## **Exercise 1-Local Variance**

(A) We can estimate local variance by using this estimator:

$$\hat{c}_{i_t}(t) = \frac{1}{2k_n + 1} \sum_{j=-k_n}^{k_n} (r_{i_{t+j}}^c)^2$$

If the jump interval  $i_t$  falls in the begin or end of market hour, we can adjust our estimator as:

$$\hat{c}_{i_t}(t) = \frac{1}{J_1 - J_2 + 1} \sum_{i=-J_2}^{J_1} (r_{i_{t+j}}^c)^2$$

with

$$J_2 = max(1, i - k_n)$$

$$J_1 = min(k_n + i, n)$$

where i is the number of interval intraday that jump time falls in, n is the number of daily observe interval,  $k_n$  is the half length of estimate windows.

The following is graph of the local variance estimator for PG and DIS on January 3, 2007.

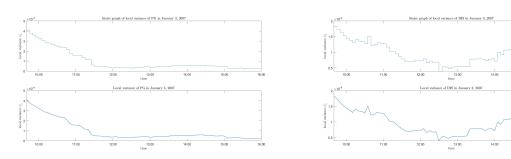


Figure 1: Estimated Local Variance of PG and DIS on January 3, 2007

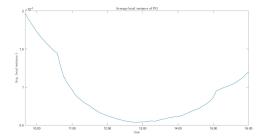
From the graphs we can find the daily local variance shows a U-shape pattern (this is much more obvious in DIS' graph): at the beginning of market hour, the local variance is very high; as the deal goes on, the local variance decreases and always reach its lowest point at the middle of market hour; in the latter part of market hour, the local variance shows a increasing trend.

As for the value of local variance, we can find DIS' local variance is much larger than the PG's, which mean the volatility of DIS' log-returns will experience larger fluctuations.

(B) We use this estimator to estimate the average local variance for each interval:

$$\bar{\hat{c}}_i = \frac{1}{T} \sum_{t=1}^{T} \hat{c}_{i_t}$$

for i = 1, 2, ... n. The following is graph of the local variance estimator for PG and DIS on January 3, 2007.



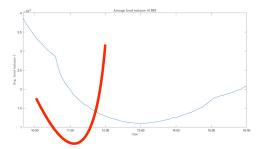


Figure 2: Estimated Average Local Variance of PG and DIS

## relation to the time-of-day factor? (-0.1)

After taking average of the local variance throughout all observe time, the U-shape pattern are much clear in for PG and DIS' local variance. From the graphs we can see, the curve shape of these two stocks are very similar, the volatility of DIS is bigger than PG's.

#### The **MATLAB** code:

Function of Local Variance

```
function [ct] = local_var(lr_c,kn)
2
   %lr_c is a n∗T diffusive return matrix
3
   %where n is the observation number each day and T is the number of
       observative days
   n=size(lr_c,1);
5
   T=size(lr_c,2);
6
   ct=zeros(n,T);
7
   for i=1:n
8
       j2=max(1,i-kn);
9
       j1=min(kn+i,n);
       ct(i,:)=sum(lr_c(j2:j1,:).^2)/((j1-j2+1)/n);
10
11
   end
12
   end
```

Scripts of Q1

```
addpath('D:\ZM—Documents\MATLAB\data','functions','scripts');
2 | [dates_PG,lp_PG]=load_stock('PG.csv','m');
 3 N_PG=sum(floor(dates_PG(1,1))==floor(dates_PG(:,1)));% number of observations
        per day
 4 T_PG=size(dates_PG,1)/N_PG;
   [rdates_PG,lr_PG]=log_return([dates_PG lp_PG],N_PG,1);
6
   days_PG=unique(floor(rdates_PG));
   %A1
8
   %local variance for n∗T intervals
9
10 a=5;
11 kn=11;
12 | [lr_c_PG,lr_d_PG]=c_d_log_returns(lr_PG,N_PG,a);
13 ct_PG=local_var(lr_c_PG,kn);
14 | %plot one day's local variance
15 | figure;
16 | subplot(2,1,1);
17 | stairs(rdates_PG(:,1),ct_PG(:,1));
18 | xlabel('time');
19 | ylabel('local variance $\hat{c_{i_t}}$');
20 | xlim([min(rdates_PG(:,1)), max(rdates_PG(:,1))]);
   title('Stairs graph of local variance of PG in January 3, 2007');
22 | datetick('x', 'keeplimits');
23
24 | subplot(2,1,2);
25 | plot(rdates_PG(:,1),ct_PG(:,1));
26 | xlabel('time');
27 | ylabel('local variance $\hat{c_{i_t}}$');
   xlim([min(rdates_PG(:,1)),max(rdates_PG(:,1))]);
29 | title('Local variance of PG in January 3, 2007');
30 | datetick('x','keeplimits');
31
32
   %1B
   %average local variance
34 | ct_avg_PG=mean(transpose(ct_PG));
   %plot
36 | figure;
37
   plot(rdates_PG(:,1),ct_avg_PG);
38 | xlabel('time');
39 | ylabel('Avg. local variance $\bar{\hat{c}}$');
40 | xlim([min(rdates_PG(:,1)), max(rdates_PG(:,1))]);
41 | title('Average local variance of PG');
42 | datetick('x','keeplimits');
```

