Problem Set 8

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1 Introduction of models

Mobile networking technology is developing rapidly. Companies choose to invest for new technologies, such as, 4g and 5g, to defend their market shares. However, the money one company has is limited at a time, the more one chooses to invest, the less one can spend today. While the investment brings profit in the future, today's expenditure is also important. Here, we are trying to model the mobile networking operator 's investment decision to achieve maximum future profits. At the beginning stage, in period one, operator will have M1 amount of money, and invest I1 into the new technology. In the end of period one, the remaining money one operator has is M1-I1, this amount will be increased to (1+) *(M1-I1) + r*I1 at the beginning stage of period two. Because the money remaining will generate interest, and we will also receive some money generated from our period one investment. For example, (1+) *(M1-I1) + r*I1= M2, (1+) *(M2-I2) +r*I2=M3, etc. Here, we will only build a two-stage model, so in period two M2 will all be used for investment, I2 = M2. And we do not consider r*I1 after period two. Which means the investment we made will only generate profit once in our model. In this scenario, we will build a dynamic programming model to solve for the best I1 to maximum our profit.

2 Preferences and model

2.1 Parameters

Rho: 0, incremental factor, rho means the remaining money in period one will be increased in period two, from M1-I1 to (1+) *(M1-I1). Because remaining money will generate interests., and rho is our interest rate. r: is sigma: coefficient for the profit function

2.2 Variables

State variable: Money as M. Operator's starting fund is limited. Control variable: Investment as I. I1 represents how much we spend money in period one. Stationary variable: Discrete time, t = 1, 2 for period one and two.

2.3 Functions

2.3.1 Profit function

$$Profitfunction : P = \frac{I^{1-\sigma}}{1-\sigma}$$

P(X) The capacity we invested today will turn into future profit. Here we assume the profit function fits into two conditions, due to the diminishing return nature of investment:

$$FOC(firstordercondition) > 0, SOC(secondordercondition) < 0$$

2.3.2 Policy function

Policy function: Specifying the control variables as a function of the state variables as following.
period one investment = I1, period one remaining = M1-I1
period two investment= (1+) *(M1-I1) + r*I1= M2
We get: I1 = (1+-r) / [(1+) * M1 +M2]

2.3.3 Value function and Bellman equation

Value function=max p(I1) + p (I2) = max p((1+-r) / [(1+) * M1 +M2]) + p (M2)
Bellman equation: V(M) max p(I) + p (I') = max p((1+-r) / [(1+) * M +M']) + p (M')
FOC: $\frac{\partial V(M)}{\partial V(M')}$= p'((1+-r) / [(1+) * M +M']) + p'(M')

