CSCI-1200 Data Structures — Spring 2016
Lecture 25 – Hash Tables

Announcements: HW 3 Due Date

- Because of the ongoing iClicker problems, we will extend the due date for homework 10 to Friday April 29 at 11:59:59 PM. You can submit hw10 until one second before midnight Friday without a late day penalty.

Announcements: Test 3 Information

- Test 3 will be held Monday, May 2 from 6-7:50pm in DCC 308 and DCC 318 No make-ups will be given except for emergency situations, and even then a written excuse from the Dean of Students or the Office of Student Experience will be required.
- Seating assignments will be posted on the homework server this weekend.
- Coverage: Lectures 1-24, Labs 1-12, HW 1-10.
- Closed-book and closed-notes *except for 1 sheet of notes on 8.5x11 inch paper (front & back) that may be handwritten or printed.* All students must bring their Rensselaer photo ID card.
- Practice problems from previous exams are available on the course website. Solutions to the problems will be posted a day or two before the exam.
- If you have an accommodation letter, and you have not sent me a copy, please do so immediately.
- If you sent me an accommodation letter, and you have not received an e-mail from me telling you when and where the test will be held, please contact me immediately.
- UPE will hold an Exam #3 Review Session on Friday 4/29/16 at 4pm in Lally 104.
- Bring to the exam room:

  Your Rensselaer photo ID card.

  Pencil(s) & eraser (pens are ok, but not recommended). The exam *will* involve handwritten code on paper (and other short answer problem solving). Neat legible handwriting is appreciated. We will be somewhat forgiving to minor syntax errors – it will be graded by humans not computers :)

  [OPTIONAL] You may bring 1 sheet of notes on 8.5x11 inch paper (front & back) that may be handwritten or printed. Learn how to print double-sided. You may not staple or tape or glue together 2 or more single-sided sheets of paper.
- Do not bring your own scratch paper. We will provide scratch paper.
- Computers, cell-phones, smart watches, calculators, music players, etc. are not permitted. Please do not bring your laptop, books, backpack, etc. to the exam room – leave everything in your dorm room. *Unless you are coming directly from another class or sports/club meeting.*

Today's Lecture

- "the single most important data structure known to mankind"

  `https://www.mtu.edu/career/students/toolbox/interviews/prepare.pdf`
- Hash Tables, Hash Functions, and Collision Resolution
- Performance of: Hash Tables vs. Binary Search Trees
- Collision resolution: separate chaining vs open addressing
- STL's `unordered_set` (and `unordered_map`)
- Using a hash table to implement a set/map
  - Hash functions as functors/function objects
  - Iterators, find, insert, and erase

25.1 Definition: What's a Hash Table?

- A table implementation with *constant time access.*
  - Like a set, we can store elements in a collection. Or like a map, we can store key-value pair associations in the hash table. But it's even faster to do find, insert, and erase with a hash table! However, hash tables *do not* store the data in sorted order.
- A hash table is implemented with an array at the top level.
- Each element or key is mapped to a slot in the array by a *hash function.*

25.2 Definition: What's a Hash Function?

- A simple function of one argument (the key) which returns an integer index (a bucket or slot in the array).
- Ideally the function will "uniformly" distribute the keys throughout the range of legal index values ($0 \rightarrow$ k-1).
- **What's a collision?**
  When the hash function maps multiple (different) keys to the same index.
- **How do we deal with collisions?**
  One way to resolve this is by storing a linked list of values at each slot in the array.

25.3 Example: Caller ID

- We are given a phonebook with 50,000 name/number pairings. Each number is a 10 digit number. We need to create a data structure to lookup the name matching a particular phone number. Ideally, name lookup should be O(1) time expected, and the caller ID system should use O(n) memory (n = 50,000).
- Note: In the toy implementations that follow we use small datasets, but we should evaluate the system scaled up to handle the large dataset.
- The basic interface:

```
// add several names to the phonebook
add(phonebook, 1111, "fred");
add(phonebook, 2222, "sally");
add(phonebook, 3333, "george");
// test the phonebook
std::cout << identify(phonebook, 2222) << " is calling!" << std::endl;
std::cout << identify(phonebook, 4444) << " is calling!" << std::endl;
```

- We'll review how we solved this problem in Lab 9 with an STL `vector` then an STL `map`. Finally, we'll implement the system with a hash table.