# Exercise 2-Jumps in the Variance

(A) To estimate the volatility before jump  $\hat{c}_{i_t}^-$ , we use:

$$\hat{c}_{i_t}^- = \frac{1}{i - J_2} \sum_{j = -J_2}^{i-1} (r_{i_{t+j}}^c)^2$$

with  $J_2 = max(1, i - k_n), i = 2, 3, ... n$ .

To estimate the volatility before jump  $\hat{c}_{it}^+$ , we use:

$$\hat{c}_{i_t}^+ = \frac{1}{J_1 - i} \sum_{j=i+1}^{J_1} (r_{i_{t+j}}^c)^2$$

with  $J_1 = min(k_n + i, n), i = 1, 2, ... n - 1.$ 

To measure the magnitude of the jump return, we use absolute value of jump returns:

$$M_{r_{t,i_t}^d} = |r_{t,i_t}^d|$$

with i = 2, 3, ... n.

The following is graph of  $\hat{c}^-_{it},\,\hat{c}^+_{it}$  and  $|r^d_{t,i_t}|$  of PG and DIS.

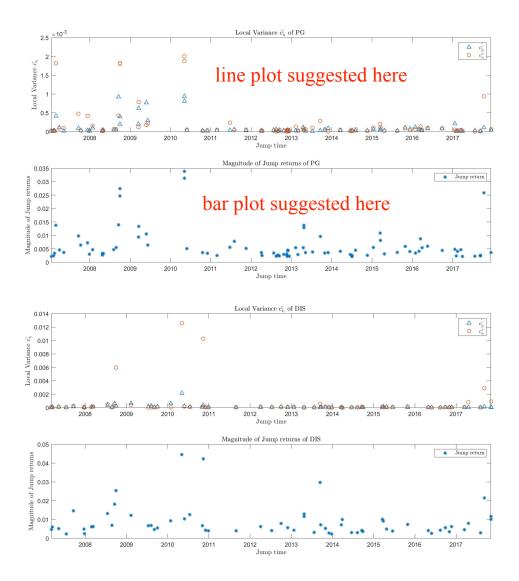


Figure 3:  $\hat{c}_{i_t}^-, \hat{c}_{i_t}^+$  and  $|r_{t,i_t}^d|$  of PG and DIS

From these graphs, we can detect the trading time with large magnitude of jump returns is consistent with the time when the difference of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$  is large, which indicates a jump local variance may exist. This finding may give us a method to detect the jump volatility in stock's returns: if there is a large magnitude of stock return in the stock price, then there is a high probability that the volatility is experiencing a jump.

(B) (i) To create the confidence interval of left and right local variance, we can use bootstrap method. In order to simplify the programming, we here just focus on those jumps fall in the middle of market hour. This simplification should be reasonable since about 95% jumps occur in the middle of market hour and the accuracy es-

timates for the beginning and end interval's local variance are undesirable due to limited available data.

The follows are the graphs of jump local variance and its confidence interval of PG and DIS.

