

1.B:

Output:

Initial factors:

```
{0: array([0.05, 0.95]), 1: array([0.03571, 0.96429]), 2: array([0.3, 0.7]), 3: array([[0.6, 0.1],
      [0.4, 0.9]]), 4: array([[0.8, 0.4],
      [0.5, 0. ]]),
      [[0.2, 0.6],
      [0.5, 1. ]]), 5: array([[0.99, 0.75],
      [0.9 , 0.5 ]]),
      [[0.65, 0.2 ],
      [0.4 , 0. ]]),
      [[0.01, 0.25],
      [0.1 , 0.5 ]],
      [[0.35, 0.8 ],
      [0.6 , 1. ]])}]
{0: ['S'], 1: ['M'], 2: ['NA'], 3: ['B', 'S'], 4: ['NH', 'M', 'NA'], 5: ['FH', 'S', 'NH', 'M']}
```

After restrict: No change

After each step of summing out HiddenVariables(Seperate by #####):

```
{0: array([0.05, 0.95]), 1: array([0.03571, 0.96429]), 2: array([0.3, 0.7]), 4: array([[0.8, 0.4],
      [0.5, 0. ]]),
      [[0.2, 0.6],
      [0.5, 1. ]]), 5: array([[0.99, 0.75],
      [0.9 , 0.5 ]]),
      [[0.65, 0.2 ],
      [0.4 , 0. ]]),
      [[0.01, 0.25],
      [0.1 , 0.5 ]],
      [[0.35, 0.8 ],
      [0.6 , 1. ]])}, 6: array([1., 1.])}
{0: ['S'], 1: ['M'], 2: ['NA'], 4: ['NH', 'M', 'NA'], 5: ['FH', 'S', 'NH', 'M'], 6: ['S']}
#####
****multiplied factors****
[[[0.028568 0.014284]
  [0.482145 0.      ]]]

[[0.007142 0.021426]
  [0.482145 0.96429 ]]]
*****
```

****multiplied factors****

```
[[[[[2.8282320e-02 1.4141160e-02]
      [3.6160875e-01 0.0000000e+00]]
```

```
      [[6.4278000e-03 1.9283400e-02]
        [2.4107250e-01 4.8214500e-01]]]]
```

```
[[[1.8569200e-02 9.2846000e-03]
     [9.6429000e-02 0.0000000e+00]]
```

```
[[2.8568000e-03 8.5704000e-03]
 [0.0000000e+00 0.0000000e+00]]]]
```

```
[[[2.8568000e-04 1.4284000e-04]
     [1.2053625e-01 0.0000000e+00]]
```

```
[[7.1420000e-04 2.1426000e-03]
 [2.4107250e-01 4.8214500e-01]]]]
```

```
[[[9.9988000e-03 4.9994000e-03]
     [3.8571600e-01 0.0000000e+00]]
```

```
[[4.2852000e-03 1.2855600e-02]
 [4.8214500e-01 9.6429000e-01]]]]]
```

```
{0: array([0.05, 0.95]), 2: array([0.3, 0.7]), 6: array([1., 1.]), 7: array([[3.8989107e-01, 1.4141160e-02],
      [2.4750030e-01, 5.0142840e-01]]],
```

```
      [[1.1499820e-01, 9.2846000e-03],
        [2.8568000e-03, 8.5704000e-03]]],
```

```
      [[1.2082193e-01, 1.4284000e-04],
        [2.4178670e-01, 4.8428760e-01]]],
```

```
      [[3.9571480e-01, 4.9994000e-03],
        [4.8643020e-01, 9.7714560e-01]]]]])}
```

```
{0: ['S'], 2: ['NA'], 6: ['S'], 7: ['FH', 'S', 'NH', 'NA']}
```

```
#####
```

