1. SYSTEM PARAMETERS

$BW = 60$ kg, $BH = 1.60$ m

$m = 0.006(BW) + 0.054 = 0.414$ kg

$l = 0.517$ m, $g = 9.8$ m/s^2

$I_{pivot} = \frac{1}{3}ml^2$(Uniform Rod)

2. LAGRANGIAN FORMULATION (SPHERICAL PENDULUM)

$$E_k = \frac{1}{2}I_{pivot}(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) = \frac{1}{6}ml^2(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta)$$

$$E_p = mgh_{com} = -mg(\frac{l}{2})\cos\theta$$

$$L = E_k - E_p = \frac{1}{6}ml^2(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + \frac{1}{2}mgl\cos\theta$$

3. EULER-LAGRANGE EQUATION

$$\frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}}) - \frac{\partial L}{\partial\theta} = \tau_{total}$$

$$\frac{\partial L}{\partial\dot{\theta}} = \frac{1}{3}ml^2\dot{\theta} \Rightarrow \frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}}) = \frac{1}{3}ml^2\ddot{\theta}$$

$$\frac{\partial L}{\partial\theta} = \frac{1}{3}ml^2\dot{\phi}^2\sin\theta\cos\theta - \frac{1}{2}mgl\sin\theta$$

4. MOTION EQUATION (SAGITTAL PLANE - FLEXION/EXTENSION)

$$\frac{1}{3}ml^2\ddot{\theta} - (\frac{1}{3}ml^2\dot{\phi}^2\sin\theta\cos\theta - \frac{1}{2}mgl\sin\theta) = \tau_{pas}$$

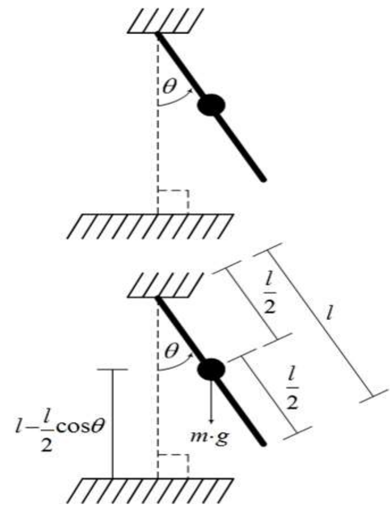
$\tau_{pas} = -B\dot{\theta} - K\theta$(Viscoelastic Model)

Divide by $\frac{1}{3}ml^2$:

$$\ddot{\theta} - \dot{\phi}^2\sin\theta\cos\theta + \frac{3}{2}\frac{g}{l}\sin\theta = \frac{3}{ml^2}(-B\dot{\theta} - K\theta)$$

Final Acceleration Equation ($\ddot{\theta}$):

$$\ddot{\theta} = \dot{\phi}^2\sin\theta\cos\theta - 1.5\frac{g}{l}\sin\theta - \frac{3B}{ml^2}\dot{\theta} - \frac{3K}{ml^2}\theta$$



Numerical Substitution:

$$\ddot{\theta} = \dot{\phi}^2\sin\theta\cos\theta - 28.43\sin\theta - D_{sag}\dot{\theta} - S_{sag}\theta$$

5. MOTION EQUATION (FRONTAL PLANE - RADIAL/ULNAR)

Assuming decoupled active/passive forces with coupled inertia:

$$\ddot{\phi} = -2\dot{\theta}\dot{\phi}\cot\theta - 1.5\frac{g}{l}\sin\phi - \frac{3B}{ml^2}\dot{\phi} - \frac{3K}{ml^2}\phi$$

6. STATE SPACE MATRIX REPRESENTATION

State vector $X = [\theta, \dot{\theta}]^T$:

$$s\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\left(\frac{1.5g}{l}\cos\theta + S_{norm}\right) & -D_{norm}\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ \dot{\phi}^2\sin\theta\cos\theta\end{bmatrix}$$