IDX G9 Math S+
Study Guide Issue S1 Midterms
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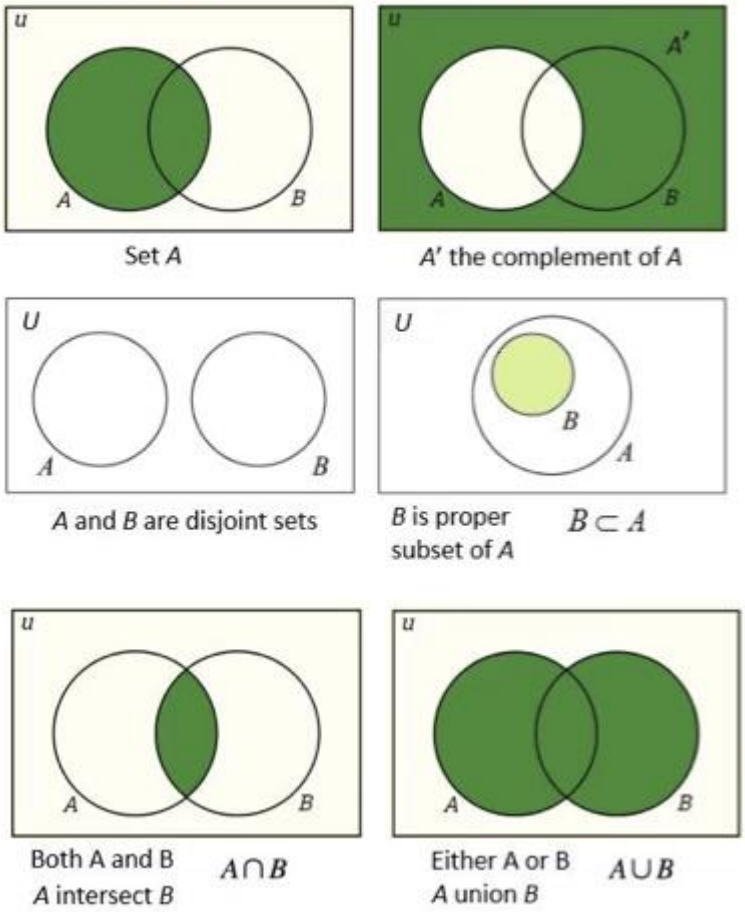
Contents:

Set

- Sets and Venn Diagrams
  - A= {1,2,3} – {} represents set / 1,2,3 represents elements
  - B= {Pencils, Pens, Rulers} – Other values that are not numbers could be elements
  - n(A) = 3 (# of elements in set A)
  - A set is a collection of distinct numbers or objects. Each object is called an element or member of the set.
- Properties of Sets:
  - An element is either in the set or not in the set.
  - The elements in a set are distinct. Same element can appear only once.
  - There is no fixed order when we describe the elements in the set.
  - Two sets are equal if they contain exactly the same elements
- **Important Number Set, Classification of Sets**
  - Important Number Sets:
    - N – Natural Numbers
    - Z – Integers
    - Z^+ or N* - Positive Integers
    - Z^- - Negative Integers
    - Q – Rational Numbers
    - R – Real numbers
  - Classification of Sets:
    - -**Finite** – {1,2,3} has particular defined value
    - -**Infinite** – {1,2,3…} doesn't has particular defined value
    - *Specific – {} or ∅
    - ∅, {0}, {∅}
      - n(∅) = 0
      - ∅ is a set for 1st set.
      - n({0}) = 1
      - ∅ ∈ {∅}
      - n({∅}) = 1
- Descriptive Method, Sets, Properties of Subsets
  - **Descriptive Method**
    - The descriptive method **uses words or mathematical rules to describe the members of the set** without listing them explicitly.