25.4 Caller ID with an STL Vector

```
// create an empty phonebook
std::vector<std::string> phonebook(10000, "UNKNOWN CALLER");

void add(std::vector<std::string> &phonebook, int number, std::string name) {
  phonebook[number] = name; }

std::string identify(const std::vector<std::string> &phonebook, int number) {
  return phonebook[number]; }
```

Exercise: What's the memory usage for the vector-based Caller ID system?
What's the expected running time for find, insert, and erase?

25.5 Caller ID with an STL Map

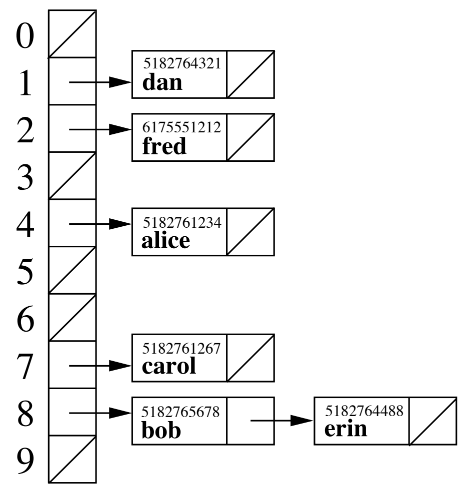
```
// create an empty phonebook
std::map<int,std::string> phonebook;

void add(std::map<int,std::string> &phonebook, int number, std::string name) {
  phonebook[number] = name; }

std::string identify(const std::map<int,std::string> &phonebook, int number) {
  map<int,std::string>::const_iterator tmp = phonebook.find(number);
  if (tmp == phonebook.end()) return "UNKNOWN CALLER"; else return tmp->second;
}
```

Exercise: What's the memory usage for the map-based Caller ID system?
What's the expected running time for find, insert, and erase?

25.6 Now let's implement Caller ID with a Hash Table




```
#define PHONEBOOK_SIZE 10

class Node {
public:
  int number;
  string name;
  Node* next;
};

// create the phonebook, initially all numbers are unassigned
Node* phonebook[PHONEBOOK_SIZE];
for (int i = 0; i < PHONEBOOK_SIZE; i++) {
  phonebook[i] = NULL;
}

// corresponds a phone number to a slot in the array
int hash_function(int number) {



}

// add a number, name pair to the phonebook
void add(Node* phonebook[PHONEBOOK_SIZE], int number, string name) {



}

// given a phone number, determine who is calling
void identify(Node* phonebook[PHONEBOOK_SIZE], int number) {



}
```

25.7 Exercise: Choosing a Hash Function

- What's a good hash function for this application?

- What's a bad hash function for this application?

25.8 Exercise: Hash Table Performance

- What's the memory usage for the hash-table-based Caller ID system?

- What's the expected running time for find, insert, and erase?

25.9 What makes a Good Hash Function?

- Goals: **fast O(1) computation** and a **random, uniform distribution of keys throughout the table**, *despite the actual distribution of keys that are to be stored.*

- For example, using: `f(k) = abs(k)%N` as our hash function satisfies the first requirement, but may not satisfy the second.

  It's not too bad if k is uniformly distributed and the load factor is low.

- Another example of a dangerous hash function on string keys is to add or multiply the ascii values of each char:

```
unsigned int hash(string const& k, unsigned int N) {
  unsigned int value = 0;
  for (unsigned int i=0; i<k.size(); ++i)
    value += k[i];  // conversion to int is automatic
  return k % N;
}
```

  The problem is that different permutations of the same string result in the same hash table location.

  AND and DAN map to the same locations.

- This can be improved through multiplications that involve the position and value of the key:

```
unsigned int hash(string const& k, unsigned int N) {
  unsigned int value = 0;
  for (unsigned int i=0; i<k.size(); ++i)
    value = value*8 + k[i];  // conversion to int is automatic
  return k % N;
}
```

- The 2nd method is better, but can be improved further. The theory of good hash functions is quite involved and beyond the scope of this course.

25.10 How do we Resolve Collisions? METHOD 1: Separate Chaining

- Each table location stores a linked list of keys (and values) hashed to that location (as shown above in the phonebook hashtable). Thus, the hashing function really just selects which list to search or modify.
- This works well when the number of items stored in each list is small, e.g., an average of 1. Other data structures, such as binary search trees, may be used in place of the list, but these have even greater overhead considering the (hopefully, very small) number of items stored per bin.