3 Results

3.1 Value function and State variable – money

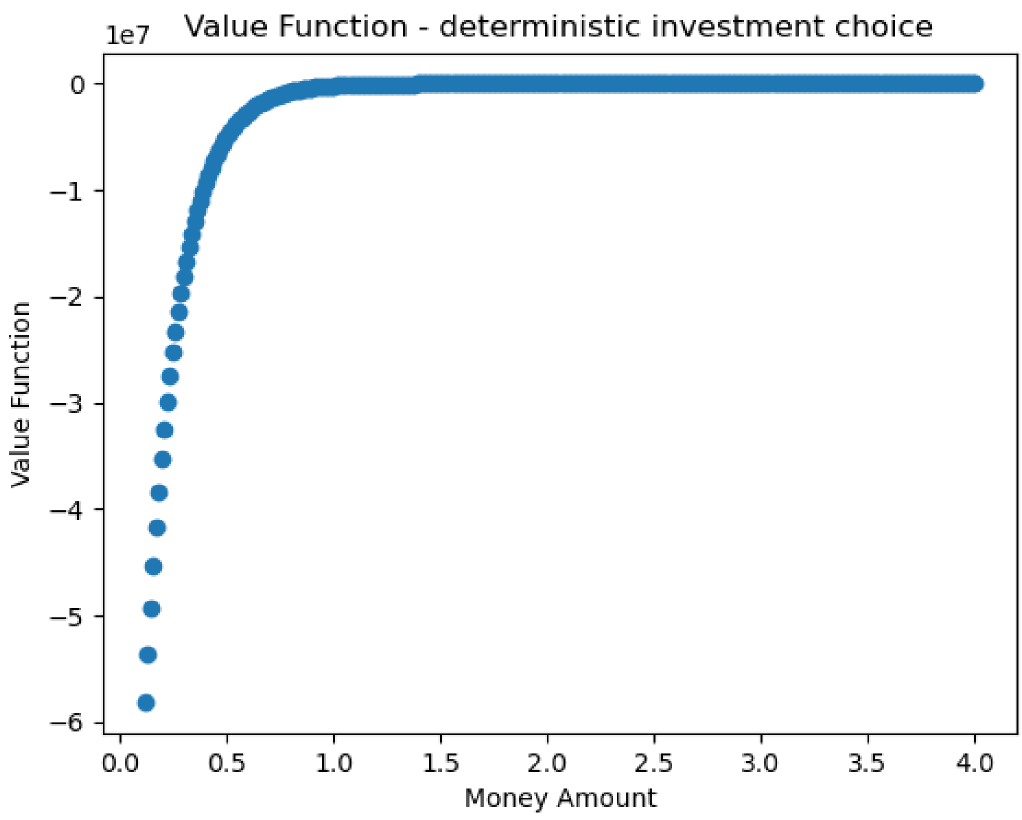

Figure 1: Value Function

In this picture, we can find the optimal value given each amount of money we have. We can see that, at the beginning stage, the value increase rapidly for each dollar we have. But it has a diminishing return pattern, which means that when

3.2 Policy function and State variable – money

4 Results

4.1 Value function and State variable – money

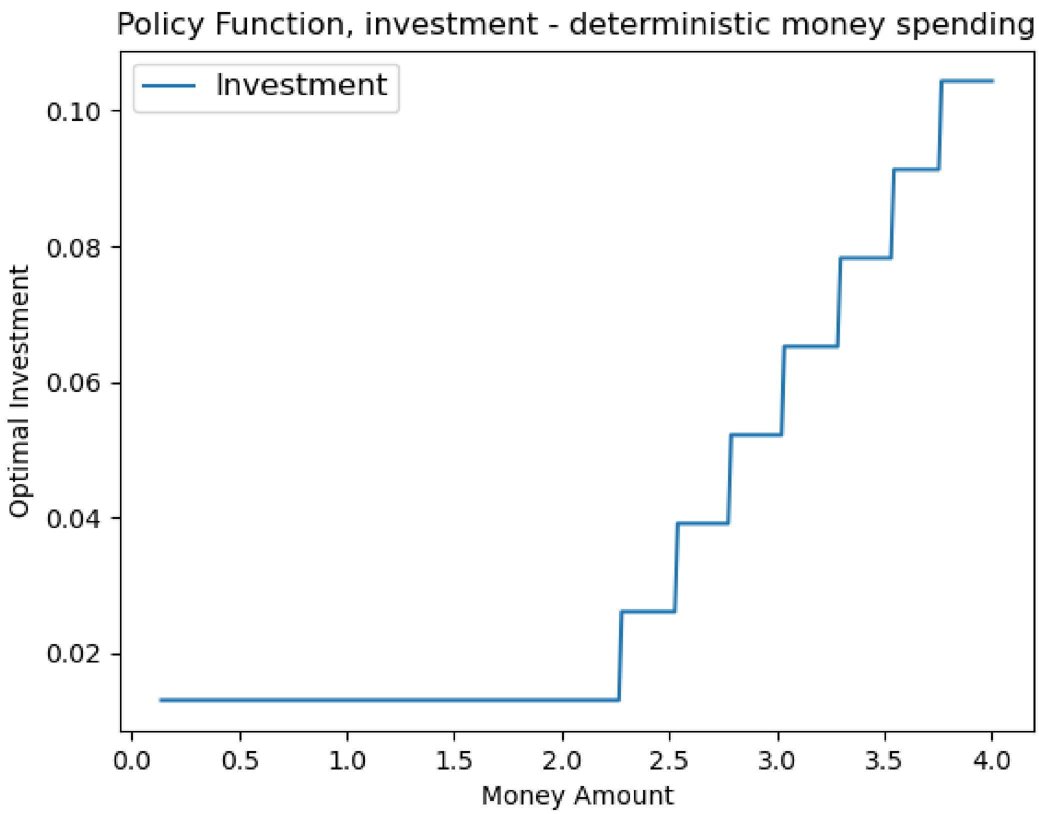

Figure 2: Policy Function

In this picture, we can find the optimal investment given each amount of money we have. It shows that when money is greater than 2.25, the more money we have the more we should choose to invest. Overall, the amount of money we have has a positive relationship with the optimal investment strategy.