Block-level Precoding via IRS-aided Hybrid Transmitter

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Abstract

This article proposes a transmission scheme for a hybrid Analog-Digital system where passive antenna array is used as analog unit. This scheme can lower the update rate of the analog unit by stacking several input vectors together. To determine the optimal phase shifts of the passive antennas, convex projection and gradient descent method are used and compared. This paper also shows experimentally that the number of passive antennas may affect the performance of this system.

I. Introduction

In the past few years, the Fifth Generation communication system (5G) is becoming more and more widespread. Massive multiple-input multiple-output (MIMO) systems are one of the crucial technologies that make ”5G” possible. It increases the channel capacity significantly without the exponential increament of the transmittion power by utilizing the diversity gain. This often leads to higher complexity and cost since each RF chain includes a power amplifier and an digital-to-analog converter. The high cost of the RF chains is a major holdback that limits the antenna number of base stations. For a time division duplex (TDM) system, if the base station has the perfect channel state information (CSI), reciprocity property can always be used. Therefore, only downlink is investigated in this paper.

Hybrid analog-digital (HAD) transmission is a rather recent technique for reducing the implementational complexity in MIMO systems. An HAD transmitter consists of a digital base-band unit and an analog unit which operates in the radio frequency (RF) domain. There are various technologies by which the analog unit can be implemented; see for instance radio frequecy beamforming networks with buttler matrix [1]. In this paper, we consider a recent proposal which implements the analog units via intelligent reflecting surfaces (IRSs).

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Intuitively, some information is encoded into the output signal by the analog unit. Therefore, the degree of freedom provided by RF chains can be reduced. In other words, the number of RF chains can be reduced. This can lower the cost and complexity of base stations. According to [**?**], the update rate of analog unit can be set slower than the update rate of the digital unit for a system with a general linear analog unit without causing any interference. To investigate this point in IRS-aided HAD transmission systems, block fading is assumed and a block-wise precoding procedure will be discussed. The IRS unit receives signals from the digital units and then reflects the signals with some tunable phase shifts. The digital unit is used to precode the user messages and tune the phase shifts of the IRS unit properly.

In the first part of this paper, the system model for the IRS-aided HAD transmission system is discussed in detail. Then in Section III, a digital baseband precoding scheme is introduced. The proposed analog beamforming scheme deals with a unit-modulus optimization problem. To address this problem, we develop three low-complexity alogorithms by means of convex projection technique, method of gradient descent and majorize-minimization (MM) algorithm. Our derivation initially consider the ideal cas with perfect CSI at the transmitter. We then extend the derivations to more realistic scenarios with imperfect CSI. The performance of the proposed algorithms are further investigated through several numerical simulations. Motivated by the findings in the first part, we show that interference free transmission is achievable at the base station and a large enough IRS is employed. A proof for this result in the system with block length one will be given. Finally, in Section V, we will give some conclusions.

II. System Model

We consider downlink transmission in a multi-user in a MIMO transmission system consisting of a base station (BS) with N antennas and K user terminals (UTs). If there are M transmitter antennas and K UTs, the vector of the received signals in this network is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{K\times M}$ describes the wireless channel from base station to all the UTs, $\mathbf{y} = \begin{bmatrix} y_1 & \dots & y_K \end{bmatrix}$ with y_k denoting the signal at the kth UT, $\mathbf{x} \in \mathbb{C}^{M\times 1}$ represents the signal at the base station and $\mathbf{n}$ is a noise vector whose entries are i.i.d. and follows Gaussian distribution with zero mean and variance σ^2. The transmitter intends to transmit data symbol s_k to the kth UT. To this end, it should precode the data using an HAD precoder into vector $\mathbf{x}$ such that

$y_k = s_K$, i.e. the received symbol at the kth UT is the same as the intended symbol for it. Since the mobile stations cannot jointly decode the signals, it can be assumed that the channel $\mathbf{H}$ is only known by the base station. Therefore, if the base intend to make the received symbols to be $\mathbf{s}$ at the mobile stations, the base station has to design $\mathbf{x}$ such that $\mathbf{y} = \mathbf{s}$.

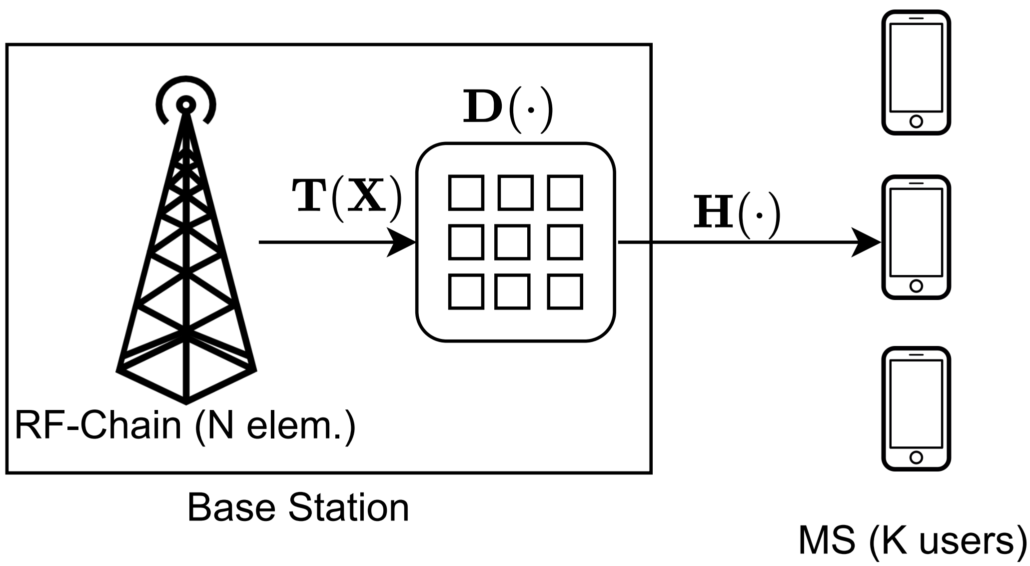

Fig. 1. Transmitter Model

Figure 1 shows the IRS-aided HAD transmission system. The BS is made up of the components inside the rectangle block. We can assume that there are N RF-chains (active antennas), M IRS reflecting elements, and K UTs. From the figure, $\mathbf{X} \in \mathbb{C}^{N\times L}$ where the rows correspond to the number of RF-chains, and columns correspond to the block length. This matrix $\mathbf{X}$ is used to represent signals sent from the RF chains in one time block. Matrix $\mathbf{T} \in \mathbb{C}^{M\times N}$ describes the internal channel from RF chains to IRS. $\mathbf{D} = diag_{matrix}\{e^{j\beta_1}, \ldots, e^{j\beta_M}\}$ is a diagonal matrix representing the phase shifts applied to the incoming signal of the IRS unit. Matrix $\mathbf{H}$ is the channel from the base station to mobile stations. Fig. 3 illustrates the internal structure of the transmitter in more details.

As is mentioned above, we are going to analyse the block-wise performance of the transmission scheme. Therefore, a buffer is added between the message source and the digital unit. It stacks L message vectors intended for the UTs into a matrix of messages symbols $\mathbf{S} = [\boldsymbol{s_1}, \boldsymbol{s_2}, \boldsymbol{...}, \boldsymbol{s_L}] \in \mathbb{C}^{K\times L}$. Similarly, in each time block, the RF chain output is also denoted as matrix $\mathbf{X} = [\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{...}, \boldsymbol{x_L}] \in \mathbb{C}^{N\times L}$. The IRS parameters $\mathbf{D}$ is tuned only once at the begining of every

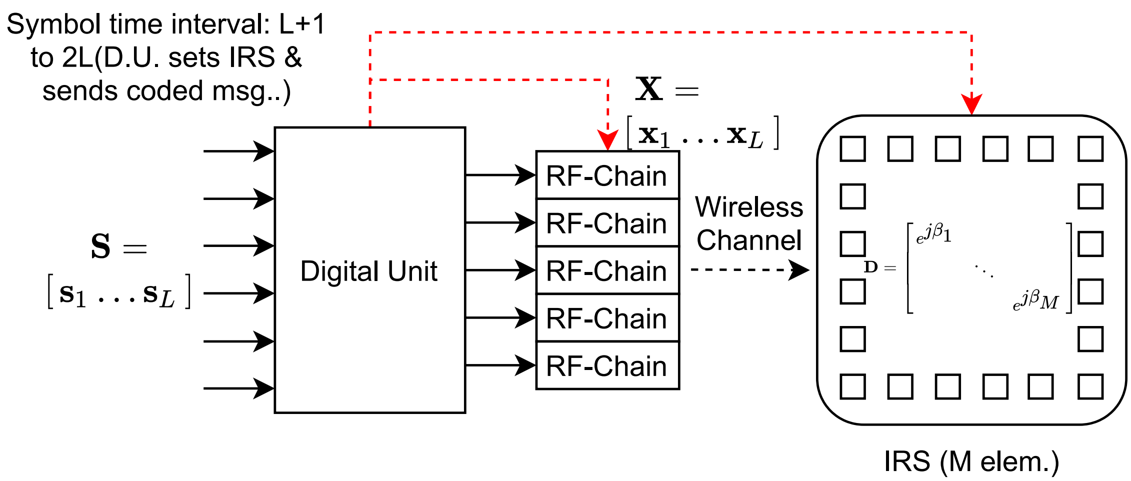

Fig. 2. Transmitter Model For The Second Block

transmittion block. According to this scheme, the final output data of the transmitter is $\mathbf{F} = \mathbf{DTX}$. Suppose that the user symbols are generated at rate R, the update rate of the passive antenna array is only R/L. This process can be concluded in a time table. Let $\mathbf{H} \in \mathbb{C}^{K \times M}$ be

