IDX G9 Math H
Study Guide S1 Midterms
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Contents:

Definitions

- **Transversal**
  - A line that intersects two or more coplanar lines at distinct points.
- **Corresponding Angles / Alternate Interior Angles / Same-Side Interior Angles / Alternate Exterior Angles / Same-Side Exterior Angles**
  - Classifications of angles formed by a transversal intersecting two lines, distinguished by their positions relative to the transversal (same side/opposite sides) and the intersected lines (interior/exterior).
- **Parallel Lines**
  - Two lines in the same plane that do not intersect, denoted by " // ".

- **Perpendicular Lines**
  - Two lines that intersect to form right angles (90°).
- **Triangle**
  - A closed planar figure formed by three line segments connected end-to-end.
  - Classified by angles:
    - Acute triangle (all angles acute)
    - Right triangle (contains one right angle, with legs and a hypotenuse)
    - Obtuse triangle (contains one obtuse angle)
    - Equiangular triangle (all angles congruent)
  - Classified by sides:
    - Equilateral triangle (all sides congruent)
    - Isosceles triangle (at least two sides congruent, with legs, a base, a vertex angle, and base angles)
    - Scalene triangle (no congruent sides).
- **Exterior Angle of a Polygon**
  - An angle formed by one side of a polygon and the extension of an adjacent side (one exterior angle per vertex)
- **Remote Interior Angles (of a Triangle)**
  - The two interior angles of a triangle that are not adjacent to a given exterior angle.
- **Polygon**
  - A closed planar figure with three or more line segments.
  - Named by the number of sides:
    - Triangle
    - Quadrilateral
    - Etc.
  - Classified by shape:
    - Convex polygon (all interior angles < 180 degrees, diagonals inside the polygon)
    - Concave polygon (contains at least one interior angle > 180 degrees)
    - Equilateral polygon (all sides congruent)
    - Equiangular polygon (all angles congruent)
    - Regular polygon (both equilateral and equiangular).
- **Congruent Polygons**
  - Polygons that can be superimposed exactly, with congruent corresponding vertices, sides, and angles. When naming congruent polygons, corresponding vertices must be listed in order

- **CPCTC**
    - **Abbreviation for "Corresponding Parts of Congruent Triangles are Congruent," used to prove that corresponding sides or angles of congruent triangles are congruent.**
- **Midpoint**
    - A point that divides a segment into two congruent parts. The coordinate formula for the midpoint of segment AB with A (x1, y1) and B (x2, y2) is
      M = (((x1 + x2) / 2), ((y1 + y2) / 2)).
- **Angle Bisector**
    - **A ray or segment that divides an angle into two congruent angles.**
- **Perpendicular Bisector**
    - **A line, ray, or segment that is both perpendicular to a segment and bisects it.**
- **Median**
    - A segment connecting a vertex to the midpoint of the opposite side.
- **Altitude**
    - A perpendicular segment from a vertex to the line containing the opposite side.
- **Angle Bisector (of a Triangle)**
    - A segment that bisects an interior angle and intersects the opposite side.
- **Circumcenter**
    - The intersection of the perpendicular bisectors of a triangle’s sides; equidistant from the triangle’s vertices (center of the circumscribed circle).
- **Incenter**
    - The intersection of the angle bisectors of a triangle’s angles; equidistant from the triangle’s sides (center of the inscribed circle).
- **Centroid**
    - The intersection of a triangle’s medians; divides each median into a 2:1 ratio (the distance from the centroid to a vertex is twice the distance from the centroid to the midpoint of the opposite side).
- **Orthocenter**
    - The intersection of a triangle’s altitudes.
- **Distance from a Point to a Line**
    - The length of the perpendicular segment from the point to the line (the shortest distance).

- **Slope**
  - A ratio representing the steepness of a line, calculated as slope = (y2 - y1)/(x2 - x1) for points (x1, y1) and (x2, y2). Parallel lines have equal slopes; the product of the slopes of perpendicular lines is -1 (for non-vertical/non-horizontal lines).