25.11 How do we Resolve Collisions? METHOD 2: Open Addressing

- In *open addressing*, when the chosen table location already stores a key (or key-value pair), a different table location is sought in order to store the new value (or pair).
- Here are three different open addressing variations to handle a collision during an *insert* operation:
  - *Linear probing:* If `i` is the chosen hash location then the following sequence of table locations is tested ("probed") until an empty location is found:

    `(i+1)%N, (i+2)%N, (i+3)%N, ...`
  - *Quadratic probing:* If `i` is the hash location then the following sequence of table locations is tested:

    `(i+1)%N, (i+2*2)%N, (i+3*3)%N,  (i+4*4)%N, ...`

    More generally, the j^{th} "probe" of the table is $(i + c_1 j + c_2 j^2) \mod N$ where c_1 and c_2 are constants.
  - *Secondary hashing*: when a collision occurs a second hash function is applied to compute a new table location. This is repeated until an empty location is found.
- For each of these approaches, the *find* operation follows the same sequence of locations as the *insert* operation. The key value is determined to be absent from the table only when an empty location is found.
- When using open addressing to resolve collisions, the *erase* function must mark a location as "formerly occupied". If a location is instead marked empty, *find* may fail to return elements in the table. With many deletions, the table may become clogged with locations maked as "formerly occupied". This may require rebuilding the table.
- Problems with open addressing:
  - Slows dramatically when the table is nearly full (e.g. about 80% or higher). This is particularly problematic for linear probing.
  - Fails completely when the table is full.
  - Cost of computing new hash values. Might require rebuilding teh table.

25.12 Hash Table in STL?

- The Standard Template Library standard and implementation of hash table have been slowly evolving over many years. Unfortunately, the names "hashset" and "hashmap" were spoiled by developers anticipating the STL standard, so to avoid breaking or having name clashes with code using these early implementations...
- STL's agreed-upon standard for hash tables: `unordered_set` and `unordered_map`
- Depending on your OS/compiler, you may need to add the `-std=c++11` flag to the compile line (or other configuration tweaks) to access these more recent pieces of STL. (And this will certainly continue to evolve in future years!) Also, for many types STL has a good default hash function, so you may not always need to specify both template parameters!

25.13 Our Copycat Version: A Set As a Hash Table

- The class is templated over both the key type and the hash function type.

```
template  < class KeyType, class HashFunc >
class ds_hashset {    ...    };
```

- We use separate chaining for collision resolution. Hence the main data structure inside the class is:

```
std::vector< std::list<KeyType> > m_table;
```

- We will use automatic resizing when our table is too full. Resize is expensive of course, so similar to the automatic reallocation that occurs inside the vector `push_back` function, we at least double the size of underlying structure to ensure it is rarely needed.

25.14 Our Hash Function Object

- Remember from a previous lecture that "function objects" or "functors" are just a class wrapper around a function, and that function is implemented as the overloaded function call operator for the class.
- Often the programmer/designer for the program using a hash function has the best understanding of the distribution of data to be stored in the hash function. Thus, they are in the best position to define a custom hash function (if needed) for the data & application.
- Here's an example of a (generically) good hash function for STL strings, wrapped up inside of a class:

```
class hash_string_obj {
public:
  unsigned int operator() (std::string const& key) const {
    // This implementation comes from
    // http://www.partow.net/programming/hashfunctions/
    unsigned int hash = 1315423911;
    for(unsigned int i = 0; i < key.length(); i++)
      hash ^= ((hash << 5) + key[i] + (hash >> 2));
    return hash;
  }
};
```

- Once our new type containing the hash function is defined, we can create instances of our hash set object containing `std::string` by specifying the type `hash_string_obj` as the second template parameter to the declaration of a `ds_hashset`. E.g.,

```
ds_hashset<std::string, hash_string_obj> my_hashset;
```

- Alternatively, we could use function pointers as a non-type template argument.
  (We don't show that syntax here!).

25.15 Hash Set Iterators

- Iterators move through the hash table in the order of the storage locations rather than the ordering imposed by (say) an `operator<`. Thus, the visiting/printing order depends on the hash function and the table size.
  - Hence the increment operators must move to the next entry in the current linked list or, if the end of the current list is reached, to the first entry in the next non-empty list.
- The declaration is nested inside the `ds_hashset` declaration in order to avoid explicitly templating the iterator over the hash function type.
- The iterator must store:
  - A pointer to the hash table it is associated with. This reflects a subtle point about types: even though the `iterator` class is declared inside the `ds_hashset`, this does not mean an iterator automatically knows about any particular `ds_hashset`.
  - The index of the current list in the hash table.
  - An iterator referencing the current location in the current list.
- Because of the way the classes are nested, the `iterator` class object must declare the `ds_hashset` class as a friend, but the reverse is unnecessary.

