Calculating Vladimirov derivatives

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1 Derivative of the χ character

Calculating Vladimirov derivative of $\chi(k_n y)$ where $k_n = p^{-n}$:

$$\begin{aligned}
D_y^s \chi(k_n y) &= \frac{1}{\Gamma_p(-s)} \int\limits_{\mathbb{Q}_p} dx \frac{\chi(p^{-n}x) - \chi(p^{-n}y)}{|x-y|_p^{1+s}} \\
&= \frac{1}{\Gamma_p(-s)} \left(\int\limits_{\mathbb{Q}_p} du \frac{\chi(p^{-n}(u+y))}{|u|_p^{1+s}} - \int\limits_{\mathbb{Q}_p} du \frac{\chi(p^{-n}y)}{|u|_p^{1+s}} \right) \\
&= \frac{1}{\Gamma_p(-s)} \chi(p^{-n}y) \left(\int\limits_{\mathbb{Q}_p} du \frac{\chi(p^{-n}u)}{|u|_p^{1+s}} - \int\limits_{\mathbb{Q}_p} du \frac{1}{|u|_p^{1+s}} \right) \\
&= \frac{1}{\Gamma_p(-s)} \chi(p^{-n}y) \left(-p^{(n-1)(s+1)-n} + \sum_{n}^{\infty} p^{ls}(1-\frac{1}{p}) - \sum_{-\infty}^{\infty} p^{ls}(1-\frac{1}{p}) \right) \\
? &= \frac{1}{\Gamma_p(-s)} \chi(p^{-n}y) \left(p^{ns}\Gamma_p(-s)\right) = p^{ns}\chi(k_n y)
\end{aligned} \tag{1}$$