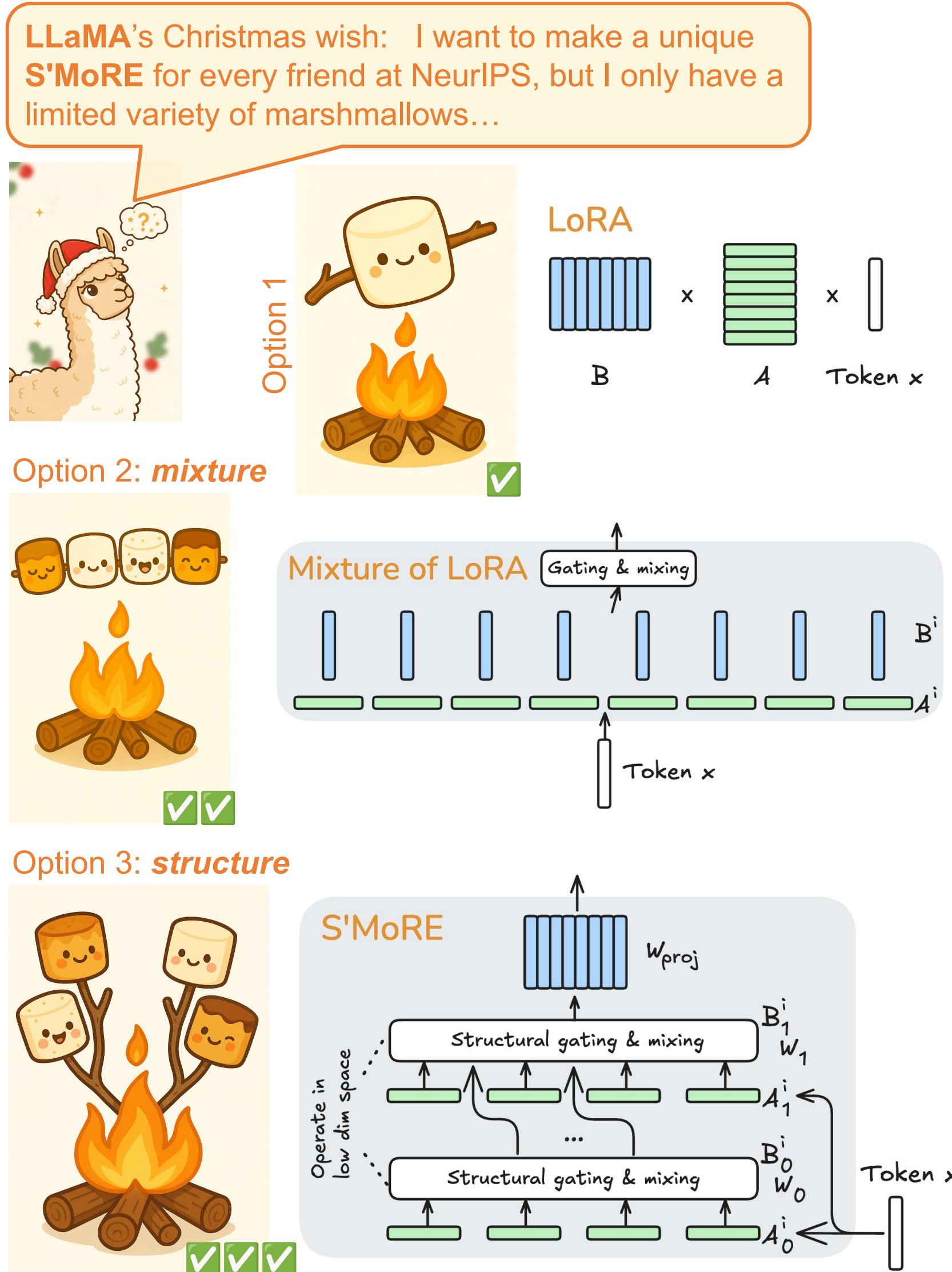




# S'MoRE: Structural Mixture of Residual Experts for Parameter-Efficient LLM Fine-Tuning

Code: <https://github.com/ZimpleX/SMoRE-LLM>

s'more /'smo(:)r/: a dessert consisting usually of toasted marshmallow and pieces of chocolate bar sandwiched between two graham crackers.