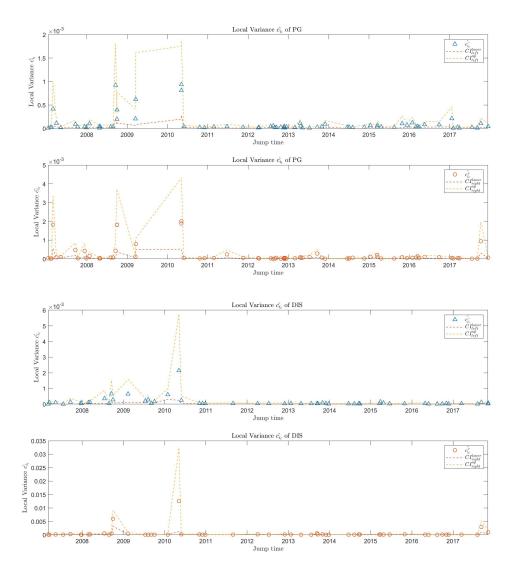


Figure 4:  $\hat{c}_{i_t}^-$ ,  $\hat{c}_{i_t}^+$  and its confidence interval of PG and DIS

The first and third graphs are the confidence interval of left limit local variance  $\hat{c}_{it}^-$  and the second and fourth graphs are the confidence interval of left limit local variance  $\hat{c}_{it}^+$ . The blue triangles in the graphs are data estimated  $\hat{c}_{it}^-$  and the red circle are data estimated  $\hat{c}_{it}^+$ .

From these graphs we can see, almost every data estimated values are bounded by the confidence interval upper boundary and lower boundary. Trough calculated, we find the cover rate of these bootstrap confidence interval is 100%.

(ii) To test whether the confidence interval of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$  intersect or not, we want to test whether  $CI_{left}^{lower} > CI_{right}^{up}$  or  $CI_{right}^{lower} > CI_{left}^{up}$ .

Here are the graphs of mixed confidence interval boundary of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$ .

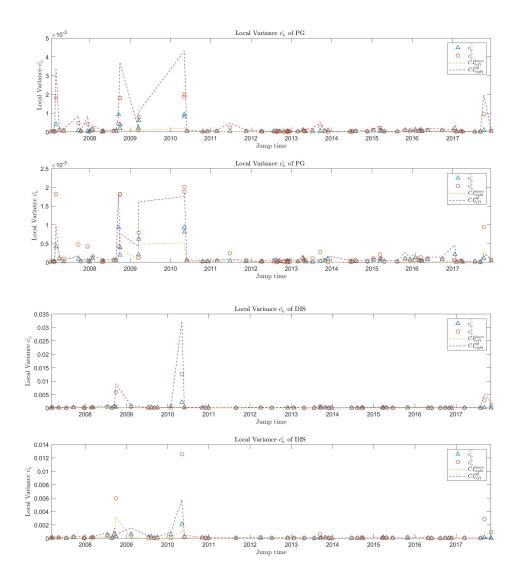


Figure 5: Mixed CI Boundary of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$  for PG and DIS

In these graphs, the purple dot lines are upper CI boundaries and the yellow dot lines are lower CI boundaries. If the lower boundaries are larger than the upper boundaries, these confidence interval of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$  will not have intersections.

From the graphs we can find, about 90% of the sample data, the lower boundaries are smaller than the upper boundaries, which indicates the confidence interval of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$  will have intersections for most of time. According to our calculations, the number and percentage of intervals that do not have intersections is 14 (17.95%) and 12 (20.69%), for PG and DIS respectively.

We can put these two confidence interval together to detect their intersections more clearly.

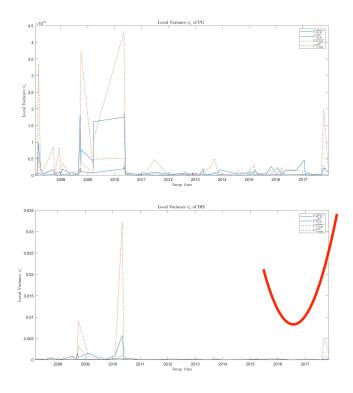


Figure 6: CI Boundary of  $\hat{c}_{i_t}^-$  and  $\hat{c}_{i_t}^+$  for PG and DIS

The blue lines are the confidence interval of  $\hat{c}_{i_t}^-$  while the red lines are the confidence interval of  $\hat{c}_{i_t}^+$ . Easily to see from the figures that these two confidence intervals have intersection for most of observations.

