

HW5

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December 18, 2025

Problem 1

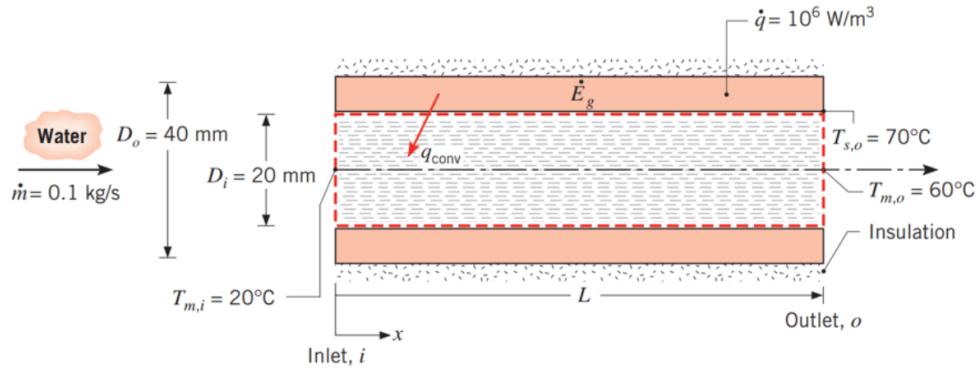


Figure 1: Channel forced convection problem

Solutions:

(a)

Firstly, we calculate the total heat absorbed by the water flow through the pipeline (\dot{Q}_{water})

$$\dot{Q}_{water} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

$$\dot{Q}_{water} = 0.1 \times 4179 \times 40 = 16716 \text{ W}$$

Then calculate the heat generated by the wall current of the pipe

$$V_{wall} = \frac{\pi}{4}(D_o^2 - D_i^2)L$$

$$\begin{aligned}\dot{Q}_{gen} &= \dot{q} \times V_{wall} \\ &= \dot{q} \frac{\pi}{4}(D_o^2 - D_i^2)L \\ &= 10^6 \times 0.0003\pi \times L \\ &= 300\pi L (\text{W})\end{aligned}$$

According to the conservation of energy, all the heat generated inside the pipe wall is transferred to the water (ignoring axial heat conduction), so we can get:

$$\dot{Q}_{gen} = \dot{Q}_{water}$$

$$300\pi L = 16716$$

$$L \approx 17.736 \text{ m}$$

To achieve the desired outlet temperature, the tube should be 17.74m.

(b)

To calculate the local convection heat transfer coefficient at the outlet, we need to calculate the heat flux q''_s of the tube.

Internal surface area A_s is:

$$\begin{aligned}A_s &= \pi D_i L \\ q''_s &= \frac{\dot{Q}_{water}}{A_s} = \frac{\dot{Q}_{water}}{\pi D_i L} \\ q''_s &= \frac{1200}{0.08} = 15000 \text{ W/m}^2\end{aligned}$$

Using Newton's law of cooling to solve h

$$q_s'' = h(T_{s,o} - T_{m,o})$$
$$h = \frac{15000}{10} = 1500 \text{ W/(m}^2 \cdot \text{K)}$$

Problem 2

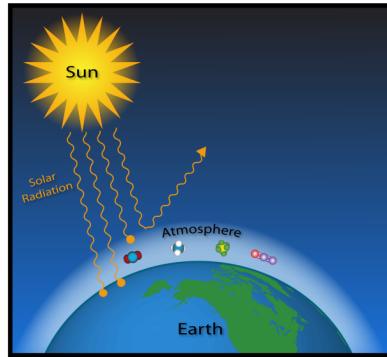


Figure 2: Sun-Earth-Atmosphere radiation problem

Solutions:

Before we calculate the Earth's surface temperature, use Python to plot a logic graph of this problem.

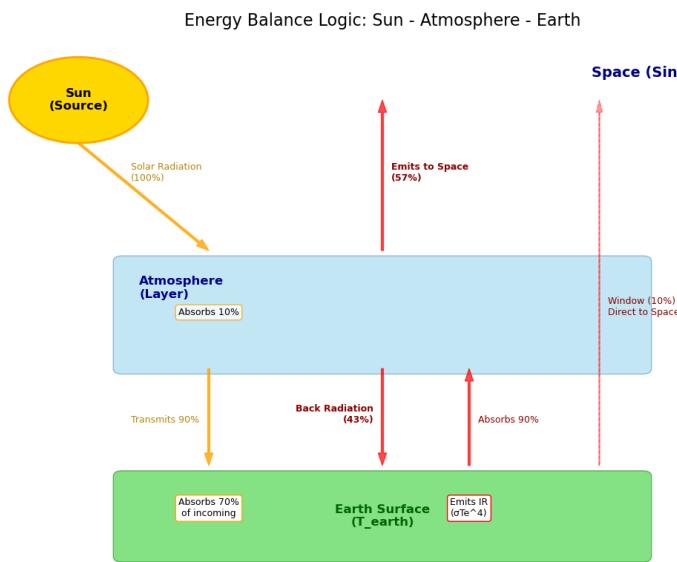


Figure 3: Sun-Earth-Atmosphere radiation logic graph

The basic information:

$$\sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4)$$

$$T_{\text{sun}} = 5800 \text{ K}$$

$$D_{\text{sun}} = 1.39 \times 10^9 \text{ m} \Rightarrow R_{\text{sun}} = 6.95 \times 10^8 \text{ m}$$

$$d = 1.5 \times 10^{11} \text{ m}$$

$$D_{\text{earth}} = 1.27 \times 10^7 \text{ m} \Rightarrow R_e = 6.35 \times 10^6 \text{ m}$$

$$T_{\text{atm}} \approx T_{\text{earth}}$$

The Surface energy balance equation:

$$E_{in,\text{solar}} + E_{in,\text{atm}} = E_{out,\text{earth}}$$

1. First we calculate the Solar Constant G_{sun}

$$P_{\text{sun}} = \sigma T_{\text{sun}}^4 \cdot (4\pi R_{\text{sun}}^2)$$

$$G_{\text{sun}} = \frac{P_{\text{sun}}}{4\pi d^2} = \sigma T_{\text{sun}}^4 \left(\frac{R_{\text{sun}}}{d} \right)^2$$

$$G_{sc} \approx 1377.5 \text{ W/m}^2$$

2. Then we analyze the solar radiation absorbed by the Earth's surface $q''_{\text{solar},\text{absorbed}}$

The actual proportion of solar energy absorbed by the surface is:

$$\eta_{\text{solar}} = 0.9 \times 0.7 = 0.63$$

So the $q''_{\text{solar},\text{absorbed}}$ is:

$$q''_{\text{solar},\text{absorbed}} = \frac{G_{sc} \times \pi R_e^2}{4\pi R_e^2} \times \eta_{\text{solar}} = \frac{G_{sc}}{4} \times 0.63$$

$$q''_{\text{solar},\text{absorbed}} = \frac{1377.5}{4} \times 0.63 \approx 344.375 \times 0.63 \approx 216.96 \text{ W/m}^2$$

3. Analyze the Long wave radiation in the atmosphere $q''_{\text{atm} \rightarrow \text{earth}}$

10% of surface launches are directly launched into space through atmospheric windows. This means the transmittance of the atmosphere to longwave radiation $\tau = 0.1$, based on Kirchhoff's law of radiation, $\alpha = 0.9$

So we can get the emissivity of the atmosphere is $\epsilon_{atm} = 0.9$

So the total radiation emission capability of the atmosphere

$$E_{atm} = \epsilon_{atm}\sigma T_e^4 = 0.9\sigma T_e^4$$

$$q''_{atm \rightarrow earth} = 43\% \times E_{atm} = 0.43 \times 0.9\sigma T_e^4 = 0.387\sigma T_e^4$$

4. Establish the surface radiation energy balance equation:

$$q''_{solar, absorbed} + q''_{atm \rightarrow earth} = E_{earth}$$

$$216.96 + 0.387\sigma T_e^4 = \sigma T_e^4$$

So we can get

$$T_e \approx 281.0 \text{ K}$$

Therefore, the surface temperature of the Earth is about $8^\circ C$

Problem 3

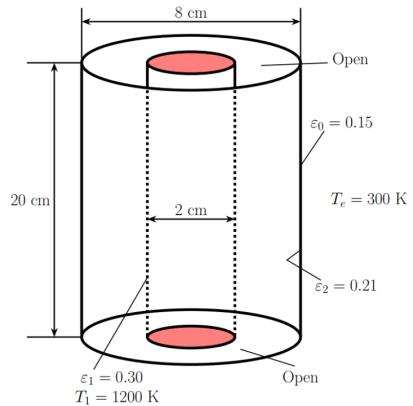
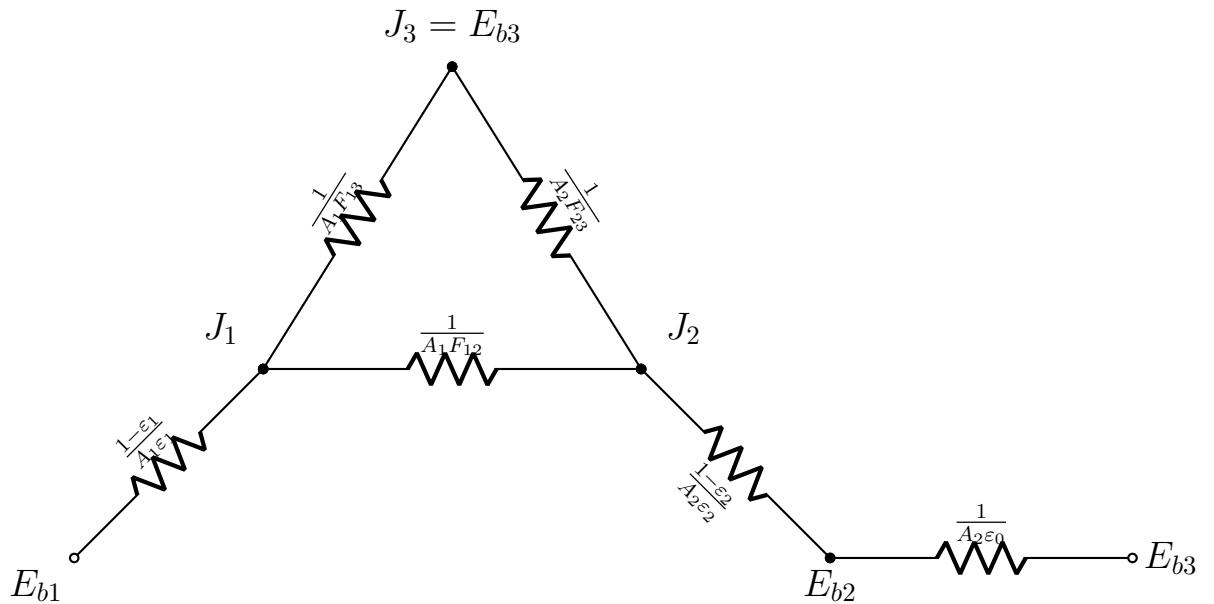


