Calculating Vladimirov derivatives

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1 Derivative of the χ character

Calculating Vladimirov derivative of $\chi(k_n y)$ where $k_n = p^{-n}$:

$$\begin{aligned}
D^s_y \chi(k_n y) &= \frac{1}{\Gamma_p(-s)} \int\limits_{\mathbb{Q}_p} dx \frac{\chi(p^{-n}x) - \chi(p^{-n}y)}{|x-y|_p^{1+s}} \\
&= \frac{1}{\Gamma_p(-s)} \left(\int\limits_{\mathbb{Q}_p} du \frac{\chi(p^{-n}(u+y))}{|u|_p^{1+s}} - \int\limits_{\mathbb{Q}_p} du \frac{\chi(p^{-n}y)}{|u|_p^{1+s}} \right) \\
&= \frac{1}{\Gamma_p(-s)} \chi(p^{-n}y) \left(\int\limits_{\mathbb{Q}_p} du \frac{\chi(p^{-n}u)}{|u|_p^{1+s}} - \int\limits_{\mathbb{Q}_p} du \frac{1}{|u|_p^{1+s}} \right) \\
&= \frac{1}{\Gamma_p(-s)} \chi(p^{-n}y) \left(-p^{(n-1)(s+1)-n} + \sum_{n}^{\infty} p^{ls}(1-\frac{1}{p}) - \sum_{-\infty}^{\infty} p^{ls}(1-\frac{1}{p}) \right) \\
? &= \frac{1}{\Gamma_p(-s)} \chi(p^{-n}y) \left(p^{ns}\Gamma_p(-s)\right) = p^{ns}\chi(k_n y)
\end{aligned} \tag{1}$$

2 Check shift invariance of a general p-adic integral

Define a general integral over p-adic number field as:

$$\int\limits_{\mathbb{Q}_p} dx f(|x|_p) = \sum_{l=-\infty}^{\infty} \int\limits_{p^l \mathbb{U}_p} dx f(p^{-l}) \tag{2}$$

A shift of the integral variable gives:

$$\int\limits_{\mathbb{Q}_p} dx f(|x|_p) = \int\limits_{\mathbb{Q}_p} d(u-y) f(|u-y|_p) \quad \text{where} \quad u = x + y \tag{3}$$

Suppose that $d(u-y) = du$ when y is constant. Then we can proceed by splitting $\mathbb{Q}_p$:

$$\int\limits_{\mathbb{Q}_p} d(u) f(|u-y|_p) = \sum_{l=-\infty}^{\infty} \int\limits_{p^l \mathbb{U}_p} du f(|u-y|_p) \tag{4}$$

Note that we cannot write $|u-y|_p$ as p^{-l} in each block any more. Suppose that $y = p^m(a_0 + a_1 p + a_2 p^2 + ...)$