CSCI-1200 Data Structures — Spring 2016
Lecture 20 – Trees, Part III

Review from Lecture 18 & 19

- Overview of the `ds_set` implementation
- `begin`, `find`, `destroy_tree`, `insert`
- In-order, pre-order, and post-order traversal; Breadth-first and depth-first tree search

```
template <class T>
void breadth_first_print (TreeNode<T> *p) {
  if (p != NULL) {
    std::list<TreeNode<T>*> current_level;
    current_level.push_back(p);
    while (current_level.size() != 0) {
      std::list<TreeNode<T>*> next_level;
      for (std::list<TreeNode<T>*>::iterator itr = current_level.begin();
           itr != current_level.end(); itr++) {
        std::cout << (*itr)->value;
        if ((*itr)->left != NULL) { next_level.push_back((*itr)->left); }
        if ((*itr)->right != NULL) { next_level.push_back((*itr)->right); }
      }
      current_level = next_level;
    }
  }
}
```

- Iterator implementation. Finding the in order successor to a node: add parent pointers *or* add a list/vector/stack of pointers to the iterator.

Today's Lecture

- Last piece of `ds_set`: removing an item, `erase`
- Tree height, longest-shortest paths, breadth-first search
- Erase with parent pointers, increment operation on iterators
- Limitations of our `ds_set` implementation, brief intro to red-black trees

20.1 `ds_set` Warmup/Review Exercises

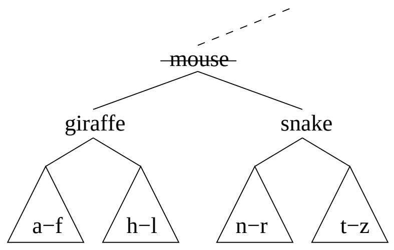
- Draw a diagram of a *possible* memory layout for a `ds_set` containing the numbers 16, 2, 8, 11, and 5. Is there only one valid memory layout for this data as a `ds_set`? Why?

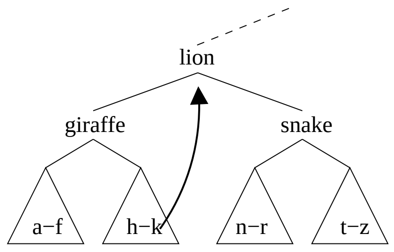
- In what order should a forward iterator visit the data?

20.2 Erase

First we need to find the node to remove. Once it is found,
the actual removal is easy if the node has no children or only one child.
Draw picture of each case! It is harder if there are two children:

- Find the node with the greatest value in the left subtree or the node with the smallest value in the right subtree.
- The value in this node may be safely moved into the current node because of the tree ordering.
- Then we recursively apply erase to remove that node — which is guaranteed to have at most one child.
  - Why?





Exercise: Write a recursive version of erase.

Exercise: How does the order that nodes are deleted affect the tree structure? Starting with a mostly balanced tree, give an erase ordering that yields an unbalanced tree.

20.3 Height and Height Calculation Algorithm

- The *height* of a node in a tree is the length of the longest path down the tree from that node to a leaf node. The height of a leaf is 1. We will think of the height of a null pointer as 0.
- The height of the tree is the height of the root node, and therefore if the tree is empty the height will be 0.

  Exercise: Write a simple recursive algorithm to calculate the height of a tree.
  - 1+max(height(left_subtree), height(right_subtree))
- What is the best/average/worst-case running time of this algorithm? What is the best/average/worst-case memory usage of this algorithm? Give a specific example tree that illustrates each case.

20.4 Shortest Paths to Leaf Node

- Now let's write a function to instead calculate the *shortest* path to a NULL child pointer.

- What is the running time of this algorithm? Can we do better? *Hint: How does a breadth-first vs. depth-first algorithm for this problem compare?*

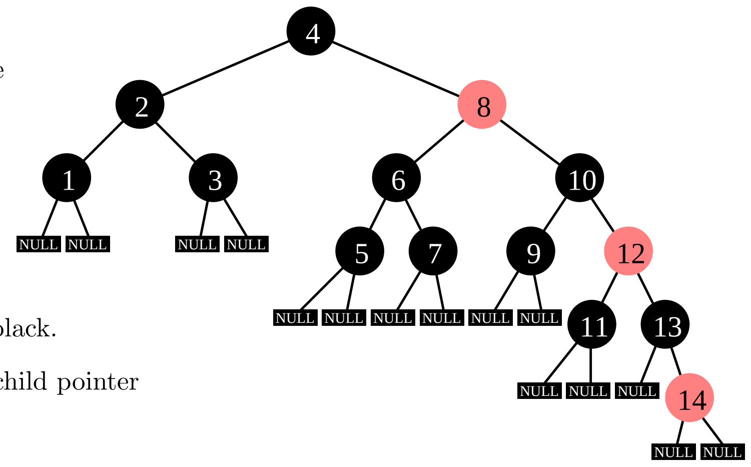
20.5 Erase (now with parent pointers)

- If we choose to use parent pointers, we need to add to the Node representation, and re-implement several `ds_set` member functions.
- **Exercise:** Study the new version of `insert`, with parent pointers.
- **Exercise:** Rewrite `erase`, now with parent pointers.

