暨大資工系　線性代數　期中考　　111.4.13

P_1 : the set of all real polynomials of degree 1 or less.

1. (10%) Consider the matrix $A=\begin{bmatrix} 5 & -2 \\ 3 & 3 \end{bmatrix}$
   (a) (4%) Find elementary matrices $E_1, E_2, \ldots, E_k$ such that $E_k \ldots E_2 E_1 A = I_2$.
   (b) (3%) Write A^{-1} as a product of elementary matrices.
   (c) (3%) Write A as a product of elementary matrices.

2. (10%) Suppose A is a $n \times n$ matrix, show that if $\det(A) = 0$, then the homogeneous system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

3. (15%) Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$
   (a) Find a basis for the column space of A. A^T
   (b) Find a basis for the null space of A.
   (c) Suppose $A\mathbf{x}=\mathbf{b}$ was consistent, where $\mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ what is the relationship between b_1, b_2 & b_3?

4. (10%) Find the inverse of $A=\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$ <u>by using the elementary row operations</u>.

5. (10%) Evaluate the determinant of the matrix A= $\begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix}$.

6. (15%) Prove that $A^{-1} = \dfrac{1}{\det(A)} adj(A)$

77
59
18

7. (10%) Find all 2x2 matrices whose nullspace is the line $3x-5y=0$.

8. (20%) Consider the bases $B = \{p_1, p_2\}$ and $B' = \{q_1, q_2\}$ for P_1, where $p_1 = 6+3x$, $p_2 = 10+2x$, $q_1 = 2$, $q_2 = 3+2x$.
   (a) (5%) Find the transition matrix from B' to B.
   (b) (5%) Find the transition matrix from B to B'.
   (c) (5%) Compute the coordinate vector $[\mathbf{p}]_B$, where $p = -4+x$, and use the result (b) to compute $[\mathbf{p}]_{B'}$.
   (d) (5%) Check your work by computing $[\mathbf{p}]_{B'}$ directly.

36 × 4
12 ×
177
− 33
144

$\begin{bmatrix} 6 & 3 \\ 10 & 2 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$

$6+3x = a(2) + b(3+2x)$

59
3
180−3 = 177

$2a + 3b = 6$

$2b = 3$, $b = \frac{3}{2}$, $2a + \frac{9}{2} = 6 \Rightarrow a = \frac{3}{4}$

$\frac{7}{3} \cdot \frac{11}{9}$

59
× 7
420 − 7 =

$2 = a(6+3x) + b(10+2x)$

$6a + 10b = 2$

$3a + 2b = 0$

$6b = 2$ $\quad 3a + \frac{2}{3} = 0$

$b = \frac{1}{3}$ $\quad a = -\frac{2}{9}$

$3+2x = c(6+3x) + d(10+2x)$

$6c + 10d = 3$

$3c + 2d = 2$

−56 −3 56
−59

$6d = -1$ $\quad 3c - \frac{2}{6} = 2$

$d = -\frac{1}{6}$ $\quad 3c = \frac{14}{6}$

$c = \frac{14}{18} \frac{7}{9}$