Figure 4: Cylindrical Shell Network radiation problem

Solutions:

(a)



Node Definitions:

- E_{b1} : Blackbody emissive power of the inner rod (σT_1^4).
- J_1 : Effective radiation (Radiosity) of the inner rod.
- J_2 : Effective radiation (Radiosity) of the inner surface of the cylinder.
- J_3 : Effective radiation (Radiosity) of the outside surface of the cylinder.
- E_{b2} : Blackbody emissive power of the cylinder wall (σT_2^4).
- E_{b3} : Emissive power of the environment/opening (σT_e^4).

Resistance Definitions:

- $R_1 = \frac{1-\epsilon_1}{\epsilon_1 A_1}$ (Surface thermal resistance of the inner rod)
- $R_2 = \frac{1-\epsilon_2}{\epsilon_2 A_2}$ (Surface thermal resistance of the cylinder inner surface)
- $R_{12} = \frac{1}{A_1 F_{12}}$ (Space thermal resistance from inner rod to cylinder)
- $R_{13} = \frac{1}{A_1 F_{13}}$ (Space thermal resistance from inner rod to opening)
- $R_{2e} = \frac{1}{A_2 F_{23}}$ (Space thermal resistance from cylinder to opening)
- R_{out} : Direct radiation from cylinder outer surface to environment, can be simplified.

(b)

Here are the four equations to calculate the temperature T_2

$$\frac{E_{b1} - J_1}{\frac{1-\epsilon_1}{A_1 \epsilon_1}} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}} \quad (1)$$

$$\frac{E_{b2} - J_2}{\frac{1-\epsilon_2}{A_2 \epsilon_2}} = \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}} + \frac{J_2 - J_1}{\frac{1}{A_1 F_{12}}} \quad (2)$$

$$J_3 = E_{b3} = \sigma T_3^4 \quad (3)$$

$$\frac{J_2 - E_{b2}}{\frac{1-\epsilon_2}{A_2 \epsilon_2}} = \epsilon_0 A_2 (E_{b2} - E_{b3}) \quad (4)$$

(c)

We already know $F_{21} = 0.225$ and $F_{22} = 0.617$

Because:

$$F_{21} + F_{22} + F_{23} = 1$$

So:

$$F_{23} = 0.158$$

And:

$$F_{21} \times A_2 = F_{21} \times A_1$$

So:

$$F_{12} = 0.9$$

Because:

$$F_{11} + F_{12} + F_{13} = 1$$

So:

$$F_{13} = 0.1$$

So all the view factors we need are:

- $F_{12} = 0.9$
- $F_{13} = 0.1$
- $F_{23} = 0.158$

(d)

$$0.3E_{b1} = J_1 - 0.63J_2 - 0.07E_{b3}$$

$$0.84E_{b2} = 2.05028J_2 - 0.711J_1 - 0.49928E_{b3}$$

$$0.21J_2 + 0.1185E_{b3} = 0.3285E_{b2}$$

$$E_{b2} = 0.64J_2 + 165.67$$

$$E_{b3} = \sigma T_e^4 = 459.27 \text{W}/(\text{m}^2 \cdot \text{K})$$

$$E_{b1} = \sigma T_1^4 = 117573.12 \text{W}/(\text{m}^2 \cdot \text{K})$$

We finally get:

$$2.254J_2 = 35626.595 + 0.75584J_2 + 195.656$$

So:

$$J_{2i} \approx 23903 \text{W/m}^2$$

$$E_{b2} \approx 15440 \text{W/m}^2$$

$$E_{b2} = \sigma T_2^4$$

$$T_2 \approx 722.6 \text{ K}$$