

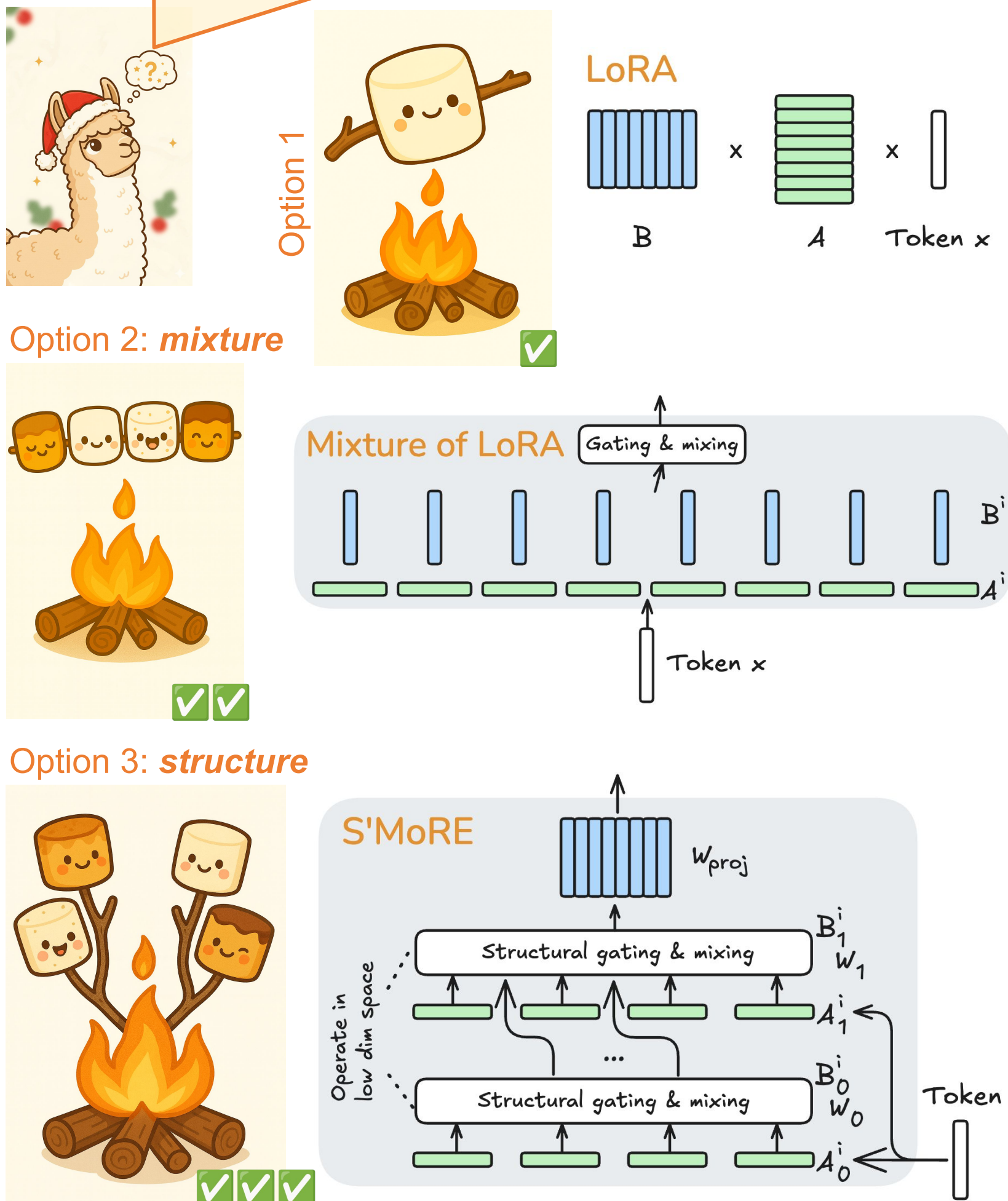


# S'MoRE: Structural Mixture of Residual Experts for Parameter-Efficient LLM Fine-Tuning

Code: <https://github.com/ZimpleX/SMoRE-LLM>

*s'more* /'smo(:)j/: a dessert consisting usually of toasted marshmallow and pieces of chocolate bar sandwiched between two graham crackers.

