微積分 (上) 第一次段考　　共 10 題，每題 10 分　　2021/11/02

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請直接在本試題紙上作答　　系級: 資工一　　姓名: 陳安綾　　學號: 110321054

1. Find $\lim_{x\to5}(2x^3-3x+1)$.

$\lim_{x\to5} 2x^3-3x+1$

$=250-15+1=236$ #

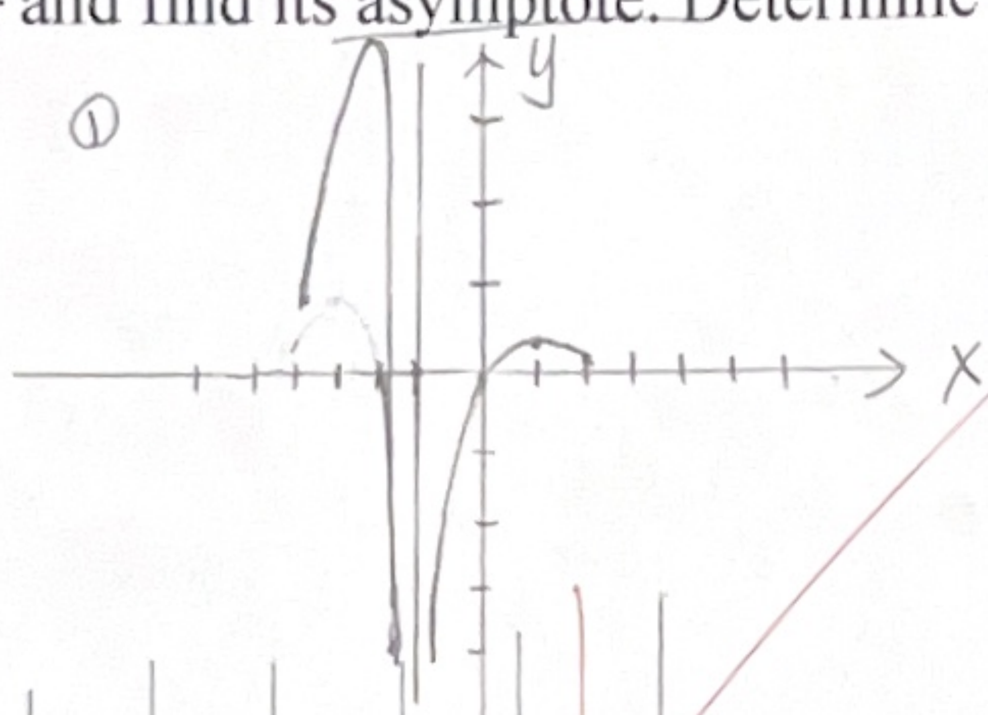
① ② 漸近線 ③

2. Sketch the graph of $\frac{2x}{(x+1)^3}$ and find its asymptote. Determine the limit $\lim_{x\to-1}\frac{2x}{(x+1)^3}$.

③ $\lim_{x\to-1}\frac{2x}{(x+1)^3}$

$=\frac{\to -2}{\to 0}=-\infty$

①



x	0	1	2	-1	-2	-3
y	0	$\frac{1}{4}$	$\frac{4}{27}$	$-\infty$	4	

② asymptote = $x=-1$ #

3. Find $\lim_{h\to0}\frac{(h-5)^2-25}{h}$.

$\lim_{h\to0}\frac{(h-5)^2-25}{h}$

$=\lim_{h\to0}\frac{(h-5-5)\cancel{(h-5+5)}}{\cancel{h}} = \lim_{h\to0}(h-10) = -10$ #

4. Apply $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ to find the derivative of $f(x)=\frac{1}{x-1}$.

$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to0}\frac{\frac{1}{(x+h)-1}-\frac{1}{x-1}}{h}$

$=\lim_{h\to0}\frac{\frac{(x-1)-(x+h-1)}{[(x+h-1)](x-1)}}{h} = \lim_{h\to0}\frac{\frac{-h}{x^2-2x+xh-h+1}}{h}$

$=\frac{-1}{x^2-2x+1}$ #

5. Find the derivative of $f(x)=(x^3-2x^2+3)(7x^2-4x)$.

$f(x)=(x^3-2x^2+3)(7x^2-4x)$

$f'(x)=(x^3-2x^2+3)'(7x^2-4x)+(x^3-2x^2+3)(7x^2-4x)'$

$=(3x^2-4x)(7x^2-4x)+(x^3-2x^2+3)(14x-4)$

$=35x^4-72x^3+24x^2+42x-12$ #

6. Find equation of the tangent line to $y=x^3-3x^2+x$ at point P (2, -2).

$f(x)=x^3-3x^2+x$

$f'(x)=3x^2-6x+1$

$f'(2)=12-12+1=1$

$y+2=x-2$

$y=x-4$

Ans: $y=x-4$ #

7. Find the derivative of $y = (4x^2 + 1)^7$.

$f(x) = (4x^2+1)^7$

$f'(x) = 7(4x^2+1)^6 (4x^2+1)'$

$= 7 \times 8x (4x^2+1)^6$

$= 56x(4x^2+1)^6$

8. Find the derivative of $f(t) = A + B\cos\left(\frac{2\pi}{T}(t - \phi)\right)$.

$\left[\frac{2\pi}{T}(t-\phi)\right]' = \frac{2\pi}{T}(t-\phi)' + \left(\frac{2\pi}{T}\right)'(t-\phi) \rightarrow 0$

$= \frac{2\pi}{T}$

$f(t) = A + B\cos\left(\frac{2\pi}{T}(t-\phi)\right)$

$f'(t) = -B\sin\left(\frac{2\pi}{T}(t-\phi)\right) \times \left(\frac{2\pi}{T}(t-\phi)\right)'$

$= -\left[B\sin\left(\frac{2\pi}{T}(t-\phi)\right) \times \frac{2\pi}{T}\right] = -\frac{2\pi}{T} B\sin\left(\frac{2\pi}{T}(t-\phi)\right)$

9. Find y'' for $x^2 + 4y^2 = 1$ by implicit differentiation.

$f(x) = x^2 + 4y^2 = 1$

$f'(x) = 2x + 8y \cdot y' = 0$

$8y \times y' = -2x$

$y' = \frac{-2x}{8y} = \frac{-x}{4y}$

$y'' = \frac{d}{dx}\left(\frac{-x}{4y}\right) = \frac{4y(-x)' - (4y)'(-x)}{16y^2}$

$= \frac{-4y + (4y)'x}{16y^2} = \frac{-4y - 4x \times \frac{-x}{4y}}{16y^2}$

$= \frac{-4y + \frac{x^2}{y}}{16y^2} = \frac{\frac{-4y^2+x^2}{y}}{16y^2} = \frac{-4y^2+x^2}{16y^3}$

10. Use a linear approximation (or differentials) of $f(x) = \sqrt{x}$ at $x = 9$ to estimate $\sqrt{9.1}$.

$f(x) = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(9) = 3$

$f'(9) = \frac{1}{6}$

$L(x) = f(9) + f'(9)(x-9) = 3 + \frac{1}{6}(x-9)$

$= 3 + \frac{1}{6}x - \frac{3}{2}$

$= \frac{1}{6}x + \frac{3}{2}$

$\sqrt{x} \approx \frac{1}{6}x + \frac{3}{2}$

$\sqrt{9.1} \approx \frac{1}{6} \times 9.1 + \frac{3}{2}$

$= 1.5167 + 1.5$

$= 3.0167$

Ans = 3.0167