20.6 Limitations of Our BST Implementation

- The efficiency of the main insert, find and erase algorithms depends on the height of the tree.
- The best-case and average-case heights of a binary search tree storing n nodes are both $O(\log n)$. The worst-case, which often can happen in practice, is $O(n)$.
- Developing more sophisticated algorithms to avoid the worst-case behavior will be covered in Introduction to Algorithms. One elegant extension to binary search tree is described below...

20.7 Red-Black Trees



In addition to the binary search tree properties, the following red-black tree properties are maintained throughout all modifications to the data structure:

1. Each node is either red or black.
2. The NULL child pointers are black.
3. Both children of every red node are black.
   Thus, the parent of a red node must also be black.
4. All paths from a particular node to a NULL child pointer contain the same number of black nodes.

What tree does our `ds_set` implementation produce if we insert the numbers 1-14 *in order*?
The tree above is the result using a red-black tree. Notice how the tree is still quite balanced.
Visit these links for an animation of the sequential insertion and re-balancing:

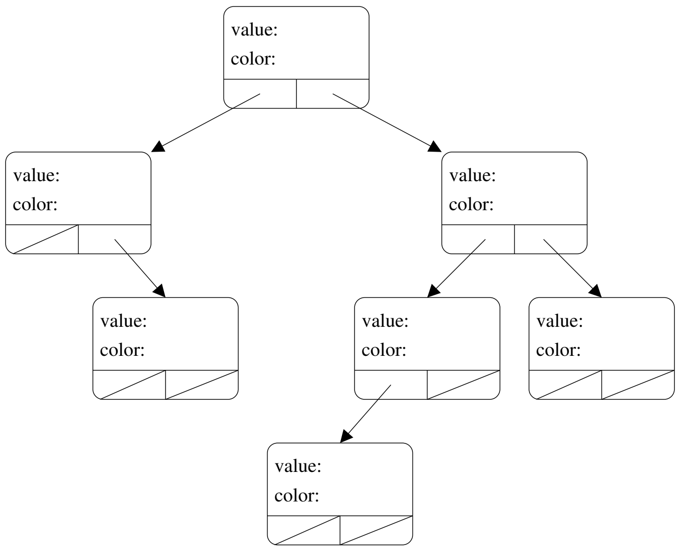
http://www.ibr.cs.tu-bs.de/courses/ss98/audii/applets/BST/RedBlackTree-Example.html
http://users.cs.cf.ac.uk/Paul.Rosin/CM2303/DEMOS/RBTree/redblack.html
http://www.youtube.com/watch?v=vDHFF4wjWYU&noredirect=1

- What is the best/worst case height of a red-black tree with n nodes?

- What is the best/worst case shortest-path from root to leaf node in a red-black tree with n nodes?

20.8 Exercise

Fill in the tree on the right with the integers 1-7 to make a binary search tree. Also, color each node "red" or "black" so that the tree also fulfills the requirements of a Red-Black tree.



