暨大資工系　線性代數　小考 1　　111. 3. 9

1. (30%) (1) and (2) are equivalent

$$\begin{cases} 4x_1 - x_2 + 3x_3 = -4 \\ 3x_1 + x_2 + 9x_3 = -1 \end{cases} \quad (1) \qquad \begin{cases} x_1 - 2x_2 - 6x_3 = -3 \\ 7x_1 + 12x_3 = -5 \end{cases} \quad (2)$$

(a) Respectively find the augmented matrices, $\overline{A}$ and $\overline{B}$, of (1) and (2)

(b) Show that $\overline{B}$ is row-equivalent to $\overline{A}$, that is $\overline{B}$ can be obtained from $\overline{A}$ by a finite sequence of elementary row operations; and $\overline{A}$ can be obtained from $\overline{B}$ by a finite sequence of elementary row operations

(c) Find the solution of (1).

2. (20%) Let the solution spaces of (3) and (4) be equal to α and β, respectively.

$$\begin{cases} 4x_1 - x_2 + 3x_3 = -4 \\ 3x_1 + x_2 + 9x_3 = -1 \end{cases} \quad (3) \qquad \begin{cases} x_2 = -1 \\ 7x_1 + 12x_3 = -5 \end{cases} \quad (4)$$

Find $\alpha \cap \beta$.

3. (30%) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

(a) (15%) Find A^{-1} by using the row operations.

(b) (15%) According to those row operations used in (a), matrix A^{-1} can be expressed as a product of elementary matrices, find those elementary matrices.

Theorem 1.4.6 If A & B are invertible, then $(AB)^{-1} = B^{-1}A^{-1}$

THEOREM 1.5.1 Row Operations by Matrix Multiplication

If the elementary matrix E results from performing a certain row operation on I_m and if A is an $m \times n$ matrix, then the product EA is the matrix that results when this same row operation is performed on A.

Theorem 1.5.2 Every elementary matrix is invertible, and the inverse is also an elementary matrix.

Theorem 1.5.3 If A is an n×n matrix, the following statements are equivalent, i.e. all true or all false.

(a) A is invertible
(b) AX=0 has only the trival solution
(c) The reduced row-echelon form of A is I_n
(d) A is expressible as a product of elementary matrices

4. (20%) In Theorem 1.5.3 (Hint: 以上一些定理會用到，用來佐證作答)

Prove that if statement (d) is true, then statement (b) is true.