IDX G10 AP Precalculus
Study Guide Issue S1 Final
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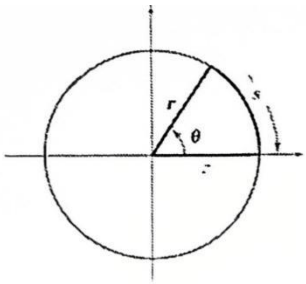
7.1 Measure of Angels

Basic Concepts

- An Angel: is formed by rotating a ray about its endpoint.
- Vertex: the endpoint of the ray
- Initial ray: the starting position of the ray
- Terminal ray: the ending position
- Degree: a common unit for measuring angels
- Revolution: a common unit for measuring vary large angels, a complete circular motion
- Radian: Angels measures can be expressed in units of degrees or in real-number units called radians. Degrees are based on fractional parts of a circular revolution. Radian measure compares the length of an arc that a central angel of a circle subtends with the radius of the circle
- 360° = 1 revolution = 2π



The measure of θ of the central angle is $\theta = \frac{s}{r}$

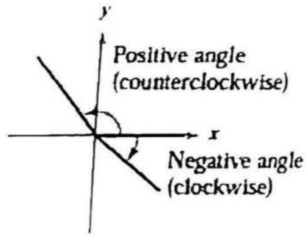
The arc length of 1 revolution is the circumference of the circle, $2\pi r$ and $\theta = 2\pi$

CONVERSIONS BETWEEN DEGREES AND RADIANS
1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180^\circ}$.
2. To convert radians to degrees, multiply radians by $\frac{180^\circ}{\pi \text{ rad}}$.
To apply these two conversion rules, use the basic relationship π rad $= 180^\circ$. (See Figure 9.11.)

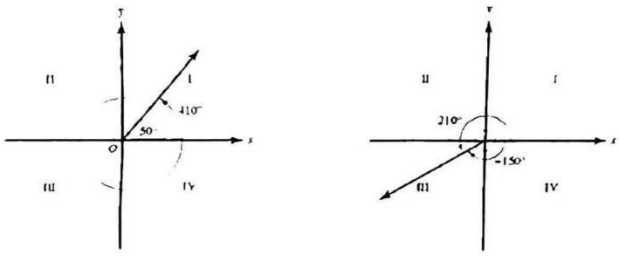
Standard Position – In coordinate plane, the angle appears with its vertex at the origin and its initial ray along the positive x-axis.

Positive measure – the rotation is counter-clockwise.

Negative measure – the rotation is clockwise.



Coterminal Angles – angles of different measures in standard position but having the same terminal side.



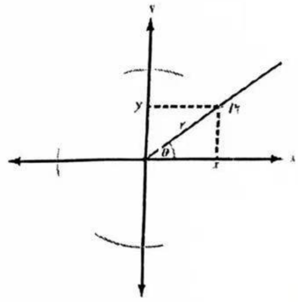
If θ is expressed in degrees, all coterminal angles have the form of $\theta + n \cdot 360^\circ$ (n is an integer)

If θ is expressed in radians, all coterminal angels have the form of $\theta + n \cdot 2\pi$ (n is an integer)

7.2 The Sine, Cosine, and Tangent Functions

➢ Sine, Cosine, and Tangent of an Angle

Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The number r denotes the distance from P to the origin.



The sine of the angle θ is the ratio of the vertical displacement of P from the x-axis to the distance between the origin point and point P: $\sin\theta = \frac{y}{r}$

The cosine of the angle θ is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin point and point P: $\cos\theta = \frac{x}{r}$

The tangent of the angle θ is the ratio of the y-coordinate to the x-coordinate of the point at which the terminal ray intersects the circle: $\tan\theta = \frac{y}{x}$

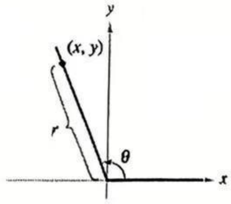
DEFINITIONS OF TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$\sin\theta = \frac{y}{r}$ $\qquad \cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x},\quad x \neq 0$ $\qquad \cot\theta = \frac{x}{y},\quad y \neq 0$