Postulates

- **Postulates — basic principles that do not require proof:**
- **Postulate 3-1 (Corresponding Angles Postulate)**
  - If a transversal intersects two parallel lines, then the corresponding angles formed are congruent.
- **Postulate 3-2 (Converse of the Corresponding Angles Postulate)**
  - If a transversal intersects two lines such that the corresponding angles formed are congruent, then the two lines are parallel.
- **Postulate 4-1 (SSS Congruence Postulate)**
  - **If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent (symbol: △ABC ≅ △DEF)**
  - Proof: A postulate based on the stability (rigidity) of triangles. When the lengths of three sides are given, the shape and size of the triangle are unique, so no proof is needed
- **Postulate 4-2 (SAS Congruence Postulate)**
  - **If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.**
  - Proof: Postulate; no proof required. Construction verification shows that "a triangle is unique when two sides and their included angle are determined."
- **Postulate 4-3 (ASA Congruence Postulate)**
  - **If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.**
  - Proof: Postulate; no proof required. Construction verification shows that "a triangle is unique when two angles and their included side are determined."

Theorems

- **Theorems — geometric conclusions that require proof:**
- **Theorem 3-1 (Alternate Interior Angles Theorem)**

- **If a transversal intersects two parallel lines, then the alternate interior angles formed are congruent.**
- Proof:
  - Given lines a // b, and a transversal t intersecting a and b at two points. Let the corresponding angles angle 1 be congruent to angle 2. (by Postulate 3-1).
  - Angle 2 and the alternate interior angle angle 3 are vertical angles, and vertical angles are congruent (Vertical angles theorem).
  - By the Transitive Property of Congruence, angle 1 is congruent to angle 3, so alternate interior angles are congruent.

- **Theorem 3-2 (Same-Side Interior Angles Theorem)**
  - **If a transversal intersects two parallel lines, then the same-side interior angles formed are supplementary (their sum is 180°).**
  - Proof:
    - Given a // b and transversal t, corresponding angles angle1 and angle 2 are congruent (by Postulate 3-1), so the measure of angle1 = measure of angle2.
    - Angle 2 and the same-side interior angle angle 3 form a straight angle, so measure of angle 2 = measure of angle 3 = 180 degrees.
    - Substituting measure of angle 1 = measure of angle 2 gives

      measure of angle 1 + measure of angle 3 = 180 degrees, so same-side interior angles are supplementary.
- **Theorem 3-3 (Alternate Exterior Angles Theorem)**
  - **If a transversal intersects two parallel lines, then the alternate exterior angles formed are congruent.**
  - Proof:
    - Given a // b and transversal t, corresponding angles angle 1 and angle 2 are congruent (by Postulate 3-1), so the measure of angle 1 = measure of angle 2.
    - Angle 2 and the alternate exterior angle 3 are vertical angles, and vertical angles are congruent.
    - By the Transitive Property of Congruence, angle 1 is congruent to angle 3.
- **Theorem 3-4 (Same-Side Exterior Angles Theorem)**
  - **If a transversal intersects two parallel lines, then the same-side exterior angles formed are supplementary.**

- Proof:
  - Given a // b and transversal t, corresponding angles angle 1 and angle 2 are congruent (by Postulate 3-1), so the measure of angle 1 = measure of angle 2.
  - Angle 2 and the same-side exterior angle angle3 form a straight angle, so measure of angle 2 + measure of angle 3 = 180 degrees.
  - Substituting measure of angle 1=measure of angle 2 gives

    measure of angle 1 + measure of angle 3=180 degrees, alternate exterior angles are supplementary

