# Calculating Vladimirov derivatives

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## 1 Derivative of the $\chi$ character

It seems to me that there are two ways to do the calculation: one is to split $\mathbb{Q}_p$ into different domains($\xi\mathbb{U}_p$) and do the integral of $\chi$; the other is to decompose the $\chi$ character function into piece-wise constant functions $\gamma_n$ and apply the V-derivatives on them.

### a) Splitting $\mathbb{Q}_p$

$$D_y^s\chi(k_ny) = \frac{1}{\Gamma_p(-s)}\int\limits_{\mathbb{Q}_p} dx\frac{\chi(p^{-n}x) - \chi(p^{-m}y_0)}{|x - p^{n-m}y_0|_p^{1+s}} \tag{1}$$

Where $k_n = p^{-n}$ and $y = p^{n-m}y_0$ with $m \geq 1$(other wise $\chi$ will be a constant function and the derivative is zero).
Consider the integral in each $x \in p^l\mathbb{U}_p$ block($x = p^lx_0$ where $x_0 \in \mathbb{U}_p$) and we have 3 parts:
First, for $l < n - m$, we have $|x - y|_p = |x|_p = p^{-l}$ and the integral becomes:

$$\begin{aligned}
&\frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1}\int\limits_{p^l\mathbb{U}_p} dx\frac{\chi(p^{-n}x) - \chi(p^{-m}y_0)}{p^{-l(1+s)}} \\
&= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1}\int\limits_{p^{l-n}\mathbb{U}_p} dz|p^n|_p\frac{\chi(z) - \chi(p^{-m}y_0)}{p^{-l(1+s)}} \\
&= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1} p^{-n}\frac{0 - \chi(p^{-m}y_0)}{p^{-l(1+s)}}(1 - \frac{1}{p})|p^{l+n}|_p \\
&= \frac{1}{\Gamma_p(-s)}\sum_{l=-\infty}^{n-m-1} -\chi(p^{-m}y_0)(1 - \frac{1}{p})p^{ls} \\
&= \frac{-\chi(p^{-m}y_0)}{\Gamma_p(-s)}\frac{p-1}{p^s-1}p^{s(n-m)-1}.
\end{aligned} \tag{2}$$

The second case, $l = n - m$, is more complicated. The derivative is:

$$\begin{aligned} &\frac{1}{\Gamma_p(-s)} \int\limits_{p^{n-m}\mathbb{U}_p} dx \frac{\chi(p^{-n}x) - \chi(p^{-m}y_0)}{|p^{n-m}(x_0 - y_0)|_p^{1+s}} \\ &= \frac{1}{\Gamma_p(-s)} \int\limits_{p^{n-m}\mathbb{U}_p} dx \frac{\chi(p^{-n}x) - \chi(p^{-m}y_0)}{|p^{n-m}(x_0 - y_0)|_p^{1+s}} \end{aligned} \tag{3}$$

The last term, $l > n - m$, is:

$$\begin{aligned} &\frac{1}{\Gamma_p(-s)} \sum_{l=n-m+1}^{\infty} \int\limits_{p^l\mathbb{U}_p} dx \frac{\chi(p^{-n}x) - \chi(p^{-m}y_0)}{|p^{n-m}y_0|_p^{1+s}} \\ &= \frac{1}{\Gamma_p(-s)} p^{(n-m)(1+s)} \sum_{l=n-m+1}^{\infty} \int\limits_{p^l\mathbb{U}_p} dx (\chi(p^{-n}x) - \chi(p^{-m}y_0)) \end{aligned} \tag{4}$$