$\sec\theta = \frac{r}{x},\quad x \neq 0$ $\qquad \csc\theta = \frac{r}{y},\quad y \neq 0$

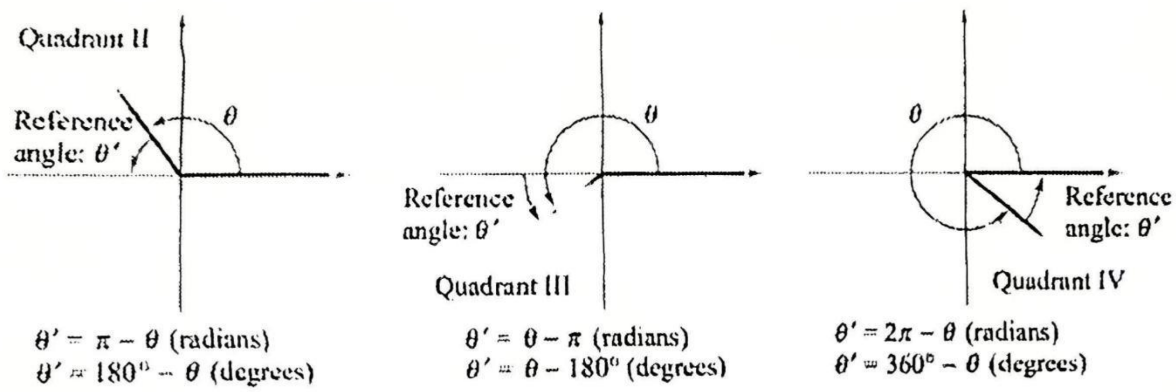


- Sine refers to the y coordinates
- Cosine refers to the x coordinates
- Tangent refers to the slope