- **Theorem 3-9 (Parallel to the Same Line Theorem)**
  - If two lines are both parallel to a third line, then the two lines are parallel to each other.
  - Proof (for coplanar case):
    - Let a // k and m // k, and a transversal t intersect a, m, and k at three points.
    - By Postulate 3-1, a // k implies, angle 2 is congruent to angle 1 (corresponding angles), and m // k implies angle 3 is congruent to angle 1 (corresponding angles).
    - By the Transitive Property of Congruence, angle 2 is congruent to angle 3. By Postulate 3-2, a // m.
- **Theorem 3-10 (Perpendicular to the Same Line Theorem)**
  - In a plane, if two lines are both perpendicular to a third line, then the two lines are parallel to each other.
  - Proof:
    - Let r perpendicular t and s perpendicular t. Then, angle1 and angle2 are both right angles (90 degrees, by definition of perpendicularity), so angle1 congruent to angle2.
    - Angle1 and angle2 are corresponding angles. By Postulate 3-2, r // s.
- **Theorem 3-11 (Perpendicular to One Parallel Line Theorem)**
  - In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.
  - Proof:
    - Let a // b and c perpendicular a; we need to prove c perpendicular b.

        - c perpendicular a implies angle 1 = 90 degrees. Since a // b and c is a transversal, corresponding angles angle 1 and angle 2 are congruent (by Postulate 3-1), so angle 2 = 90 degrees. Thus, c is perpendicular to b.
- **Theorem 3-12 (Triangle Angle-Sum Theorem)**
    - The sum of the measures of the angles of a triangle is 180 °.
    - Proof:
        - Draw a line CP // AB through vertex C of triangleABC.
        - By the property of parallel lines, angle1 congruent to angleA (alternate interior angles) and angle2 congruent to angleB (alternate interior angles).
        - Angle1, angle 2, and angle 3 form a straight angle, so measure angle 1 + measure angle 2 + measure angle 3 = 180 degrees.
        - Substituting gives measure angle A + measure angle B + measure angle C = 180 degrees.
- **Theorem 3-13 (Triangle Exterior Angle Theorem)**
    - **The measure of each exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.**
    - Proof:
        - Let angle 1 be an exterior angle of triangle ABC (formed by extending side BC). Then, angle ACB and angle 1 form a straight angle, so measure angle ACB + measure angle1 = 180 degrees.
        - By the Triangle Angle-Sum Theorem, measure angle A + measure angle B + measure angle ACB = 180 degrees.
        - Equating the two expressions gives measure angle 1 = measure angle A + measure angle B.

- **Theorem 3-14 (Polygon Angle-Sum Theorem)**
    - The sum of the measures of the interior angles of an n-sided polygon with n greater than 3 is (n - 2) * 180 degrees.
    - Proof:
        - Draw (n – 3) diagonals from one vertex of the n-sided polygon, dividing the polygon into (n - 2) triangles.

        - The sum of the interior angles of one triangle is 180 degrees, so the sum of the interior angles of the n-sided polygon is (n - 2) * 180 degrees.
- **Theorem 3-15 (Polygon Exterior Angle-Sum Theorem)**
    - The sum of the measures of the exterior angles of any polygon (one exterior angle at each vertex) is 360 °(regardless of the number of sides).
    - Proof:
        - At each vertex of an n-sided polygon, the interior angle and its corresponding exterior angle are supplementary. Thus, the sum of "interior angles + exterior angles" for all n vertices is n * 180 degrees.
        - Substituting the interior angle sum (n - 2) * 180 degrees, the exterior angle sum = n * 180 degree - (n - 2) * degree = 360 degree.
- **Theorem 4-1 (Third Angles Theorem)**
    - If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the two triangles are also congruent.
    - Proof:
        - Let angle A congruent to angle D and angle B congruent to angle in triangle ABC and triangle DEF, respectively.
        - By the Triangle Angle-Sum Theorem, measure angle C = 180 – measure angle A – measure angle B and measure angle F = 180 – measure angle D – measure angle E.
        - Since measure angle A = measure angle D and measure angle B = measure angle E, it follows that measure angle C = measure angle F, so angle C is congruent to angle F.
- **Theorem 4-2 (AAS Congruence Theorem)**
    - **If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the two triangles are congruent.**
    - Proof:
        - Let angle A be congruent to angle D, angle B be congruent to angle E, and BC be congruent to EF in triangle ABC and triangle DEF, respectively.
        - By Theorem 4-1, angle C congruent to angle F.