**LLaMA's Question:** How to make a unique s'more for everyone using limited ingredients?



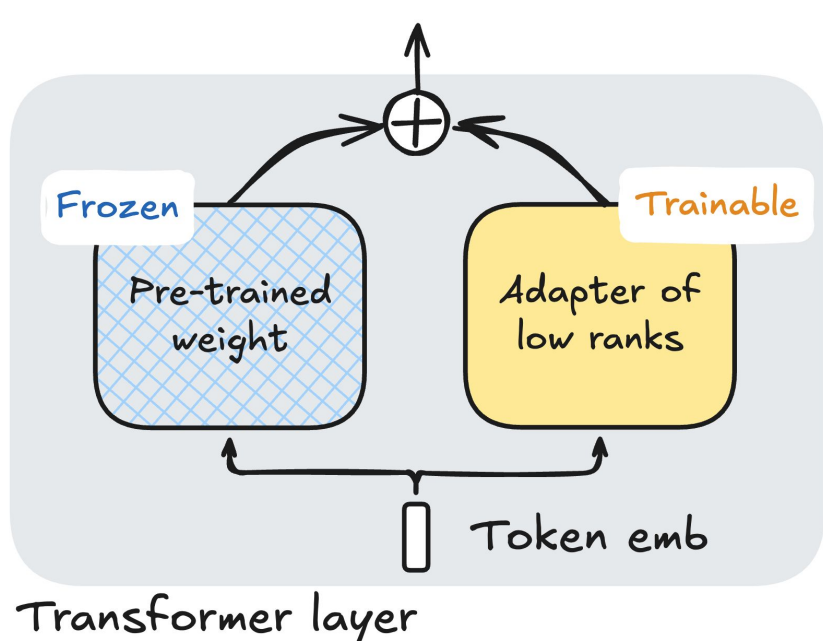
Comparison on the 3 options of adapters:

- Similar **efficiency**: same parameters of  $\text{---}$  &  $\text{||}$
- Higher **expressivity**: LoRA < Mixture of LoRA < S'MoRE (measured via *structural flexibility*)

## Problem Setup

Parameter-efficient fine-tuning (PEFT) on pre-trained LLM

- Adapt to downstream tasks
- Freeze pre-trained weight; Update low-rank adapter parameters

