1103-1054 陳姿綾 資工一

CSIE Probability Exam II

Mon, May 2, 2022

Attempt all the questions. Justify your answers unless otherwise specified. Give your answer in terms of Standard Normal CDF $\Phi(\cdot)$, Standard Normal Complementary CDF $Q(\cdot)$, $\exp(\cdot)$, $\log(\cdot)$, fractions, etc., if needed. Please write down your name and student ID number on your answer sheet.

1. At a base station, the number X of the messages it receives during 6:00-6:20am is a Poisson random variable with $\mathrm{E}[X] = 2$. Find $\mathrm{E}[X^2]$. (15pts)

good!

$T => 0$

$E[X] = Var[X] = \alpha = \lambda T$ $\quad \alpha = 2$

$E[X] = 2 = \lambda$ $\quad 2 = 20\lambda \Rightarrow \lambda = \frac{1}{10}$

15/15

$Var[X] = E[X^2] - (E[X])^2$

$2 = E[X^2] - 4$

$E[X^2] = 6$

Ans: 6 ✗

2. Discrete random variable X has the PMF

$$P_X(x) = C_x^5 (\frac{1}{2})^5.$$

Find $\mathrm{E}[X] = \mu_X$, $\mathrm{Var}[X] = \sigma_X^2$, and $\mathrm{E}[X^2]$. (20pts)

Binomial

$n = 5$, $p = \frac{1}{2}$

$E[X] = \mu_X = np = 5 \times \frac{1}{2} = \frac{5}{2}$ ✓

$Var[X] = \sigma_X^2 = np(1-p) = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$ ✓

20

$\frac{5}{4} = E[X^2] - (E[X])^2$

$\frac{5}{4} + \frac{25}{4} = E[X^2]$

$\frac{30}{4} = \frac{15}{2} = E[X^2]$ ✓

A: $E[X] = \frac{5}{2}$

$Var[X] = \frac{5}{4}$

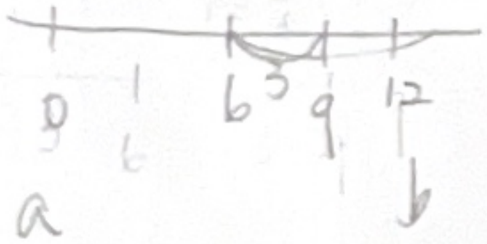
$E[X^2] = \frac{15}{2}$ ✗

3. X is a uniform continuous random variable with $\mathrm{E}[X] = \mu_X = 6$ and $\mathrm{P}[6 \leq X \leq 9] = 0.25$. Find $\mathrm{Var}[X] = \sigma_X^2$ and $\mathrm{E}[X^2]$. (20pts)

$E[X] = \mu_X = 6 = \frac{a+b}{2}$

$P[6 \leq X \leq 9] = 0.25$

$a + b = 12$



$\mathrm{Var}[X] = \frac{(b-a)^2}{12} = \frac{12 \times 12}{12} = 12$

$E[X^2]$

$12 = E[X^2] - (E[X])^2$

$12 = E[X^2] - 36$

$48 = E[X^2]$

$\mathrm{Var}[X] = 12$

Ans: $E[X^2] = 48$

4. T is a Gaussian random variable with $[T < -30] = 0.5$ and $\mathrm{Var}[T] = 100$. Find $\mathrm{P}[T \geq 0]$. (15pts)

$P[T < -30] = 0.5$

$\Phi\left(\frac{-30-\mu}{\sigma}\right) = \frac{1}{2}$

$\mu = -30$

$\mathrm{Var}[T] = 100$

$\sigma = 10$

Gaussian $(-30, 10)$

$P[T \geq 0]$

$Q\left(\frac{0+30}{10}\right)$

$= Q(3) = 1 - \Phi(3)$

Ans: $1 - \Phi(3)$

5. X is a Gaussian random variable with $\mathrm{E}[X] = 0$ and $\mathrm{P}[|X| \leq 10] = 0.2$. Find $\mathrm{P}[X < 10]$. (15pts)

Gaussian $(0, \sigma)$

$P[|x| \leq 10] = 0.2$

$P[-10 \leq x \leq 10] = 0.2$

$\Phi\left(\frac{10}{\sigma}\right) - \Phi\left(\frac{-10}{\sigma}\right) = 0.2$

$\Phi\left(\frac{10}{\sigma}\right) - \left(1 - \Phi\left(\frac{10}{\sigma}\right)\right) = 0.2$

$2\Phi\left(\frac{10}{\sigma}\right) = 1.2$

$\Phi\left(\frac{10}{\sigma}\right) = 0.6$

$\frac{10}{\sigma} = 0.27$

$\frac{10}{0.27} = \sigma$

$P[X < 10]$

$= \Phi\left(\frac{10}{\sigma}\right) = 0.6$

Ans: 0.6