	cos	sin	tan
QI	+	+	+
QII	-	+	-
QIII	-	-	+
QIV	+	-	-

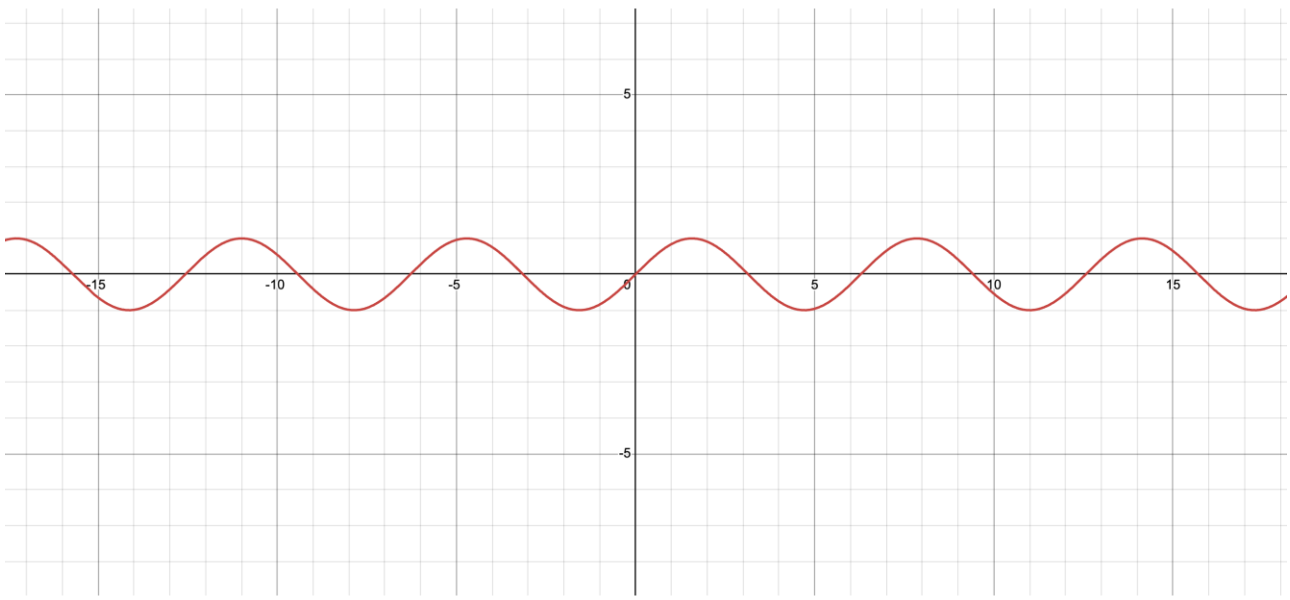
Reference Angel

- Let theta be an angel in standard position. Its reference angel is the **acute angel** formed by the terminal side of theta and the **horizontal axis**



7.3 Graphs of Sine and Cosine Functions

Sine Curve

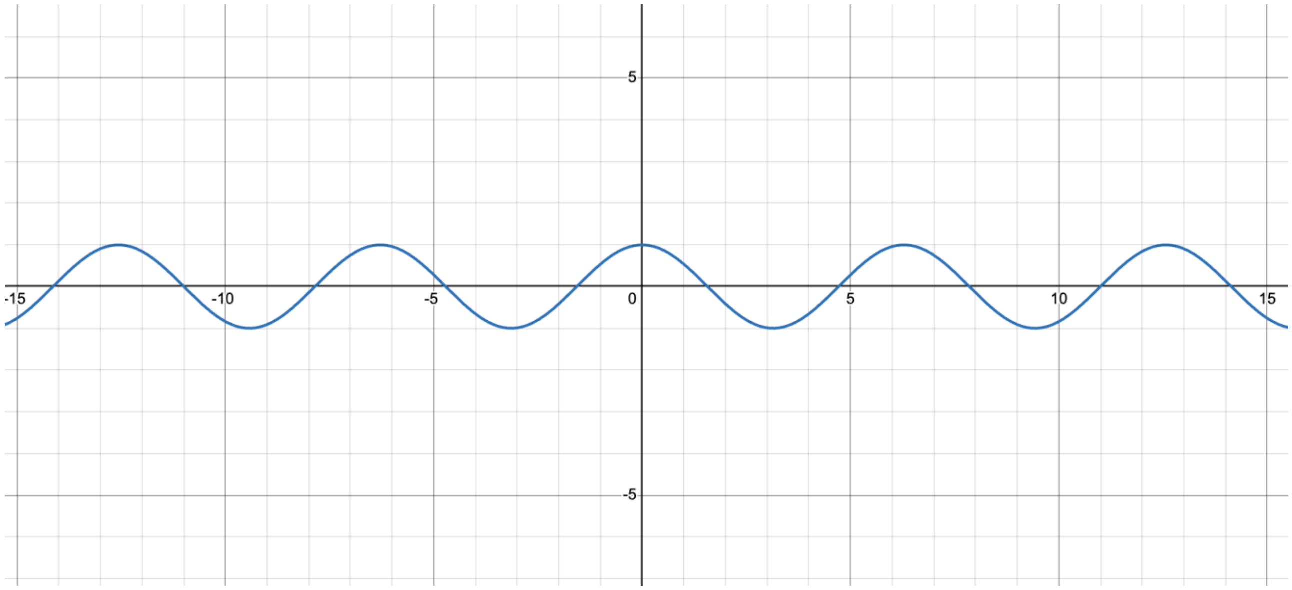


- One cycle of the sine curve represents one period of the function
- Repeats infinitely in the positive and negative directions
- Domain: all real numbers
- Range: [-1,1]
- Amplitude: 1
- Period: $2k\pi$ (k is positive integer)
- Frequency: $1/(2\pi)$
- Symmetric
  - odd function

- line: $x=(2k+1)\pi/2$ k is integer
- point: $(k\pi,0)$ k is integer

- Concavity
  - concave up: $((2k-1)\pi, 2k\pi)$
  - concave down: $(2k\pi, (2k+1)\pi)$