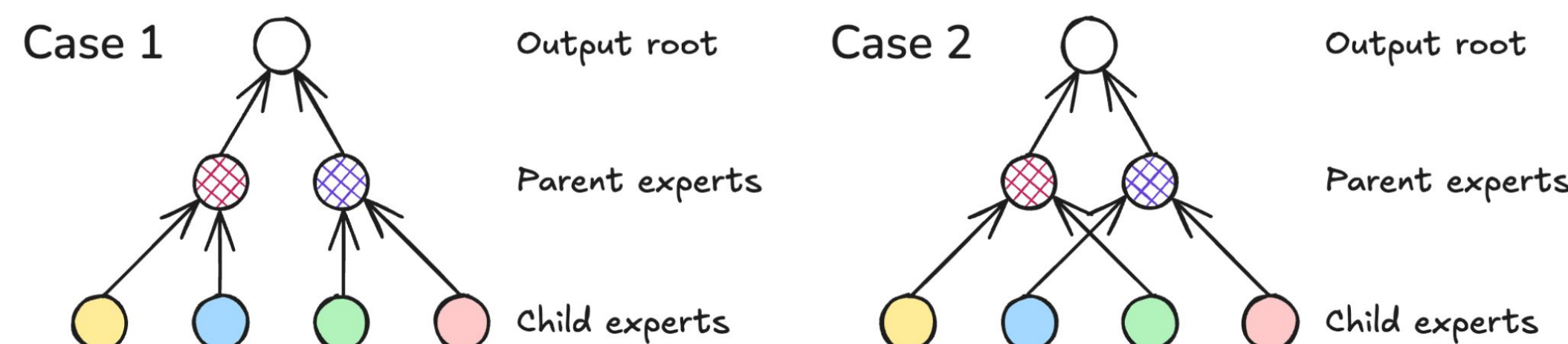


## How Does Structure Help?

MoE routing problem

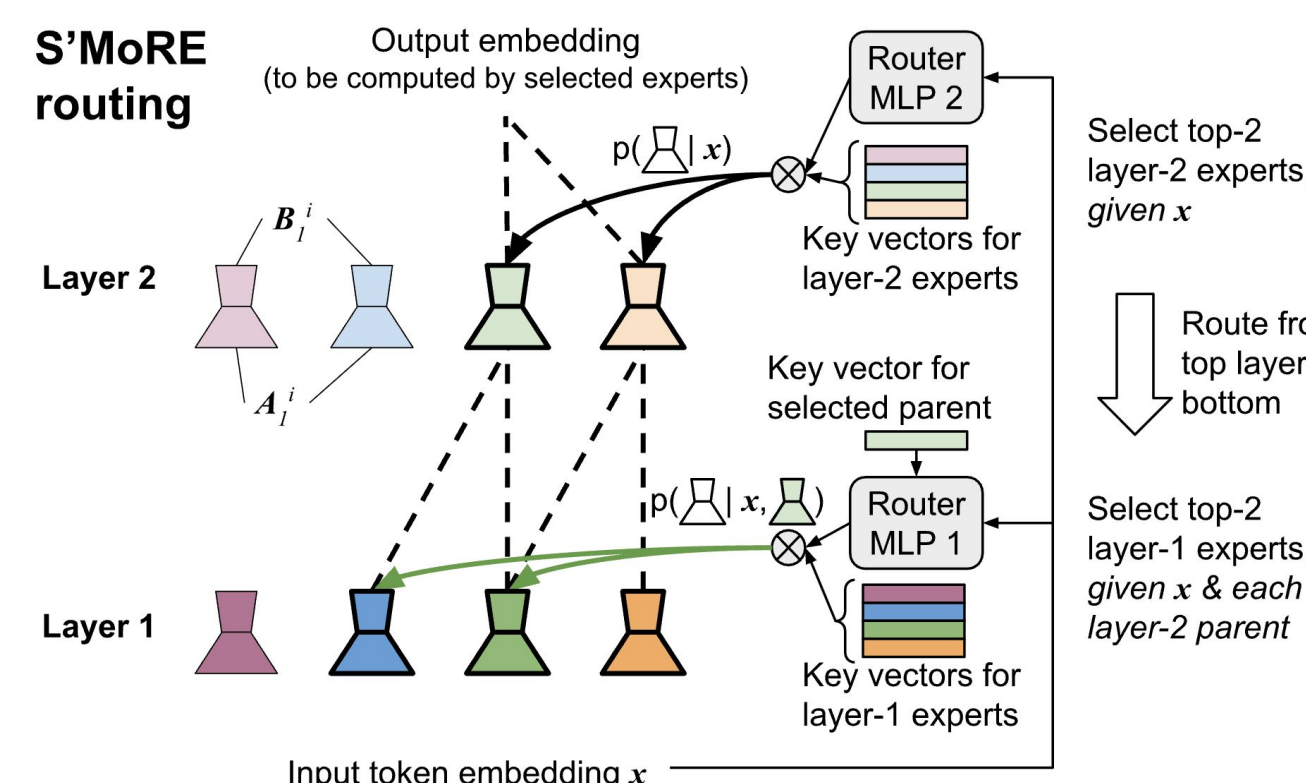
- What experts to activate?  $\Rightarrow$  Existing works
- How to connect activated experts?  $\Rightarrow$  S'MoRE & structural scaling

The **same** set of experts can form **exponentially many** different structures!

