注意事項：
1. 禁止使用計算機、翻譯機、禁止攜帶計算紙；**手機請關機**。
2. **計算與證明題需要計算過程方予計分。**
3. 當然不可以作弊。
4. 請於答案卷左上角填上題號以方便閱卷。
5. 使用兩張答案卷的同學記得兩張都要寫名字，並將之合併在一起交回。
6. 請努力作答。

一、簡答題 (48%)

1. (1-2 no.11) Three small towns, designated by A, B, and C, are interconnected by a system of two-way roads, as shown in Fig. 1.

   a) In how many ways can Linda travel from town A to town C? (2%)

   b) How many different round trips can Linda travel from town A to town C and back to town A? (2%)

   c) How many of the round trips in part (b) are such that the return trip (from town C to town A) is at least partially different from the route Linda takes from town A to town C? (For example, if Linda travels from town A to town C along toads R_1 and R_6, then on her return she might take roads R_6 and R_3, or roads R_7 and R_2, or road R_9, among other possibilities, but she does not travel on roads R_6 and R_1.) (2%)
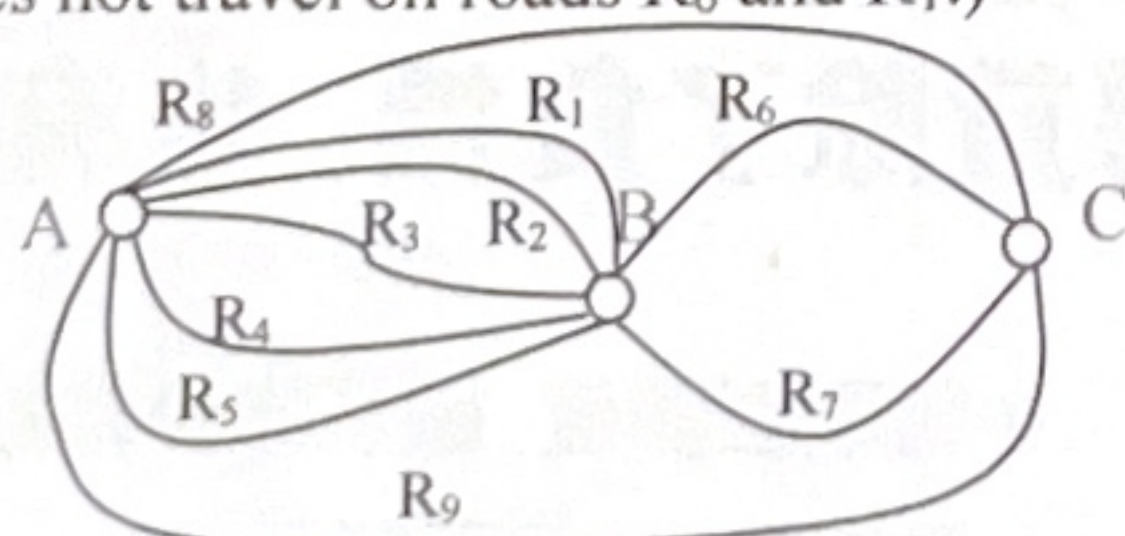
2. (1-2 no.35) a) In how many ways can eight people be arranged about a circular table if two of the people insist on sitting next to each other? (2%)

   b) If two of the people insist on sitting next to each other, how many arrangements are possible? (2%)

3. (1-3 no.23b, 25d, no.26a) Determine the coefficient of

   a) $x^8 y^2$ in $(x + 2y)^{10}$ (2%)

   b) xyz^{-2} in $(2x - 2y + 3z^{-1})^4$ (2%)

   c) $w^2 x^2 y^2 z^2$ in $(w + x + y + z - 2)^{10}$. (2%)

4. (Ch 1 no.23) Francesca has 24 different books but the shelf in her dormitory residence will hold only 16 of them.

a) In how many ways can Francesca line up 16 of these books on her bookshelf? (3%)

b) How many of the arrangements in part (a) include Francesca's four books on tennis? (3%)

5. (2-3 no.12) Write each of the following arguments in symbolic form. Then establish the validity of the argument or give a counterexample to show that it is invalid. (8%)

a) If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the supervisor's position or she did not work hard.

b) If there is a chance of rain or her red headband is missing, then Lois will not mow her lawn. Whenever the temperature is over 80°F, there is no chance for rain. Today the temperature is 85°F and Lois is wearing her red headband. Therefore (sometime today) Lois will mow her lawn.

6. (2-4 no.8) Let $p(x)$, $q(x)$, and $r(x)$ denote the following open statements.

$$p(x): \quad x^2 - 8x + 15 = 0$$
$$q(x): \quad x \text{ is odd}$$
$$r(x): \quad x > 0$$

For the universe of all integers, determine the truth or falsity of the following statements. If a statement is false, give a counterexample.

a) $\forall x \, [p(x) \to q(x)]$ (2%)

b) $\forall x \, [q(x) \to p(x)]$ (2%)

c) $\forall x \, [(p(x) \vee q(x)) \to r(x)]$ (2%)

d) $\exists x \, [p(x) \to (p(x) \wedge r(x))]$ (2%)

7. (2-4 no. 16, 17) Write the negation of each of the following true statements. For (a) the universe consists of all the students at the university where Professor Lenhart teaches; for (b) the universe comprises all real numbers.

