Kaluza-Klein theory with ultrametric compact dimensions

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1 Action

$$S = \int_{R^{3,1}} dx^4 \int_{\mathbb{Z}_p} dy \quad \frac{1}{2}\phi(\partial^2 + D_y^s)\phi + V(\phi) \tag{1}$$

2 Consistent Truncation

A certain type of dimensional reductions are consistent truncations:

$\mathbb{Z}_p \to p\mathbb{Z}_p \to p^2\mathbb{Z}_p \to ... \to 0$?

Two types of consistent truncation:

- Kaluza-Klein dimensional reduction without changing dof of each space-time point?
- Introducing constraints that reduce independent dof.

What will happen if we truncate at $|k|_p = 3$? Plugging in

$$\phi(x) = \phi_0(x) + \phi_{-\frac{2}{3}}(x)\chi\left(-\frac{2y}{3}\right) + \phi_{-\frac{1}{3}}(x)\chi\left(-\frac{y}{3}\right) + \phi_{\frac{1}{3}}(x)\chi\left(\frac{y}{3}\right) + \phi_{\frac{2}{3}}(x)\chi\left(\frac{2y}{3}\right) \tag{2}$$

to equation 1 and completing the $\mathbb{Z}_p$ integral, one obtains various terms:

- $\phi(x)\phi''(x)$ becomes

  $$3^s\left(\phi_{\frac{2}{3}}(x) + \phi_{-\frac{1}{3}}(x)\right)\left(\phi''_{\frac{1}{3}}(x) + \phi''_{-\frac{2}{3}}(x)\right) + 3^s\left(\phi_{\frac{1}{3}}(x) + \phi_{-\frac{2}{3}}(x)\right)\left(\phi''_{\frac{2}{3}}(x) + \phi''_{-\frac{1}{3}}(x)\right) + \phi_0(x)\phi''_0(x) \tag{3}$$

  We note $\left(\phi_{\frac{1}{3}}(x) + \phi_{-\frac{2}{3}}(x)\right)$ as ϕ_1, then $\left(\phi_{\frac{2}{3}}(x) + \phi_{-\frac{1}{3}}(x)\right)$ is ϕ_1^*(because ϕ is real). Then we have

  $$\phi\partial^2\phi = 3^s(\phi_1^*\partial^2\phi_1 + \phi_1\partial^2\phi_1^*) + \phi_0\partial^2\phi_0 \tag{4}$$

- $\phi(x)^2$ becomes

  $$\phi_0(x)^2 + 2\left(\phi_{\frac{1}{3}}(x) + \phi_{-\frac{2}{3}}(x)\right)\left(\phi_{\frac{2}{3}}(x) + \phi_{-\frac{1}{3}}(x)\right) = \phi_0(x)^2 + 2\phi_1\phi_1^* \tag{5}$$

- $\phi(x)^3$ becomes

$$\phi_0^3 + 6\phi_1\left(\phi_1\right)^*\phi_0 + \phi_1^3 + \left(\left(\phi_1\right)^*\right)^3 \tag{6}$$

- $\phi(x)^4$ becomes

$$\phi_0^4 + 12\phi_1\left(\phi_1\right)^*\phi_0^2 + 4\left(\phi_1^3 + \left(\left(\phi_1\right)^*\right)^3\right)\phi_0 + 6\phi_1^2\left(\left(\phi_1\right)^*\right)^2 \tag{7}$$

- $\phi(x)^5$ becomes

$$\phi_0^5 + 20\phi_1\left(\phi_1\right)^*\phi_0^3 + 10\left(\phi_1^3 + \left(\left(\phi_1\right)^*\right)^3\right)\phi_0^2 + 30\phi_1^2\left(\left(\phi_1\right)^*\right)^2\phi_0 + 5\phi_1\left(\phi_1\right)^*\left(\phi_1^3 + \left(\left(\phi_1\right)^*\right)^3\right) \tag{8}$$