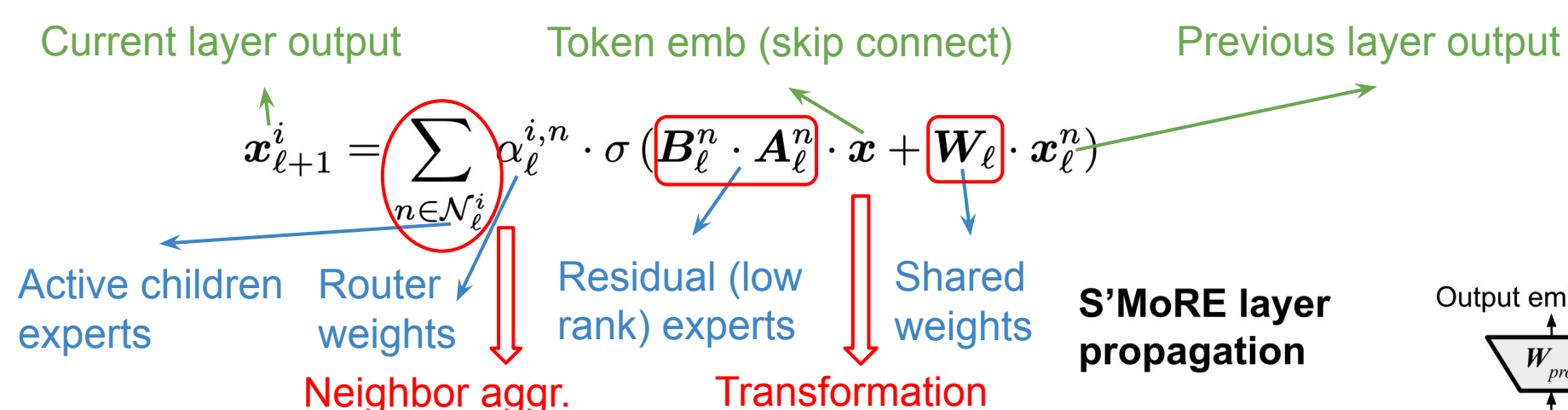


## S'MoRE Routing

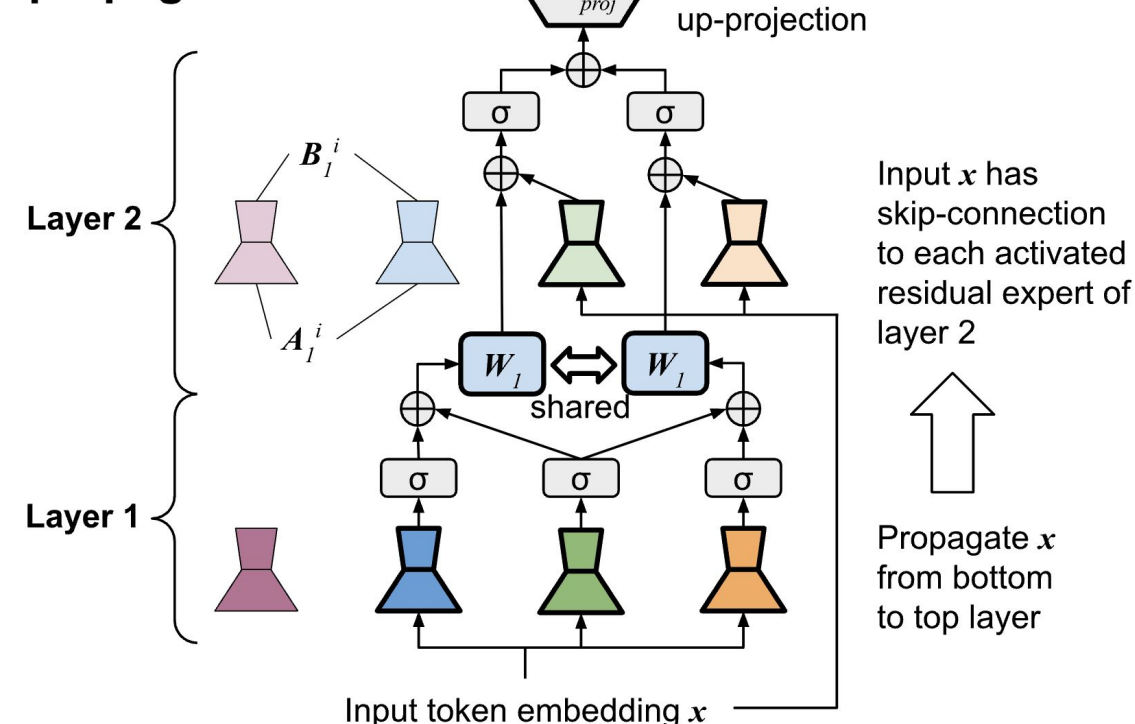
- Hierarchical routing (**top-down**)
- Router computes conditional probability by
  - active ancestors
  - input token
- "Token-expert" similarity based on key-query dot product
- Query embedding: by Router's compact MLP
- Key embedding: learnable for each residual expert



