

Homework 1

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PHY 539: Introduction to String Theory

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Problem 1.

$$-\det g_{\alpha\beta} = 1 - \frac{1}{\Gamma_3(-s)}$$

Problem 2.

- (1) Prove: If $[a] \in \mathbb{Z}_n$ is a unit, then $[a]$ is not a zero divisor.
- (2) Prove: If $[b] \in \mathbb{Z}_n$ is a zero divisor, then $[b]$ is not a unit.

Proof.

□

Problem 3.

Show that every nonzero element of \mathbb{Z}_n is either a unit or a zero divisor.

Proof.

□

Problem 4.

Suppose that $[a]$ is a unit in \mathbb{Z}_n and $[b]$ is an element of \mathbb{Z}_n . Prove that the equation $[a]x = b$ has exactly one solution in \mathbb{Z}_n .

Proof.

□

Problem 5.

Suppose that $[a]$ and $[b]$ are both units in \mathbb{Z}_n . Show that the product $[a] \cdot [b]$ is also a unit in \mathbb{Z}_n . (Note that this confirms closure under multiplication in the group U_n).

Proof.

□

Problem 6.

Which of the following are Groups? Which of the following are not groups, and why?

- (1) $G = \{2, 4, 6, 8\}$ in \mathbb{Z}_{10} . Where $a \star b = ab$
- (2) $G = \mathbb{Q}^*$, where $a \star b = \frac{a}{b}$
- (3) $G = \mathbb{Z}$, where $a \star b = a - b$
- (4) $G = \{2^x \mid x \in \mathbb{Q}\}$, where $a \star b = ab$

Proof.

□

Problem 7.

Consider the set $Q = \{ \pm 1, \pm i, \pm j, \pm k \}$ of the complex matrices as follows:

$$1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$i = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

$$j = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$k = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

Show that Q is a group under matrix multiplication by writing out its multiplication table. (Note: Q is called the quaternion group).

Proof.

□