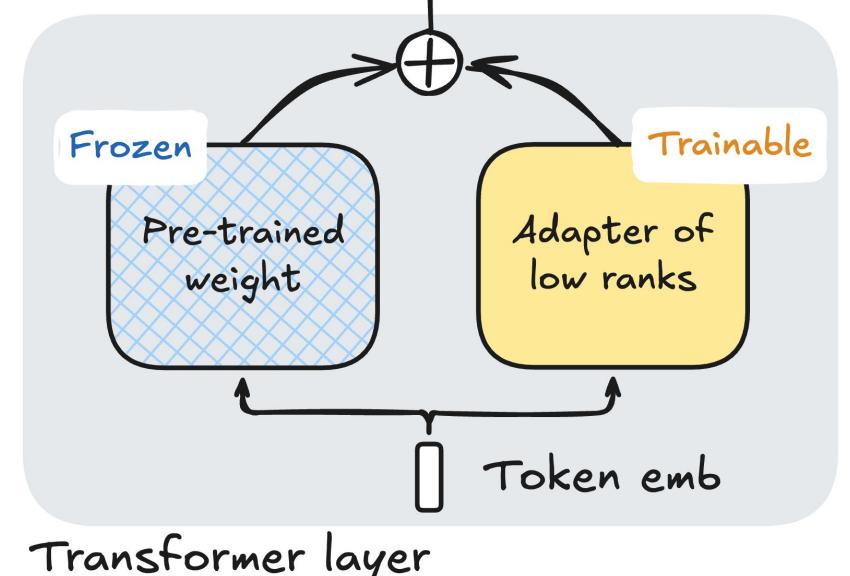
Comparison on the 3 options of adapters:

- Similar **efficiency**: same parameters of &
- Higher **expressivity**: LoRA < Mixture of LoRA < S'MoRE (measured via *structural flexibility*)

## Problem Setup

Parameter-efficient fine-tuning (PEFT) on pre-trained LLM

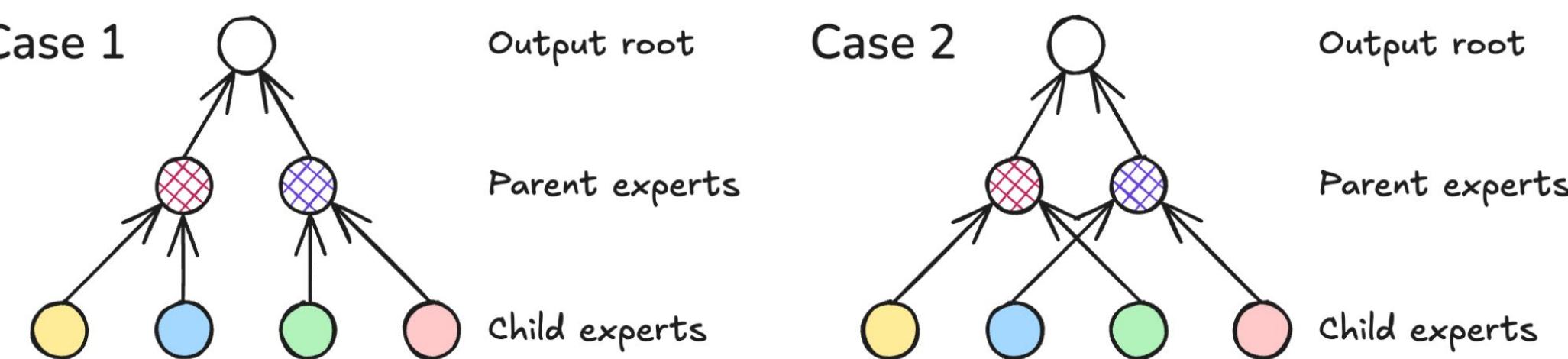
- Adapt to downstream tasks
- Freeze pre-trained weight; Update low-rank adapter parameters