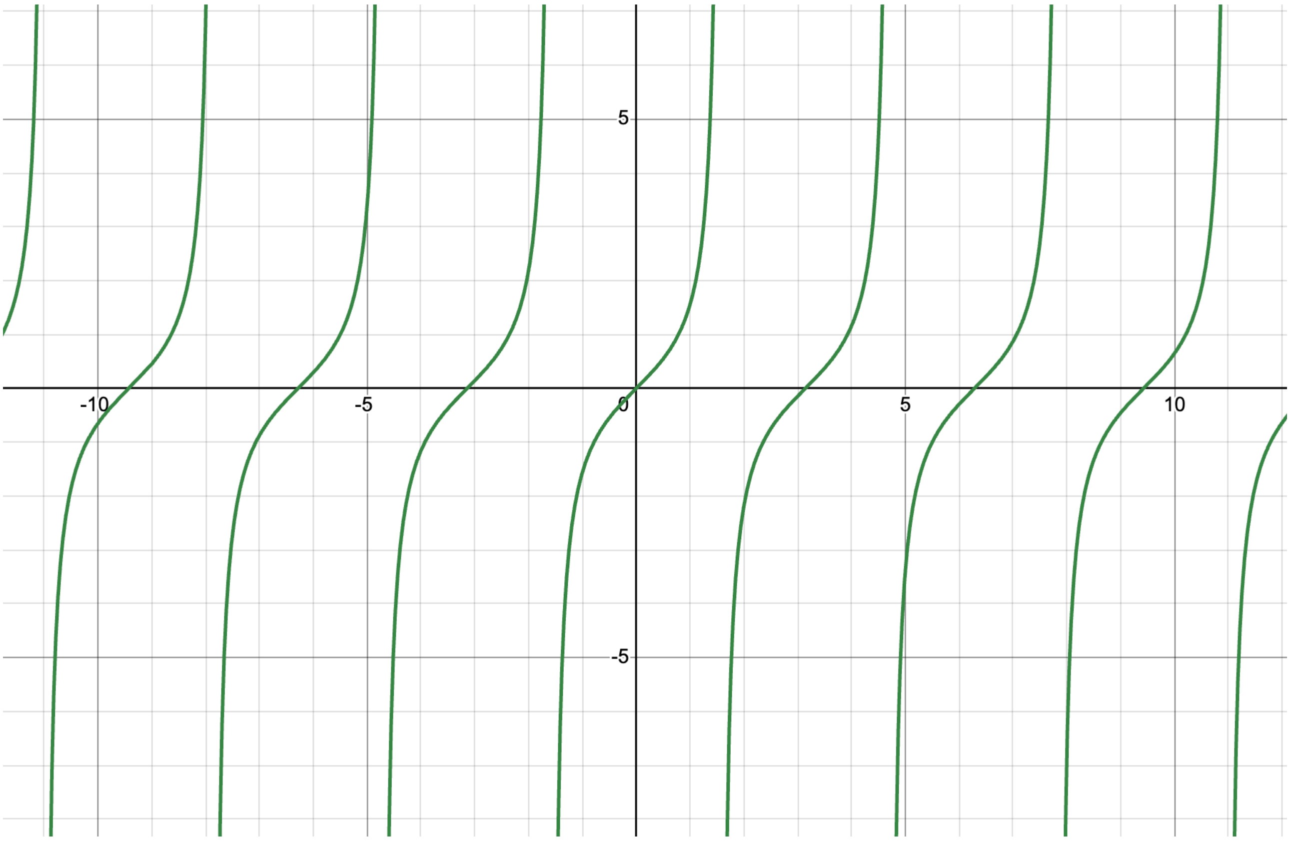
Cosine function



- One cycle of the cosine curve represents one period of the function
- Repeats infinitely in the positive and negative directions
- Domain: all real numbers
- Range: [-1,1]
- Amplitude: 1
- Period: $2k\pi$ (k is positive integer)
- Frequency: $1/(2\pi)$
- Symmetric
  - even function
  - line: $x=k\pi$ k is integer
  - point: $((2k+1)\pi/2 ,0)$ k is integer
- Concavity
  - concave up: $(2k\pi-3\pi/2, 2k\pi-\pi/2)$
  - concave down: $((2k+1)\pi-3\pi/2, (2k+1)\pi-\pi/2)$