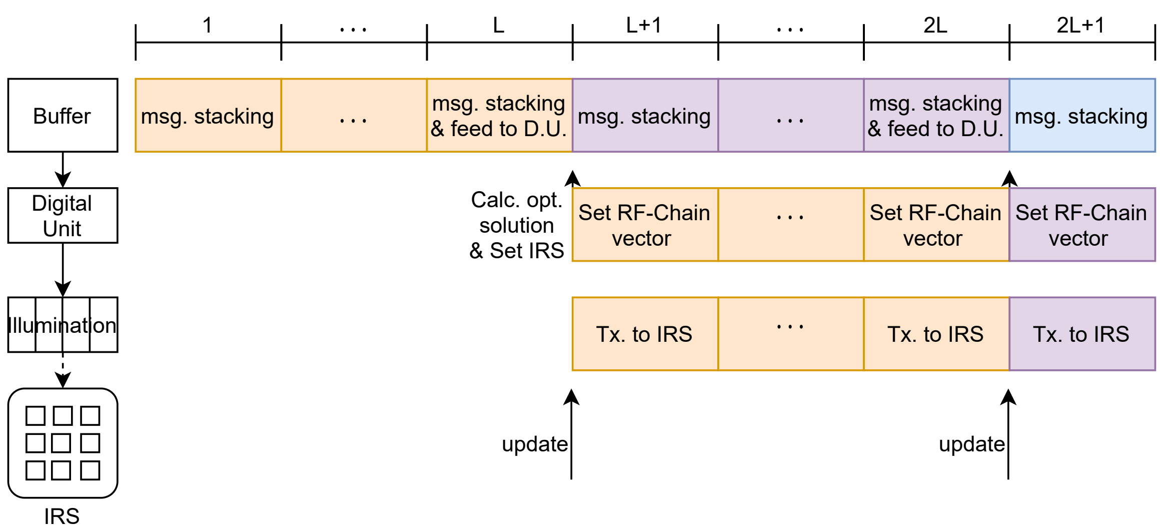

Fig. 3. Transmitter Time Table

the channel, and $\mathbf{N}$ be a K by L noise random matrix where the entries are i.i.d. zero mean Gaussian random variables with variance σ_N^2 respectively. The received signals of the UTs can be described as $\mathbf{Y} = \mathbf{HDTX} + \mathbf{N}$.

III. Method

A. Mathematical representation of the precoding problem

As is mentioned before, in symbol-wise transmission, we want to design a precoding algorithm such that $\mathbf{s}$ is as close as possible with $\mathbf{y}$, i.e. we want to make the difference between received symbols at UTs and the intended symbols as small as possible. Similarly, when designing a block-wise tranmission scheme, we want to close the gap between $\mathbf{S}$ and $\mathbf{Y}$. This distance can be defined as a frobenius norm, which states

$$d(\mathbf{S},\mathbf{Y}) = tr\{(\mathbf{Y}-\mathbf{S})(\mathbf{Y}-\mathbf{S})^H\} \tag{2}$$

To design a precoding algorithm, we need to calculate the optimal value for $\mathbf{D}$ and $\mathbf{X}$ (IRS parameters and RF chains output). In the transmitter's point of view, the equation (2) is a function containing noise $\mathbf{N}$ as a random matrix. To simplify the problem, we can try to minimize the mean value of equation (2) with respect to $\mathbf{D}$ and $\mathbf{X}$. A cost function can be defined based on this idea.

$$\begin{aligned} cost(\mathbf{D},\mathbf{X}) =& \mathcal{E}_{\mathbf{N}}\{d(\mathbf{S},\mathbf{Y})\} - \mathcal{E}_{\mathbf{N}}\{tr\{\mathbf{N}\mathbf{N}^H\}\} \\ =& tr\{(\mathbf{HDTX}-\mathbf{S})(\mathbf{HDTX}-\mathbf{S})^H\}. \end{aligned} \tag{3}$$

The term $\mathcal{E}_{\mathbf{N}}\{tr\{\mathbf{N}\mathbf{N}^H\}\}$ is subtracted, because it is a constant term and have no effect on the problem of optimizing $\mathbf{D}$ and $\mathbf{X}$. The optimization problem can then be formulated as follow,

$$\begin{aligned} \text{Objective: } & tr\{(\mathbf{HDTX}-\mathbf{S})(\mathbf{HDTX}-\mathbf{S})^H\} \\ \text{Variables: } & \mathbf{D},\mathbf{X} \\ \text{Constraints: } & \max_i |x_i|^2 < P_{peak} \\ & tr\{\mathbf{X}\mathbf{X}^H\} \leq P_{total} \end{aligned} \tag{4}$$

By introducing a regularizer λ, the optimization problem can be further simplified as

$$\begin{aligned} \text{Objective: } & tr\{(\mathbf{HDTX}-\mathbf{S})(\mathbf{HDTX}-\mathbf{S})^H\} + \lambda tr\{\mathbf{X}\mathbf{X}^H\} \\ \text{Variables: } & \mathbf{D},\mathbf{X} \\ \text{Constraints: } & \max_i |x_i|^2 < P_{peak} \end{aligned} \tag{5}$$

B. General solution for the problem

The problem stated in (5) is generally not a convex problem since the domain of $\mathbf{D}$ is not a convex set. Iterative methods can be adopted to obtain a suboptimal solution. The pseudocode below dipicts a general procedure. A termiation condition for the loop should be chosen according

Algorithm 1 General Procedure for finding ($\mathbf{D}$, $\mathbf{X}$)

1: $(\mathbf{D}, \mathbf{X}) \leftarrow initial\ values$
2: **while** terminate condition not met **do**
3: $\quad \mathbf{X} = \arg\min_{\mathbf{X}}\ tr\{(\mathbf{HDTX} - \mathbf{S})(\mathbf{HDTX} - \mathbf{S})^H\} + \lambda tr\{\mathbf{XX}^H\}$
4: $\quad \mathbf{D} = \arg\min_{\mathbf{D}}\ tr\{(\mathbf{HDTX} - \mathbf{S})(\mathbf{HDTX} - \mathbf{S})^H\}$
5: **end while**

to system requirements. For example, let the loop be executed until the difference between the cost function values of two consequent loops is smaller than a certain threshold. The value of the cost function decreases after each iteration and it is upper bounded by zero. Thus, this algorithm must converge to a certain local optimal value after enough iterations. In this algorithm, step 3 can be solved by convex optimization procedures, since $tr\{(\mathbf{HDTX} - \mathbf{S})(\mathbf{HDTX} - \mathbf{S})^H\} + \lambda tr\{\mathbf{XX}^H\}$ is a convex function with respect to $\mathbf{X}$.

Proof.

$$\begin{aligned} &\arg\min_{\mathbf{X}}\ tr\{(\mathbf{HDTX} - \mathbf{S})(\mathbf{HDTX} - \mathbf{S})^H\} + \lambda tr\{\mathbf{XX}^H\} \\ = &\arg\min_{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_L}}\ (\mathbf{HDT}\boldsymbol{x_1} - \boldsymbol{s_1})^H(\mathbf{HDT}\boldsymbol{x_1} - \boldsymbol{s_1}) + \lambda \boldsymbol{x_1}^H \boldsymbol{x_1} + \\ &+(\mathbf{HDT}\boldsymbol{x_2} - \boldsymbol{s_2})^H(\mathbf{HDT}\boldsymbol{x_2} - \boldsymbol{s_2}) + \lambda \boldsymbol{x_2}^H \boldsymbol{x_2} + \\ &+ \ldots + \\ &+(\mathbf{HDT}\boldsymbol{x_L} - \boldsymbol{s_L})^H(\mathbf{HDT}\boldsymbol{x_L} - \boldsymbol{s_L}) + \lambda \boldsymbol{x_L}^H \boldsymbol{x_L}. \end{aligned} \tag{6}$$

Since the sum of convex function is still convex function, hence the original objective function is convex.

