IDX G10 [Math][H]

Study Guide Issue #1

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Contents (Related to the important parts of the textbook and homework assigned by teachers.)

Chapter 1-5: Complex Numbers

Real Numbers

- The real numbers con sist of zero and all positive and negative integers, rational numbers, and irrational numbers.
- Their squares are never negative.

Complex Numbers

- **A number that it's squares are negative. Represented with the term "*i*"**
  - $i = \sqrt{-1}$ and $i^2 = -1.$
- Define the square root of any negative number as:
  - If $a > 0$, $\sqrt{-a} = i\sqrt{a}.$
- Any number of **the form a + bi**, where a and b are real numbers and i is the imaginary unit, is called a **complex number.**
  - **"*a*" is called the real part, and "*b*" is called the imaginary part.**
    - **In equations you can factor/separate the real and imaginary BUT REMEMBER TO REMOVE THE "*i*" IN THE EQUATION. *Got it?***
    - An example would be "Find real numbers x and y such that (3x – 4y) + (6x + 2y)*i* = 5*i*"
    - And the way you would solve it is having 3x – 4y = 0 and 6x + 2y = 5. (Where you remove the imaginary number to find real numbers)
    - Another example is where you want to "Find the square roots of 3 + 4*i*"
    - So, you make (a + b*i*)^2 = 3 + 4i, then you separate the real and imaginary like the past example. So that a^2 + 2ab*i* – b^2 = 3 + 4*i*, and you separate a^2 – b^2 = 3 and 2ab = 4. Then solve.
- Complex Conjugates, if z = a + b*i*, then other z = a – b*i* and this applies to roots. (Chap 1.6)

Addition:	$(a+bi)+(c+di)= (a+c)+(b+d)i$
Subtraction:	$(a+bi)-(c+di)= (a-c)+(b-d)i$
Multiplication:	$(a+bi)(c+di)= ac+bic+dia-bd$
Division:	$\dfrac{a+bi}{c+di} = \dfrac{(a+bi)(c-di)}{(c+di)(c-di)} = \dfrac{(ac+bic-dia+bd)}{c^2+d^2}$

<u>Chapter 1-6: Solving Quadratic Equations</u>

- Any equation that can be written in the form ax^2 + bx + c = 0 (a ≠ 0), is called a **quadratic equation.** A **root**, or solution, of a quadratic equation is a value of the variable that satisfies the equation.
  - Solved through Factoring, Completing the Square, and the Quadratic Formula.

Factoring

- Whenever the product of two factors is zero, at least one of the factors must be zero.
- A quadratic equation must be written in the standard form ax^2 + bx + c = before it can be solved by factoring. *(We all basically know factoring, right?)*

Completing the Square *(Personally haven't seen most questions with this use, but still important)*

- The method of transforming a quadratic equation so that one side is a perfect square trinomial is called **Completing the Square**.
  - An example would be "Solve **2x^2 – 12x – 7 = 0**"
  1) You'd divide it by 2 first to make it easier: **x^2 – 6x – 7/2 = 0**
  2) Subtract the constant term from both sides: **x^2 – 6x = 7/2**
  3) Add the square of one half the coefficient of x: **x^2 – 6x + (-3)^2 = 7/2 + (-3)^2**
  4) Take the square root of both sides and solve for x.

The Quadratic Formula

- The equation we all love: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- The Discriminant $b^2 - 4ac$:

The Nature of the Discriminant

Given the quadratic equation $ax^2 + bx + c = 0$, where a, b, and c are real numbers:

If $b^2 - 4ac < 0$, there are two conjugate imaginary roots.
If $b^2 - 4ac = 0$, there is one real root (called a *double root*).
If $b^2 - 4ac > 0$, there are two different real roots.

Losing or Gaining a Root

- It is possible to lose a root by dividing both sides of an equation by a common factor.
  - 4x (x – 1) = 3(x – 1)^2 and you divide both sides by (x – 1). (*That's how they get you.)*

- It is possible to gain a root by squaring both sides of an equation. Another possible way to gain a root is by multiplying both sides of an equation by an expression.

Solve $\frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{8-4x}{x^2-4}$.

Multiply both sides of the equation by $(x+2)(x-2)$.

$$(x+2)^2 + (x-2)^2 = 8 - 4x$$
$$(x^2 + 4x + 4) + (x^2 - 4x + 4) = 8 - 4x$$
$$2x^2 + 8 = 8 - 4x$$
$$2x^2 + 4x = 0$$
$$2x(x+2) = 0$$
$$x = 0, -2$$

Check: $x = 0$

$$\frac{0+2}{0-2} + \frac{0-2}{0+2} \stackrel{?}{=} \frac{8-0}{0^2-4}$$
$$-1 + (-1) = -2$$

Thus, 0 is a root.

$x = -2$

$$\frac{-2+2}{-2-2} + \frac{-2-2}{-2+2} \stackrel{?}{=} \frac{8-4(-2)}{(-2)^2-4}$$

Since two denominators are zero, the equation is meaningless. Thus, -2 is *not* a root of the original equation.

Therefore, the solution is $x = 0$.

Chapter 1-7: Quadratic Functions and Their Graphs

- The graph of the quadratic function f(x) = ax^2 + bx + c, where a ^ 0, is the set of points (x, y) that satisfy the equation y = ax^2 + bx + c.
  - The graph for this is a curve. Properly pronounced parabola.
  - Axis of Symmetry: A vertical imaginary line where you can fold the parabola symmetrically. The equation x = -b/2a.
  - Vertex: The point where the axis of symmetry intersects the parabola. Where you input x into the equation to get it.

Find the axis and vertex of the parabola as follows:

Method 1. The equation of the axis is $x = -\frac{b}{2a}$. Substitute this value of x into $y = ax^2 + bx + c$ to find the y-coordinate of the vertex.

Method 2. Rewrite the equation in the form

$$y = a(x-h)^2 + k.$$

The vertex is (h, k) and the axis is $x = h$.

- If a > 0, the parabola opens upward, and the function has a minimum value.
- If a < 0, the parabola opens downward, and the function has a maximum value.
- The bigger $|a|$ is. the narrower the parabola is.

Chapter 2-1: Polynomials

- A **polynomial** in x is an expression that can be written in the form:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

  - The **terms** are the individual parts of the polynomials without the signs. Like $a_n x^n$ to a_0.
  - The **coefficients** are the numbers besides the x, like a_n, a_2 or a_0.
  - The **leading term** is the term containing the highest power of x.
  - The **degree** is the power of x in the leading term.

Degree	*Name*	*Example*
0	constant	5
1	linear	$3x + 2$
2	quadratic	$x^2 - 4$
3	cubic	$x^3 + 2x + 1$
4	quartic	$-3x^4 + x$
5	quintic	$x^5 + \pi x^4 - 3.1x^3 + 11$

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