    - Now, we have angle B is congruent to angle E, BC is congruent to EF, and angle C is congruent to angle F (satisfying the ASA condition), so triangle ABC is congruent to triangle DEF.

- **Theorem 4-3 (Isosceles Triangle Theorem)**
  - **If a triangle has two congruent sides (an isosceles triangle), then the angles opposite those two sides (the base angles) are congruent.**
  - Proof:
    - Let triangle XYZ have XY congruent to XZ. Draw the angle bisector XB of the vertex angle YXZ, intersecting YZ at B.
    - Angle 1 congruent to angle 2 (Definition of angle bisector), XY congruent to XZ (given), and XB congruent to XB (Reflexive Property of Congruence).
    - By SAS, triangle XYB is congruent to triangle XZB. Thus, angle Y is congruent to angle Z (CPCTC).
- **Theorem 4-4 (Converse of the Isosceles Triangle Theorem)**
  - If a triangle has two congruent angles, then the sides opposite those two angles are congruent.
  - Proof:
    - Let triangle PQR have angle P congruent to angle Q. Draw the angle bisector RC of angle PRQ, intersecting PQ at C.
    - Angle 1 is congruent to angle 2 (Definition of angle bisector), angle P congruent to angle Q (given), and RC congruent to RC (Reflexive Property of Congruence).
    - By AAS, triangle PRC is congruent to triangle QRC. Thus, PR is congruent to QR (CPCTC).
- **Theorem 4-5 (Vertex Angle Bisector Theorem for Isosceles Triangle)**
  - **The angle bisector of the vertex angle of an isosceles triangle is also the perpendicular bisector of the base.**
  - Proof:
    - Let triangle ABC have AB congruent to AC, and AD bisecting angle BAC.

        - By Theorem 4-3, angle B congruent to angle C. Angle BAD is congruent to angle CAD (Definition of angle bisector), and AB is congruent to AC (given).
        - By ASA, triangle ABD is congruent to triangle ACD. Thus, BD is congruent to CD (bisecting the base) and angle ADB is congruent to angle ADC (CPCTC).
        - Angle ADB + angle ADC = 180 degrees (straight angle), so angle ADB = 90 degrees, and AD is perpendicular to BC.

- **Theorem 4-6 (HL Congruence Theorem)**
    - **If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent.**
    - Proof:
        - Let Rt. triangle PQR (right angle at Q) and Rt. Triangle XYZ (right angle at Y) have PR congruent to XZ (hypotenuses) and PQ congruent XY (legs).
        - Extend ZY to S in triangle XYZ such that YS = QR. By SAS, triangle PQR is congruent to triangle XYS (PQ congruent XY, angle Q = angle Y = 90 degrees, QR = YS).
        - Thus, PR is congruent to XS (CPCTC). Since PR is congruent to XZ, it follows that XS is congruent to XZ, and triangle XYS is congruent to triangle XYZ (by AAS).
        - By the Transitive Property of Congruence, triangle PQR is congruent to triangle XYZ (SSA, HL).
- **Theorem 5-1 (Triangle Midsegment Theorem)**
    - If a segment joins the midpoints of the two sides of a triangle, then the segment is parallel to the third side, and is half its length.
- **Theorem 5-2 (Perpendicular Bisector Theorem)**
    - **If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the two endpoints of the segment.**
    - Proof:

        - Let line L be the perpendicular bisector of segment AB, and let P lie on L. Let M be the midpoint of AB (so L is perpendicular to AB and AM is congruent to MB).
        - Angle PMA = angle PMB = 90 degrees. AM is congruent to MB, and PM is congruent to PM (Reflexive Property of Congruence).
        - By SAS, triangle PMA is congruent to triangle PMB. Thus, PA is congruent to PB (CPCTC).
- **Theorem 5-3 (Converse of the Perpendicular Bisector Theorem)**
    - **If a point is equidistant from the two endpoints of a segment, then the point lies on the perpendicular bisector of the segment.**
    - Proof:
        - Let PA be congruent to PB, and let M be the midpoint of AB (so AM is congruent to MB).
        - PA is congruent to PB, AM is congruent to MB, and PM is congruent to PM (Reflexive Property of Congruence). By SSS, triangle PMA is congruent to triangle PMB.
        - Angle PMA is congruent to angle PMB (CPCTC), and angle PMA + angle PMB = 180 degrees, so angle PMA = 90 degrees, and PM perpendicular AB. Thus, P lies on the perpendicular bisector of AB.
- **Theorem 5-4 (Angle Bisector Theorem)**
    - **If a point lies on the bisector of an angle, then the point is equidistant from the two sides of the angle.**
    - Proof:
        - Let ray AD bisect angle BAC, and let P lie on AD. Let PE be perpendicular to AB and PF be perpendicular to AC (definition of distance from a point to a line).
        - Angle PAE is congruent to angle PAF (Definition of angle bisector), so angle PEA = angle PFA = 90 degree, and PA is congruent to PA (Reflexive Property of Congruence).
        - By AAS, triangle PEA is congruent to triangle PFA. Thus, PE is congruent to PF (CPCTC).
- **Theorem 5-5 (Converse of the Angle Bisector Theorem)**

- If a point is equidistant from the two sides of an angle, then the point lies on the bisector of the angle.
- Proof:
  - Let P be equidistant from the sides AB and AC of angle BAC (so PE = PF,where PE perpendicular AB and PF perpendicular AC).
  - Angle PEA = angle PFA = 90 degree, PE = PF, and PA congruent to PA (Reflexive Property of Congruence). By HL Rt. Triangle PEA congruent to Rt. triangle PFA.
  - Angle PAE congruent to angle PAF (by CPCTC), P lies on the bisector ofangle BAC.

- **Theorem 5-6 (Concurrent Perpendicular Bisectors Theorem)**
  - **The perpendicular bisectors of the sides of a triangle intersect at a single point (the circumcenter), which is equidistant from all three vertices of the triangle.**
  - Proof:
    - Let the perpendicular bisector L1 of AB and the perpendicular bisector L2 of BC in triangle ABC intersect at O.
    - By Theorem 5-2, OA = OB (since O lies on L1) and OB = OC (since O lies on L2), so OA = OC.
    - By Theorem 5-3, O lies on the perpendicular bisector of AC. Thus, the perpendicular bisectors of the three sides intersect at O, and OA = OB = OC.
- **Theorem 5-7 (Concurrent Angle Bisectors Theorem)**
  - **The angle bisectors of the angles of a triangle intersect at a single point (the incenter), which is equidistant from all three sides of the triangle.**
  - Proof:
    - Let the bisectors of angle A and angle B in triangle ABC intersect at I.
    - By Theorem 5-4, I is equidistant from AB and AC (since I lies on the bisector of angle A) and equidistant from AB and BC (since I lies on thebisector of angle B).
    - Thus, I is equidistant from AC and BC. By Theorem 5-5, I lies on the bisector of angle C. Thus, the angle bisectors of the three angles intersect at I, and I is equidistant from the three sides.
- **Theorem 5-8 (Concurrent Medians Theorem)**

