $x_0 = \dot{x}_0 = 0$):

$$x(t) = \frac{\delta_{st}}{1-r^2}\left[2\sin\frac{\omega+\omega_n}{2}t\ \sin\frac{\omega_n-\omega}{2}t\right]$$

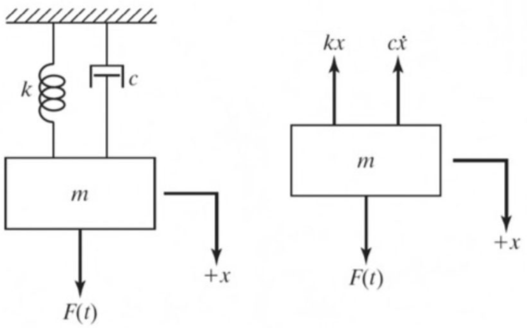
If $\omega_n - \omega = 2\varepsilon$ is a small positive quantity, then $\omega_n \approx \omega$ and

$$\omega+\omega_n \approx 2\omega, \quad \omega_n^2-\omega^2 \approx 4\varepsilon\omega \quad \Rightarrow \quad x(t) = \underbrace{\left(\frac{F_0/m}{2\varepsilon\omega}\sin\varepsilon t\right)}_{X \text{ Envelope (dashed line)}}\sin\omega t$$



單自由度阻尼受迫振动

Response of a damped system under harmonic force



(a) (b) Free-body diagram

$$m\ddot{x} + c\dot{x} + kx = F_0\cos\omega t$$

$\Downarrow$ Laplace transform

$$\mathcal{L}(x) = \frac{F_0}{m}\frac{s}{(s^2+\omega^2)(s^2+2\zeta\omega_n s+\omega_n^2)} + \frac{x_0(s+2\zeta\omega_n)+\dot{x}_0}{s^2+2\zeta\omega_n s+\omega_n^2}, \quad \omega_n = \sqrt{\frac{k}{m}}, \zeta = \frac{c}{2\sqrt{mk}}$$

Partial fraction of steady-state part: $F(s) = \mathcal{L}(x_p(t))$

求解太困难了

$$F(s) = \frac{F_0}{m}\left(\frac{a_1 s + a_2}{s^2 + \omega^2} + \frac{a_3 s + a_4}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$$

$$a_1 = \frac{\omega_n^2 - \omega^2}{(2\zeta\omega_n)^2\omega^2 + (\omega_n^2 - \omega^2)^2} \qquad a_3 = -\frac{\omega_n^2 - \omega^2}{(2\zeta\omega_n)^2\omega^2 + (\omega_n^2 - \omega^2)^2}$$

$$a_2 = \frac{2\zeta\omega_n\omega^2}{(2\zeta\omega_n)^2\omega^2 + (\omega_n^2 - \omega^2)^2} \qquad a_4 = -\frac{2\zeta\omega_n^3 \; \cancel{\omega_n^2 - \omega^2}}{(2\zeta\omega_n)^2\omega^2 + (\omega_n^2 - \omega^2)^2}$$

Partial fraction of complete solution: $X(s) = \mathcal{L}(x(t))$

力的频率 steady

$$X(s) = \frac{F_0}{m}\frac{1}{(2\zeta\omega_n)^2\omega^2 + (\omega_n^2 - \omega^2)^2}\Bigg[(\omega_n^2 - \omega^2)\left(\frac{s}{s^2 + \omega^2}\right) + (2\zeta\omega_n\omega)\left(\frac{\omega}{s^2 + \omega^2}\right)$$

$$- (\omega_n^2 - \omega^2)\left(\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) - (2\zeta\omega_n)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)\Bigg]$$

自然频率 transient

Total solution:

$$x(t) = \boxed{\frac{F_0}{m}\frac{1}{(2\zeta\omega_n)^2 + (\omega_n^2 - \omega^2)^2}[(\omega_n^2 - \omega^2)\cos\omega t + 2\zeta\omega_n\omega\sin\omega t}$$