□

Step 4 cannot generally be categorized as a convex optimization problem. In order to optimize $\mathbf{D}$, we will first introduce convex projection method and gradient descent method.

C. Convex projection method

The idea of convex projection method is to transform the problem stated in step 4 to a convex problem by first expanding the domain of $\mathbf{D}$ to a convex set and then after all the iterations project the final value of $\mathbf{D}$ onto the unit circle. It can be shown that the function $g(\mathbf{D}) = tr\{\mathbf{HDQD}^H\mathbf{H}^H\}$ is a convex function if its domain is a convex set.

Proof. To varify the the convexity of the function $g(\mathbf{D}) = tr\{\mathbf{HDQD}^H\mathbf{H}^H\}$, we first consider the function

$$\begin{gathered} f(\mathbf{X}) = tr\{\mathbf{XQX}^H\}, \\ where, \mathbf{X} \in \mathbb{C}^{K\times M}, \mathbf{Q} \in \mathbb{S}_+^M. \end{gathered} \tag{7}$$

The set $\mathbb{S}_+^M$ is defined as the set of all positive semidefinite matrices of M dimension. It is implied in equation (7) that $f(\mathbf{X})$ is a convex function. The original function $g(\mathbf{D})$ can be constructed by $f(\mathbf{X})$ as follow,

$$g(\mathbf{D}) = f(\mathbf{HD}) = tr\{\mathbf{HDQD}^H\mathbf{H}^H\}. \tag{8}$$

Assume $\theta \in [0, 1]$, then

$$\begin{aligned} g(\theta\mathbf{X} + (1-\theta)\mathbf{Y}) &= f(\mathbf{H}(\theta\mathbf{X} + (1-\theta)\mathbf{Y})) \\ &= f(\theta\mathbf{HX} + (1-\theta)(HY)) \\ &\leq \theta f(\mathbf{HX}) + (1-\theta)f(\mathbf{HY}) \\ &= \theta g(\mathbf{X}) + (1-\theta)g(\mathbf{Y}) \end{aligned} \tag{9}$$

This is exactly the definition of convex function. □

Based on this proof, let's define $\mathbf{D}' \in \{X \in \mathbb{C}^{M\times M} | X\ \ is\ \ diagonal\}$ to be any diagonal matrix. The following table illustrates the convex projection method. In this algorithm, we first do the iterative optimization in a relaxed domain and project the matrix $\mathbf{D}$ to its original domain after the iterations.

D. Gradient descent method

Another way of optimizing $\mathbf{D}$ is gradient descent method. Suppose $\mathbf{B} = \arg(\mathbf{D})$ is a real diagonal matrix which represents the phase shifts of the passive antenna arrays,

$$\mathbf{B} = diag_{matrix}\{[\beta_1, \beta_2, ..., \beta_M]\}. \tag{10}$$

Algorithm 2 Convex projection method for finding ($\mathbf{D}$, $\mathbf{X}$)

1: $(\mathbf{D}', \mathbf{X}) \leftarrow initial\ values$
2: **while** termination condition not met **do**
3: $\quad \mathbf{X} = convex\{\arg\min_{\mathbf{X}}\ tr\{(\mathbf{HD'TX} - \mathbf{S})(\mathbf{HD'TX} - \mathbf{S})^H\} + \lambda tr\{\mathbf{XX}^H\}\}$
4: $\quad \mathbf{D}' = convex\{\arg\min_{\mathbf{D}'}\ tr\{(\mathbf{HD'TX} - \mathbf{S})(\mathbf{HD'TX} - \mathbf{S})^H\}\}$
5: **end while**
6: $\mathbf{D} \leftarrow e^{j \times arg\{\mathbf{D}'\}}$
7: $\mathbf{X} = convex\{\arg\min_{\mathbf{X}}\ tr\{(\mathbf{HDTX} - \mathbf{S})(\mathbf{HDTX} - \mathbf{S})^H\} + \lambda tr\{\mathbf{XX}^H\}\}$

The gradient of the cost function with respect to the phase matrix $\mathbf{B}$ can be represented as,

$$\frac{\partial cost(\mathbf{D}, \mathbf{X})}{\partial \mathbf{B}}. \tag{11}$$

This gradient consists of a quadratic term and a first-order term. If $\mathbf{Q}$ is symmetric, then these two terms have the form of

$$diag(\frac{\partial tr\{\mathbf{HDQD}^H\mathbf{H}^H\}}{\partial \mathbf{B}}) = diag(-2\Im\{\mathbf{QD^H H^H HD}\}); \tag{12}$$

and

$$diag(\frac{\partial (tr\{\mathbf{HDA}\} + tr\{\mathbf{A}^H\mathbf{D}^H\mathbf{H}^H\})}{\partial \mathbf{B}}) = diag(-2\Im\{\mathbf{AHD}\}). \tag{13}$$

The pseudocode for gradient descend is very similar to algorithm (2), except that convex optimization method and gradient descend method are used in step 3 and step 4 respectively.

IV. Imperfect CSI

A. *Mathematical model of IRS-aided HAD transmission in imperfect CSI conditions*

In more general cases, the perfect CSI is often not available. In order to investigate the impact brought by the channel estimation error, the following channel model is adopted,

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{e}, \tag{14}$$

where $\mathbf{H} \in \mathbb{C}^{K \times M}$ denotes the real channel, $\hat{\mathbf{H}}$ is the estimated channel, and $\mathbf{e}$ is the error. It is assumed that the entries in $\mathbf{e}$ are i.i.d. zero mean Gaussian random variable with variance σ_e^2.

The distance between intended message symbols and received symbols at the UTs and objective function can be formulized as follow,

$$\begin{aligned} d(\mathbf{S},\mathbf{Y}) =& tr\{(\mathbf{Y}-\mathbf{S})^H(\mathbf{Y}-\mathbf{S})\} \\ cost(\mathbf{D},\mathbf{X}) =& \mathcal{E}_{\mathbf{e},\mathbf{N}} d(\mathbf{S},\mathbf{Y}) \\ =& \mathcal{E}_{\mathbf{e}}\{tr\{((\hat{\mathbf{H}}+\mathbf{e})\mathbf{DTX}-\mathbf{S})^H((\hat{\mathbf{H}}+\mathbf{e})\mathbf{DTX}-\mathbf{S})\}+\lambda tr\{\mathbf{X}^H\mathbf{X}\}\}. \end{aligned} \tag{15}$$

Following the assumption for $\mathbf{e}$ above that the errors of the estimated channel are i.i.d. and zero mean, the objective function Equation 15 can be simplified as

$$cost(\mathbf{D},\mathbf{X}) = \left|\left|\hat{\mathbf{H}}\mathbf{DTX}-\mathbf{S}\right|\right|_{fro}^2 + \lambda \left|\left|\mathbf{X}\right|\right|_{fro}^2 + tr\{\mathbf{X}^H\mathbf{T}^H\mathbf{D}^H \begin{bmatrix} K\cdot\sigma_e^2 & & \\ & \ddots & \\ & & K\cdot\sigma_e^2 \end{bmatrix} \mathbf{DTX}\} \tag{16}$$

Equation (16) is very similar to the previous problem, where perfect CSI is assumed except for the last term. The idea of designing $\mathbf{D}$ and $\mathbf{X}$ is to minimize the cost with respect to $\mathbf{X}$ and $\mathbf{D}$ iteratively which is exactly the same as before where we assume perfect CSI. In addition to the previously mentioned methods, we also adopt MM-algorithm which is a bench mark method in this area.

B. MM-algorithm

In MM-Algorithm, a surrogate function needs to be constructed, which touches the cost function at one point but is always larger or equal to every other point on it within the domain. The surrogate function also need to be simpler than the original cost function.

