

Homework 3

Ziming Ji

PHY 539: Introduction to String Theory

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1 Problem 1

a) It is not hard to show that under $z \rightarrow \frac{az+b}{cz+d}$, the cross ratio becomes

$$\frac{\left(\frac{az_1+b}{cz_1+d} - \frac{az_2+b}{cz_2+d}\right) \left(\frac{az_3+b}{cz_3+d} - \frac{az_4+b}{cz_4+d}\right)}{\left(\frac{az_1+b}{cz_1+d} - \frac{az_3+b}{cz_3+d}\right) \left(\frac{az_2+b}{cz_2+d} - \frac{az_4+b}{cz_4+d}\right)} = \frac{(z_1 - z_2)(z_3 - z_4)(bc - ad)^2}{(z_1 - z_3)(z_2 - z_4)(bc - ad)^2} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_3)(z_2 - z_4)}. \quad (1)$$

b) We set the transformation to be $z \rightarrow \frac{az+b}{cz+d}$. Solving the set of equations

$$az_1 + b = 0 \wedge az_2 + b = cz_2 + d \wedge cz_3 + d = 0 \wedge ad - bc = 1, \quad (2)$$

we have

$$\begin{aligned} a &\rightarrow -\frac{\sqrt{z_3 - z_2}}{\sqrt{(z_1 - z_2)(z_1 - z_3)}}, b \rightarrow \frac{z_1 \sqrt{z_3 - z_2}}{\sqrt{(z_1 - z_2)(z_1 - z_3)}}, \\ c &\rightarrow \frac{z_2 - z_1}{\sqrt{(z_1 - z_2)(z_1 - z_3)}\sqrt{z_3 - z_2}}, d \rightarrow \frac{(z_1 - z_2)z_3}{\sqrt{(z_1 - z_2)(z_1 - z_3)}\sqrt{z_3 - z_2}} \end{aligned} \quad (3)$$

or

$$\begin{aligned} a &\rightarrow \frac{\sqrt{z_3 - z_2}}{\sqrt{(z_1 - z_2)(z_1 - z_3)}}, b \rightarrow -\frac{z_1 \sqrt{z_3 - z_2}}{\sqrt{(z_1 - z_2)(z_1 - z_3)}}, \\ c &\rightarrow \frac{z_1 - z_2}{\sqrt{(z_1 - z_2)(z_1 - z_3)}\sqrt{z_3 - z_2}}, d \rightarrow \frac{(z_2 - z_1)z_3}{\sqrt{(z_1 - z_2)(z_1 - z_3)}\sqrt{z_3 - z_2}}. \end{aligned} \quad (4)$$