****multiplied factors****

```
[[[[[1.16967321e-01 9.89881200e-03]
      [7.42500900e-02 3.50999880e-01]]
```

```
      [[3.44994600e-02 6.49922000e-03]
        [8.57040000e-04 5.99928000e-03]]]]
```

```
[[[3.62465790e-02 9.99880000e-05]
     [7.25360100e-02 3.39001320e-01]]
```

```
[[1.18714440e-01 3.49958000e-03]
 [1.45929060e-01 6.84001920e-01]]]]
```

```
{0: array([0.05, 0.95]), 6: array([1., 1.]), 8: array([[0.12686613, 0.42524997],
      [0.04099868, 0.00685632]]],
```

```
      [[0.03634657, 0.41153733],
        [0.12221402, 0.82993098]]]]})
```

```
{0: ['S'], 6: ['S'], 8: ['FH', 'S', 'NH']}
```

```
#####
```

```

{0: array([0.05, 0.95]), 6: array([1., 1.]), 9: array([[0.5521161, 0.047855 ],
[0.4478839, 0.952145 ]])}
{0: ['S'], 6: ['S'], 9: ['FH', 'S']}
#####
***multiplied factors***
[0.05 0.95]
*****
***multiplied factors***
[[0.02760581 0.04546225]
 [0.02239419 0.90453775]]
*****
{10: array([0.07306806, 0.92693194])}
{10: ['FH']}
#####
--

```

After Multiply out to one factor:

```

{10: array([0.07306806, 0.92693194])}
--

```

After Normalize: [0.07306806 0.92693194]

Result for 1B : 0.07306806

1.C:

Initial factors:

```
{0: array([0.05, 0.95]), 1: array([0.03571, 0.96429]), 2: array([0.3, 0.7]), 3: array([[0.6, 0.1],
[0.4, 0.9]]), 4: array([[0.8, 0.4],
[0.5, 0. ]]),

[[0.2, 0.6],
[0.5, 1. ]]), 5: array([[0.99, 0.75],
[0.9 , 0.5 ]]),

[[0.65, 0.2 ],
[0.4 , 0. ]]),

[[[0.01, 0.25],
[0.1 , 0.5 ]],

[[0.35, 0.8 ],
[0.6 , 1. ]]]]}
{0: ['S'], 1: ['M'], 2: ['NA'], 3: ['B', 'S'], 4: ['NH', 'M', 'NA'], 5: ['FH', 'S', 'NH', 'M']}
```

After Restrict:

```
{0: array([0.05, 0.95]), 1: array([0.03571]), 2: array([0.3, 0.7]), 3: array([[0.6, 0.1],
[0.4, 0.9]]), 4: array([[0.8, 0.4],
[0.2, 0.6]]), 5: array([[0.99, 0.9 ],
[0.65, 0.4 ]])}
{0: ['S'], 1: [], 2: ['NA'], 3: ['B', 'S'], 4: ['NH', 'NA'], 5: ['S', 'NH']}
```

After each step of summing out HiddenVariables(Seperate by #####):

****multiplied factors****

[[0.24 0.28]

[0.06 0.42]]

{0: array([0.05, 0.95]), 1: array([0.03571]), 3: array([[0.6, 0.1],
[0.4, 0.9]]), 5: array([[0.99, 0.9],
[0.65, 0.4]]), 6: array([0.52, 0.48])}

{0: ['S'], 1: [], 3: ['B', 'S'], 5: ['S', 'NH'], 6: ['NH']}

#####

****multiplied factors****

[[0.5148 0.432]

[0.338 0.192]]

{0: array([0.05, 0.95]), 1: array([0.03571]), 3: array([[0.6, 0.1],
[0.4, 0.9]]), 7: array([0.9468, 0.53])}

{0: ['S'], 1: [], 3: ['B', 'S'], 7: ['S']}

#####

{0: array([0.05, 0.95]), 1: array([0.03571]), 7: array([0.9468, 0.53]), 8: array([1., 1.])}

{0: ['S'], 1: [], 7: ['S'], 8: ['S']}

#####

50 00000000 0 00000000

After Multiply out to one factor:

```
****multiplied factors****  
[0.0017855 0.0339245]  
*****  
****multiplied factors****  
[0.00169051 0.01797999]  
*****  
****multiplied factors****  
[0.00169051 0.01797999]  
*****
```

After Normalize:

[0.08594147 0.91405853]

Answer: 0.08594147

1.D:

Initial factors:

```
{0: array([0.05, 0.95]), 1: array([0.03571, 0.96429]), 2: array([0.3, 0.7]), 3: array([[0.6, 0.1],
      [0.4, 0.9]]), 4: array([[0.8, 0.4],
      [0.5, 0. ]],
      [[0.2, 0.6],
      [0.5, 1. ]]), 5: array([[0.99, 0.75],
      [0.9 , 0.5 ]],
      [[0.65, 0.2 ],
      [0.4 , 0. ]]),
      [[0.01, 0.25],
      [0.1 , 0.5 ]],
      [[0.35, 0.8 ],
      [0.6 , 1. ]])}]
{0: ['S'], 1: ['M'], 2: ['NA'], 3: ['B', 'S'], 4: ['NH', 'M', 'NA'], 5: ['FH', 'S', 'NH', 'M']}
```

After restrict:

```
{0: array([0.05, 0.95]), 1: array([0.03571]), 2: array([0.3, 0.7]), 3: array([0.6, 0.1]), 4: array([[0.8, 0.4],
      [0.2, 0.6]]), 5: array([[0.99, 0.9 ],
      [0.65, 0.4 ]])}
{0: ['S'], 1: [], 2: ['NA'], 3: ['S'], 4: ['NH', 'NA'], 5: ['S', 'NH']}
```

After each step of summing out HiddenVariables(Seperate by #####):

****multiplied factors****

```
[[0.24 0.28]
 [0.06 0.42]]
```

```
{0: array([0.05, 0.95]), 1: array([0.03571]), 3: array([0.6, 0.1]), 5: array([[0.99, 0.9 ],
      [0.65, 0.4 ]]), 6: array([0.52, 0.48])}
```

```
{0: ['S'], 1: [], 3: ['S'], 5: ['S', 'NH'], 6: ['NH']}
```

#####

****multiplied factors****

```
[[0.5148 0.432 ]
 [0.338  0.192 ]]
```

```
{0: array([0.05, 0.95]), 1: array([0.03571]), 3: array([0.6, 0.1]), 7: array([0.9468, 0.53 ])}
```

```
{0: ['S'], 1: [], 3: ['S'], 7: ['S']}
```

#####

After Multiply out to one factor:

```
|
****multiplied factors****
[0.0017855 0.0339245]
*****
****multiplied factors****
[0.0010713 0.00339245]
*****
****multiplied factors****
[0.00101431 0.001798 ]
*****
```

After Normalize:

```
[0.36066739 0.63933261]
```

Answer: 0.36066739

1.E:

Initial factors:

```
{0: array([0.05, 0.95]), 1: array([0.03571, 0.96429]), 2: array([0.3, 0.7]), 3: array([[0.6, 0.1],
[0.4, 0.9]]), 4: array([[0.8, 0.4],
[0.5, 0. ]]),
[[0.2, 0.6],
[0.5, 1. ]]), 5: array([[0.99, 0.75],
[0.9 , 0.5 ]]),
[[0.65, 0.2 ],
[0.4 , 0. ]]),
[[[0.01, 0.25],
[0.1 , 0.5 ]],
[[0.35, 0.8 ],
[0.6 , 1. ]]]}]
{0: ['S'], 1: ['M'], 2: ['NA'], 3: ['B', 'S'], 4: ['NH', 'M', 'NA'], 5: ['FH', 'S', 'NH', 'M']}
```


After restrict:

```
{0: array([0.05, 0.95]), 1: array([0.03571]), 2: array([0.3]), 3: array([0.6, 0.1]), 4: array([0.8, 0.2]), 5: array([[0.99, 0.9 ],
[0.65, 0.4 ]])}
{0: ['S'], 1: [], 2: [], 3: ['S'], 4: ['NH'], 5: ['S', 'NH']}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
****multiplied factors****
[[0.792 0.18 ]
 [0.52  0.08 ]]
*****
{0: array([0.05, 0.95]), 1: array([0.03571]), 2: array([0.3]), 3: array([0.6, 0.1]), 6: array([0.972, 0.6  ])}
{0: ['S'], 1: [], 2: [], 3: ['S'], 6: ['S']}
#####
```

After Multiply out to one factor:

```
****multiplied factors****
[0.0017855 0.0339245]
*****
****multiplied factors****
[0.00053565 0.01017735]
*****
****multiplied factors****
[0.00032139 0.00101773]
*****
****multiplied factors****
[0.00031239 0.00061064]
*****
```

After Normalize:

```
[0.33844011 0.66155989]
```

Answer: 0.33844011

Problem 2

2.a:

$P(G|W)$

Initial factors:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]],
[[0.05, 0.9 ],
[0.1 , 0.95]]]), 3: array([[0.8, 0.4],
[0.2, 0.6]]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After restrict:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]],
[[0.05, 0.9 ],
[0.1 , 0.95]]]), 3: array([0.8, 0.4]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['A'], 4: ['G', 'A']}
```

After each step of summing out HiddenVariables(Seperate by #####):

****multiplied factors****

```
[[[0.095 0.09 ]
[0.09  0.045]]
```

```
[[[0.005 0.81 ]
[0.01  0.855]]]
```

```
{1: array([0.05, 0.95]), 3: array([0.8, 0.4]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]]), 5: array([[0.185, 0.135],
[0.815, 0.865]])}
```

```
{1: ['E'], 3: ['A'], 4: ['G', 'A'], 5: ['A', 'E']}
```

#####

```

****multiplied factors****
[[0.00925 0.12825]
 [0.04075 0.82175]]
*****
{3: array([0.8, 0.4]), 4: array([[0.4 , 0.05],
                                [0.6 , 0.95]]), 6: array([0.1375, 0.8625])}
{3: ['A'], 4: ['G', 'A'], 6: ['A']}
#####

```

```

****multiplied factors****
[[0.32 0.02]
 [0.48 0.38]]
*****
****multiplied factors****
[[0.044 0.01725]
 [0.066 0.32775]]
*****
{7: array([0.06125, 0.39375])}
{7: ['G']}
#####

```

After Multiply out to one factor:

```
[0.06125 0.39375]
```

After Normalize:

```
[0.13461538 0.86538462]
```

Answer: 0.13461538

1. $P(G|\neg W)$

Initial factors:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]],
[[0.05, 0.9 ],
[0.1 , 0.95]]]), 3: array([[0.8, 0.4],
[0.2, 0.6]]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After Restrict:

```
def restrict(factor, variables, values):
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]],
[[0.05, 0.9 ],
[0.1 , 0.95]]]), 3: array([0.2, 0.6]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['A'], 4: ['G', 'A']}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
#####
****multiplied factors****
[[[0.095 0.09 ]
[0.09 0.045]]

[[0.005 0.81 ]
[0.01 0.855]]]
#####
{1: array([0.05, 0.95]), 3: array([0.2, 0.6]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]]), 5: array([[0.185, 0.135],
[0.815, 0.865]])}
{1: ['E'], 3: ['A'], 4: ['G', 'A'], 5: ['A', 'E']}
#####
```

```

****multiplied factors****
[[0.00925 0.12825]
 [0.04075 0.82175]]
*****
{3: array([0.2, 0.6]), 4: array([[0.4 , 0.05],
      [0.6 , 0.95]]), 6: array([0.1375, 0.8625])}
{3: ['A'], 4: ['G', 'A'], 6: ['A']}
#####

```

```

****multiplied factors****
[[0.08 0.03]
 [0.12 0.57]]
*****
****multiplied factors****
[[0.011    0.025875]
 [0.0165   0.491625]]
*****
{7: array([0.036875, 0.508125])}
{7: ['G']}
#####

```

After Multiply out to one factor:

```

-----\
[0.036875 0.508125]

```

After Normalize:

```

[0.06766055 0.93233945]

```

Answer: 0.06766055

So they are not equal.

2.b

$$P(B|W \wedge G \wedge A)$$

Initial factors:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
               [0.9 , 0.05]],
               [[0.05, 0.9 ],
                [0.1 , 0.95]]), 3: array([[0.8, 0.4],
               [0.2, 0.6]]), 4: array([[0.4 , 0.05],
               [0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After Restrict:

```
*****multiplied factors*****
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
               [0.9 , 0.05]]), 3: array([0.8]), 4: array([0.4])}
{0: ['B'], 1: ['E'], 2: ['E', 'B'], 3: [], 4: []}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
*****multiplied factors*****
[[0.0475 0.005 ]
 [0.855  0.0475]]
*****
{0: array([0.1, 0.9]), 3: array([0.8]), 4: array([0.4]), 5: array([0.9025, 0.0525])}
{0: ['B'], 3: [], 4: [], 5: ['B']}
#####
```

After Multiply out to one factor:

```

****multiplied factors****
[0.08 0.72]
*****
****multiplied factors****
[0.032 0.288]
*****
****multiplied factors****
[0.02888 0.01512]
*****

```

After Normalize:

[0.65636364 0.34363636]

Answer: 0.65636364

$P(B|A)$

Initial factors:

```

{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
               [0.9 , 0.05]]),
  [[0.05, 0.9 ],
   [0.1 , 0.95]]), 3: array([[0.8, 0.4],
   [0.2, 0.6]]), 4: array([[0.4 , 0.05],
   [0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After Restrict:

```
return factor[slices]
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
               [0.9 , 0.05]]), 3: array([0.8, 0.2]), 4: array([0.4, 0.6])}
{0: ['B'], 1: ['E'], 2: ['E', 'B'], 3: ['W'], 4: ['G']}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
****multiplied factors****
[[0.0475 0.005 ]
 [0.855  0.0475]]
*****
{0: array([0.1, 0.9]), 3: array([0.8, 0.2]), 4: array([0.4, 0.6]), 5: array([0.9025, 0.0525])}
{0: ['B'], 3: ['W'], 4: ['G'], 5: ['B']}
#####
{0: array([0.1, 0.9]), 3: array([0.8, 0.2]), 5: array([0.9025, 0.0525]), 6: 1.0}
{0: ['B'], 3: ['W'], 5: ['B'], 6: []}
#####
{0: array([0.1, 0.9]), 5: array([0.9025, 0.0525]), 6: 1.0, 7: 1.0}
{0: ['B'], 5: ['B'], 6: [], 7: []}
#####
```

After Multiply out to one factor:

```
return factor[slices]
****multiplied factors****
[0.09025 0.04725]
*****
****multiplied factors****
[0.09025 0.04725]
*****
****multiplied factors****
[0.09025 0.04725]
*****
```

After Normalize:

[0.65636364 0.34363636]

Answer: 0.65636364

So they are equal.

2.c

$$P(B|A \wedge G \wedge W)$$

Initial factors:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]]),
[0.05, 0.9 ],
[0.1 , 0.95]]), 3: array([[0.8, 0.4],
[0.2, 0.6]]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After restrict:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]]), 3: array([0.8]), 4: array([0.4])}
{0: ['B'], 1: ['E'], 2: ['E', 'B'], 3: [], 4: []}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
****multiplied factors****
[[0.0475 0.005 ]
 [0.855  0.0475]]
*****
{0: array([0.1, 0.9]), 3: array([0.8]), 4: array([0.4]), 5: array([0.9025, 0.0525])}
{0: ['B'], 3: [], 4: [], 5: ['B']}
#####
```

After Multiply out to one factor:

```

. . . . . [-----]
****multiplied factors****
[0.08 0.72]
*****
****multiplied factors****
[0.032 0.288]
*****
****multiplied factors****
[0.02888 0.01512]
*****

```

After Normalize:

[0.65636364 0.34363636]

Answer: 0.65636364

P (B|W)

Initial factors:

```

{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
    [0.9 , 0.05]],
    [[0.05, 0.9 ],
    [0.1 , 0.95]]]), 3: array([[0.8, 0.4],
    [0.2, 0.6]]), 4: array([[0.4 , 0.05],
    [0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After Restrict:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
      [0.9 , 0.05]]),
      [[0.05, 0.9 ],
      [0.1 , 0.95]]]), 3: array([0.8, 0.4]), 4: array([[0.4 , 0.05],
      [0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['A'], 4: ['G', 'A']}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
learn factor [steps]
****multiplied factors****
[[[0.0475 0.005 ]
  [0.855  0.0475]]

 [0.0025 0.045 ]
  [0.095  0.9025]]]
*****
{0: array([0.1, 0.9]), 3: array([0.8, 0.4]), 4: array([[0.4 , 0.05],
      [0.6 , 0.95]]), 5: array([[0.9025, 0.0525],
      [0.0975, 0.9475]])}
{0: ['B'], 3: ['A'], 4: ['G', 'A'], 5: ['A', 'B']}
#####
****multiplied factors****
[[0.32 0.02]
 [0.48 0.38]]
*****
****multiplied factors****
[[[0.2888 0.0168 ]
  [0.00195 0.01895]]