  - **The medians of a triangle intersect at a single point (the centroid), which divides each median into a ratio of 2:1 (the distance from the centroid to a vertex is twice the distance from the centroid to the midpoint of the opposite side).**
  - Proof:
    - Let the median BD (D is the midpoint of AC) and the median CE (E is the midpoint of AB) in triangle ABC intersect at G. Connect DE.
    - DE is a midline of triangle ABC, so DE // BC and DE = ½ BC. Thus, triangle GDE is similar to triangle GBC (AA Similarity).
    - The similarity ratio is 1:2, so DG:GB = 1:2 and EG:GC = 1:2. Similarly, it can be proven that the third median passes through G, meaning the three medians intersect at G and divide each median into a 2:1 ratio.
- **Theorem 5-9 (Concurrent Altitudes Theorem)**
  - **The altitudes of a triangle (or their extensions) intersect at a single point (the orthocenter).**
  - Proof (for acute triangles):
    - Let the altitudes BD and CE of triangle ABC intersect at H. Connect AH and extend it to intersect BC at F.
    - BD is perpendicular to AC and CE is perpendicular to AB, so angle ADB = angle AEH = 90 degrees. Thus, triangle ADH is similar to triangle AEH (sharing angle DAH).
    - Similarly, it can be proven that angle HDC = angle HFB = 90 degrees, and AF is perpendicular to BC. Thus, the three altitudes intersect at H.
- **Theorem 5-10 (Angle-Side Relationship Theorem)**
  - **If two sides of a triangle are not congruent, then the larger side is opposite the larger angle.**
  - Proof (Indirect Proof):
    - Let BC > AC in triangle ABC. Assume angle A <= angle B.
    - If angle A is congruent to angle B, then by Theorem 4-4, BC = AC (contradiction). If angle A < angle B, construct AD = AC (with D on BC).

        - Then angle ADC = angle ACD. Since angle ADC is an exterior angle of triangle ABD, angle ADC > angle B, so angle ACD > angle B, which contradicts angle A < angle B.
        - Thus, angle A > angle B.
- **Theorem 5-11 (Converse of the Angle-Side Relationship Theorem)**
    - **If two angles of a triangle are not congruent, then the larger angle is opposite the larger side.**
    - Proof (Indirect Proof):
        - Let angle A > angle B in triangle ABC. Assume BC <= AC.
        - If BC = AC, then by Theorem 4-3, angle A = angle B (contradiction). If BC < AC, then by Theorem 5-10, angle A < angle B (contradiction).
        - Thus, BC > AC.
- **Theorem 5-12 (Triangle Inequality Theorem)**
    - **The sum of the lengths of any two sides of a triangle is greater than the length of the third side.**
    - Proof:
        - Let there be triangle ABC. Extend BA to D such that AD = AC, and connect CD.
        - Since AD = AC, angle ADC = angle ACD. Thus, angle BCD = angle ACB + angle ACD > angle ADC.
        - By Theorem 5-11, BD > BC. Since BD = BA + AD = BA + AC, it follows that BA + AC > BC. Similarly, it can be proven that AB + BC > AC and AC + BC > AB.

Corollaries

- **Corollaries — conclusions directly derived from theorems:**
- **Corollary to Theorem 3-13 (Triangle Exterior Angle Corollary)**
    - The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles.
- **Corollary to Theorem 4-3 (Equilateral ⇒ Equiangular Corollary)**
    - If a triangle is equilateral (all three sides are congruent), then it is equiangular (all three angles are congruent, each measuring 60°).

- **Corollary to Theorem 4-4 (Equiangular ⇒ Equilateral Corollary)**
    - If a triangle is equiangular (all three angles are congruent), then it is equilateral (all three sides are congruent).

1.1 Sets and Venn Diagrams

Key points:

- Definition and properties of sets, subsets, elements
- Representation
- Operations

Sets and Elements

- **Set - A collection of distinct numbers or objects *(A,B,C,……)***
- **Object - each object of the set *(a,x,y,……)***
- **∈ - is an element of (), is in**
- **∈/ - is not an element of (), is not in**
    - Ex. In set A {1,2,3}, 1, 2, and 3 are all elements: 3∈A, 4∈/A

Number Set types:

- ℤ-Integers
- ℕ-Natural numbers set (integer>=0)
- ℤ+ or ℕ∗-Positive integers
- ℚ-Rational numbers
- ℝ-Real numbers
- ℝ>ℚ>ℤ>ℕ>ℤ+ or ℕ∗ (from largest range to smallest)