The MM-algorithm is like a variation of gradient descent. Both of the methods includes iterative procedures. However, in MM-algorithm we don't need to state the step size explicitly. In MM-algorithm, a surrogate function is at first constructed at a random point $\mathbf{D}_0$ within the domain of the original cost function. Then we find the optimal $\mathbf{D}_1$ that minimize the surrogate function. After that, we construct another surrogate function at that optimal point $\mathbf{D}_1$. If there is a unique minimum point on the surrogate function of every point in the domain of the cost function. This process should be repeated until the IRS matrix $\mathbf{D}$ converges to a certain point

$\mathbf{D}_\infty$. We first try to construct the surrogate function for the quadratic term of the cost function. We can define the following auxilary variables to simplify derivation. Let

$$\begin{aligned}
&\mathbf{A} = \mathbf{T}\mathbf{X};\\
&\mathbf{Y} = \mathbf{D}\mathbf{A};\\
&\mathbf{Q} = \hat{\mathbf{H}}^H\hat{\mathbf{H}} + \begin{bmatrix} K\cdot\sigma_e^2 & & \\ & \ddots & \\ & & K\cdot\sigma_e^2 \end{bmatrix}.
\end{aligned} \tag{17}$$

By following the definitions in equation (17), the quadratic term of the cost function can be simplified to

$$\mathbf{X}^H\mathbf{T}^H\mathbf{D}^H\left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \begin{bmatrix} K\cdot\sigma_e^2 & & \\ & \ddots & \\ & & K\cdot\sigma_e^2 \end{bmatrix}\right)\mathbf{D}\mathbf{T}\mathbf{X} = \mathbf{Y}^H\mathbf{Q}\mathbf{Y} \tag{18}$$

With this simplification, the problem of optimizing matrix $\mathbf{D}$ is converted to the problem of optimizing $\mathbf{Y}$. An auxilary matrix $\mathbf{R}$ can be defined such that $\mathbf{R}-\mathbf{Q}$ is positive definite. The idea of introducing this auxilary matrix is to remove the quadratc terms in the surrogate function while keep the new srrogate function always at least above the original cost function. For the quadratic term $\mathbf{Y}^H\mathbf{Q}\mathbf{Y}$, its surrogate function can then be constructed as

$$\begin{aligned}
&tr\left\{\mathbf{Y}^H\mathbf{Q}\mathbf{Y}\right\}\\
=&tr\left\{\mathbf{Y}_0^H\mathbf{Q}\mathbf{Y}_0 + (\mathbf{Y}-\mathbf{Y}_0)^H\mathbf{Q}\mathbf{Y}_0 + \mathbf{Y}_0^H\mathbf{Q}(\mathbf{Y}-\mathbf{Y}_0) + (\mathbf{Y}-\mathbf{Y}_0)^H\mathbf{Q}(\mathbf{Y}-\mathbf{Y}_0)\right\}\\
\leq&tr\left\{\mathbf{Y}_0^H\mathbf{Q}\mathbf{Y}_0 + (\mathbf{Y}-\mathbf{Y}_0)^H\mathbf{Q}\mathbf{Y}_0 + \mathbf{Y}_0^H\mathbf{Q}(\mathbf{Y}-\mathbf{Y}_0) + (\mathbf{Y}-\mathbf{Y}_0)^H\mathbf{R}(\mathbf{Y}-\mathbf{Y}_0)\right\}\\
=&tr\left\{\mathbf{Y}^H\mathbf{R}\mathbf{Y} + \mathbf{Y}^H(\mathbf{Q}-\mathbf{R})\mathbf{Y}_0 + \mathbf{Y}_0^H(\mathbf{Q}-\mathbf{R})\mathbf{Y} + \mathbf{Y}_0^H(\mathbf{Q}-\mathbf{R})\mathbf{Y}_0\right\},
\end{aligned} \tag{19}$$

where $\mathbf{Y}_0$ can be any value of $\mathbf{Y}(\mathbf{D}) = \mathbf{D}\mathbf{T}\mathbf{X}$. Since the only condition for the auxilary matrix $\mathbf{R}$ is that $\mathbf{R}-\mathbf{Q}$ is positive definite, it is safe to define $\mathbf{R} = \lambda_{max}^{\mathbf{Q}}\mathbf{I}$, where $\lambda_{max}^{\mathbf{Q}}$ is the largest eigenvalue of $\mathbf{Q}$. We also have

$$\mathbf{Y}^H\mathbf{R}\mathbf{Y} = \lambda_{max}^{\mathbf{Q}}\mathbf{A}^H\mathbf{D}^H\mathbf{I}\mathbf{D}\mathbf{A} = \lambda_{max}^{\mathbf{Q}}\mathbf{A}^H\mathbf{A}. \tag{20}$$

It is a constant term for $\mathbf{D}$ as well as the last term $\mathbf{Y}_0^H(\mathbf{Q}-\mathbf{R})\mathbf{Y}_0$ in equation (19), thus they can be omitted when we are optimizing $\mathbf{D}$. The surrogate function for the entire cost function is equivalent to

$$\begin{aligned}
&\overline{cost(\mathbf{D})}\\
=&tr\left\{\mathbf{Y}^H(\mathbf{Q}-\mathbf{R})\mathbf{Y}_0 + \mathbf{Y}_0^H(\mathbf{Q}-\mathbf{R})\mathbf{Y} - \mathbf{S}^H\mathbf{H}\mathbf{D}\mathbf{T}\mathbf{X} - \mathbf{X}^H\mathbf{T}^H\mathbf{D}^H\mathbf{H}^H\mathbf{S}\right\}
\end{aligned} \tag{21}$$

In order to keep the equation simple, the following variables are introduced,

$$
\begin{aligned}
\mathbf{B} &= \mathbf{S}^H\mathbf{H} \\
\mathbf{R} &= \mathbf{A}^H\mathbf{D}_0^H(\mathbf{Q}-\lambda_{max}^{\mathbf{Q}}\mathbf{I})-\mathbf{B} \\
\mathbf{G} &= \mathbf{A}\mathbf{R} \\
\mathbf{F} &= -\mathbf{G}^H.
\end{aligned}
\tag{22}
$$

To find the minimum of the surrogate function (21), we need to minimze

$$
\overline{cost(\mathbf{D})} = tr\{\mathbf{G}\mathbf{D}+\mathbf{D}^H\mathbf{G}^H\} \tag{23}
$$

with respect to $\mathbf{D}$. Since additive constant terms do not affect the outcome of optimization problems. The terms $tr\{\mathbf{D}^H\mathbf{D}\}$ and $tr\{\mathbf{G}\mathbf{G}^H\}$ can be added to the surrogate function. We then have

$$
\begin{aligned}
& \arg\min_{\mathbf{D}}(tr\{\mathbf{G}\mathbf{D}+\mathbf{D}^H\mathbf{G}^H\}) \\
\Longleftrightarrow\ & \arg\min_{\mathbf{D}}(tr\{\mathbf{G}\mathbf{D}+\mathbf{D}^H\mathbf{G}^H+\mathbf{D}^H\mathbf{D}+\mathbf{G}\mathbf{G}^H\}) \\
\Longleftrightarrow\ & \arg\min_{\mathbf{D}}(tr\{(\mathbf{D}-(-\mathbf{G})^H)^H(\mathbf{D}-(-\mathbf{G})^H)\}) \\
\Longleftrightarrow\ & \arg\min_{\mathbf{D}}(||\mathbf{D}-\mathbf{F}||_{fro}^2) \\
\Longrightarrow\ & \mathbf{D}_{opt} = diag_{matrix}(\arg\mathbf{F})
\end{aligned}
\tag{24}
$$

C. Gradient descent method

The gradient descent method is exactly the same as before. We can define the phases in matrix $\mathbf{D}$ as

$$
\mathbf{\Omega} = \arg\ \mathbf{D}. \tag{25}
$$

By expanding the cost function (16), and simplifying it with the same notation in equations (17) and (22) we have

$$
cost(D) = tr\{\mathbf{A}^H\mathbf{D}^H\mathbf{Q}\mathbf{D}\mathbf{A}-(\mathbf{A}^H\mathbf{D}^H\mathbf{B}^H+\mathbf{B}\mathbf{D}\mathbf{A}+\mathbf{S}^H\mathbf{S})\} \tag{26}
$$

By using the same derivation methods as before, the partial derivertives are

$$
\begin{aligned}
\frac{\partial tr\{\mathbf{A}^H\mathbf{D}^H\mathbf{Q}\mathbf{D}\mathbf{A}\}}{\partial\mathbf{\Omega}} &= -2diag_{matrix}(\Im\{\mathbf{D}\mathbf{A}\mathbf{A}^{\mathbf{H}}\mathbf{D}^{\mathbf{H}}\mathbf{Q}\}) \\
\frac{\partial tr\{\mathbf{A}^H\mathbf{D}^H\mathbf{B}^H+\mathbf{B}\mathbf{D}\mathbf{A}\}}{\partial\mathbf{\Omega}} &= -2diag_{matrix}(\Im\{\mathbf{D}\mathbf{A}\mathbf{B}\})
\end{aligned}
\tag{27}
$$

Based on these derivertives, the gradient method can be implemented without difficulties.

V. Evaluation

Residual sum of squares (RSS) measure the interference power of the received signal and is defined as,

$$RSS = tr\{(\mathbf{HDTX} - \mathbf{S})(\mathbf{HDTX} - \mathbf{S})^H\}$$
$$meanRSS = RSS/(K \times L) \tag{28}$$

This transmission scheme is simulated by computer and RSS to PAPR behavior is evaluated. The setup is as follow,

$$K = 8; N = 4; L \in 1, 3, 5; M = 64; \lambda = 0.5, \tag{29}$$

the entries of matrix $\mathbf{T}$ are uniformly distributed on a complex unit circle, and the entries of $\mathbf{H}$ are zero mean complex gaussian random variables with variance one. In order to compare the performance of convex projection and gradient descend method, both methods are simulated and the results are plotted.