### The MATLAB:

Scripts of Q2

```
addpath('D:\ZM—Documents\MATLAB\data','functions','scripts');
```

```
[dates_PG,lp_PG]=load_stock('PG.csv','m');
 3
   N_PG=sum(floor(dates_PG(1,1))==floor(dates_PG(:,1)));% number of observations
        per day
   T_PG=size(dates_PG,1)/N_PG;
   [rdates_PG,lr_PG]=log_return([dates_PG lp_PG],N_PG,1);
   days_PG=unique(floor(rdates_PG));
6
 7
8
   %2A
9 a=5;
10 | [lr_c_PG, lr_d_PG]=c_d_log_returns(lr_PG, N_PG-1,a);
11 indi_PG=find(lr_d_PG~=0);
12 | n=size(lr_c_PG,1);
13 kn=11;
14 %left and right limit local variance
15 ct1=zeros(n,T_PG);
16 | ct2=zeros(n,T_PG);
17 | for i=2:n-1
18
        j2=max(1,i-kn);%start
19
        j1=min(kn+i,n);%stop
20
        ct1(i,:)=sum(lr_c_PG(j2:i,:).^2,1)/((i-j2)/n);%left
21
        ct2(i,:)=sum(lr_c_PG(i:j1,:).^2,1)/((j1-i)/n);%right
22
   end
23
24
   %plot jump left and right local variance at jump return time
25 | figure;
26 | subplot(2,1,1);
   plot(rdates_PG(indi_PG),ct1(indi_PG),'^','linewidth',0.8);
   hold on;
   plot(rdates_PG(indi_PG),ct2(indi_PG),'o','linewidth',0.8);
29
30
   xlabel('Jump time');
   ylabel('Local Variance $\hat{c_{i_t}}$');
   legend('$\hat{c_{i_t}^-}$','$\hat{c_{i_t}^+}$');
   xlim([min(rdates_PG(indi_PG)), max(rdates_PG(indi_PG))]);
35 | title('Local Variance $\hat{c_{i_t}}$ of PG ');
36
   datetick('x','keeplimits');
37
38 | subplot(2,1,2);
39 | plot(rdates_PG(indi_PG),abs(lr_d_PG(indi_PG)),'*','linewidth',1);
40 | xlabel('Jump time');
   ylabel('Magnitude of Jump returns');
42 | legend('Jump return');
43 | xlim([min(rdates_PG(indi_PG)), max(rdates_PG(indi_PG))]);
44 | title('Magnitude of Jump returns of PG');
```

```
datetick('x','keeplimits');
45
46
47
48 %2B
   %To simplify the program ,we here just consider the jumps fall in the
   %middle intervals
50
51 | indi_PG=find(lr_d_PG~=0);
52 | a1=find(indi_PG.*(mod(indi_PG,77)<=11));</pre>
53 | a2=find(indi_PG.*(mod(indi_PG,77)>=66));
54 | a=[a1; a2];
  indi_PG(a)=[];
56 | j1_PG=indi_PG-11;
57
   j2_PG=indi_PG+11;
58
59 left_sample_PG=zeros(11,size(indi_PG,1));
60 | right_sample_PG=zeros(11, size(indi_PG,1));
61
  for i=1:size(indi_PG,1)
   left_sample_PG(:,i)=lr_c_PG(j1_PG(i):indi_PG(i)-1);
   right_sample_PG(:,i)=lr_c_PG(indi_PG(i)+1:j2_PG(i));
64
   end
65
66
   %bootstrap sample
67
68 r=1000;
69 | sct1_PG=zeros(r,size(indi_PG,1));
70 | sct2_PG=zeros(r,size(indi_PG,1));
71
   parfor i=1:r
72
       newsample1=sbsample(left_sample_PG,kn,size(indi_PG,1));
73
       newsample2=sbsample(right_sample_PG,kn,size(indi_PG,1));
74
       sct1_PG(i,:)=sum(newsample1.^2,1)/(kn/n);%left
75
      sct2_PG(i,:)=sum(newsample2.^2,1)/(kn/n);%right
76 end
   ct1_low_PG = quantile(sct1_PG, 0.025);
   ct1_up_PG = quantile(sct1_PG, 0.975);
79
   ct2_low_PG = quantile(sct2_PG, 0.025);
80
   ct2_up_PG = quantile(sct2_PG, 0.975);
81
   Notinter_PG=sum((ct1_low_PG>ct2_up_PG)|(ct1_up_PG<ct2_low_PG));</pre>
82
83
84
85
86 | num_in_PG_d=sum(CI_low_PG <=TV_PG & CI_up_PG >=TV_PG);
   % cover_rate_PG_d=num_in_PG_d/T_PG;
88 | figure;
```

```
subplot(2,1,1);
 90 |plot(rdates_PG(indi_PG),ct1(indi_PG),'^','linewidth',0.8);
