CSIE Probability Exam II Solution

1. At a base station, the number X of the messages it receives during 6:00-6:20am is a Poisson random variable with $\mathrm{E}[X] = 2$. Find $\mathrm{E}[X^2]$. (15pts)

   $\mathrm{E}[X] = \mu_X = 2 = \alpha$. Thus, $\mathrm{Var}[X] = \sigma_X^2 = \alpha = 2$, and $\mathrm{E}[X^2] = \mu_X^2 + \sigma_X^2 = 6$.

2. Discrete random variable X has the PMF

   $$P_X(x) = C_x^5 (\frac{1}{2})^5.$$

   Find $\mathrm{E}[X] = \mu_X$, $\mathrm{Var}[X] = \sigma_X^2$, and $\mathrm{E}[X^2]$. (20pts)

   X is binomial with $n = 5$, $p = 1/2$. $\mathrm{E}[X] = \mu_X = np = 2.5$. $\mathrm{Var}[X] = \sigma_X^2 = np(1-p) = 1.25$. $\mathrm{E}[X^2] = \mu_X^2 + \sigma_X^2 = 7.5$.

3. X is a uniform continuous random variable with $\mathrm{E}[X] = \mu_X = 6$ and $\mathrm{P}[6 \le X \le 9] = 0.25$. Find $\mathrm{Var}[X] = \sigma_X^2$ and $\mathrm{E}[X^2]$. (20pts)

   Assume X is distributed on $[a, b]$. $\mathrm{E}[X] = \mu_X = 6 = (a+b)/2$.

   $\mathrm{P}[6 \le X \le 9] = 0.25 = 3/(b-a)$. Therefore, $a = 0$, $b = 12$.

   $\mathrm{Var}[X] = \sigma_X^2 = (b-a)^2/12 = 12$. $\mathrm{E}[X^2] = \mu_X^2 + \sigma_X^2 = 48$.

4. T is a Gaussian random variable with $[T < -30] = 0.5$ and $\mathrm{Var}[T] = 100$. Find $\mathrm{P}[T \ge 0]$. (15pts)

   T is Gaussian(-30,10). $\mathrm{P}[T \ge 0 = Q(\frac{0-(-30)}{10}) = Q(3) \simeq 0.001349898$.

5. X is a Gaussian random variable with $\mathrm{E}[X] = 0$ and $\mathrm{P}[|X| \le 10] = 0.2$. Find $\mathrm{P}[X < 10]$. (15pts)

   $\mathrm{P}[|X| \le 10] = \mathrm{P}[-10 \le X \le 10] = 0.2$.

   By symmetry, $\mathrm{P}[-10 \le X \le 0] = \mathrm{P}[0 \le X \le 10] = 0.1$.

   Hence, $\mathrm{P}[X < 10] = \mathrm{P}[X \le 0] + \mathrm{P}[0 < X < 10] = 0.5 + 0.1 = 0.6$.