## S'MoRE Layer Propagation

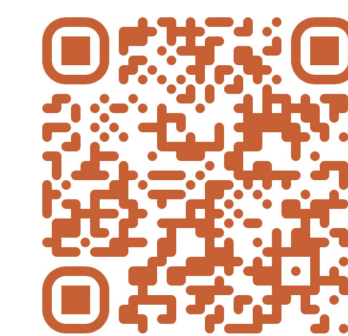


### S'MoRE layer propagation



- Selected experts form a residual tree
- Token emb propagates: leaves  $\rightarrow$  root
- Each layer: aggregation + transformation  $\Rightarrow$  GNN
- Craft each layer's output dim for parameter & computation efficiency
- $\sigma$  &  $W$  theoretically ensures expressive power

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## Summary of Theoretical Properties

Parameter & computation efficiency:

- Similar to vanilla LoRA

Recovering MoE baseline:

- By simply setting activation  $\sigma$  as identity

Expressive power w.r.t. "**structural flexibility**"  $\Gamma \Rightarrow$  Graph isomorphism test

- Definition:** Given token,  $\Gamma$  = num distinct outputs that different expert structures can generate

**Theorem 3.3.** The structural flexibility of **MoMOR** is upper-bounded by  $\Gamma_{\text{MoMOR}} = \max_{x, \Theta} \text{dist}(x; \Theta) \leq \binom{s_L-1}{f_L-1} \cdot \prod_{\ell=0}^{L-2} \left( \sum_{i=f_\ell}^{\min\{F_\ell, s_\ell\}} \binom{s_\ell}{i} \right)$ . (i.e., 1-layer MoE) **Sum over fanout**

**Theorem 3.4.** Setting  $\sigma(\cdot)$  as an MLP, there exists some  $\Theta'$  such that the structural flexibility of **S'MoRE** is:  $\Gamma_{\text{S'MoRE}} = \min_x \text{dist}(x; \Theta') = \prod_{\ell=0}^{L-1} \binom{s_\ell}{f_\ell}^{F_{\ell+1}}$  where we define  $F_L := 1$ . **Exponent over fanout**

## Experiments

Setup

- 7 benchmarks
- 2 model families & 3 model scales
- 3 gating types
- 4 or 8 total number of experts

Main observations

- Accuracy boost due to structural mixture
- Comparable parameter size due to low-dim aggr.
- Consistent gains across model families
- Structure improves scaling on math & coding