25.16 Implementing `begin()` and `end()`

- `begin()`: Skips over empty lists to find the first key in the table. It must tie the iterator being created to the particular `ds_hashset` object it is applied to. This is done by passing the `this` pointer to the iterator constructor.
- `end()`: Also associates the iterator with the specific table, assigns an index of `-1` (indicating it is not a normal valid index), and thus does not assign the particular list iterator.
- **Exercise:** Implement the `begin()` function.

25.17 Iterator Increment, Decrement, & Comparison Operators

- The increment operators must find the next key, either in the current list, or in the next non-empty list.
- The decrement operator must check if the iterator in the list is at the beginning and if so it must proceed to find the previous non-empty list and then find the last entry in that list. This might sound expensive, but remember that the lists should be very short.
- The comparison operators must accommodate the fact that when (at least) one of the iterators is the `end`, the internal list iterator will not have a useful value.

25.18 Insert & Find

- Computes the hash function value and then the index location.
- If the key is already in the list that is at the index location, then no changes are made to the set, but an iterator is created referencing the location of the key, a pair is returned with this iterator and `false`.
- If the key is not in the list at the index location, then the key should be inserted in the list (at the front is fine), and an iterator is created referencing the location of the newly-inserted key a pair is returned with this iterator and `true`.
- **Exercise:** Implement the `insert()` function, ignoring for now the `resize` operation.
- Find is similar to insert, computing the hash function and index, followed by a `std::find` operation.

25.19 Erase

- Two versions are implemented, one based on a key value and one based on an iterator. These are based on finding the appropriate iterator location in the appropriate list, and applying the list erase function.

25.20 Resize

- Must copy the contents of the current vector into a scratch vector, resize the current vector, and then re-insert each key into the resized vector. **Exercise:** Write `resize()`