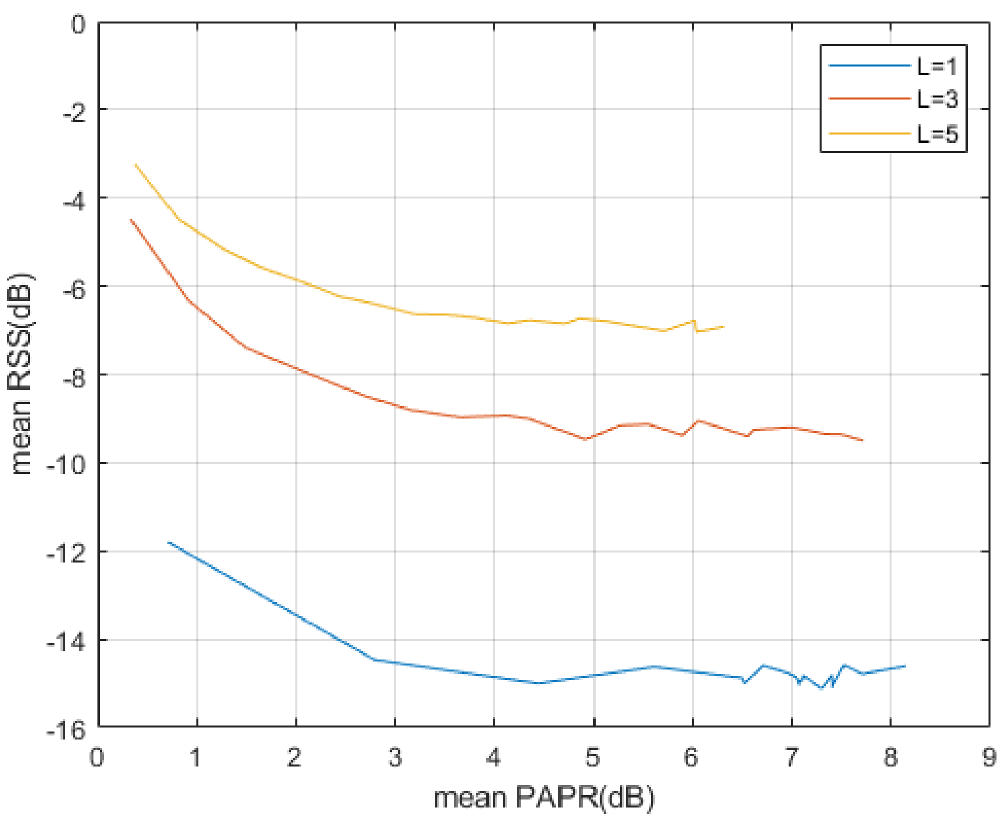

Fig. 4. RSS-PAPR with convex projection

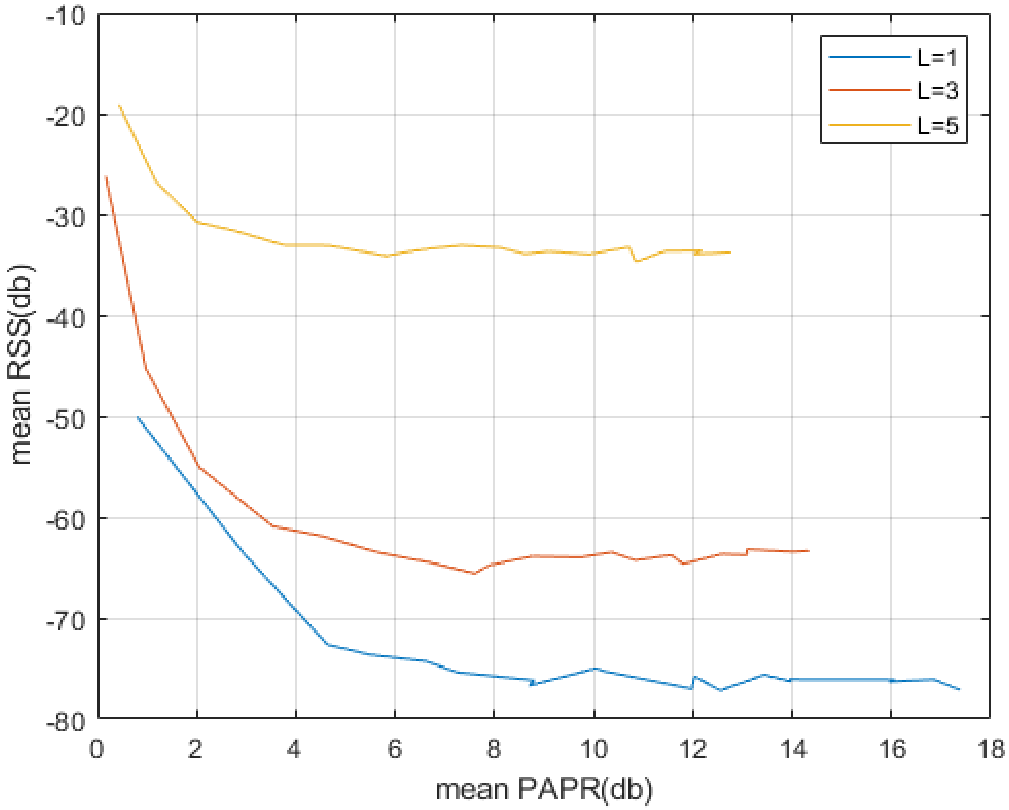

Fig. 5. RSS-PAPR with gradient descend

The result shows that gradient descend method performs better than convex projection. The graph of gradient descend method also indicates that RSS will be very close to zero if $L \leq N$, which will later be stated in more details. It can be also observed that RSS performance is better for the systems with $L = 1$ than the ones with $L > 1$. A theoretical explaination for this phenomenon is as follow.

Proof.

$$
\begin{aligned}
&\underset{\mathbf{D},\mathbf{X}}{\arg\min}\ tr\{(\mathbf{HDTX}-\mathbf{S})(\mathbf{HDTX}-\mathbf{S})^H\}+\lambda tr\{\mathbf{XX}^H\}\\
=&\underset{\mathbf{D},\boldsymbol{x_1},\boldsymbol{x_2},\ldots,\boldsymbol{x_L}}{\arg\min}\ (\mathbf{HDT}\boldsymbol{x_1}-\boldsymbol{s_1})^H(\mathbf{HDT}\boldsymbol{x_1}-\boldsymbol{s_1})+\lambda\boldsymbol{x_1}^H\boldsymbol{x_1}+\\
&+(\mathbf{HDT}\boldsymbol{x_2}-\boldsymbol{s_2})^H(\mathbf{HDT}\boldsymbol{x_2}-\boldsymbol{s_2})+\lambda\boldsymbol{x_2}^H\boldsymbol{x_2}+\\
&+\ldots+\\
&+(\mathbf{HDT}\boldsymbol{x_L}-\boldsymbol{s_L})^H(\mathbf{HDT}\boldsymbol{x_L}-\boldsymbol{s_L})+\lambda\boldsymbol{x_L}^H\boldsymbol{x_L}\\
\geq&\min_{\mathbf{D}_1,\boldsymbol{x_1},\boldsymbol{x_2},\ldots,\boldsymbol{x_L}}\{(\mathbf{HD_1T}\boldsymbol{x_1}-\boldsymbol{s_1})^H(\mathbf{HD_1T}\boldsymbol{x_1}-\boldsymbol{s_1})+\lambda\boldsymbol{x_1}^H\boldsymbol{x_1}\}+\\
+&\min_{\mathbf{D_2},\boldsymbol{x_1},\boldsymbol{x_2},\ldots,\boldsymbol{x_L}}\{(\mathbf{HD_2T}\boldsymbol{x_2}-\boldsymbol{s_2})^H(\mathbf{HD_2T}\boldsymbol{x_2}-\boldsymbol{s_2})+\lambda\boldsymbol{x_2}^H\boldsymbol{x_2}\}+\\
&+\ldots+\\
+&\min_{\mathbf{D_L},\boldsymbol{x_1},\boldsymbol{x_2},\ldots,\boldsymbol{x_L}}\{(\mathbf{HD_LT}\boldsymbol{x_L}-\boldsymbol{s_L})^H(\mathbf{HD_LT}\boldsymbol{x_L}-\boldsymbol{s_L})+\lambda\boldsymbol{x_L}^H\boldsymbol{x_L}\}\\
=&\min_{\mathbf{D_1},\boldsymbol{x_1}}\{(\mathbf{HD_1T}\boldsymbol{x_1}-\boldsymbol{s_1})^H(\mathbf{HD_1T}\boldsymbol{x_1}-\boldsymbol{s_1})+\lambda\boldsymbol{x_1}^H\boldsymbol{x_1}\}+\\
+&\min_{\mathbf{D_2},\boldsymbol{x_2}}\{(\mathbf{HD_2T}\boldsymbol{x_2}-\boldsymbol{s_2})^H(\mathbf{HD_2T}\boldsymbol{x_2}-\boldsymbol{s_2})+\lambda\boldsymbol{x_2}^H\boldsymbol{x_2}\}+\\
&+\ldots+\\
+&\min_{\mathbf{D_L},\boldsymbol{x_L}}\{(\mathbf{HD_LT}\boldsymbol{x_L}-\boldsymbol{s_L})^H(\mathbf{HD_LT}\boldsymbol{x_L}-\boldsymbol{s_L})+\lambda\boldsymbol{x_L}^H\boldsymbol{x_L}\}
\end{aligned}
\tag{30}
$$