Gate	Method	ARC-c Acc.	ARC-c Param.	ARC-e Acc.	ARC-e Param.	CSQA Acc.	CSQA Param.	OBQA Acc.	OBQA Param.	Winogrande Acc.	Winogrande Param.	Avg Acc.	Avg Param.
LLaMA 3.2 1B	Base LoRA	32.54	0	66.31	0	23.67	0	43.80	0	50.75	0	43.41	0
	HydraLoRA (4)	35.93	0.006	73.54	0.023	66.34	0.002	71.60	0.023	50.75	0.012	59.63	0.013
	HydraLoRA (8)	35.93	0.012	72.31	0.007	62.08	0.042	71.60	0.012	50.99	0.012	58.58	0.017
	MixLoRA (4)	39.66	0.021	72.84	0.134	65.44	0.134	70.40	0.134	51.30	0.007	59.93	0.086
	MixLoRA (8)	39.32	0.021	74.78	0.270	66.42	0.069	69.60	0.134	51.14	0.037	60.25	0.106
	S'MoRE (2-2)	<b>40.00</b>	0.017	<b>75.31</b>	0.085	<b>66.99</b>	0.037	<b>72.20</b>	0.085	<b>52.01</b>	0.015	<b>61.30</b>	0.048
	S'MoRE (4-4)	39.66	0.017	74.43	0.085	<b>67.32</b>	0.045	<b>72.80</b>	0.202	<b>52.01</b>	0.168	61.24	0.103
	MixLoRA (4)	39.32	0.037	71.96	0.069	64.70	0.134	70.00	0.134	51.46	0.069	59.49	0.089
	MixLoRA (8)	37.97	0.069	72.84	0.270	65.03	0.134	70.80	0.270	51.46	0.069	59.62	0.162
	S'MoRE (2-2)	<b>39.66</b>	0.029	73.19	0.135	64.95	0.135	70.00	0.102	51.54	0.029	59.87	0.086
LLaMA 3.2 3B	Base LoRA	80.34	0	89.77	0	70.35	0	73.80	0	59.91	0	74.83	0
	HydraLoRA (4)	<b>83.39</b>	0.013	91.53	0.160	81.82	0.013	88.20	0.082	83.82	0.160	85.75	0.086
	HydraLoRA (8)	81.69	0.079	91.53	0.015	81.49	0.024	86.60	0.015	84.14	0.297	85.09	0.086
	MixLoRA (4)	81.69	0.026	<b>92.24</b>	0.247	81.24	0.033	89.40	0.478	84.06	0.247	85.73	0.206
	MixLoRA (8)	82.37	0.132	91.71	0.247	81.00	0.033	88.60	0.075	85.40	0.478	85.82	0.193
	S'MoRE (2-2)	82.37	0.090	<b>92.24</b>	0.190	<b>81.90</b>	0.037	89.40	0.054	<b>88.24</b>	0.480	<b>86.83</b>	0.170
	S'MoRE (4-4)	82.71	0.190	91.89	0.247	<b>81.90</b>	0.033	<b>90.00</b>	0.480	<b>85.48</b>	0.247	86.40	0.157
	MixLoRA (4)	82.37	0.075	91.53	0.247	80.75	0.075	87.80	0.075	82.00	0.478	84.89	0.190
	MixLoRA (8)	<b>83.39</b>	0.950	91.53	0.247	80.67	0.075	88.40	0.247	83.19	0.478	85.44	0.399
	S'MoRE (2-2)	82.37	0.305	91.36	0.090	81.82	0.104	88.20	0.047	83.27	0.190	85.40	0.147
LLaMA 3.2 8B	Base LoRA	82.37	0.104	<b>91.71</b>	0.305	<b>82.06</b>	0.047	<b>90.00</b>	0.480	<b>85.48</b>	0.174	<b>86.32</b>	0.330
	MixLoRA (4)	82.37	0.132	<b>92.95</b>	0.478	81.08	0.047	88.80	0.478	84.53	0.247	85.95	0.276
	MixLoRA (8)	82.03	0.033	91.71	0.132	81.24	0.047	88.60	0.247	85.95	0.950	85.91	0.282
	S'MoRE (2-2)	83.05	0.133	92.24	0.061	81.82	0.029	<b>89.80</b>	0.076	<b>86.42</b>	0.247	86.67	0.109
	S'MoRE (4-4)	<b>83.39</b>	0.076	92.42	0.305	<b>82.15</b>	0.047	<b>89.80</b>	0.305	85.87	0.305	<b>86.73</b>	0.208

Table 4: Results on **Gemma 2-9B**. We evaluate on representative benchmarks due to limited resources.

