Calculating Vladimirov derivatives

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1 Derivative of the χ character

It seems to me that there are two ways to do the calculation: one is to split $\mathbb{Q}_p$ into different domains($\xi\mathbb{U}_p$) and do the integral of χ; the other is to decompose the χ character function into piece-wise constant functions γ_n and apply the V-derivatives on them.

a) Splitting $\mathbb{Q}_p$

$$D_y^s\chi(k_n y) = \frac{1}{\Gamma_p(-s)}\int\limits_{\mathbb{Q}_p} dx \frac{\chi(p^{-n}x)-\chi(p^{-m}y_0)}{|x-p^{n-m}y_0|_p^{1+s}} \tag{1}$$

Where $k_n = p^{-n}$ and $y = p^{n-m}y_0$ with $m \geq 1$(other wise χ will be a constant function and the derivative is zero).
Consider the integral in each $x \in p^l\mathbb{U}_p$ block and we here discuss 2 cases:
First, for$l < n-m$,we have $|x-y|_p = |x|_p = p^{-l}$ and the integral becomes:

$$\begin{aligned}
\frac{1}{\Gamma_p(-s)}\int\limits_{\mathbb{Q}_p} dx \frac{\chi(p^{-n}x)-\chi(p^{-m}y_0)}{|x|_p^{1+s}} &= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1}\int\limits_{p^l\mathbb{U}_p} dx\frac{\chi(p^{-n}x)-\chi(p^{-m}y_0)}{p^{-l(1+s)}} \\
&= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1}\int\limits_{p^{l-n}\mathbb{U}_p} dz|p^n|_p\frac{\chi(z)-\chi(p^{-m}y_0)}{p^{-l(1+s)}} \\
&= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1} p^{-n}\frac{0-\chi(p^{-m}y_0)}{p^{-l(1+s)}}(1-\frac{1}{p})|p^{l+n}|_p \qquad (2) \\
&= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1} -\chi(p^{-m}y_0)(1-\frac{1}{p})p^{ls} \\
&= \frac{-\chi(p^{-m}y_0)}{\Gamma_p(-s)}\frac{p-1}{p^s-1}p^{s(n-m)-1}.
\end{aligned}$$