 92 |%plot(rdates_PG(indi_PG),ct2(indi_PG),'linewidth',0.8);
 93 hold on;
    plot(rdates_PG(indi_PG),ct1_low_PG,'—');
 95 hold on;
 96 | plot(rdates_PG(indi_PG),ct1_up_PG,'—');
97
    hold off;
98
99 | xlabel('Jump time');
100 | ylabel('Local Variance $\hat{c_{i_t}}$');
101 | legend('\$\hat{c_{i_t}^-}', '\$CI_{left}^{lower}', '\$CI_{left}^{up}$');
102 | xlim([min(rdates_PG(indi_PG)), max(rdates_PG(indi_PG))]);
103 | title('Local Variance $\hat{c_{i_t}}$ of PG ');
104 | datetick('x','keeplimits');
106 \%---right
107
    subplot(2,1,2);
108
    plot(rdates_PG(indi_PG),ct2(indi_PG),'o','linewidth',0.8,'color',[0.8500
        0.3250 0.0980]);
109 hold on;
110
    %plot(rdates_PG(indi_PG),ct2(indi_PG),'linewidth',0.8);
111 hold on;
112 | plot(rdates_PG(indi_PG),ct2_low_PG,'—');
    plot(rdates_PG(indi_PG),ct2_up_PG,'—');
114
115 hold off;
116
117
118 | xlabel('Jump time');
119
    ylabel('Local Variance $\hat{c_{i_t}}$');
120 | legend('$\hat{c_{i_t}^+}$','$CI_{right}^{lower}$','$CI_{right}^{up}$');
    xlim([min(rdates_PG(indi_PG)), max(rdates_PG(indi_PG))]);
    title('Local Variance $\hat{c_{i_t}}$ of PG ');
123
    datetick('x','keeplimits');
124
125 | %put together
126 | figure;
127 | subplot(2,1,1);
    plot(rdates_PG(indi_PG),ct1(indi_PG),'^','linewidth',0.8);
129
130
    plot(rdates_PG(indi_PG),ct2(indi_PG),'o','linewidth',0.8);
131
    hold on;
```

```
plot(rdates_PG(indi_PG),ct1_low_PG,'—');
133
       hold on;
134
      plot(rdates_PG(indi_PG),ct2_up_PG,'—');
      hold off;
136
       xlabel('Jump time');
137
138
      ylabel('Local Variance $\hat{c_{i_t}}$');
       legend('$\hat{c}_{i_t}^-)$', '$\hat{c}_{i_t}^+)$', '$CI_{left}^{lower}$', '$CI_{
139
             right}^{up}$');
140
       xlim([min(rdates_PG(indi_PG)), max(rdates_PG(indi_PG))]);
       title('Local Variance $\hat{c_{i_t}}$ of PG ');
142 | datetick('x','keeplimits');
143
144 | %----right
145 | subplot(2,1,2);
146 | plot(rdates_PG(indi_PG),ct1(indi_PG),'^','linewidth',0.8);
147
      hold on;
148
       plot(rdates_PG(indi_PG),ct2(indi_PG),'o','linewidth',0.8);
       hold on;
150
       plot(rdates_PG(indi_PG),ct2_low_PG,'—');
151 hold on;
152
       plot(rdates_PG(indi_PG),ct1_up_PG,'—');
153 hold off;
154
155 | xlabel('Jump time');
       ylabel('Local Variance $\hat{c_{i_t}}$');
157
       legend('\$\hat{c}_{i_t}^-)\$', '\$\hat{c}_{i_t}^+\}\$', '\$CI_{right}^{lower}\$', '\$CI_{right}^{lower}^{lower}
             left}^{up}$');
158
       xlim([min(rdates_PG(indi_PG)), max(rdates_PG(indi_PG))]);
       title('Local Variance $\hat{c_{i_t}}$ of PG ');
160
       datetick('x','keeplimits');
161
162 %CI together
      figure;
163
164 | plot(rdates_PG(indi_PG),ct1_low_PG,'-','color',[0 0.4470 0.7410]);
165
       hold on;
166 | plot(rdates_PG(indi_PG),ct1_up_PG,'-','color',[0 0.4470 0.7410]);
167
       hold on;
168 |plot(rdates_PG(indi_PG),ct2_low_PG,'—','color',[0.8500 0.3250 0.0980]);
169 hold on;
       plot(rdates_PG(indi_PG),ct2_up_PG,'—','color',[0.8500 0.3250 0.0980]);
171
      hold off;
172
173 | xlabel('Jump time');
```