        - Examples
            - B = {X / X is a weekday} (Descriptive Method)
            - B = {Monday, Tuesday, Wednesday, Thursday, Friday} (Roster Method)
    - **Sets**
        - A=B Same Elements
        - A⊆B Fewer Elements – A is a subset of B
        - A⊂B – A is a proper subset of B
        - A = {1,2,3} B = {1,2,3,4…}
        - A⊂B: B must have all the elements that are in set A.
    - **Properties of Subsets**
        - A⊆A
        - A = B ó A⊆B, B⊆A
        - ∅ is a subset of every sets.
        - ∈ - Belongs to, within. (A∈ B : A is an element of B)
- Set Operations & Venn Diagram
    - **Transversal Property of Subset**
        - A⊆B, B⊆C => A⊆C
    - **Reflexive Property of Subset**
        - A⊆A

Set Operations and Venn Diagrams



- Intersection and Union Properties
    - ∩ **Properties**
        - A∩B = B∩A
        - A∩A = A
        - A∩∅ = ∅
        - A∩B ⊆A, A∩B ⊆B
        - A∩B = A if and only if A⊆B
        - (A∩B)∩C = A∩(B∩C)
    - **∪ Properties**
        - A∪B = B∪A
        - A∪A = A
        - A∪∅ = ∅
        - A∪B ⊆A, A∪B ⊆B
        - A∪B = A if and only if A⊆B
        - (A∪B) ∪C = A∪ (B∪C)
- Bracket Notations

- [a, b] → closed
-] a, b [→ open
- [a, b [→ closed at a, open at b
-] a, b] → open at a, closed at b

1-1 Patterns and Inductive Reasoning

- **Inductive Reasoning**
  - Reasoning that is based on patterns you observe. If you observe a pattern in a sequence, you can use inductive reasoning to tell what the next terms in the sequence will be.
- A conclusion you reach using inductive reasoning is called a **conjecture**.
- Not all conjectures are true, you can prove that the conjecture is false by finding a counterexample. A **counterexample** of a conjecture is an example for which the conjecture is incorrect.

1-3 Points, Lines, and Planes

- In geometry, words such as *point, line,* and *plane* are undefined. In order to define these words, it is necessary to use words that need further defining. It is important to have general descriptions of their meanings.
- A **point** has no size. It is represented by a small dot and is named by a capital letter. A geometric figure is a set of points, and **space** is defined as the set of all points.
- A **line** is a series of points that extend in two opposite directions without end. A name of a line could be represented by any two points on the line, such as line AB. Another way to name a line is with a single lowercase letter, such as line l. Points that lie on the same line are **collinear points.**
- A **plane** is a flat surface that has no thickness. A plane contains many lines and extends without end in the directions of all its lines. You can name a plane by either a single capital letter or by at least three of its noncollinear points. Points and lines in the same plane are **coplanar** (all collinear points are coplanar).
- A **postulate** or **axiom** is an accepted statement of fact.
  - Postulate 1-1: Through any two points there is exactly one line.
  - Postulate 1-2: If two lines intersect, then they intersect in exactly one point.
  - Postulate 1-3: If two planes intersect, then they intersect in exactly one line.

- Postulate 1-4: Through any three noncollinear points there is exactly one plane.

1-4 Segments, Rays, Parallel Lines and Planes

- A **segment** is the part of a line consisting of two endpoints and all points between them.
- A **ray** is the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.
- **Opposite rays** are two collinear rays with the same endpoint. Opposite rays always form a line.
- Lines that do not intersect may or may not be coplanar.
- **Parallel lines** are coplanar lines that do not intersect. **Skew lines** are noncoplanar: therefore, they are not parallel and do not intersect.
- Segments or rays are parallel if they lie in parallel lines. They are skew if they lie in skew lines.

1-5 Measuring Segments

- Postulate 1-5 **Ruler Postulate**
  - The points of a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers.
- Congruent segments are equal in length and similar in shape.
- Postulate 1-6 **Segment Addition Postulate**
  - If three points A, B, and C are collinear and B is between A and C, then AB + BC = AC.
- A **midpoint** of a segment is a point that divides the segment into two congruent segments. A midpoint, or any line, ray, or other segment through a midpoint, is said to bisect the segment.

