

Homework 1

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PHY 539: Introduction to String Theory

September 24, 2018

1 Problem 1

Adopting the static gauge, we have:

$$-\det g_{\alpha\beta} = 1 - \partial_0 X^i \partial_0 X_i + \partial_1 X^i \partial_1 X_i + \partial_2 X^i \partial_2 X_i \quad (1) \quad \boxed{1}$$

Define $g'_{\alpha\beta} = \partial_\alpha X^i \partial_\beta X_i$, then

$$-\det g_{\alpha\beta} = 1 + g'_{\alpha\beta} \eta^{\alpha\beta} \quad (2) \quad \boxed{2}$$

Consider derivatives of $X^i(\sigma^0, \sigma^1, \sigma^2)$ as small quantities and expand action to the fourth order of them:

$$S = -\mathcal{T} \int d^3\sigma \left(1 + \frac{1}{2} g'_{\alpha\beta} \eta^{\alpha\beta} - \frac{1}{8} (g'_{\gamma\delta} \eta^{\gamma\delta})^2 \right) \quad (3) \quad \boxed{3}$$

Contract the stress-energy tensor with $\eta^{\alpha\beta}$, we have $T_{\alpha\beta} \eta^{\alpha\beta} = -\frac{1}{2} g'_{\alpha\beta} \eta^{\alpha\beta}$. So the fourth order term is:

$$-\frac{1}{8} (g'_{\alpha\beta} \eta^{\alpha\beta})^2 = -\frac{1}{2} (T_\beta^\alpha)^2 \quad (4) \quad \boxed{4}$$

2 Problem 2

a) This is the same as two straight open strings that their endpoints coincide. Adopting the open string solution:

$$X^0 = A\tau, X^1 = A \cos \tau \cos \sigma, X^2 = A \sin \tau \cos \sigma \quad (5) \quad \boxed{5}$$

but let σ runs from 0 to 2π .

b) At any endpoint, for example $\sigma = 0$, we have:

$$\sqrt{\left(\frac{dX^1}{dX^0}\right)^2 + \left(\frac{dX^2}{dX^0}\right)^2} = \sqrt{(-\sin \tau)^2 + (\cos \tau)^2} = 1 \quad (6) \quad \boxed{6}$$

So it moves at the speed of light.

c) Energy can be obtained by:

$$E = P^0 = \mathcal{T} \int_0^{2\pi} d\sigma \frac{dX^0}{d\tau} = 2\pi A \mathcal{T} \quad (7) \quad \boxed{7}$$

While angular momentum is:

$$J = \mathcal{T} \int_0^{2\pi} d\sigma \left(X^1 \frac{dX^2}{d\tau} - X^2 \frac{dX^1}{d\tau} \right) = \pi A^2 \mathcal{T} \quad (8) \quad \boxed{8}$$

So we have $J = \frac{1}{4\pi T} E^2$. The slope is half of the open string case.

d) One rotating period is 2π . So we have:

$$2\pi \hbar n = \mathcal{T} \int_0^{2\pi} d\sigma \int_0^{2\pi} d\tau (-\partial_\tau X^\mu \partial_\tau X_\mu + \partial_\sigma X^\mu \partial_\sigma X_\mu) = 4\pi^2 A \mathcal{T} \quad (9) \quad \boxed{9}$$