The last term of this formula represents the result of this communication scheme with $L = 1$. And this indicates that if the best $(\mathbf{D}, \mathbf{X})$ pairs are applied for the schemes with $L = 1$ and $L > 1$ respectively, then the transmission scheme with $L = 1$ must have better performance. □

A. Relation between block length and RF chain number

It can be observed from the graphes (6) that for any L, if $L \leq N$, the mean RSS grow smaller as M grows larger. But from the graph (7), there seem to be a boundary for L below which the mean RSS will converge to a small value It has been proven in a previous article [2] that if the domain of $\mathbf{D}$ is expanded to $\mathbb{C}^{M\times M}$, i.e. if a more generalized analog unit is used instead of IRS, the mean RSS can always reach zero as long as $L \leq N$. The reason behind is that the RSS will be zero if all the eigenvectors corresponding to zero eigenvalues of the matrix $\mathbf{HDTT^HD^HH^H}$ lies in the linear space spanned by eigenvectors corresponding to the zero eigenvalues of matrix $\mathbf{SS^H}$. One requirement for this solution to be achievable is $L \leq N$, since the matrix $\mathbf{HDTT^HD^HH^H}$ has $K-N$ zero eigenvalues and there are $K-L$ zero eigenvalues in $\mathbf{SS^H}$. A mathematical proof for the phenomenon that RSS will tend to zero as M grows larger can be given by using the property of F-distribution and mathematical induction.

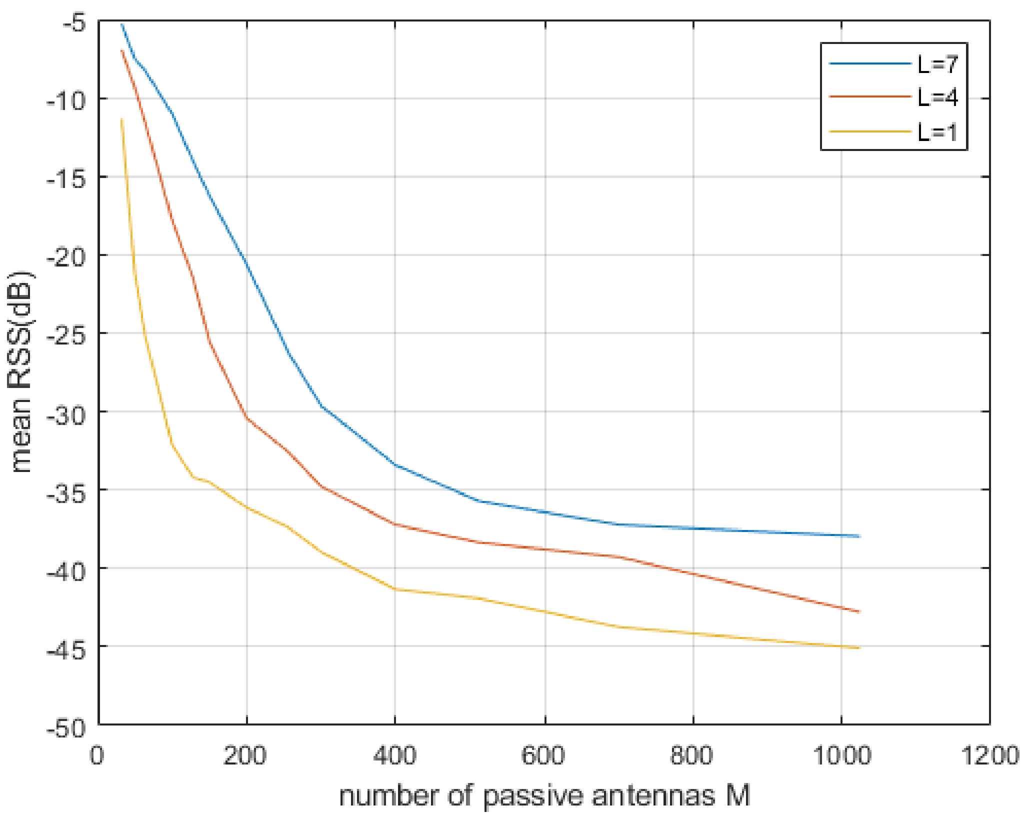

Fig. 6. RSS-M for block length $L \in \{1, 4, 7\}$

B. Relation between mean RSS and number of passive antennas

It can be proved that even in extreme case where there is only one RF chain, the probability that there exist no solution for RSS to equal to zero is a higher order infinitesimal of the function $f(M) = (1/M)^k$, where $k \in \mathbb{N}$ if $L = 1$.

Suppose that there is only one RF-chain namely $N = 1$. This means the only RF-chain serves only as a power source, and the information is largely embeded in the phaes. W.l.og. since the RF-chain can regulate the power such that every entries in the channel matrix $\mathbf{H}$ has the same power as the entries in the message matrix, we can assume that the entries in these two matrices have the same variance $\sigma^2 = 1$. The term $\mathbf{HDt}x$ can be rewriten into $\mathbf{Hd}$, where $\mathbf{d}$ is diag$\{\mathbf{D}\}$

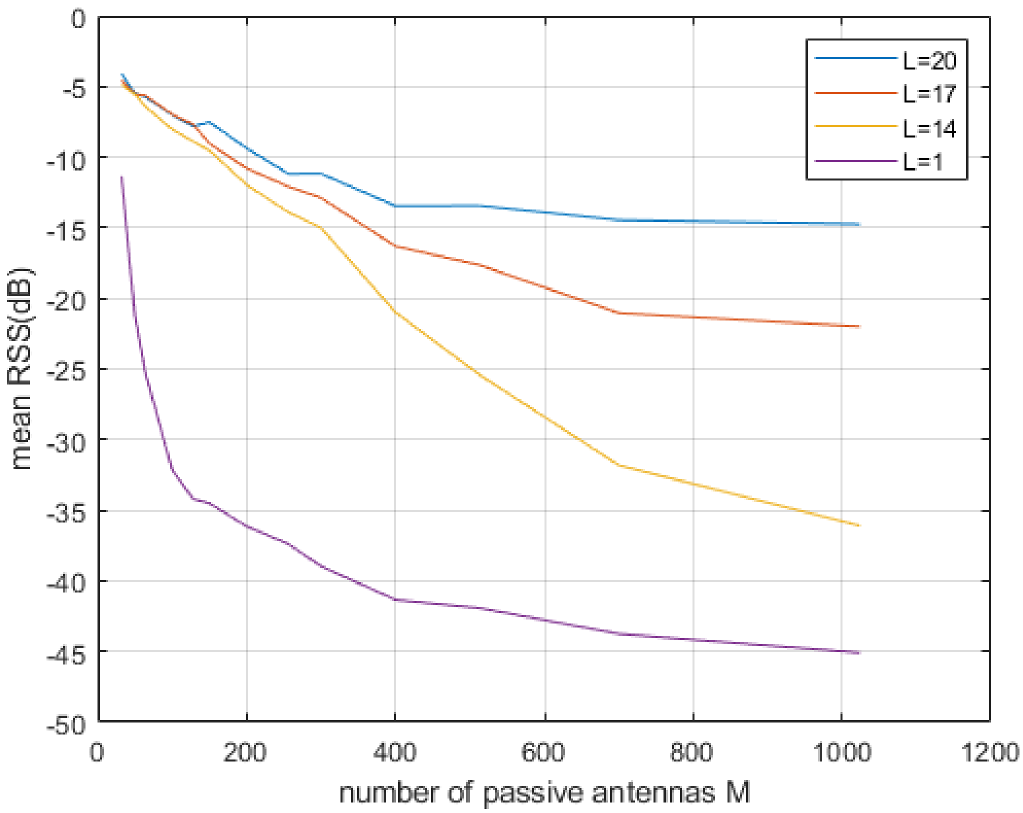

Fig. 7. RSS-M full result

if the internal channel matrix $\mathbf{t}$ is an all-one vector, and the entries in $\mathbf{H}$ are i.i.d. complex normal gaussian random variables. A theorem can be proven.