1.6 Measuring Angles

- Postulate 1-7 **Protractor Postulate**
  - Let ray OA and ray OB be opposite rays in a plane. Ray OA, ray OB, and all the rays with endpoint O that can be drawn on one side of line AB can be paired with the real numbers from 0 to 180 so that
    - Ray OA is paired with 0 and ray OB is paired with 180.

        - If ray OC is paired with x and ray OD is paired with y, then angle COD = |x - y |
- Acute Angle: 0<X<90
- Right Angle: X=90
- Obtuse Angle: 90<X<180
- Straight Angle: X=180
- Congruent Angle: Angles with same measure.
- Postulate 1-8 **Angle Addition Postulate:**
    - If point B is in the interior of angle AOC, then angle AOB + angle BOC = angle AOC. If angle AOC is a straight angle, then angle AOB + angle BOC = 180.

2.1 Conditional Statements

- You have heard *if-then* statements such as this one:
    - If you are not completely satisfied, then your money will be refunded.
- Another name for an if-then statement is a **conditional**. Every conditional has two parts. The part following if is the **hypothesis**, and the part following then is the **conclusion**.
    - A conditional can have a **truth value** of *true or false*. To show that a conditional is true, show that every time the hypothesis is true, the conclusion is also true. To show that a conditional is false, you need to find only one counterexample for which the hypothesis is true, and the conclusion is false.
    - The **converse** of a statement switches the hypothesis and conclusion.

2.2 Biconditional and Definitions

- If both conditional and its converse are true, you can combine them as a true **biconditional**. This is the statement you get by connecting the conditional and its converse with the word *and*. You can write a biconditional more concisely, however, by joining the two parts of each conditional with the phrase *if and only if*.
- A biconditional contains q=>p and p=>q as q<=>p.
- A good definition…
    - Is a statement that can help you identify or classify an object.
    - Has several important components.
    - Uses clearly understood terms. The terms should be commonly understood or already defined.
    - Is precise. Good definitions avoid words such as *large, sort of,* and *almost*.

- Is reversible. That means that you can write a good definition as a true biconditional.

2.3 Deductive Reasoning

- **Deductive reasoning** (or logical reasoning) is the process of reasoning logically from given statements to conclusion. If the given statements are true, deductive reasoning produces a true conclusion.
- **Law of Detachment:** If a conditional is true and its hypothesis is true, then its conclusion is true. (If p->q is a true statement and p is true, then q is true.)
- **Law of Syllogism:** If p->q and q->r are true statements, then p->r is a true statement.

5-4 Inverses, Contrapositives, and Indirect Reasoning

- The negation of a statement has the opposite truth value. Represented in ~, for example, ~P.
- The **inverse** of a conditional statement negates both the hypothesis and the conclusion. The **contrapositive** of a conditional switches the hypothesis and the conclusion and negates both.
- Equivalent statements have the same truth value.
- Conditional and contrapositive statements share truth value and inverse and converse statements shares same truth value.
- This type of reasoning is called **indirect reasoning**. In indirect reasoning, all possibilities are considered and then all but one are proved false. The remaining possibility must be true.
- A proof involving indirect reasoning is an **indirect proof**. In an indirect proof, a statement and its negation often are the only possibilities.
- Writing an Indirect Proof
    - Step 1. State as an assumption the opposite of what you want to prove.
    - Step 2. Show that this assumption leads to a contradiction.
    - Step 3. Conclude that the assumption must be false and that you want to prove must be true.