Draw two other red-black binary search trees with the values 1-7.

```
// -------------------------------------------------------------------
// TREE NODE CLASS
template <class T> class TreeNode {
public:
  TreeNode() : left(NULL), right(NULL), parent(NULL) {}
  TreeNode(const T& init) : value(init), left(NULL), right(NULL), parent(NULL) {}
  T value;
  TreeNode* left;
  TreeNode* right;
  TreeNode* parent; // to allow implementation of iterator increment & decrement
};
template <class T> class ds_set;
// -------------------------------------------------------------------
// TREE NODE ITERATOR CLASS
template <class T> class tree_iterator {
public:
  tree_iterator() : ptr_(NULL), set_(NULL) {}
  tree_iterator(TreeNode<T>* p, const ds_set<T> * s) : ptr_(p), set_(s) {}
  // operator* gives constant access to the value at the pointer
  const T& operator*() const { return ptr_->value; }
  // comparions operators are straightforward
  bool operator== (const tree_iterator& rgt) { return ptr_ == rgt.ptr_; }
  bool operator!= (const tree_iterator& rgt) { return ptr_ != rgt.ptr_; }
  // increment & decrement operators
  tree_iterator<T> & operator++() {
    if (ptr_->right != NULL) { // find the leftmost child of the right node
      ptr_ = ptr_->right;
      while (ptr_->left != NULL) { ptr_ = ptr_->left; }
    } else { // go upwards along right branches...  stop after the first left
      while (ptr_->parent != NULL && ptr_->parent->right == ptr_) { ptr_ = ptr_->parent; }
      ptr_ = ptr_->parent;
    }
    return *this;
  }
  tree_iterator<T> operator++(int) { tree_iterator<T> temp(*this);  ++(*this); return temp; }
  tree_iterator<T> & operator--() {
    if (ptr_ == NULL) { // so that it works for end()
      assert (set_ != NULL);
      ptr_ = set_->root_;
      while (ptr_->right != NULL) { ptr_ = ptr_->right; }
    } else if (ptr_->left != NULL) { // find the rightmost child of the left node
      ptr_ = ptr_->left;
      while (ptr_->right != NULL) { ptr_ = ptr_->right; }
    } else { // go upwards along left brances... stop after the first right
      while (ptr_->parent != NULL && ptr_->parent->left == ptr_) { ptr_ = ptr_->parent; }
      ptr_ = ptr_->parent;
    }
    return *this;
  }
  tree_iterator<T> operator--(int) { tree_iterator<T> temp(*this); --(*this); return temp; }
private:
  // representation
  TreeNode<T>* ptr_;
  const ds_set<T>* set_;
};
// -------------------------------------------------------------------
// DS_ SET CLASS
template <class T> class ds_set {
public:
  ds_set() : root_(NULL), size_(0) {}
  ds_set(const ds_set<T>& old) : size_(old.size_) { root_ = this->copy_tree(old.root_,NULL); }
  ~ds_set() { this->destroy_tree(root_); root_ = NULL; }
  ds_set& operator=(const ds_set<T>& old) {
    if (&old != this) {
      this->destroy_tree(root_);
      root_ = this->copy_tree(old.root_,NULL);
      size_ = old.size_;
    }
    return *this;
  }
  typedef tree_iterator<T> iterator;
  friend class tree_iterator<T>;
  int size() const { return size_; }
  bool operator==(const ds_set<T>& old) const { return (old.root_ == this->root_); }
  // FIND, INSERT & ERASE
  iterator find(const T& key_value) { return find(key_value, root_); }
  std::pair< iterator, bool > insert(T const& key_value) { return insert(key_value, root_, NULL); }
  int erase(T const& key_value) { return erase(key_value, root_); }
  // ITERATORS
  iterator begin() const {
    if (!root_) return iterator(NULL,this);
    TreeNode<T>* p = root_;
    while (p->left) p = p->left;
    return iterator(p,this);
  }
  iterator end() const { return iterator(NULL,this); }
private:
  // REPRESENTATION
  TreeNode<T>* root_;
  int size_;
  // PRIVATE HELPER FUNCTIONS
  TreeNode<T>*  copy_tree(TreeNode<T>* old_root, TreeNode<T>* the_parent) {
    if (old_root == NULL) return NULL;
    TreeNode<T> *answer = new TreeNode<T>();
    answer->value = old_root->value;
    answer->left = copy_tree(old_root->left,answer);
    answer->right = copy_tree(old_root->right,answer);
    answer->parent = the_parent;
    return answer;
  }
  void destroy_tree(TreeNode<T>* p) {
    if (!p) return;
    destroy_tree(p->right);
    destroy_tree(p->left);
    delete p;
  }
  iterator find(const T& key_value, TreeNode<T>* p) {
    if (!p) return end();
    if      (p->value > key_value) return find(key_value, p->left);
    else if (p->value < key_value) return find(key_value, p->right);
    else                           return iterator(p,this);
  }
  std::pair<iterator,bool> insert(const T& key_value, TreeNode<T>*& p, TreeNode<T>* the_parent) {
    if (!p) {
      p = new TreeNode<T>(key_value);
      p->parent = the_parent;
      this->size_++;
      return std::pair<iterator,bool>(iterator(p,this), true);
    }
    else if (key_value < p->value)
      return insert(key_value, p->left, p);
    else if (key_value > p->value)
      return insert(key_value, p->right, p);
    else
      return std::pair<iterator,bool>(iterator(p,this), false);
  }
  int erase(T const& key_value, TreeNode<T>* &p) {
    /* Implemented in Lecture 20 */
  }
};
```