## How Does Structure Help?

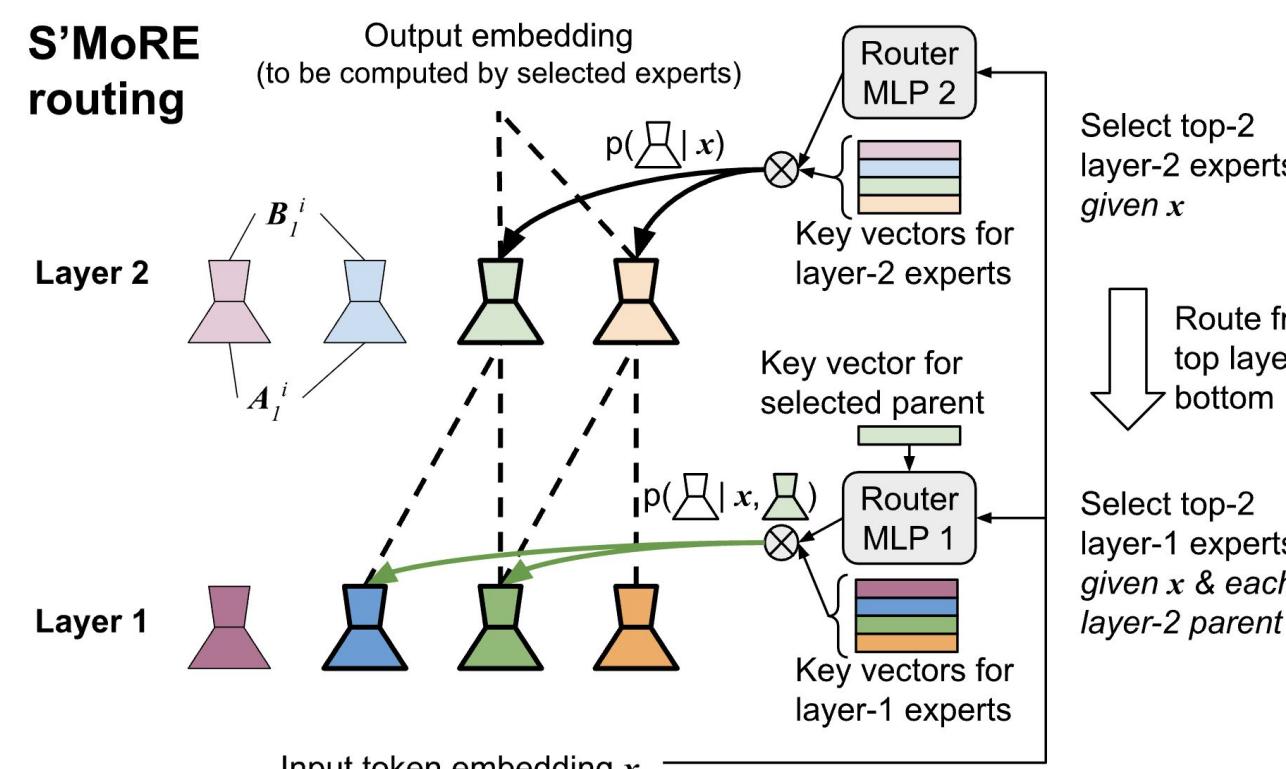
MoE routing problem

- What experts to activate?  $\Rightarrow$  Existing works
  - How to connect activated experts?  $\Rightarrow$  S'MoRE & structural scaling
- The **same** set of experts can form **exponentially many** different structures!