Reasoning in Algebra

- Properties of Equality
    - Addition Property – If a=b, then a+c=b+c
    - Subtraction Property – If a=b, then a-c=b-c
    - Multiplication Property – If a=b, then ac=bc

    - Division Property – If a=b and c is not 0, then a/c=b/c
    - Reflexive Property – a=a
    - Symmetric Property – If a=b, then b=a
    - Transitive Property – If a=b and b=c, then a=c
    - Substitution Property – If a=b, then a could substitute b at any equations
    - Distributive Property – a(b+c) = ab+ac
- Properties of Congruence
    - Reflexive Property – a ≅a
    - Symmetric Property – If a ≅b, then b ≅a
    - Transitive Property – If a ≅b and b ≅c, then a ≅c

Proving Angles Congruent

- Theorem 2-1 Vertical Angles Theorem
    - Vertical angles are congruent
- Theorem 2-2 Congruent Supplements Theorem
    - If two angles are supplements of the same angle, (or congruent angles), then the two angles are congruent.
- Theorem 2-3 Congruent Complements Theorem
    - If two angles are complements of the same angle, (or congruent angles), then the two angles are congruent.
- Theorem 2-4
    - All right angles are congruent.
- Theorem 2-5
    - If two angles are congruent and supplementary, then each is right angle.

Properties of Parallel Lines

- Terms
    - Transversal line: A line that intersects two or more lines at different points.
    - Alternate interior angles: Angles formed on opposite sides of the transversal and inside the two lines.

    - Alternate exterior angles: Angles formed on opposite sides of the transversal and outside the two lines.
    - Same-side interior angles: Angles formed on the same side of the transversal and inside the two lines.
    - Corresponding angles: Angles that occupy the same relative position at each intersection where a transversal crosses two lines.
- Postulate 3-1 Corresponding Angles Postulate
    - If a transversal intersects two parallel lines, then corresponding angles are congruent.
- Theorem 3-1 Alternate Interior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate interior angles are congruent.
- Theorem 3-2 Same-Side Interior Angles Theorem
    - If a transversal intersects two parallel lines, then the same-side interior angles are supplementary.
- Theorem 3-3 Alternate Exterior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate exterior angles are congruent.
- Theorem 3-4 Same-Side Exterior Angles Theorem
    - If a transversal intersects two parallel lines, then the same-side exterior angles are supplementary.

Proving Lines Parallel

- Postulate 3-2 Converse of the Corresponding Angles Postulate
    - If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.
- Theorem 3-5 Converse of the Alternate Interior Angles Postulate
    - If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.
- Theorem 3-6 Converse of the Same-Side Interior Angles Postulate
    - If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

- Theorem 3-7 Converse of the Alternate Exterior Angles Postulate
    - If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.
- Theorem 3-8 Converse of the Same-Side Exterior Angles Postulate
    - If two lines and a transversal form same-side exterior angles that are supplementary, then the two lines are parallel.

Parallel and Perpendicular Lines

- Theorem 3-9
    - If two lines are parallel to the same line, then they are parallel to each other.
- Theorem 3-10
    - In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.
- Theorem 3-11
    - In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.

Triangle Angle-Sum Theorem

- Theorem 3-12 Triangle Angle-Sum Theorem
    - The sum of measures of the angles of a triangle is 180.
- Theorem 3-12 Triangle Exterior Angle Theorem
    - The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Congruent Figures

- Congruent polygons have congruent corresponding parts-their matching sides and angles. Matching vertices are corresponding vertices. When you name congruent polygons, always list corresponding vertices in the same order.
- Theorem 4-1
    - If two angles of one triangle are congruent to two angles of another triangles, then the third angles are congruent.

Triangle Congruence by SSS and SAS

- Postulate 4-1 Side-Side-Side (SSS) Postulate
  - If the three sides of one triangle are congruent to the three side of another triangle, then the two triangles are congruent.
- Postulate 4-2 Side-Angle-Side (SAS) Postulate
  - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Triangle Congruence by ASA and AAS

- Postulate 4-3 Angle-Side-Angle (ASA) Postulate
  - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
- Theorem 4-2 Angle-Angle-Side Theorem
  - If the angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

Using Congruent Triangles: CPCTC

- CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
  - With SSS, SAS, ASA, and AAS, you know how to use three parts of triangles to show that the triangles are congruent. Once you have triangles congruent, you can make conclusions about their other parts because, by definition, corresponding parts of congruent triangles are congruent. You can abbreviate this as CPCTC.