 [0.4332 0.0252 ]
  [0.03705 0.36005]]]
*****
{0: array([0.1, 0.9]), 6: array([[0.29075, 0.03575],
      [0.47025, 0.38525]])}
{0: ['B'], 6: ['G', 'B']}
#####
```

```
{0: array([0.1, 0.9]), 7: array([0.761, 0.421])}
{0: ['B'], 7: ['B']}
#####
-----
```

After Multiply out to one factor:

```
return factor[0:0.5]
****multiplied factors****
[0.0761 0.3789]
*****
```

After Normalize:

[0.16725275 0.83274725]

Answer: 0.16725275

So they are not equal.

2.d

$P(E|A \wedge B)$

Initial factors:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
[0.9 , 0.05]],
[[0.05, 0.9 ],
[0.1 , 0.95]]]), 3: array([[0.8, 0.4],
[0.2, 0.6]]), 4: array([[0.4 , 0.05],
[0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After Restrict:

```
return factor[0:1000]
{0: array([0.1]), 1: array([0.05, 0.95]), 2: array([0.95, 0.9 ]), 3: array([0.8, 0.2]), 4: array([0.4, 0.6])}
{0: [], 1: ['E'], 2: ['E'], 3: ['W'], 4: ['G']}
```

After each step of summing out Hidden Variables (Separated by #####):

```
return factor[0:1000]
{0: array([0.1]), 1: array([0.05, 0.95]), 2: array([0.95, 0.9 ]), 3: array([0.8, 0.2]), 5: 1.0}
{0: [], 1: ['E'], 2: ['E'], 3: ['W'], 5: []}
#####
{0: array([0.1]), 1: array([0.05, 0.95]), 2: array([0.95, 0.9 ]), 5: 1.0, 6: 1.0}
{0: [], 1: ['E'], 2: ['E'], 5: [], 6: []}
#####
```

After Multiply out to one factor:

```
****multiplied factors****
[0.005 0.095]
*****
****multiplied factors****
[0.00475 0.0855 ]
*****
****multiplied factors****
[0.00475 0.0855 ]
*****
****multiplied factors****
[0.00475 0.0855 ]
*****
```

After Normalize:

[0.05263158 0.94736842]

Answer: 0.05263158

$P(E|A)$

Initial factors:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
      [0.9 , 0.05]]),
      [[0.05, 0.9 ],
      [0.1 , 0.95]]]), 3: array([[0.8, 0.4],
      [0.2, 0.6]]), 4: array([[0.4 , 0.05],
      [0.6 , 0.95]])}
{0: ['B'], 1: ['E'], 2: ['A', 'E', 'B'], 3: ['W', 'A'], 4: ['G', 'A']}
```

After Restrict:

```
{0: array([0.1, 0.9]), 1: array([0.05, 0.95]), 2: array([[0.95, 0.1 ],
      [0.9 , 0.05]]), 3: array([0.8, 0.2]), 4: array([0.4, 0.6])}
{0: ['B'], 1: ['E'], 2: ['E', 'B'], 3: ['W'], 4: ['G']}
```

After each step of summing out HiddenVariables(Seperate by #####):

```
****multiplied factors****
[[0.095 0.09 ]
 [0.09  0.045]]
#####
{1: array([0.05, 0.95]), 3: array([0.8, 0.2]), 4: array([0.4, 0.6]), 5: array([0.185, 0.135])}
{1: ['E'], 3: ['W'], 4: ['G'], 5: ['E']}
#####
{1: array([0.05, 0.95]), 3: array([0.8, 0.2]), 5: array([0.185, 0.135]), 6: 1.0}
{1: ['E'], 3: ['W'], 5: ['E'], 6: []}
#####
{1: array([0.05, 0.95]), 5: array([0.185, 0.135]), 6: 1.0, 7: 1.0}
{1: ['E'], 5: ['E'], 6: [], 7: []}
#####
[0.06777777 0.02222222]
```

After Multiply out to one factor:

```
****multiplied factors****  
[0.00925 0.12825]  
*****  
****multiplied factors****  
[0.00925 0.12825]  
*****  
****multiplied factors****  
[0.00925 0.12825]  
*****  
--
```

After Normalize:

[0.06727273 0.93272727]

Answer: 0.06727273

So they are not equal.