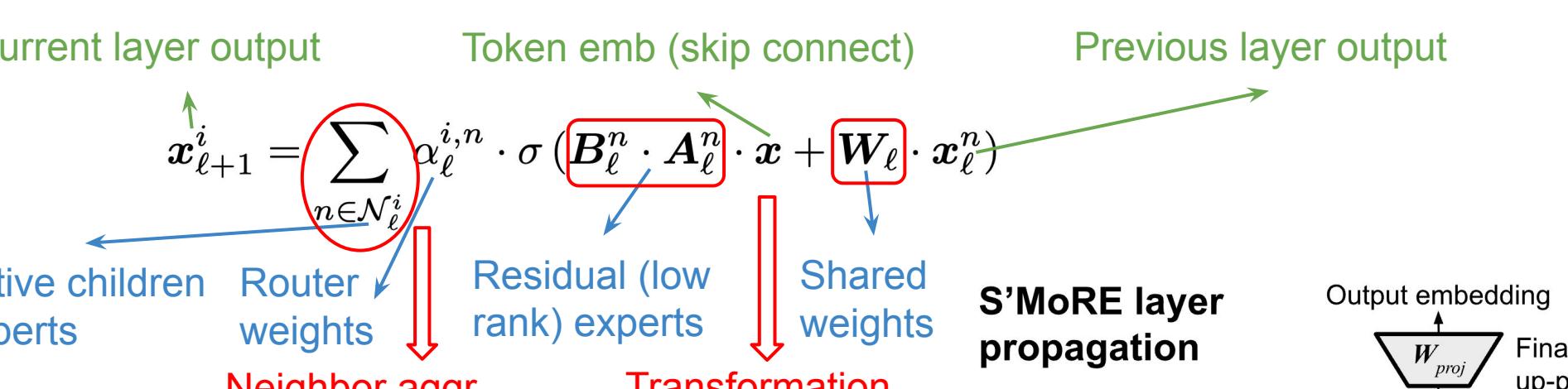


## S'MoRE Routing

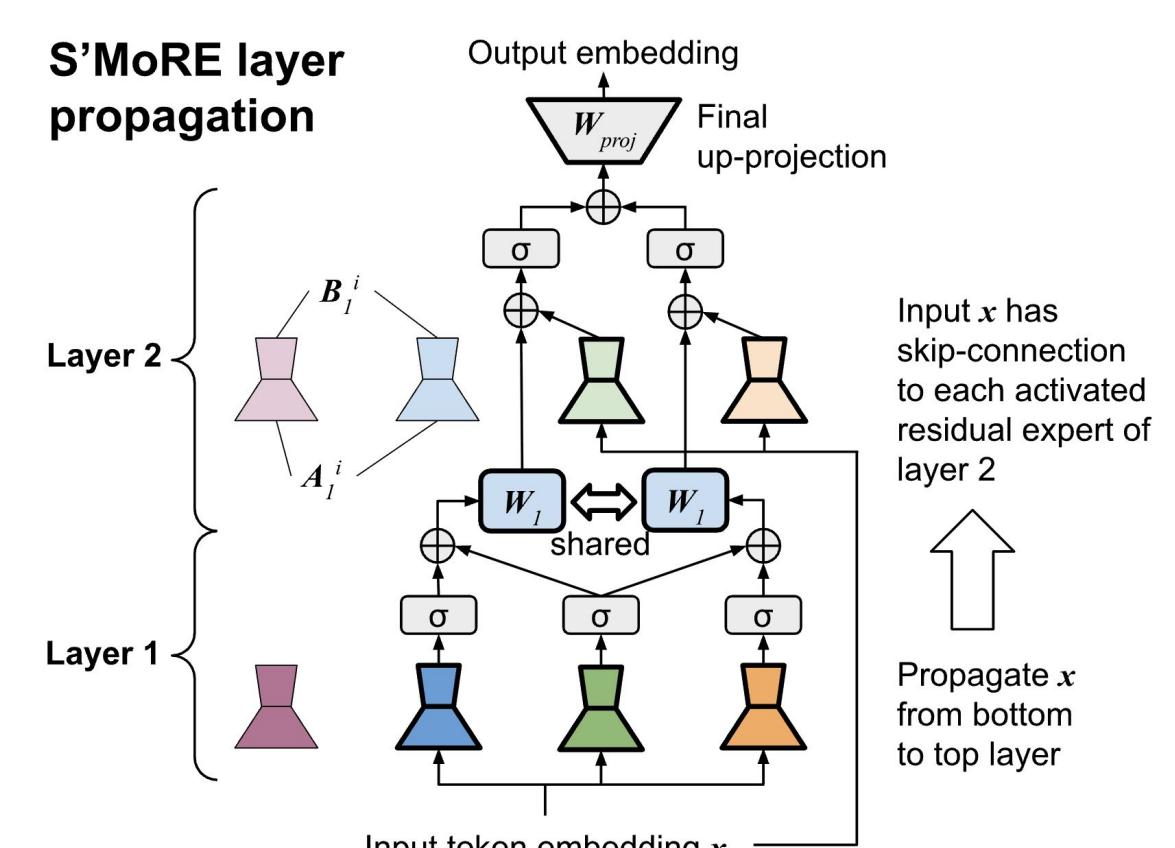
- Hierarchical routing (**top-down**)
- Router computes conditional probability by
  - active ancestors
  - input token
- "Token-expert" similarity based on key-query dot product
- Query embedding: by Router's compact MLP
- Key embedding: learnable for each residual expert



