A1 Assignment - Zino Meyer

1.1)

For this exercise, we work our way backwards from the the SVD results to the matrix A. We use the SVD equation $A = UDV^t$.

To focus on the first column, we will use the first column of the identity matrix i_1.

That way, we can compute $A_1 = UDV^t \cdot i_1$.

Starting from the right:

$$V^T \cdot e_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Next, we multiply by D:

$$D \cdot (V^T \cdot i_1) = \begin{pmatrix} 1.414214 & 0 \\ 0 & 1.414214 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1.414214 \\ 0 \end{pmatrix}$$

Finally, we calculate $U \cdot (D \cdot V^T \cdot i_1)$:

$$U \cdot (D \cdot V^T \cdot i_1) = \begin{pmatrix} -0.7071068 & -0.7071068 \\ -0.7071068 & 0.7071068 \end{pmatrix} \begin{pmatrix} -1.414214 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Successfully recovered the first column: $A_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

1.10)

Given: we need to balance the equation $C_3H_8 + xO_2 \to yCO_2 + zH_2O$

Solution:

Carbon: $1 \times 3 = y \Rightarrow y = 3$

Hydrogen: $1 \times 8 = 2z \Rightarrow z = 4$

Oxygen: $2x = 2y + z$

Substituting C and H into O:

⇒ $2x = 2 \times 3 + 4$

⇒ $2x = 10$

⇒ $x = 5$

Thus, the balanced equation is: $C_3H_8 + 5O_2 \to 3CO_2 + 4H_2O$

2.4 a)

To proof, I will express one row as a linear combination of the others explicitly. For the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

the third row can be expressed as a combination of the first two:

$$a \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + b \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

This equation has a solution for $a = -1$ and $b = 2$, showing that row 3 (vector 3) is a linear combination of row 1 & 2 (vectors 1 & 2), thus, the rows are linearly dependant

b)

Since row 3 can be expressed by row one and two, we reduce the system of equations by row three and end up with:

1. $x_1 + 2x_2 + 3x_3 = 0$
2. $4x_1 + 5x_2 + 6x_3 = 0$

Treating $x3$ as arbitraty value, we can rewrite the system like this:

1. $x_1 + 2x_2 = -3x_3$
2. $4x_1 + 5x_2 = -6x_3$

where x_3 is our arbitrary value, and x_1 and x_2 are variables.

c)

Solve the two equations for x_1 and x_2 and express them in terms of x_3 :

1. $x_1 + 2x_2 = -3x_3$
2. $4x_1 + 5x_2 = -6x_3$

Use elimination to solve for x_2:

$4 \times 1. - 2.$

⇒ $3x_2 = -6x_3$

⇒ $x_2 = -2x_3$

Substitude x_2 into $1.$:

$x_1 + 2(-2x_3) = -3x_3$

⇒ $x_1 - 4x_3 = -3x_3$

⇒ $x_1 = x_3$

So the solutions are:

$x_1 = x_3$

$x_2 = -2x_3$

$x_3 = x_3$

2.5)

Given: we need to balance the equation $2C_2H_6 + xO_2 \to yCO_2 + zH_2O$

Solution:

Carbon: $2 \times 2 = y \Rightarrow 4 = y$

Hydrogen: $2 \times 6 = 2z \Rightarrow 6 = z$

Oxygen: $2x = 2y + z$

Substituting C and H into the Oxygen equation:

$\Rightarrow 2x = 2(2 \times 2) + \frac{1}{2}(2 \times 6)$

$\Rightarrow 2x = 8 + 6$

$\Rightarrow x = 7$

The balanced equation: $2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$

2.7)

Given matrix & vector:

$$A = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}, u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

a)

$$\mathbf{Au} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \times 1 + 4 \times 1 \\ -2 \times 1 + (-7) \times 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \end{bmatrix}$$

Since $\mathbf{Au} \neq 4 \times \mathbf{u}$, where $4 \times \mathbf{u} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, it shows that $\mathbf{u}$ is not an eigenvector of $\mathbf{A}$. The vector is not only scaled, but also changes its direction.