Theorem 1. *If $\mathbf{H}$ is an complex normal gaussian random matrix with independent entries and the length of vector $\mathbf{d}$ can be arbitary long, then it is almost certain that there is a $\mathbf{d}$ such that $\mathbf{H} \cdot \mathbf{d} = \mathbf{0}$.*

For the simple case where $\mathbf{H}$ is a row vector, namely there is only one user $K = 1$. Let $\mathbf{h}^T = H$, then probability $Pr(\exists \mathbf{d}, (\mathbf{h}^T \mathbf{d} = 0))$ is desired. This can be translated into geometry language. In a complex plain, each entry in vector $\mathbf{h}^T$ is an vector whose length is a RV following standard gaussian distribution and phase is uniformly distributed between 0 to 2π. The vector $\mathbf{d}$ is a rotation vector which can rotate each entry of $\mathbf{h}^T$ with an arbitary angle. Therefore, the proposition $\exists \mathbf{d}, (\mathbf{h}^T \mathbf{d} = 0)$ can be interpreted as finding an rotation angle for each of the entries in matrix $\mathbf{h}^T$ such that these rotated vectors add up to zero. We have

$$\begin{aligned}&\exists \mathbf{d}, (\mathbf{h}^T\mathbf{d} = 0)\\ \Leftrightarrow &\max(|h_1|, \ldots, |h_M|) \leq |h_1| + \cdots + |h_M| - \max(|h_1|, \ldots, |h_M|)\end{aligned} \tag{31}$$

It can also be observed that $|h_1| > |h_2| + \cdots + |h_M|$ and $|h_2| > |h_1| + \cdots + |h_M|$ will never happen simultaneously. Therefore, we can write the above probability in negation form.

$$\begin{aligned}P_1 =&Pr(\exists \mathbf{d}, (\mathbf{h}^T\mathbf{d} = 0))\\ =&1 - (M \times Pr(|h_1| > |h_2| + \cdots + |h_M|))\end{aligned} \tag{32}$$

The properties of F-distribution can be used to determin the upper bound of $Pr(|h_1| > |h_2| + \cdots + |h_M|)$.

$$\begin{aligned}&M \times Pr(|X| > |X_1| + \cdots + |X_{M-1}|)\\ \leq&M \times Pr(|X|^2 > |X_1|^2 + \cdots + |X_{M-1}|^2)\\ \leq&M \times Pr(|X|^2 + |Y|^2 > |X_1|^2 + \cdots + |X_{M-1}|^2)\\ =&M \times Pr\left(\frac{2}{M-1} > \frac{(|X_1|^2 + \cdots + |X_{M-1}|^2)/(M-1)}{(|X|^2 + |Y|^2)/2}\right)\\ =&M \times F\left(\frac{2}{M-1}; M-1, 2\right) = M \times I_{\frac{1}{2}}\left(\frac{M-1}{2}, 1\right)\\ =&M \times \left(\frac{1}{2}\right)^{\frac{M-1}{2}}\end{aligned} \tag{33}$$

The first less equal sign holds because $|X|^2 > (|X_1|^2 + \cdots + |X_{M-1}|^2)^2 > |X_1|^2 + \cdots + |X_{M-1}|^2$. For the multi-user case where $K \neq 1$, the matrix $\mathbf{H}$ has more than one rows. A solution for $\mathbf{d}$ can be constructed recursively, which means we can build a sufficient condition for the existence of rotation vector $\mathbf{d}$. The idea here is to find a recursive expression for the upper bound of the probability for the sufficient condition to not hold. As is mentioned above we want to find a solution for $\mathbf{d}$ recursively such that $\mathbf{Hd} = \mathbf{0}$. Suppose we already know how to build $\mathbf{d}$ if $K = k-1$, namely for the case where there are $k-1$ rows in $\mathbf{H}$. Let that matrix be denoted as $\mathbf{H}^{(k-1)}$. Then we have $\mathbf{H}^{(k-1)} \cdot \mathbf{d} = \mathbf{0}$. This also implies that $(\mathbf{H}^{(k-1)} \cdot \mathbf{d}) \cdot e^{j\varphi} = \mathbf{0}$, where φ is an arbitary number. Based on these assumptions and discussions, for the case where $K = k$, we can divide $\mathbf{H}^{(k)}$ in to several $(k-1)$-row submatrix with smaller column numbers and divide $\mathbf{d}$ respectively. This is illustrated as figure (8). We first try to find $\mathbf{d}_1, \ldots, \mathbf{d}_N$ such that $\forall i \in \{1, \ldots, N\}, \mathbf{H}_i\mathbf{d}_i = \mathbf{0}$. Then for the last row of matrix $\mathbf{H}^{(k)}$, we know $\mathbf{h}_i^T \cdot \mathbf{d}_i$ is a scalar value. However, for each subvector $\mathbf{d}_i$ there is at least one degree of freedom left. We can

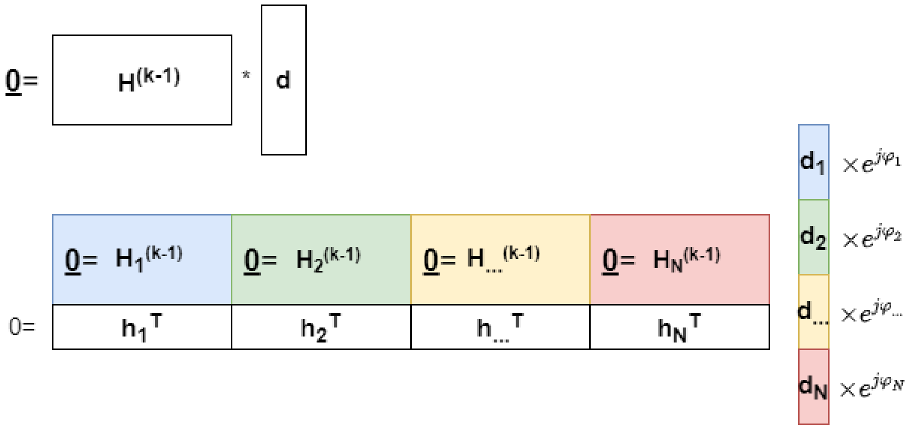

Fig. 8. Idea of construction

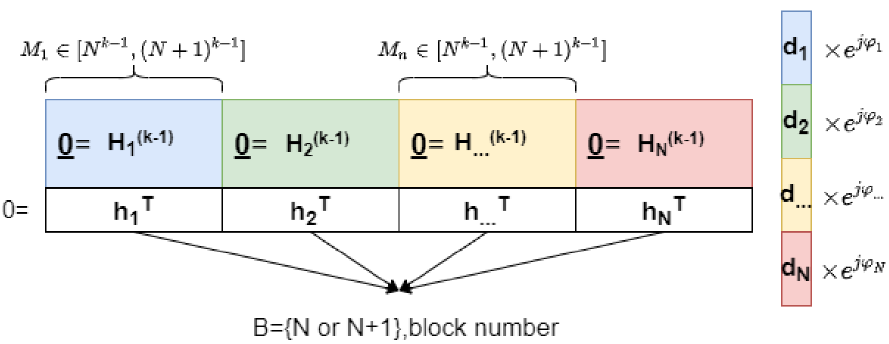

Fig. 9. Submatrix Construction

apply an overall rotation $e^{j\varphi_i}$ to each of the vectors, and the colored parts in figure (8) stay $\mathbf{0}$. Therefore, the problem is transformed back to the previous one, where we want to find rotation angles $\varphi_1, \dots, \varphi_N$ such that

$$\begin{bmatrix} \mathbf{h}_1^T \mathbf{d}_1 & \dots & \mathbf{h}_N^T \mathbf{d}_N \end{bmatrix} \cdot \begin{bmatrix} e^{j\varphi_1} \\ \vdots \\ e^{j\varphi_N} \end{bmatrix} = 0 \tag{34}$$

This is the general idea for the proof. Suppose $\mathbf{H}^{(k)}$ has M columns. We can find at least one $N \in \mathbb{N}$, such that $M \in [N^k, (N+1)^k]$. Therefore, the matrix $\mathbf{H}^{(k)}$ can be divided as figure (9). The proposition that $\mathbf{H}^{(k)}$ cannot be rotated to zero implies that one of the colored blocks cannot be rotated to zero or the final row cannot be made zero. The probability that the colored part cannot be made zero can be calculated by using recursion. Let's consider the last row. We can convert this probabilty to F-distribution if the variance of $\mathbf{h}_i^T \cdot \mathbf{d}_i$ can be made to 1. Define

$$
\begin{aligned}
&B \in \{N, N+1\} : \text{block number} \\
&M_{1 \ldots B} : \text{column number the blocks.}
\end{aligned}
\tag{35}
$$