## S'MoRE Layer Propagation

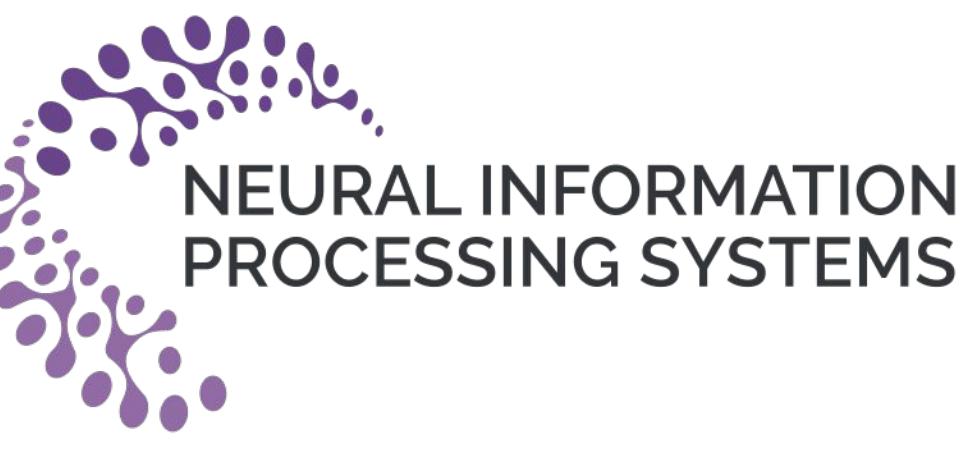
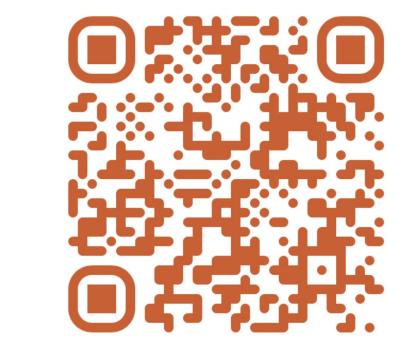


- Selected experts form a residual tree
- Token emb propagates: leaves  $\rightarrow$  root
- Each layer: aggregation + transformation  $\Rightarrow$  GNN
- Craft each layer's output dim for parameter & computation efficiency
- $\sigma$  &  $W$  theoretically ensures expressive power



Hanqing Zeng, Yinglong Xia, Zhuokai Zhao, Chuan Jiang, Qiang Zhang, Jiayi Liu, Qunshu Zhang, Lizhu Zhang, Xiangjun Fan, Benyu Zhang

Meta MRS



## Summary of Theoretical Properties

Parameter & computation efficiency:

- Similar to vanilla LoRA

Recovering MoE baseline:

- By simply setting activation  $\sigma$  as identity

Expressive power w.r.t. **"structural flexibility"**  $\Gamma \Rightarrow$  Graph isomorphism test

- Definition:** Given token,  $\Gamma = \text{num distinct outputs that different expert structures can generate}$

(i.e., 1-layer MoE)

**Theorem 3.3.** The structural flexibility of  $\text{MoMOR}$  is upper-bounded by  $\Gamma_{\text{MoMOR}} = \max_{x, \Theta} \text{dist}(x; \Theta) \leq \binom{s_{L-1}}{f_{L-1}} \cdot \prod_{\ell=0}^{L-2} \left( \sum_{i=f_\ell}^{\min\{F_{\ell+1}, s_\ell\}} \binom{s_\ell}{i} \right)$ . **Sum over fanout**

**Theorem 3.4.** Setting  $\sigma(\cdot)$  as an MLP, there exists some  $\Theta'$  such that the structural flexibility of S'MoRE is:  $\Gamma_{S'MoRE} = \min_x \text{dist}(x; \Theta') = \prod_{\ell=0}^{L-1} \binom{s_\ell}{f_{\ell+1}}$  where we define  $F_L := 1$ .

**Exponent over fanout**

$r$	$L$	$d_L$	$2 \cdot d \cdot d_L$	$\Delta$	Overhead ratio
2	64	0.5M	0.005M	1.0%	
8	3	96	0.8M	0.014M	1.8%
4	128	1.0M	0.031M	2.9%	
2	128	1.0M	0.020M	2.0%	
16	3	192	1.6M	0.057M	3.6%
4	256	2.1M	0.123M	5.9%	

## Experiments

### Setup

- 7 benchmarks
- 2 model families & 3 model scales
- 3 gating types
- 4 or 8 total number of experts

### Main observations

- Accuracy boost due to structural mixture
- Comparable parameter size due to low-dim aggr.
- Consistent gains across model families
- Structure improves scaling on math & coding