Steady-state $x_p(t)$

$$+ \frac{(\omega_n^2 - \omega^2)}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1 - \zeta^2}\, t - \phi) \qquad \phi = \tan^{-1}\left(\frac{1 - \zeta^2}{\zeta}\right)$$

$$- \frac{(2\zeta\omega_n^2)}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1 - \zeta^2}\, t)]$$

Transient $x_h(t)$ (decays to 0 if $\zeta > 0$)

$$x_p(t) = \frac{F_0}{m}\frac{1}{(2\zeta\omega_n)^2 + (\omega_n^2 - \omega^2)^2}[(\omega_n^2 - \omega^2)\cos\omega t + 2\zeta\omega_n\omega\sin\omega t]$$

$$= \underbrace{\frac{F_0}{\sqrt{c^2\omega^2 + (k - m\omega^2)^2}}}_{X}\cos(\omega t - \phi) \qquad \phi = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right)$$

对比（有/无阻尼受迫振动

```matlab
%% Transfer Function 1
    clc; close all;
    x0=1;x0_dot=1;  %Initial condition
    L1=TransFunc(0,1);
    L2=TransFunc(0.5,1);
    L3=TransFunc(1,1);
    L4=TransFunc(2,1);
    figure
    impulse(L1)
    xlim([0 20])
    hold on
    impulse(L2)
    impulse(L3)
    impulse(L4)
    legend("ksi = 0","ksi = 0.5","ksi = 1","ksi = 2")
    title("Impulse Response when omega = 1")
    hold off

%% Transfer Function 2 无阻尼受迫振动
    x0=0.1;x0_dot=0;      %Initial condition
    F0 = 10;
    m = 4;
    w = 5; %稳态频率（力的频率）
    wn = 2; %瞬态频率（物体自然频率）

    num_static=[1 0];
    den_static=[1 0 w^2];
    Ls=F0/(m*(wn^2-w^2))*tf(num_static,den_static)

    num_transient1=[1 0];
    den_transient1=[1 0 wn^2];
    num_transient2=[wn];
    den_transient2=[1 0 wn^2];
    Lt=(-F0/(m*(wn^2-w^2))+x0)*tf(num_transient1,den_transient1)+x0_dot/wn*tf(num_transient2,den_transient2)

    L=Ls+Lt
    figure
    hold on
    impulse(Ls)
    impulse(Lt)
    impulse(L)
    xlim([0 20])
    ylim([-0.5 0.5])
    legend("Ls","Lt","L")


%% Transfer Function3 阻尼受迫振动
    x0=0.1;x0_dot=0;      %Initial condition
    F0 = 10;
    m = 4;
    w = 5; %稳态频率（力的频率）
    wn = 2; %瞬态频率（物体自然频率）
    zeta = 0.1; %阻尼系数

    num_s1=[1 0];
    den_s1=[1 0 w^2];
    num_s2=[w];
    den_s2=[1 0 w^2];
    Ls=F0/m*((wn^2-w^2)/((2*zeta*wn)^2*w^2+(wn^2-w^2)^2))*tf(num_s1,den_s1)+...
    F0/m*((2*zeta*wn*w)/((2*zeta*wn)^2*w^2+(wn^2-w^2)^2))*tf(num_s2,den_s2);

    num_t1=[1 0];
    den_t1=[1 2*zeta*wn wn^2];
    num_t2=[wn^2];
    den_t2=[1 2*zeta*wn wn^2];
    Lt=-F0/m*((wn^2-w^2)/((2*zeta*wn)^2*w^2+(wn^2-w^2)^2))*tf(num_t1,den_t1)-...
    F0/m*((2*zeta*wn)/((2*zeta*wn)^2*w^2+(wn^2-w^2)^2))*tf(num_t2,den_t2);

    L=Ls+Lt
    figure
    hold on
    impulse(Ls)
    impulse(Lt)
    impulse(L)
    xlim([0 20])
    ylim([-0.5 0.5])
    legend("Ls","Lt","L")

function [L] = TransFunc(xi,w)
    x0=1;x0_dot=1;      %Initial condition
    num=[x0 x0_dot+2*xi*w*x0];
    den=[1 2*xi*w  w^2];
    L=tf(num,den);
end
```