離 散 數 學 第 二 次 期 中 考

2021/12/15

注意事項:
1. 禁止使用計算機、翻譯機、禁止攜帶計算紙；**手機請關機**。
2. **計算與證明題需要計算過程方予計分。**
3. 當然不可以作弊。
4. 請於答案卷左上角填上題號以方便閱卷。
5. 使用兩張答案卷的同學記得兩張都要寫名字，並將之合併在一起交回。
6. 請努力作答。

一、簡答題 (32%)

1. (3-1 no.5) Determine all of the elements in the following set: $\{n^3 + n^2 | n \in \{1, 2, 3, 4, 5\}$ (2%)

2. (3-4 no.5) The Tuesday night dance club is made up of eight married couples and two of these fourteen members must be chosen to find a dance hall for an upcoming fund raiser. sixteen 16
   (a) If the two members are selected at random, what is the probability they are both women? (3%)
   (b) If Joan and Douglas are one of the couples in the club, what is the probability at least one of them is among the two who are chosen? (3%)

3. (Ch3 no.11) Let $\mathcal{U} = \mathbf{R}$ and let the index set $I = \mathbf{Q}^+$. For each $q \in \mathbf{Q}^+$, let $A_q = [0, 3q]$ and $B_q = (0, 4q]$. Determine
   (a) $A_{8/3}$ (b) $A_4 \Delta B_3$ (c) $\cup_{q \in I} A_q$ (d) $\cap_{q \in I} B_q$. $A_q = [0, 3q]$ (8%)

4. (4-3 no.17) Convert each of the following binary numbers to base 10 and 16.
   (a) 11000110 (3%)
   (b) 01011010 (3%)

5. (4-5 no.11) How many positive integers n divide $100135n + 256864608$? (5%)

6. (4-5 no.17) Find the smallest positive integer n for which the product $1890 \times n$ is a perfect cube. (5%)

二、計算與證明(須寫出完整計算過程方予計分) (68%)

7. (3-1 no.14) (a) How many subsets of $\{1, 2, 3, \ldots, 10\}$ contain at least one even integer? (3%)
   (b) How many subsets of $\{1, 2, 3, \ldots, 11\}$ contain at least one even integer? (2%)
   (c) Generalize the results of parts (a) and (b). (2%)

8. (3-2 no.17b) Using the laws of set theory, simplify: (10%)
$$(A \cap B) \cup (A \cap B \cap \overline{C}) \cup (\overline{A} \cap B).$$

9. (3-3 no.5) How many permutations of the digits 0, 1, 2, …, 8 either start with a 3 or end with a 8? (10%)

10. (4-1 no.4) A wheel of fortune has the integers from 1 to 25 places on it in a random manner. Show that regardless of how the numbers are positioned on the wheel, there are three adjacent numbers whose sum is at least 39. (10%)

11. (4-2 no.14) L_n denote the nth Lucas number. When $n \geq 1$, prove that: (10%)
$$L_1^2 + L_2^2 + L_3^2 + \cdots + L_n^2 = L_n L_{n+1} - 2.$$

12. (4-3 no.10) If $n \in \mathbf{Z}^+$, and n is 奇 odd, prove that $8|(n^2 - 1)$. (5%)

13. (4-4 no.1) For $a = 2021$, $b = 1215$, determine $\gcd(a, b)$ and express it as a linear combination of a, b. (10%)

14. (Ch4 no.14) Determine all $a, b \in \mathbf{Z}$ such that $\dfrac{a}{7} + \dfrac{b}{12} = \dfrac{1}{84}$. (6%)

註: ① The *Fibonacci numbers* may be defined recursively by

1) $F_0 = 0$, $F_1 = 1$; and
2) $F_n = F_{n-1} + F_{n-2}$, for $n \in Z^+$ with $n \geq 2$.

② The *Lucas numbers*: defined recursively by

1) $L_0 = 2$, $L_1 = 1$; and
2) $L_n = L_{n-1} + L_{n-2}$, for $n \in Z^+$ with $n \geq 2$.