This is also illustrated in figure (9). Each block in the last row $\mathbf{h}_i^T$ will be multiplied with corrspondence vector block $\mathbf{d}_i$. This is complex gaussian random variable, $\mathbf{h}_i^T \mathbf{d}_i \; CN(0, M_i)$. Therefore, the following steps are performed to transform the problem back to the one where we have standard gaussian variables. Define

$$
\begin{aligned}
&\forall b \in \{1, \ldots, B\}, Y_b = \mathbf{h}_b^T \mathbf{d}_b \; CN(0, M_b) \\
&\forall b \in \{1, \ldots, B\}, X_b \; CN(0, 1)
\end{aligned}
\tag{36}
$$

Then we have,

$$
\begin{aligned}
&\sum_{b=1}^{B} Pr(|Y_b| > |Y_1| + \cdots + |Y_{b-1}| + |Y_{b+1}| + \cdots + |Y_B|) \\
\leq &\sum_{b=1}^{B} Pr(|Y_b| > \sqrt{M_{min}}(\frac{|Y_1|}{\sqrt{M_i}} + \cdots + \frac{|Y_{b-1}|}{\sqrt{M_{b-1}}} + \frac{|Y_{b+1}|}{\sqrt{M_{b+1}}} + \cdots + \frac{|Y_B|}{\sqrt{M_B}})) \\
= &\sum_{b=1}^{B} Pr(\frac{\sqrt{M_b}}{\sqrt{M_{min}}} \frac{|Y_b|}{\sqrt{M_b}} > |X_1| + \cdots + |X_{B-1})| \\
\leq &B \times Pr(\frac{\sqrt{M_{max}}}{\sqrt{M_{min}}} |X| > |X_1| + \cdots + |X_{B-1}|) \\
\leq &B \times \left(\frac{M_{max}}{M_{max} + M_{min}} \right)^{\frac{B-1}{2}}
\end{aligned}
\tag{37}
$$

For larger M, we have

$$
\lim_{M \to +\infty} \frac{M_{max}}{M_{max} + M_{min}} \leq \lim_{N \to +\infty} \frac{(N+1)^{k-1}}{2 \times N^{k-1}} = \frac{1}{2} \leq \frac{2}{3} \tag{38}
$$

Therefore,

$$
Pr \left(\nexists (\varphi_1, \cdots, \varphi_B). \begin{bmatrix} \mathbf{h}_1^T \mathbf{d}_1 & \ldots & \mathbf{h}_B^T \mathbf{d}_B \end{bmatrix} \cdot \begin{bmatrix} e^{j\varphi_1} \\ \vdots \\ e^{j\varphi_B} \end{bmatrix} = 0 \right) \leq (N+1) \times \left(\frac{2}{3} \right)^{\frac{N-1}{2}} \tag{39}
$$

The complete probability can then be obtained

$$
\begin{aligned}
P_k &= Pr(\nexists \mathbf{d}, (\mathbf{H}^{(k)T} \mathbf{d} = 0)) \\
&\leq (N+1) \times P_{k-1} + (N+1) \times \left(\frac{2}{3} \right)^{\frac{N-1}{2}}
\end{aligned}
\tag{40}
$$

Based on this inequality, one can define an auxilary sequence recursively

$$\begin{aligned} P_1' &= (N+1) \times \left(\frac{2}{3}\right)^{\frac{N-1}{2}} \\ P_k' &= (N+1) \times P_{k-1} + (N+1) \times \left(\frac{2}{3}\right)^{\frac{N-1}{2}} \end{aligned} \tag{41}$$

By using mathematical induction, it can be proven that $\forall k \in \mathbb{N}, P_k < P_k'$. This sequence can also be writen in analytic form by solving difference equation,

$$P_k' = \frac{(N+1)^{k+1} - (N+1)}{N} \left(\frac{2}{3}\right)^{\frac{N-1}{2}} \tag{42}$$

It can be observed that this upper bound will converge to zero if N tend to infinity.

For the original problem with block length $L = 1$ and RF chain number $N = 1$, we want to know whether there is a matrix $\mathbf{D}$ such that $\mathbf{s} = \mathbf{HDt}x$. Define $\mathbf{H}' = \mathbf{H} \times diag_{matrix}\{x\mathbf{t}\}$, then the problem will be transformed to finding vector $\mathbf{d} = diag_{vector}\{\mathbf{D}\}$ such that $\mathbf{s} = \mathbf{H}'\mathbf{d}$. In order to make the left side zero, define

$$\begin{aligned} \mathbf{H}'' &= \begin{bmatrix} \mathbf{H}', & \mathbf{s} \end{bmatrix} \\ \mathbf{d}' &= \begin{bmatrix} \mathbf{d} \\ d_{M+1} \end{bmatrix} \end{aligned} \tag{43}$$

According the the theorem above, a solution for $\mathbf{d}'$ can be found such that $\mathbf{0} = \mathbf{H}''\mathbf{d}'$. Therefore the IRS parameters should be $\mathbf{D} = diag_{matrix}\{-d_{M+1}^{-1}\mathbf{d}\}$.

The M to L behavior can be approximated as the following graph for a constant mean RSS value $meanRSS = 0.025$, it seems to be approximately linear.

VI. Conclusions

This paper discusses the performance of a Hybrid A-D transmitter system with passive antenna array. In order to lower the update rate of the passive antennas, data vectors of multiple time steps can be stacked into a matrix. By doing so, the system will suffer a performance loss. However, from the experiments, if $L < N$, this loss can be compensated by increasing the number of passive antennas. Although gradient descend method can only reach a suboptimal solution, the mean RSS can still reach zero if the number of passive antennas is high. There are still some work to be done. The asymptotic behavior of large passive antenna number M is proven only for the case where block length $L = 1$. We need to varify whether the mean RSS will still tend to zero if M tends to zero with larger block lengths. We only consider the Gaussian channel

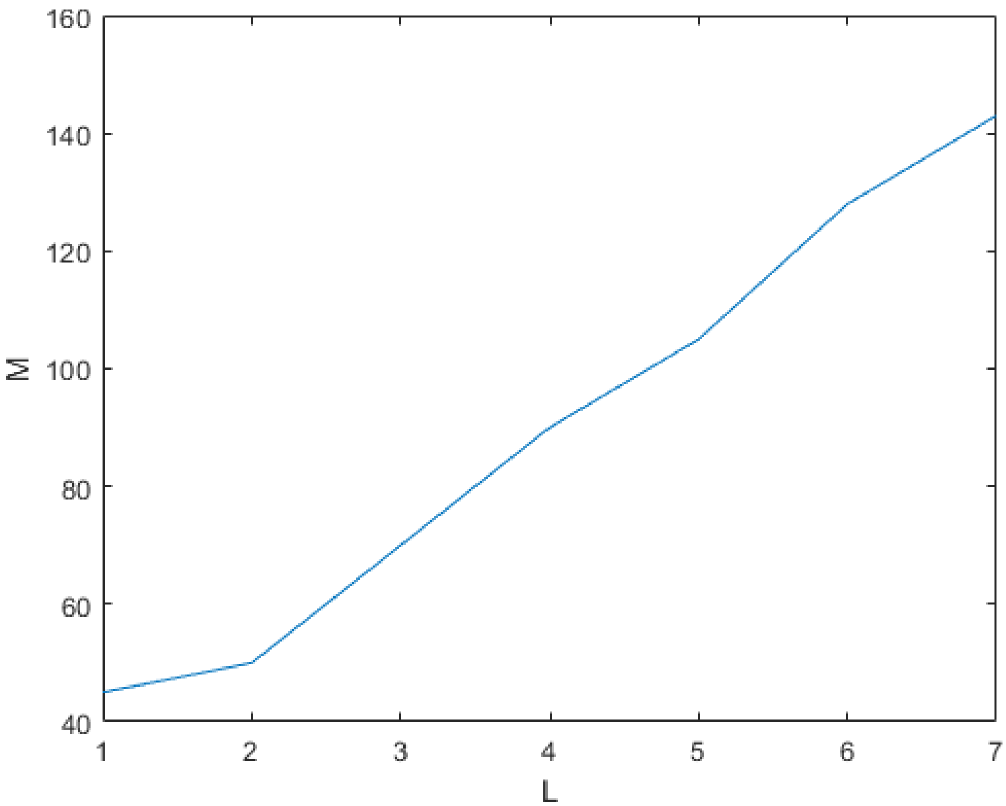

Fig. 10. M-L for constant mean RSS

here. It may be better if we investigate the design of the precoding method by using Kronecker channel model.

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