178

datetick('x','keeplimits');

```
174  | ylabel('Local Variance $\hat{c_{i_t}}$');
175  | legend('$CI_{left}^{lower}$','$CI_{left}^{up}$','$CI_{right}^{lower}$','$CI_{right}^{up}$');
176  | xlim([min(rdates_PG(indi_PG)),max(rdates_PG(indi_PG))]);
177  | title('Local Variance $\hat{c_{i_t}}$ of PG ');
```

# Exercise 3-Jump Regression(Jump Beta)

(A) By using detector  $I'_n \equiv \{i : |\Delta_i^n X_1| > \alpha \Delta_n^{0.49} \sqrt{\tau_i B V_t} \}$ , we detect there are total 116 jumps in market log-returns. Here is the summary table sheet for jump times and return magnitude from year 2007 to 2017.

Stock	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
#Jump	11	5	7	8	4	17	9	14	8	18	15
Avg.Size	0.0047	0.0095	0.0063	0.0051	0.0067	0.0034	0.0045	0.0024	0.0040	0.0027	0.0019

(B) Here is the scatter plot of PG( and DIS)'s log-returns against the market jump returns.



Figure 7:  $\hat{c}_{i_t}^-, \, \hat{c}_{i_t}^+$  and  $|r_{t,i_t}^d|$  of PG and DIS

We can see, the data mostly concentrate around the origin point (0,0) and the linear relationship between individual stock's log-return and market jump log-return is obvious in these figures.

(C) We can get the estimator of jump beta by using the OLS regression:

$$\hat{\beta} \equiv \frac{\sum_{p=1}^{P_n} \Delta_{i_p}^n X_1 \Delta_{i_p}^n X_2}{\sum_{p=1}^{P_n} (\Delta_{i_p}^n X_1)^2}$$

The estimate jump beta of PG and DIS:

Stock	$\hat{eta}$
PG	0.8269
DIS	0.9942

Having the value of  $\hat{\beta}$ , we can use this regression relation to estimate out stock's returns when there is a jump in the market return:

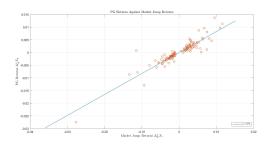
$$Y_{stock_{i_p}} = \hat{\beta}_{stock} X_{market_{i_p}} + \epsilon_{i_p}$$

where  $i_p$  is the detected jump time of market. The larger the  $\beta$  is, the bigger jump will occur at stock's return.

For PG,  $\hat{\beta} = 0.8269$ , which means when the market return jumps for 1%, we will expected PG's return will also experience a jump and the jump size is  $\hat{\beta}\%=0.8269\%$ .

For DIS,  $\hat{\beta}=0.9942$ , which means when the market return jumps for 1%, we will expected DIS' return will also experience a jump and the jump size is  $\hat{\beta}\%=0.9942\%$ .

(D) These are the plots of stocks returns against market return with estimated regression lines.



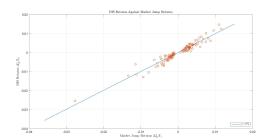


Figure 8: Stock Return v.s. Market return and Regression Line

According to the graph, we can easily see the this linear regression is good since most of our data fall on or near the regression line.

(E) We use this estimator to estimate the local variance for jump beta of PG and DIS:

$$\hat{V}_{\beta} = \frac{\sum_{p=1}^{Pn} (\Delta_{i_p}^n X_1)^2 \hat{c}_{e,t_p}}{(\sum_{p=1}^{Pn} (\Delta_{i_p}^n X_1)^2)^2}$$

The estimated value of  $\sqrt{\Delta_{i_p}^n \hat{V}_{\beta}}$  for PG and DIS:

Stock	$(\Delta_{i_p}^n \hat{V}_\beta)^{\frac{1}{2}}$
PG	0.0393
DIS	0.0505

(F) We can create a 95% confidence interval of jump beta by using it asymptotic distribution and the confidence interval can be represented as:

$$CI_{low} = \hat{\beta} - c * \sqrt{\Delta_{i_p}^n \hat{V}_{\beta}}$$

$$CI_{up} = \hat{\beta} + c * \sqrt{\Delta_{i_p}^n \hat{V}_{\beta}}$$

where c = 1.96 for a 95% confidence level interval.

