MGRECON 548Q: Empirical Economic Analysis

Fuqua School of Business, Duke University
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Help Section Week#2

Decide whether the following statements are true or false. Explain your reasoning.

Asumptions of Odinary Least Square (OLS) Regression:

- A1 (linearity): The regression model is linear in the coefficients and the error term

$$\mathrm{y} = \mathrm{X}\beta + \epsilon$$

- A2 (Orthogonal): The error term has zero expectation and is (weakly) orthogonal with X

$$\mathrm{E}[\epsilon] = 0 \quad \text{and} \quad \mathrm{E}[\epsilon \mathrm{X}] = 0$$

  By A2, we have $\mathrm{cov}(\epsilon, \mathrm{X}) = 0 \implies$ all X are uncorrelated with the ϵ

- A3 (No perfect multicollinearity): No independent variable is a perfect linear function of other explanatory variables
- A4 (No heteroscedasticity): The error term has a constant variance

$$V(\epsilon) = \sigma^2$$

- A5 (Normality of error): The error term is normally distributed (optional)

$$\epsilon \sim \mathcal{N}(0, \sigma^2)$$

Interpretation:

(without interaction term)

- Only intercept

$$Y = \alpha \mathbf{1} + e$$

- Dummy variable D

$$Y = \alpha + e + \beta D$$

- Dummy variable D and category variable J (J=0,1,2 ... K)

$$Y = \alpha + \beta D + e + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[J = j]$$

- Dummy variable D, category variable J and discrete/continuous variable X

$$Y = \alpha + \beta D + e + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[J = j] + \delta X$$

- specific example:

$$\text{Salary} = \alpha + \beta \text{College} + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[\text{Race} = j] + \delta \text{Experience} + e$$

Salary(continuous): Annual salary in dollar

College(Dummy): 1=graduated from college

Race(Category): 0=While, 1=Black, 2=Asian, 3=Mexican, 4=Mix

Experiment (Discrete): Working experience (year)

(1) Intercept α: The expected annual salary(Y) for individual who did not graduate from college (D=0) and whose race is Mix (Race=4) and with zero-year working experience (Experience=0), keeping other variables constant or Ceteris Paribus.

(2) β: The expected marginal effect of graduating (D) from college upon annual salary(Y) on individual, keeping other variables constant or Ceteris Paribus.

(3) γ_0: The expected difference upon annual salary (Y) between individual whose race is while(Race=0) versus individual whose race is Mix (Race=4), keeping other variables constant or Ceteris Paribus.

(4) δ: The expected marginal effect of working experience(Experience) upon annual salary (Y), keeping other variables constant or Ceteris Paribus.

(with interaction term)

- Basic Model

  $Y = \alpha + \beta D + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[J = j] + \delta X + e$

- Adding interaction between D and J

  $Y = \alpha + \beta D + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[J = j] + \delta X + e + \sum_{j=0}^{K-1} \theta_j \mathbb{1}[J = j] * D$

- Adding interaction between D and X

  $Y = \alpha + \beta D + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[J = j] + \delta X + e + \sum_{j=0}^{K-1} \theta_j \mathbb{1}[J = j] * D + \eta X * D$

- Adding interaction between J and X
  $Y = \alpha + \beta D + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[J = j] + \delta X + e + \sum_{j=0}^{K-1} \theta_j \mathbb{1}[J = j] * D + \eta X * D + \sum_{j=0}^{K-1} \lambda_j \mathbb{1}[J = j] * X$

- specific example:

$$\text{Salary} = \alpha + \beta\text{College} + \sum_{j=0}^{K-1} \gamma_j \mathbb{1}[\text{Race} = j] + \delta\text{Experience} + \sum_{j=0}^{K-1} \theta_j \mathbb{1}[\text{Race} = j] * \text{College}$$
$$+ \eta\text{Experience} * \text{College} + \sum_{j=0}^{K-1} \lambda_j \mathbb{1}[\text{Race} = j] * \text{Experience} + e$$

  Salary(continuous): Annual salary in dollar

  College(Dummy): 1=graduated from college

  Race(Category): 0=While, 1=Black, 2=Asian, 3=Mexican, 4=Mix

  Experiment (Discrete): Working experience (year)

  (5) θ_0(White*College): The expected difference in the marginal effect of graduating from college (D=1) upon annual salary (Y) for individuals whose Race is White(J=0) versus those race is Mix(J=4), keeping other variables constant or Ceteris Paribus.

  (Alternative: Given individuals whose Race are White, θ_0 is the expected difference in the marginal effect upon annual salary (Y) for individuals who graduated from college (D=1) versus those did not graduate from college(D=0), keeping other variables constant or Ceteris Paribus.)

  (6) η(Experience*College): The expected difference in the marginal effect of working experience(X) upon annual salary (Y) for individuals who graduated from college (D=1) versus those did not graduate from college(D=0), keeping other variables constant or Ceteris Paribus.

  (7) λ_0(Experience*White): The expected difference in the marginal effect of working experience(X) upon annual salary (Y) for individuals whose Race is White(J=0) versus those race is Mix(J=4), keeping other variables constant or Ceteris Paribus.

Relationship between $\mathrm{t_{stat}}, \hat{\sigma_{\mathrm{e}}}, \mathrm{se}(\beta)$ and Type II error:

- Recall:

$$
\begin{aligned}
t_{stat} &= \frac{\hat{\beta} - 0}{se(\hat{\beta})} \; (H_0 : \beta = 0) \\
se(\hat{\beta}) &= \hat{\sigma}_e \times \frac{1}{\sqrt{N}} \times \frac{1}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2}} \times \frac{1}{\sqrt{(1 - R_X^2)}} \\
\text{Type I error} &= \Pr(|\mathrm{t_{stat}}| > \mathrm{t_{crit}} | \beta = 0) \\
\text{Type II error} &= \Pr(|\mathrm{t_{stat}}| < \mathrm{t_{crit}} | \beta \neq 0)
\end{aligned}
$$

- $\hat{\sigma}_{\mathrm{e}} \uparrow \implies \mathrm{se}(\hat{\beta}) \uparrow \implies \mathrm{t_{stat}} \downarrow \implies$ Type II error $\uparrow$
- $\hat{\sigma}_{\mathrm{e}} \downarrow \implies \mathrm{se}(\hat{\beta}) \downarrow \implies \mathrm{t_{stat}} \uparrow \implies$ Type II error $\downarrow$