Gate	Method	ARC-e Acc.	ARC-e Param.	CSQA Acc.	CSQA Param.	OBQA Acc.	OBQA Param.	Winogrande Acc.	Winogrande Param.	Avg Acc.	Avg Param.
Dense	Base LoRA	32.54	0	66.31	0	23.67	0	43.80	0	50.75	0
Dense	LoRA	36.27	0.004	74.78	0.002	63.80	0.063	70.21	0.031	59.09	0.008
Noisy top-k	HydraLoRA (4)	35.93	0.006	73.54	0.023	66.34	0.023	71.60	0.023	50.75	0.013
Noisy top-k	HydraLoRA (8)	35.93	0.012	72.31	0.007	62.08	0.042	71.60	0.012	58.63	0.017
Noisy top-k	MixLoRA (4)	39.66	0.021	72.84	0.134	65.44	0.134	70.40	0.134	51.30	0.007
Noisy top-k	MixLoRA (8)	39.32	0.021	74.78	0.270	66.42	0.069	69.60	0.134	51.14	0.037
Noisy top-k	S'MoRE (2-2)	<b>40.00</b>	0.017	<b>75.31</b>	0.085	66.99	0.037	72.20	0.085	<b>52.01</b>	0.015
Noisy top-k	S'MoRE (4-4)	39.66	0.017	74.43	0.085	<b>67.32</b>	0.045	<b>72.80</b>	0.202	<b>52.01</b>	0.168
Switch	MixLoRA (4)	39.32	0.037	71.96	0.069	67.00	0.134	70.00	0.134	51.46	0.069
Switch	MixLoRA (8)	37.97	0.069	72.84	0.270	65.03	0.134	70.80	0.270	51.46	0.069
Switch	S'MoRE (2-2)	<b>39.66</b>	0.029	73.19	0.135	64.95	0.135	70.00	0.102	51.54	0.029
Switch	S'MoRE (4-4)	39.66	0.037	<b>74.96</b>	0.135	66.26	0.102	<b>71.40</b>	0.135	<b>52.27</b>	0.273
LLaMA 3.2B	Base	80.34	0	89.77	0	70.35	0	73.80	0	74.83	0
LLaMA 3.2B	LoRA	81.69	0.028	91.36	0.028	81.00	0.028	87.00	0.028	81.77	0.028
LLaMA 3.2B	HydraLoRA (4)	<b>83.39</b>	0.013	91.53	0.160	81.82	0.013	88.20	0.082	83.82	0.160
LLaMA 3.2B	HydraLoRA (8)	81.69	0.079	91.53	0.015	81.49	0.024	86.60	0.015	84.14	0.297
LLaMA 3.2B	MixLoRA (4)	81.69	0.026	<b>92.24</b>	0.247	81.24	0.033	89.40	0.478	84.06	0.247
LLaMA 3.2B	MixLoRA (8)	82.37	0.132	91.71	0.247	85.40	0.075	85.40	0.478	85.82	0.193
LLaMA 3.2B	S'MoRE (2-2)	82.37	0.090	92.24	0.190	<b>81.90</b>	0.037	89.40	0.054	<b>88.24</b>	0.480
LLaMA 3.2B	S'MoRE (4-4)	82.71	0.190	91.89	0.247	<b>81.90</b>	0.033	<b>90.00</b>	0.047	<b>86.83</b>	0.247
LLaMA 3.8B	Base	82.37	0.132	<b>92.95</b>	0.478	81.08	0.047	88.80	0.478	84.53	0.247
LLaMA 3.8B	LoRA	82.03	0.033	91.71	0.132	81.24	0.047	88.60	0.247	85.91	0.282
LLaMA 3.8B	MixLoRA (4)	82.37	0.135	92.24	0.061	81.82	0.104	88.20	0.047	83.27	0.190
LLaMA 3.8B	MixLoRA (8)	82.03	0.135	92.24	0.061	81.82	0.076	<b>89.80</b>	0.245	86.62	0.247
LLaMA 3.8B	S'MoRE (2-2)	82.37	0.104	91.71	0.305	<b>82.16</b>	0.047	<b>89.80</b>	0.305	85.87	0.305
LLaMA 3.8B	S'MoRE (4-4)	<b>83.39</b>	0.076	92.42	0.305	<b>82.15</b>	0.047	<b>89.80</b>	0.305	<b>86.73</b>	0.208

Table 4: Results on Gemma 2-9B. We evaluate on representative benchmarks due to limited resources.

Method	ARC-e Accuracy	CSQA Accuracy	Winogrande Accuracy	HumanEval Pass@1	Acc. / Pass@1	Param. (B)	Avg. Acc.	Avg. Param.

<tbl\_r cells="9" ix="1" maxcspan="1" maxrspan="1" usedcols="