1 The Derivative Explanation of Formulas 2.9 to 2.13

- loss function

$$L(Y, f(X)) = (Y - f(X))^2 \tag{1}$$

calculate the expectation of loss function

$$\begin{aligned} EPE(f) &= \int_{x,y} (y - f(x))^2 Pr(X = x, Y = y) dx dy \\ &= \int_x (\int_y (y - f(x))^2 Pr(Y = y|X = x)) Pr(X = x) dx \\ &= E_x(E_{Y|X}(Y - f(X))^2) \end{aligned} \tag{2}$$

$$\hat{f} = \arg\min_f E_x(E_{Y|X}(Y - f(X))^2) \tag{3}$$

Notice that, in equation (2), the inner integral is nonnegtive. we want to minimize the sum of the inner term, that is equivalent to minimize every inner term.

$$\begin{aligned} \hat{f} &= \underset{f, const X = x}{\arg\min} \; E_{Y|X}([Y - f(X)]^2) \\ &= \underset{c, const X = x}{\arg\min} \; E_{Y|X}([Y - c]^2) \end{aligned} \tag{4}$$

$$E_{Y|X}([Y - c]^2) = \int_y (y - c)^2 Pr(Y = y|X = x) dy \tag{5}$$

$$\begin{aligned} \frac{\partial E_{Y|X}([Y - c]^2)}{\partial c} &= \int_y 2(c - y) Pr(Y = y|X = x) dy \\ &= 2(\int_y c Pr(Y = y|X = x) dy - \int_y y Pr(Y = y|X = x) dy) \quad = 2(c - E[Y|X]) \end{aligned} \tag{6}$$

we set equation (6) to 0 to get the minimimum $c = E[Y|X]$