Physics 539 Fall 2018

Problem Set 2

Due on Oct. 10

1. Becker, Becker and Schwarz, Problem 2.6.

In covariantly quantized open string theory, derive the form of the space-time Lorentz generators $J^{\mu\nu}$ in terms of the operators x^μ, p^μ and α_n^μ. Calculate their commutators and verify that they satisfy the Poincaré algebra.

2. Consider the states at the first massive level of the $D = 26$ open string:

$$(s_{\mu\nu}\alpha_{-1}^\mu \alpha_{-1}^\nu + v_\mu \alpha_{-2}^\mu)|0;k\rangle$$

a) Write down the equations for $s_{\mu\nu}$, v_μ and k^μ implied by the physical state conditions.

b) Write down the gauge transformations implied by the existence of null states.

c) Show that these gauge invariances reduce the number of physical states to the 324 dimensional representation of $SO(25)$. It is convenient to work in the particle rest frame.

3. Repeat the calculation for open string states at the second massive level. Show that the physical states form the representations **2900** and **300** of $SO(25)$.

4. Consider the open bosonic string theory quantized in the light-cone gauge

$$\frac{1}{\sqrt{2}}\left(X^0 + X^{D-1}\right) = x^+ + 2\alpha' p^+ \tau \ .$$

a) Derive the form of the Lorentz generators J^{i-} in terms of x^i, p^i, p^+, x^-, and the transverse oscillators α_n^i, $n \neq 0$, where $i = 1, \ldots D-2$.

b) Using the representation $x^- = -i\frac{\partial}{\partial p^+}$ calculate the commutator

$$[J^{j-}, J^{k-}] \ .$$

Use the result to determine the critical dimension D where the theory is Poincaré invariant.