Physics 539 Fall 2018

Problem Set 1

Due on Sept. 26

1. Consider the world volume action of a relativistic membrane in D-dimensional flat space-time with coordinates X^μ:

$$S = -\mathcal{T} \int d^3\sigma \sqrt{-\det\, g_{\alpha\beta}}\ ,$$

where the induced metric is

$$g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}\ .$$

The directions X^1 and X^2 are circular, and the membrane is wrapped around them. Adopting the static gauge

$$\sigma^0 = X^0\ , \quad \sigma^1 = X^1 \quad \sigma^2 = X^2\ ,$$

derive the expansion of S up to the terms of fourth order in derivatives of $X^i(\sigma^0, \sigma^1, \sigma^2)$, where $i = 3, \ldots, D-1$.

Can you express the fourth order term in terms of the stress-energy tensor

$$T_{\alpha\beta} = \partial_\alpha X^i \partial_\beta X^i - \frac{1}{2}\eta_{\alpha\beta}\eta^{\gamma\delta}\partial_\gamma X^i \partial_\delta X^i\ .$$

2. a) Find a classical solution describing the folded straight closed string spinning around its stationary center. Note that, in the conformal gauge, it has to satisfy both the equation of motion

$$\partial_+ \partial_- X^\mu = 0$$

and the constraints

$$T_{++} = T_{--} = 0\ .$$

b) What is the speed with which the folds move?

c) Derive the relation between the angular momentum and the energy of this string. How does it compare with the corresponding relation for an open string?

d) Using the semi-classical quantization condition that the action for one period equals $2\pi\hbar n$, where n is a positive integer, determine how the angular momentum of such a string is quantized.

3. Also in conformal gauge, find a periodic pulsating circular string solution. Using the semi-classical quantization condition determine how the energy of such a string is quantized.

4. Consider a flat D-dimensional space-time where direction X^1 is a large circle of length L. A string of tension T wraps this circle once.

a) Calculate the $O(1/L)$ term in the energy spectrum of the transverse vibrations of the string. What are the degeneracies of the first three energy eigenstates?

b) Can you write down the exact formula for the energy spectrum of the transverse vibrations?