CSIE Probability Exam I

Mon, Mar 28, 2022

Attempt all the questions. Justify your answers unless otherwise specified. Give your answer in terms of fractions, exp(·), etc., if needed.

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1. Two archers X and Y are shooting the target. Archer X shoots the bull's eye with 20% accuracy, and Archer Y shoots with 80% accuracy. Pick one of the two archers randomly and let the archer shoot the target twice. B_1 is the event that the first shot hits the bull's eye. B_2 is the event that the second shot hits the bull's eye. Verify if B_1 and B_2 are independent. (15pts)

X shoot the bull's eye $\Rightarrow P[X] = \frac{20}{100}$

Y : $\Rightarrow P[Y] = \frac{80}{100}$

$P[B_1] = \frac{1}{2} \times \frac{20}{100} + \frac{1}{2} \times \frac{80}{100}$

$= \frac{1}{10} + \frac{4}{10} = \frac{1}{2}$

$P[B_2] = \frac{1}{2} \times \frac{80}{100} \times \frac{20}{100} + \frac{1}{2} \times \frac{20}{100} \times \frac{80}{100} = \frac{8}{100} + \frac{8}{100} = \frac{16}{100}$

$P[B_1B_2] = \frac{1}{2} \times \frac{20}{100} \times \frac{20}{100} + \frac{1}{2} \times \frac{80}{100} \times \frac{80}{100}$

$= \frac{2}{100} + \frac{32}{100} = \frac{34}{100}$

$\because P[B_1B_2] \neq P[B_1] \cdot P[B_2]$

$\therefore B_1$ and B_2 are not independent.

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2. At NCNU, 60% of the students are undergrads(U), and among the graduate students, the master students(M) is seven times the doctoral students(D), P[M] = 7P[D]. 60% of the students at NCNU have no computer(C_0). Two-thirds of the undergrads have no computer, P[$C_0|U$] = $\frac{2}{3}$. Among the students with one or more computers(C_1), Three-eighths of them are master students, P[$M|C_1$] = $\frac{3}{8}$. Every doctoral student possesses one or more computers, P[$C_1|D$] = 1. Finish the following table, and find the probability when you meet a graduate student, she or he has no computer P[$C_0|M \cup D$]. (20pts)

		U	M	D
$\frac{60}{100}$	C_0	$\frac{2}{5}$	$\frac{1}{5}$	0
$\frac{40}{100}$	C_1	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{20}$

$P[U] = \frac{60}{100}$

$P[M] = 7P[D]$

$P[C_0|U] = \frac{2}{3}$

$P[M|C_1] = \frac{3}{8}$

$P[C_1|D] = 1$

$\frac{P[C_0U]}{P[U]} \times P[U] = P[C_0U]$

$\frac{2}{3} \times \frac{60}{100} = \frac{4}{10}$

$\frac{P[MC_1]}{P[C_1]} \times P[C_1] = P[MC_1]$

$= \frac{3}{8} \times P[C_1] = P[MC_1]$

$\frac{3}{8} \times \frac{40}{100}$

$P[C_1D] = 1 \times P[D]$

$\frac{20}{100} + \frac{15}{100} + x = \frac{40}{100}$

$\frac{P[M]}{P[D]} = 7$

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$P[C_0|M \cup D]$

$= \frac{P[C_0M] \cup P[C_0D]}{P[M \cup D]} = \frac{\frac{1}{5}}{\frac{7}{20} + \frac{1}{20}} = \frac{1}{5} \times \frac{20}{8}$

Ans: $P[C_0|M \cup D] = \frac{1}{2}$

3. Roll a fair die twice. The outcome is independent from roll to roll. Let X_1 and X_2 be the first and the second outcomes. I is an indicator random variable such that $I = 1$ when $\max\{X_1, X_2\} = 6$; otherwise $I = 0$. Find P[$I = 0$], P[$I = 1$], and E[I]. (15pts)

Bernoulli $I=1$ $\max\{X_1, X_2\} = 6$ $P[I=1] = \cancel{\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}} = \cancel{\frac{5}{72}}$

$P[I=0] = \frac{67}{72}$

$E[I] = \frac{5}{72}$

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4. You roll two fair dice until you get doubles. (a) X is the number of the rolls. Find P[$X > 3$]. (15pts)

$P[\text{get double}] = \frac{6 \times 1}{36} = \frac{1}{6}$

$P[\text{not get double}] = \frac{5}{6}$

$1 - P[X \le 3]$

$= 1 - P[X=1] - P[X=2] - P[X=3]$

$= 1 - \frac{1}{6} - \frac{5}{6} \times \frac{1}{6} - \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = 1 - \frac{1}{6} - \frac{5}{36} - \frac{25}{216} = 1 - \left(\frac{36 + 30 + 25}{216}\right)$

$= 1 - \frac{91}{216} = \frac{125}{216}$

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(b) You continue rolling the dice until you get doubles for the fifth time. Y is the number of the total rolls. Find E[Y]. (10pts)

Pascal

$E[5] = \frac{5}{\frac{1}{6}} = 30$

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5. At a base station, the number X of the messages it receives during 6:00-6:20am is a Poisson random variable with E[X] = 2. Find the probability that one or more messages show up during 6:00-6:05am. (15pts)

≥ 1

5 min

$\alpha = \lambda T$

$T = 20$ min

$E[X] = 2 = \alpha$

$2 = \cancel{20}\, 10\, \lambda$

$\lambda = 0.1$

$P_X(x) = \alpha^x e^{-\alpha} / x!$

$\alpha = 0.1 \times 5 = 0.5$

OK

$1 - P_X(0)$

$= 1 - 0.5^0 e^{-2} / 0!$

$= 1 - e^{-2}$

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