1 properties of the red-black trees

A red-black tree is a binary tree that satisfies the following red-black properties:

- either node is black or red
- the root is black
- all the leaves are black
- if the node is red, it's childs are black
- For each node, all simple path from the node to decendent leaves contain the same number of black node

1.1 Lemma

A red-black tree has n internal node at least have height $2lg(n+1)$

proof by induction. we need first to show that a subtree rooted at x has at least $2^{bh(x)}-1$ internal nodes.

first, base case: if x is at height 0, $bh(x) = 0$, $2^0 - 1 == 0$ inductive step: supppose x is a internal node and have two child. The child of x has black-height either bh(x) or bh(x) - 1 depending on the color of the child. In the meantime, the internal nodes that the x has is more that its child. So, the child of x have at least $2^{bh(x)-1}-1$ internal nodes. then, the internal nodes of x equal to $2^{bh(x)-1}-1+2^{bh(x)-1}-1+1 = 2^{bh*(x)-1}$. Leamma prove. So the root of the tree will have at least $2^{bh(root)}-1 = n$ nodes.