

CSIE Probability Homework 2 Solution

1. You roll two fair dice until you get doubles for the fifth time. X is the number of the rolls. Find $E[X] = \mu_X$, $\text{Var}[X] = \sigma_X^2$, and $E[X^2]$. (20pts)

X is Pascal with $p = 1/6, k = 5$. $E[X] = \mu_X = k/p = 30$. $\text{Var}[X] = \sigma_X^2 = k(1-p)/p^2 = 150$. $E[X^2] = \mu_X^2 + \sigma_X^2 = 1050$.

2. At a base station, the number X of the messages it receives during 6:00-6:20am is a Poisson random variable with $E[X] = 2$. Y is the number of the messages showing up during 6:00-6:05am. (a) Find $E[Y] = \mu_Y$, $\text{Var}[Y] = \sigma_Y^2$, and $E[Y^2]$. (20pts)

$E[X] = 2 = 20\lambda$, $\lambda = 0.1$. The number Y of the messages in the first 5 minutes is a Poisson random variable with parameter $\alpha = 5\lambda = 0.5$. Thus, $E[Y] = \mu_Y = \alpha = 0.5$, $\text{Var}[Y] = \sigma_Y^2 = \alpha = 0.5$, and $E[Y^2] = \mu_Y^2 + \sigma_Y^2 = 0.75$.

- (b) $Z = (Y - \mu_Y)/\sigma_Y$. Find $E[Z]$ and $\text{Var}[Z]$. (15pts)

Z is a standardized random variable with $E[Z] = 0$ and $\text{Var}[Z] = 1$.