25.21 Hash Table Iterator Invalidation

- Any insert operation invalidates *all* `ds_hashset` iterators because the insert operation could cause a resize of the table. The erase function only invalidates an iterator that references the current object.

```cpp
#ifndef ds_hashset_h_
#define ds_hashset_h_
// The set class as a hash table instead of a binary search tree.  The
// primary external difference between ds_set and ds_hashset is that
// the iterators do not step through the hashset in any meaningful
// order.  It is just the order imposed by the hash function.
#include <iostream>
#include <list>
#include <string>
#include <vector>

// The ds_hashset is templated over both the type of key and the type
// of the hash function, a function object.
template < class KeyType, class HashFunc >
class ds_hashset {
private:
  typedef typename std::list<KeyType>::iterator hash_list_itr;

public:
  // =================================================================
  // THE ITERATOR CLASS
  // Defined as a nested class and thus is not separately templated.

  class iterator {
  public:
    friend class ds_hashset;   // allows access to private variables
  private:

    // ITERATOR REPRESENTATION
    ds_hashset* m_hs;
    int m_index;               // current index in the hash table
    hash_list_itr m_list_itr;  // current iterator at the current index

  private:
    // private constructors for use by the ds_hashset only
    iterator(ds_hashset * hs) : m_hs(hs), m_index(-1) {}
    iterator(ds_hashset* hs, int index, hash_list_itr loc)
      : m_hs(hs), m_index(index), m_list_itr(loc) {}

  public:
    // Ordinary constructors & assignment operator
    iterator() : m_hs(0), m_index(-1)  {}
    iterator(iterator const& itr)
      : m_hs(itr.m_hs), m_index(itr.m_index), m_list_itr(itr.m_list_itr) {}
    iterator&  operator=(const iterator& old) {
      m_hs = old.m_hs;
      m_index = old.m_index;
      m_list_itr = old.m_list_itr;
      return *this;
    }

    // The dereference operator need only worry about the current
    // list iterator, and does not need to check the current index.
    const KeyType& operator*() const { return *m_list_itr; }

    // The comparison operators must account for the list iterators
    // being unassigned at the end.
    friend bool operator== (const iterator& lft, const iterator& rgt)
    { return lft.m_hs == rgt.m_hs && lft.m_index == rgt.m_index &&
        (lft.m_index == -1 || lft.m_list_itr == rgt.m_list_itr); }
    friend bool operator!= (const iterator& lft, const iterator& rgt)
    { return lft.m_hs != rgt.m_hs || lft.m_index != rgt.m_index ||
        (lft.m_index != -1 && lft.m_list_itr != rgt.m_list_itr); }

    // increment and decrement
    iterator& operator++() {
      this->next();
      return *this;
    }
    iterator operator++(int) {
      iterator temp(*this);
      this->next();
      return temp;
    }
    iterator & operator--() {
      this->prev();
      return *this;
    }
    iterator operator--(int) {
      iterator temp(*this);
      this->prev();
      return temp;
    }

  private:
    // Find the next entry in the table
    void next() {
      ++ m_list_itr;  // next item in the list

      // If we are at the end of this list
      if (m_list_itr == m_hs->m_table[m_index].end()) {
        // Find the next non-empty list in the table
        for (++m_index;
             m_index < int(m_hs->m_table.size()) && m_hs->m_table[m_index].empty();
             ++m_index) {}

        // If one is found, assign the m_list_itr to the start
        if (m_index != int(m_hs->m_table.size()))
          m_list_itr = m_hs->m_table[m_index].begin();

        // Otherwise, we are at the end
        else
          m_index = -1;
      }
    }

    // Find the previous entry in the table
    void prev() {
      // If we aren't at the start of the current list, just decrement
      // the list iterator
      if (m_list_itr != m_hs->m_table[m_index].begin())
        m_list_itr -- ;

      else {
        // Otherwise, back down the table until the previous
        // non-empty list in the table is found
        for (--m_index; m_index >= 0 && m_hs->m_table[m_index].empty(); --m_index) {}

        // Go to the last entry in the list.
        m_list_itr = m_hs->m_table[m_index].begin();
        hash_list_itr p = m_list_itr; ++p;
        for (; p != m_hs->m_table[m_index].end(); ++p, ++m_list_itr) {}
      }
    }
  };
  // end of ITERATOR CLASS
  // =================================================================
```

```
private:
  // =================================================================
  // HASH SET REPRESENTATION
  std::vector< std::list<KeyType> > m_table;  // actual table
  HashFunc m_hash;                            // hash function
  unsigned int m_size;                        // number of keys

public:
  // =================================================================
  // HASH SET IMPLEMENTATION

  // Constructor for the table accepts the size of the table.  Default
  // constructor for the hash function object is implicitly used.
  ds_hashset(unsigned int init_size = 10) : m_table(init_size), m_size(0) {}

  // Copy constructor just uses the member function copy constructors.
  ds_hashset(const ds_hashset<KeyType, HashFunc>& old)
    : m_table(old.m_table), m_size(old.m_size) {}

  ~ds_hashset() {}

  ds_hashset& operator=(const ds_hashset<KeyType,HashFunc>& old) {
    if (&old != this)
      *this = old;
  }

  unsigned int size() const { return m_size; }


  // Insert the key if it is not already there.
  std::pair< iterator, bool > insert(KeyType const& key) {
    const float LOAD_FRACTION_FOR_RESIZE = 1.25;

    if (m_size >= LOAD_FRACTION_FOR_RESIZE * m_table.size())
      this->resize_table(2*m_table.size()+1);

    // implemented in lecture or lab













  }

  // Find the key, using hash function, indexing and list find
  iterator find(const KeyType& key) {
    unsigned int hash_value = m_hash(key);
    unsigned int index = hash_value % m_table.size();
    hash_list_itr p = std::find(m_table[index].begin(),
                                m_table[index].end(), key);
    if (p == m_table[index].end())
      return this->end();
    else
      return iterator(this, index, p);
  }

  // Erase the key
  int erase(const KeyType& key) {
    // Find the key and use the erase iterator function.
    iterator p = find(key);
    if (p == end())
      return 0;
    else {
      erase(p);
      return 1;
    }
  }

  // Erase at the iterator
  void erase(iterator p) {
    m_table[ p.m_index ].erase(p.m_list_itr);
  }

  // Find the first entry in the table and create an associated iterator
  iterator begin() {
    // implemented in lecture or lab







  }

  // Create an end iterator.
  iterator end() {
    iterator p(this);
    p.m_index = -1;
    return p;
  }

  // A public print utility.
  void print(std::ostream & ostr) {
    for (unsigned int i=0; i<m_table.size(); ++i) {
      ostr << i << ": ";
      for (hash_list_itr p = m_table[i].begin(); p != m_table[i].end(); ++p)
        ostr << ' ' << *p;
      ostr << std::endl;
    }
  }

private:
  // resize the table with the same values but a
  void resize_table(unsigned int new_size) {
    // implemented in lecture or lab











  }
};
#endif
```