a) At least one student in Professor Lenhart's C++ class is a history major (2%)

b) For all real numbers x, if $|x - 4| < 7$, then $-3 < x < 11$. (2%)

8. (Chap 2 no.13) Consider the open statement

$$p(x, y): y - x = 2y + x^2$$

where the universe for each of the variables x, y comprises all integers. Determine the truth value for each of the following statements.

a) $p(0, 1)$ (2%)

b) $\exists y \, p(1, y)$ (2%)

c) $\exists x \, \forall y \, p(x, y)$ (2%)

二、計算與證明, <u>須有計算過程或說明方予計分</u> (52%)

1. (1-4 no.27) Frannie tosses a coin 10 times and gets four heads and six tails. In how many ways can these tosses result in

    a) two runs of heads and one run of tails;

    b) three runs;   (4%)

    c) equal numbers of runs of heads and runs of tails?   (4%)

                                                                                                       (4%)

2. (2-1 no.17) After baking a pie for the two nieces and two nephews who are visiting her, Aunt Nellie leaves the pie on her kitchen table to cool. Then she drive to the mall to close her boutique for the day. Upon the return she finds that someone has eaten one-quarter of the pie. Since no one was in her house that day —except for the four visitors—Aunt Nellie questions each niece and nephew about who ate the piece of pie. The four "suspects" tell her the following:

    Charles: Dawn ate the piece of pie.
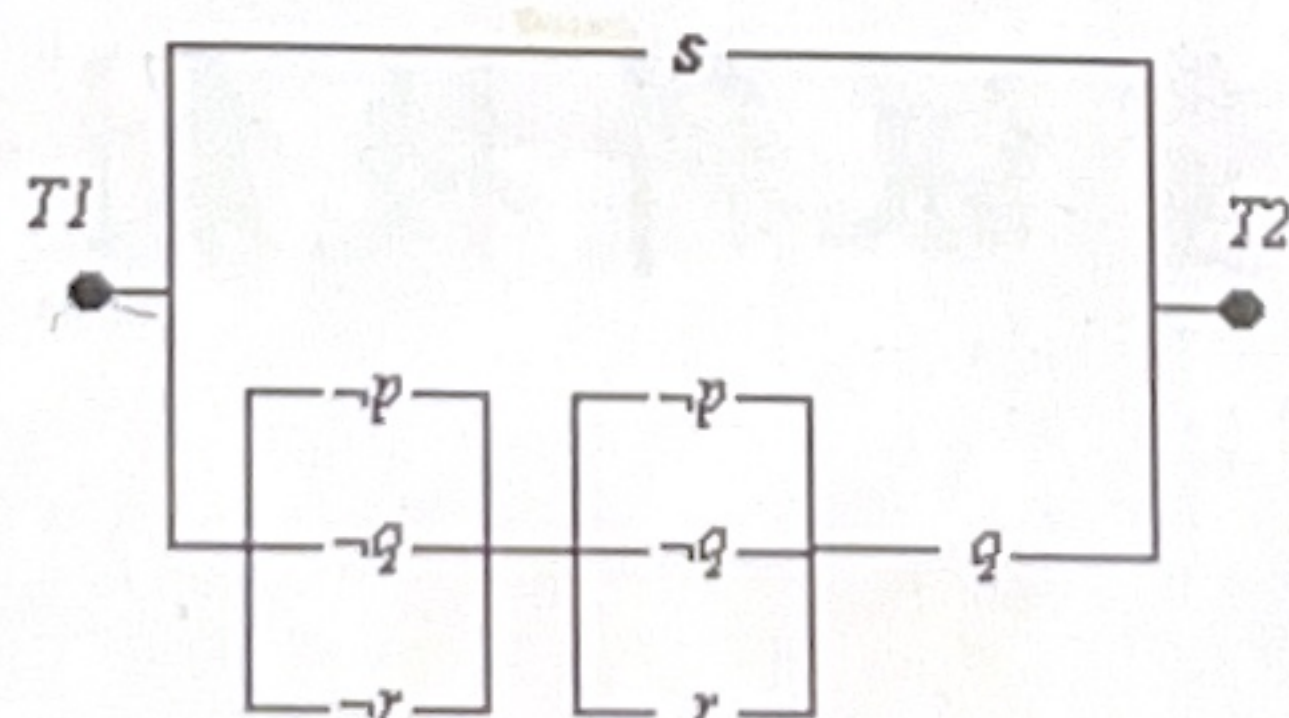
    Dawn: Tyler ate the pie.

    Kelly: If Tyler did not ate the pie then Charles ate the pie.

    Tyler: I did not eat the piece of pie.

    If only one of these four statements is true and only one of the four committed this heinous crime, who is the vile culprit that Aunt Nellie will have to punish severely? Whose statement is true?

                                                        (8%) (需說明原因才給分)

3. (2-2 EX.2.18) Simplify the network shown below: (10%)

4. (2-5 no.17) Prove the following result in three ways (as in Theorem 2.4): If n is an even integer, then $31n + 12$ is even.   (12%)

5. (Ch2 no.11) Prove or disprove each of the following, where p, q, and r are any statements.

    a) $[(p \veebar q) \veebar r] \Leftrightarrow [p \veebar (q \veebar r)]$ (5%)

    b) $[p \veebar (q \rightarrow r)] \Leftrightarrow [(p \veebar q) \rightarrow (p \veebar r)]$ (5%)