- A set is finite when n(A)/amount of elements is a defined value.
    - Ex. A={0,1,2} n(A)=3
- Otherwise, it's an infinite set.
    - Ex. A={x∈ℕ|x} (A contains all natural numbers)
- **The empty set ∅ or { } is a set which contains no elements**
- ∅-empty set, {0}-set with an element "0", {∅}-set with an element "empty set"

Bracket representation

- [x,y] represents a closed interval {a|x<=a<=y}
- (x,y) represents an open interval {a|x<a<y}
- When an interval extends for no endpoints.
    - Ex. {x|x<=a} we write (-∞,a]

Properties of sets

- Any element is either in or not in a set.
- The same element can only appear once in a set (Ex. {1,2,2} is not a set)
- There is no requirement for a fixed order of the elements in the set.
- Two sets are equal as long as they contain the same elements.
    - Ex. {1,2,3}={3,2,1}

Relationships between sets

- **For any $x \in A \Longrightarrow x \in B$, then we can call set A is subset of B, or we could write as "A⊆B"**
- If $A \subseteq B$ and there exists $x \in B \Longrightarrow x \notin A$ (or $A \neq B$), then we call set A is the proper subset of B, or we could write as "A⊂B"
- Operations:
- **Intersection: In both sets ($A \cap B = \{x | x \in A \text{ and } x \in B\}$)**
    - we call A and B mutually exclusive if $A \cap B = \emptyset$
- **Union: Either in the sets ($A \cup B = \{x | x \in A \text{ or } x \in B\}$)**
- **Complement: All elements not part of a set (A' or $\bar{A} = \{x | x \in U \text{ and } x \notin A\}$, U is the universal set of all elements)**

Venn diagrams

- A Venn diagram consists of a universal set U, and subsets in U.

1.2 Inductive reasoning

- We use inductive reasoning to prove what the next terms in a sequence will be based on patterns we observe
- Conjecture-conclusion you reach by inductive reasoning
- **Explicit formula-defining a only with respect to n (All terms can be found through explicit formula)**
- **Recursive formula-defining a(n) using preceding terms (a1, a2,…,an-1)**
- **Counterexample-an example for which the conjecture is incorrect**

1.3 Logic statement

- **Statement-declarative sentence which's either true or false, not both.**
    - Composed with hypothesis and conclusion (we could transfer it into a conditional statement: if a, then b"

- Conditional statement
  - If “if a, then b” is defined true, then we could say a implies b, a is sufficient for b, b is necessary for a.
- Equivalent statement
  - If p->q and while q->p, we could conclude p⇔q, which could be written as p is equivalent to q

- **Law of detachment-If the conditional is true and its hypothesis is true, then its conclusion is true (if p->q is a true statement and p is true, then q is true)**
- **Law of syllogism-If p->q and q->r are true, then p->r is a true statement.**
- For a conditional statement “if p then q “:
  - **Negation: if q then not q**
  - **Converse: if q then p**
  - **Inverse: if not p, then not q**
  - **Contrapositive: if not q, then not p (conditional and contrapositive are equivalent statements)**

1.4 Deductive Reasoning

- Deductive Reasoning: A logical reasoning process that moves from general statements to a conclusion.

  o Example questions: Given some statements with limited amount of true statements, determine which ones are correct or observe some facts.

1.5 Basic Geometric Elements

- **Points: A location; no size; represented by a small dot. Notation: capital letters like A, B, C, …**
- **Lines: A set of points that extend in two different directions without end.**
  - Notation: lowercase letter like a, b, c, l, … (A∈a)
- **Segments: Part of a line consisting of two endpoints and all the points between them.**
  - Notation: a bar on two capital letters representing each endpoint of the segment
- **Ray: The part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.**
  - Notation: an arrow on letter A representing the one endpoint and B representing the other point on the segment

