

Kaluza-Klein theory with ultrametric compact dimensions

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1 Action

$$S = \int_{R^{3,1}} dx^4 \int_{\mathbb{Z}_p} dy \quad \frac{1}{2} \phi (\partial^2 + D_y^s) \phi + V(\phi) \quad (1)$$

2 Consistent Truncation

A certain type of dimensional reductions are consistent truncations:

$$\mathbb{Z}_p \rightarrow p\mathbb{Z}_p \rightarrow p^2\mathbb{Z}_p \rightarrow \dots \rightarrow 0?$$

Two types of consistent truncation:

- Kaluza-Klein dimensional reduction without changing dof of each space-time point?
- Introducing constraints that reduce independent dof.

What will happen if we truncate at $|k|_p = 3$? Plugging in

$$\phi(x) = \phi_0(x) + \phi_{-\frac{2}{3}}(x) \chi\left(-\frac{2y}{3}\right) + \phi_{-\frac{1}{3}}(x) \chi\left(-\frac{y}{3}\right) + \phi_{\frac{1}{3}}(x) \chi\left(\frac{y}{3}\right) + \phi_{\frac{2}{3}}(x) \chi\left(\frac{2y}{3}\right) \quad (2)$$

to equation 1 and completing the \mathbb{Z}_p integral, one obtains various terms:

- $\phi(x)\phi''(x)$ becomes

$$3^s \left(\phi_{\frac{2}{3}}(x) + \phi_{-\frac{1}{3}}(x) \right) \left(\phi_{\frac{1}{3}}''(x) + \phi_{-\frac{2}{3}}''(x) \right) + 3^s \left(\phi_{\frac{1}{3}}(x) + \phi_{-\frac{2}{3}}(x) \right) \left(\phi_{\frac{2}{3}}''(x) + \phi_{-\frac{1}{3}}''(x) \right) + \phi_0(x) \phi_0''(x) \quad (3)$$

We note $\left(\phi_{\frac{1}{3}}(x) + \phi_{-\frac{2}{3}}(x) \right)$ as ϕ_1 , then $\left(\phi_{\frac{2}{3}}(x) + \phi_{-\frac{1}{3}}(x) \right)$ is ϕ_1^* (because ϕ is real).

Then we have

$$\phi \partial^2 \phi = 3^s (\phi_1^* \partial^2 \phi_1 + \phi_1 \partial^2 \phi_1^*) + \phi_0 \partial^2 \phi_0 \quad (4)$$

- $\phi(x)^2$ becomes

$$\phi_0(x)^2 + 2 \left(\phi_{\frac{1}{3}}(x) + \phi_{-\frac{2}{3}}(x) \right) \left(\phi_{\frac{2}{3}}(x) + \phi_{-\frac{1}{3}}(x) \right) = \phi_0(x)^2 + 2\phi_1 \phi_1^* \quad (5)$$

- $\phi(x)^3$ becomes

$$\phi_0^3 + 6\phi_1 (\phi_1)^* \phi_0 + \phi_1^3 + ((\phi_1)^*)^3 \quad (6)$$

- $\phi(x)^4$ becomes

$$\phi_0^4 + 12\phi_1 (\phi_1)^* \phi_0^2 + 4(\phi_1^3 + ((\phi_1)^*)^3) \phi_0 + 6\phi_1^2 ((\phi_1)^*)^2 \quad (7)$$

- $\phi(x)^5$ becomes

$$\phi_0^5 + 20\phi_1 (\phi_1)^* \phi_0^3 + 10(\phi_1^3 + ((\phi_1)^*)^3) \phi_0^2 + 30\phi_1^2 ((\phi_1)^*)^2 \phi_0 + 5\phi_1 (\phi_1)^* (\phi_1^3 + ((\phi_1)^*)^3) \quad (8)$$