The estimated 95% confidence interval of PG and DIS.

Stock	Estimated bet	a C	lower bound	CI upper bound
PG	0.8296		0.7499	0.9039
DIS	0.9942		0.8952	1.0932

For PG, the confidence interval is  $CI_{\hat{\beta}_{PG}} = [0.7499, 0.9003]$ , which means when there is a 1% jump in market log-return, we will expect at lease 0.7499% jump and at most 0.9003% jump in PG's return.

For DIS, the confidence interval is  $CI_{\hat{\beta}_{DIS}} = [0.8952, 1.9032]$ , which means when there is a 1% jump in market log-return, we will expect at lease 0.8952% jump and at most 1.9032% jump in PG's return. Notice that  $CI_{\hat{\beta}_{DIS}}$  includes the value of 1, which indices PG might perfectly co-movement with the market.

(G) Assume you expect the market price will jump down by 1% and  $V_{stock}$  is the value of existing portfolio,  $V_{mkt}$  is the value of market future index you plan to short sell, then we can use a table to show how to create a hedge strategy.

Asset	Position Value at $t_0$	Position Value at $t_1$	Cost of Position	P&L
Stock	$V_{stock}$	$(1 - \beta_{stock}\%)V_{stock}$	$V_{stock}$	$-\beta_{stock}\%V_{stock}$
Future	0	$V_{mkt}$	$(1-1\%)V_{mkt}$	$1\%V_{mkt}$

According to this table, if you want to perfectly hedge your current position, you need to short sell market index future for  $V_{mkt} = \beta_{stock} V_{mkt}$ , so that the P&L for your position at time  $t_1$  will equal to zero.

Here is the estimated short sell value confidence interval of PG and DIS:

Stock	estimated value	Value lower	bound	Value upper bound
PG	165.3727	149.9		180.7726
DIS	198.8345	179.03	36	218.6354

(H) The estimated jump beta and beta variance of PG and DIS.

Stock	$\beta_1$	$\beta_2$	$\hat{V}_{\beta 1}$	$\hat{V}_{eta 2}$
PG	0.7573	0.9827	0.2784	0.0474
DIS	0.9520	1.0957	0.4510	0.0468

(I) We can create a 95% confidence interval of  $\hat{\beta}_1 - \hat{\beta}_2$  by using it asymptotic distribution and the confidence interval can be represented as:

$$CI_{low} = \hat{\beta}_1 - \hat{\beta}_2 - c * \sqrt{\Delta_{i_p}^n \hat{V}_{\beta}}$$

$$CI_{up} = \hat{\beta}_1 - \hat{\beta}_2 + c * \sqrt{\Delta_{i_p}^n \hat{V}_\beta}$$

where c = 1.96 for a 95% confidence level interval.

The estimated 95% confidence interval of PG and DIS:

Stock	$\hat{\beta_1} - \hat{\beta_2}$	CI lower bound	CI upper bound
PG	-0.2255	-0.3530	-0.0979
DIS	-0.1437	-0.3013	0.0139

### not contain zero in PG (-0.1)

According to the table, PG's confidence interval contains zero, which means there is of 95% probability that  $\hat{V}_{\beta 1}$  and  $\hat{V}_{\beta 2}$  are not equal.

DIS' confidence interval contains zero, which means there is a probability that DIS'  $\hat{V}_{\beta 1}$  and  $\hat{V}_{\beta 2}$  are equal.

This result cannot be a strong evidence to against or support our assumption of jump beta is constant.