- **Opposite rays: two collinear rays with the same end point with opposite direction that always form a line.**
- **Collinear points: points that are on the same line.**
- **Planes: a flat surface that has no thickness and extends without end in the directions of all the lines on it. It contains infinitely many lines.**
    - Notation: lowercase Greek letter like $\alpha, \beta, \gamma\ldots$
- Acute angles - angles with degree between 0 and 90
- Right angles - angles with degree of 90
- Obtuse angles - angles with degree between 90 and 180
- Straight angles - angles with degree of 180
- Vertical angles - two angles with opposite rays as sides
- Adjacent angles - coplanar angles with a common side, a common vertex with no shared interior points
- Complementary angles - two angles whose measures have the sum 90
- Supplementary angles - two angles whose measures have the sum 180
- **Postulates: An accepted statement of fact (don't need to prove)**
    - 1. Only one straight line can be drawn through two given points.
    - 2. Three non-collinear points can determine a plane.
    - 3. If two lines intersect, they intersect in exactly one point.
    - 4. If two distinct points on a line are on a plane, then the line is on the plane.
    - 5. If two planes intersect, they intersect in exactly one line.
    - 6. If two distinct planes have one point in common, they have only one line in common which contains this common point.
    - 7. If A, B, C are collinear, and B is between A and C, then AB+BC=AC
    - 8. If B is an interior point of AOC, then m $\angle$AOC = m$\angle$AOB + m$\angle$BOC
- **Theorem: statement that needs proof**
    - **Vertical angles theorem: vertical angles are congruent.**
    - **Congruent complement/supplement theorem: if two angles are the complement supplement of the same angle, they are congruent.**
- Reasonings for proof
    - **Properties of Equality**
        - Addition P.O.E: If a=b, then a + c = b + c.
        - Subtraction P.O.E: If a=b, then a - c = b-c.
        - Multiplication P.O.E: If a=b, then a • c = b • c.
        - Division P.O.E: If a=b and c≠0, then a/c=b/c
        - Reflexive P.O.E: a=a

        - Symmetric P.O.E: If a=b, then b=a.
        - Transitive P.O.E: If a=b and b=c, then a=c.
        - Substitution P.O.E: If a = b, then b can replace a in any expression.
        - Distributive P.O.E: a(b + c) = ab + ac

    - **Properties of Congruence**
        - Reflexive P.O.C: AB = AB; ∠A =∠A
        - Symmetric P.O.C: If AB = CD, then CD = AB; If ∠A = ∠B, then∠B = ∠A.
        - Transitive P.O.C: If AB = CD and CD = EF, then AB = EF; If ∠A = ∠B and ∠B = ∠C, then ∠A = ∠C.

1.6 Intersecting and Parallel Lines – Concepts:

- **Transversal: A line that intersects two different lines at two different points**
- **Corresponding angles: The pair of angles on the same side of the transversal, and on the same side of the two lines**
- **Alternate interior angle: The pair of angles on the opposite sides of the transversal and are outside of the two lines**
- **Alternate exterior angle: The pair of angles on the opposite sides of the transversal and between the two lines**
- **Same-side interior angle: The pair of angles on the same side of the transversal and between the two lines**
- **Same-side exterior angle: The pair of angles on the same sides of the transversal and are outside of the two lines** –
- **Corresponding angle postulate: If a transversal intersects two parallel lines, the corresponding angles formed are congruent (Converse is also correct)**
- Theorems (If a transversal intersects two parallel lines):
    - **Alternate Interior Angles Theorem: Alternate interior angles are congruent**
    - **Same-Side Interior Angles Theorem: Same-side interior angles are supplementary**
    - **Alternate Exterior Angles Theorem: Alternate exterior angles are congruent**

- **Same-Side Exterior Angles Theorem: Same-side exterior angles are supplementary**
- Converse of the above four theorems are all correct

- Properties of parallel lines
  - If two lines are parallel to the same line, then they are parallel.
  - For two parallel lines, the distance from any point on one line to the other is a constant, and it’s defined as the distance between the two parallel lines.