Method	ARC-c Accuracy	ARC-c Param. (B)	ARC-e Accuracy	ARC-e Param. (B)	CSQA Accuracy	CSQA Param. (B)	Winogrande Accuracy	Winogrande Param. (B)	HumanEval Accuracy	HumanEval Param. (B)	Avg Acc. / Pass@1	Avg Param. (B)
LoRA	79.72	0.289	85.91	0.145	87.06	0.145	43.29	0.072	74.00	0.163		
HydraLoRA (4)	85.54	0.059	85.83	0.096	88.79	0.169	43.29	0.096	75.86	0.105		
MixLoRA (4)	83.07	0.168	85.83	0.096	89.19	0.315	44.51	0.168	75.65	0.187		
S'MoRE (2-2)	86.24	0.042	<b>86.40</b>	0.169	<b>90.13</b>	0.169	44.51	0.096	76.82	0.119		
S'MoRE (4-4)	<b>86.60</b>	0.169	86.32	0.060	<b>90.13</b>	0.315	<b>46.34</b>	0.060	<b>77.35</b>	0.151		

Table 3: **LLaMA 3-8B** model Accuracy / Pass@1, and the best-performing models' trainable parameters (B).

Gate	Method	Accuracy	GSM8K Param. (B)	HumanEval Pass@1	HumanEval Param. (B)
Dense	Base model LoRA	55.95	0	26.22	0
	HydraLoRA (4)	62.47	0.317	40.85	0.082
	HydraLoRA (8)	62.24	0.297	<b>44.51</b>	0.079
	MixLoRA (4)	61.11	0.132	39.02	0.026
	MixLoRA (8)	59.36	0.132	40.85	0.033
	S'MoRE (2-2)	62.40	0.104	42.07	0.090
	S'MoRE (4-4)	<b>65.20</b>	0.957	43.90	0.104
	MixLoRA (4)	59.67	0.047	42.68	0.075
	MixLoRA (8)	61.56	0.247	39.63	0.247
	S'MoRE (2-2)	62.47	0.133	<b>45.73</b>	0.190
Switch	S'MoRE (4-4)	<b>63.91</b>	0.957	42.07	0.090

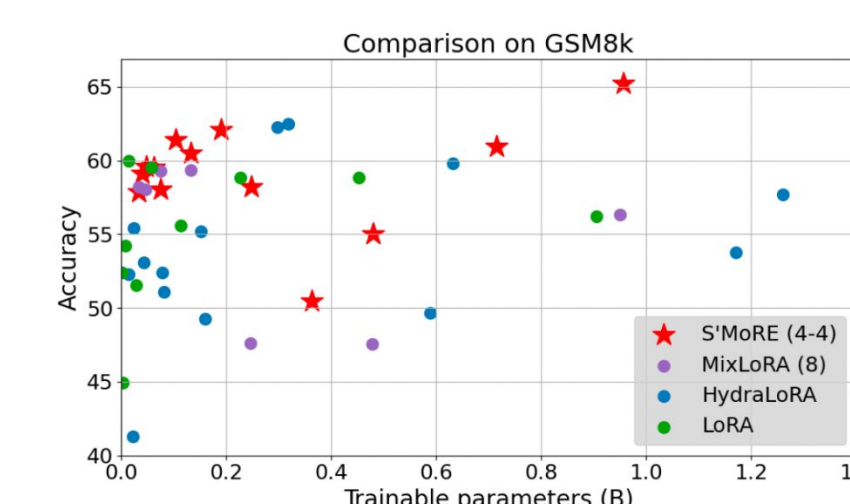


Figure 4: Change of accuracy w.r.t. trainable parameters, corresponding to models in Table 3.