### The MATLAB: Scripts of Q3

```
addpath('D:\ZM—Documents\MATLAB\data','functions','scripts');
   [dates_PG,lp_PG]=load_stock('PG.csv','m');
   N_PG=sum(floor(dates_PG(1,1))==floor(dates_PG(:,1))); number of observations
3
        per day
   T_PG=size(dates_PG,1)/N_PG;
   [rdates_PG,lr_PG]=log_return([dates_PG lp_PG],N_PG,1);
   days_PG=unique(floor(rdates_PG));
6
 7
 8
   [dates_SPY,lp_SPY]=load_stock('SPY.csv','m');
9
   N_SPY=sum(floor(dates_SPY(1,1))==floor(dates_SPY(:,1)));% number of
       observations per day
   T_SPY=size(dates_SPY,1)/N_SPY;
   [rdates_SPY,lr_SPY]=log_return([dates_SPY lp_SPY],N_SPY,1);
11
12 | days_SPY=unique(floor(rdates_SPY));
```

```
13
14 a=5;
15 kn=11;
16 | n=77;
17 | [lr_c_PG,lr_d_PG]=c_d_log_returns(lr_PG,N_PG-1,a);
   [lr_c_SPY,lr_d_SPY]=c_d_log_returns(lr_SPY,N_SPY-1,a);
18
19
20
   %Q3
21
   %A
22
   %find the jump time index In'
23 | Pn=find(lr_d_SPY~=0); %focus on jumps
24 | num_jump=size(Pn,1);
25 | jd_PG=datetime(rdates_PG(Pn), 'ConvertFrom', 'datenum');
26 | times_PG=zeros(11,1);%jump times per year
27
  size_PG=zeros(11,1);
28 | for i=1:11
29
      times_PG(i)=sum(jd_PG.Year==i+2006);
30
       k=abs(lr_d_SPY(Pn)).*(jd_PG.Year==i+2006);
       size_PG(i)=mean(k(k\sim=0));%average magnitude
32
   end
33 | %B plot x1 and x2(stock)
34 | figure;
35 | scatter(lr_SPY(Pn),lr_PG(Pn));
36 | xlabel('Market Jump Returns $\Delta_{ip}^n X_1$');
   ylabel('PG Returns $\Delta_{ip}^n X_2$');
38
   title('PG Returns against Market Jump Returns');
   %C
39
40
   beta_hat_PG=sum(lr_PG(Pn).*lr_SPY(Pn))/sum(lr_SPY(Pn).^2);
41
42 | x=linspace(1.3*min(lr_SPY(Pn)),1.3*max(lr_SPY(Pn)),200);
43 | y=beta_hat_PG*x;
44 | figure;
45 | plot(x,y);
46 | hold on;
47 | scatter(lr_SPY(Pn),lr_PG(Pn));
  grid on;
49 | xlabel('Market Jump Returns $\Delta_{ip}^n X_1$');
50 | ylabel('PG Returns $\Delta_{ip}^n X_2$');
51 | legend('y=$\beta$*x','Location','southeast');
52 | title('PG Returns Against Market Jump Returns');
53
   %E
54 et_PG=lr_PG-beta_hat_PG*lr_SPY;%local residual
  V_beta_PG=beta_var(lr_SPY,lr_PG,n,a,kn);
56 | sd_beta_PG=sqrt((1/n)*V_beta_PG);
```

```
57
   %F
58
   %CI by asym distribution
59
   c=-norminv(0.025);
60 | CI_l_beta_PG=beta_hat_PG-c*sqrt((1/n)*V_beta_PG);
   CI_u_beta_PG=beta_hat_PG+c*sqrt((1/n)*V_beta_PG);
61
62
63
   %G
64
   V_PG=beta_hat_PG*200;
   Vl_PG=CI_l_beta_PG*200;
66
   Vu_PG=CI_u_beta_PG*200;
67
68
   %Н
   daytime=datetime(rdates_PG, 'ConvertFrom', 'datenum');
69
   Y=sum(daytime(:).Year<=2011);
70
71
72
   [V_beta1_PG,beta1_PG]=beta_var(lr_SPY(1:Y),lr_PG(1:Y),77,5,11);
73
   [V_beta2_PG, beta2_PG] = beta_var(lr_SPY(Y+1:end), lr_PG(Y+1:end), 77,5,11);
74
75
   %I
   V12_PG=V_beta1_PG+V_beta2_PG;
76
77
  beta12_PG=beta1_PG—beta2_PG;
  CI_l_PG=(beta1_PG-beta2_PG)-1.96*sqrt(V12_PG/n);
78
79 | CI_u_PG=(beta1_PG-beta2_PG)+1.96*sqrt(V12_PG/n);
```