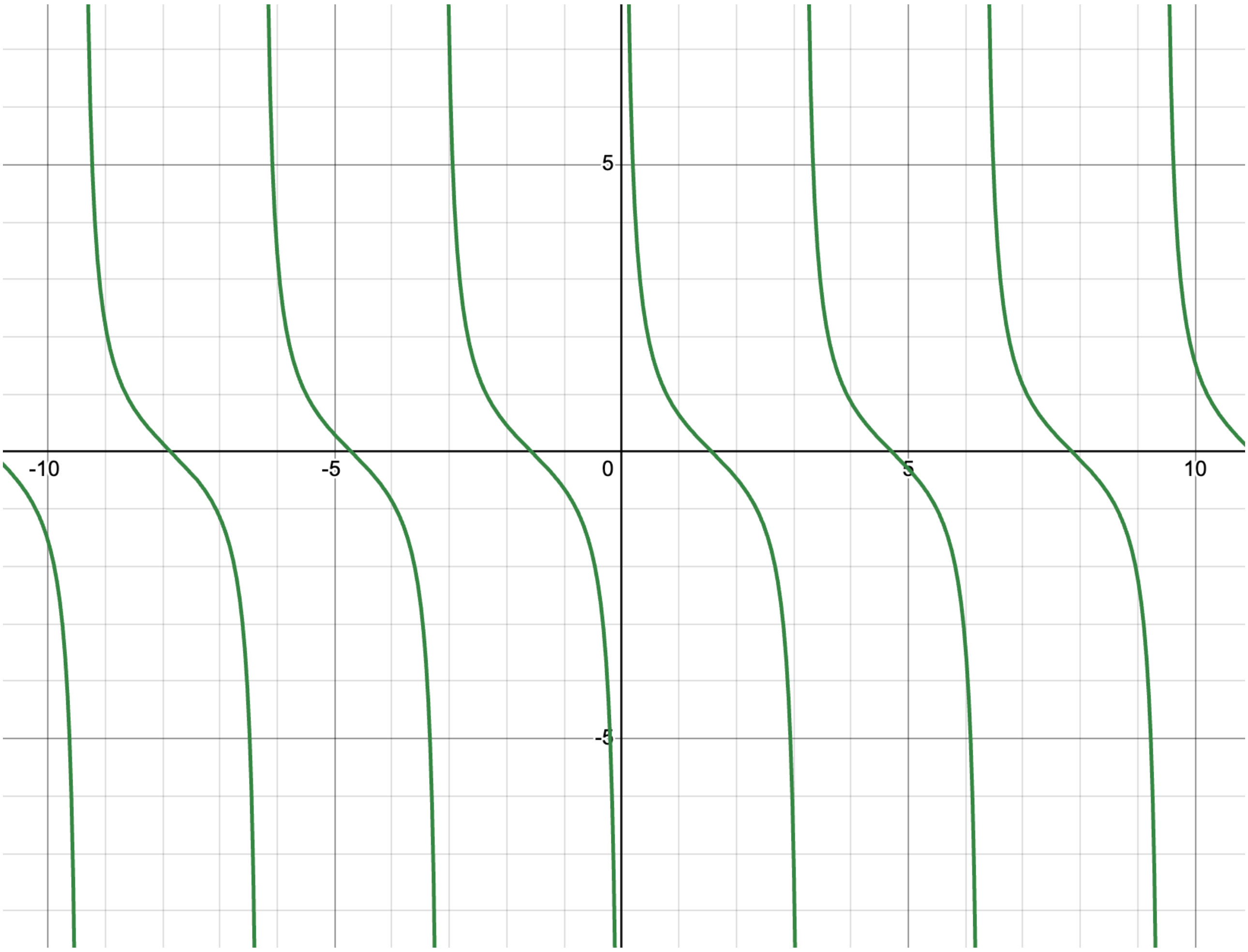
7.4 Other Trigonometric Functions

Tangent Function



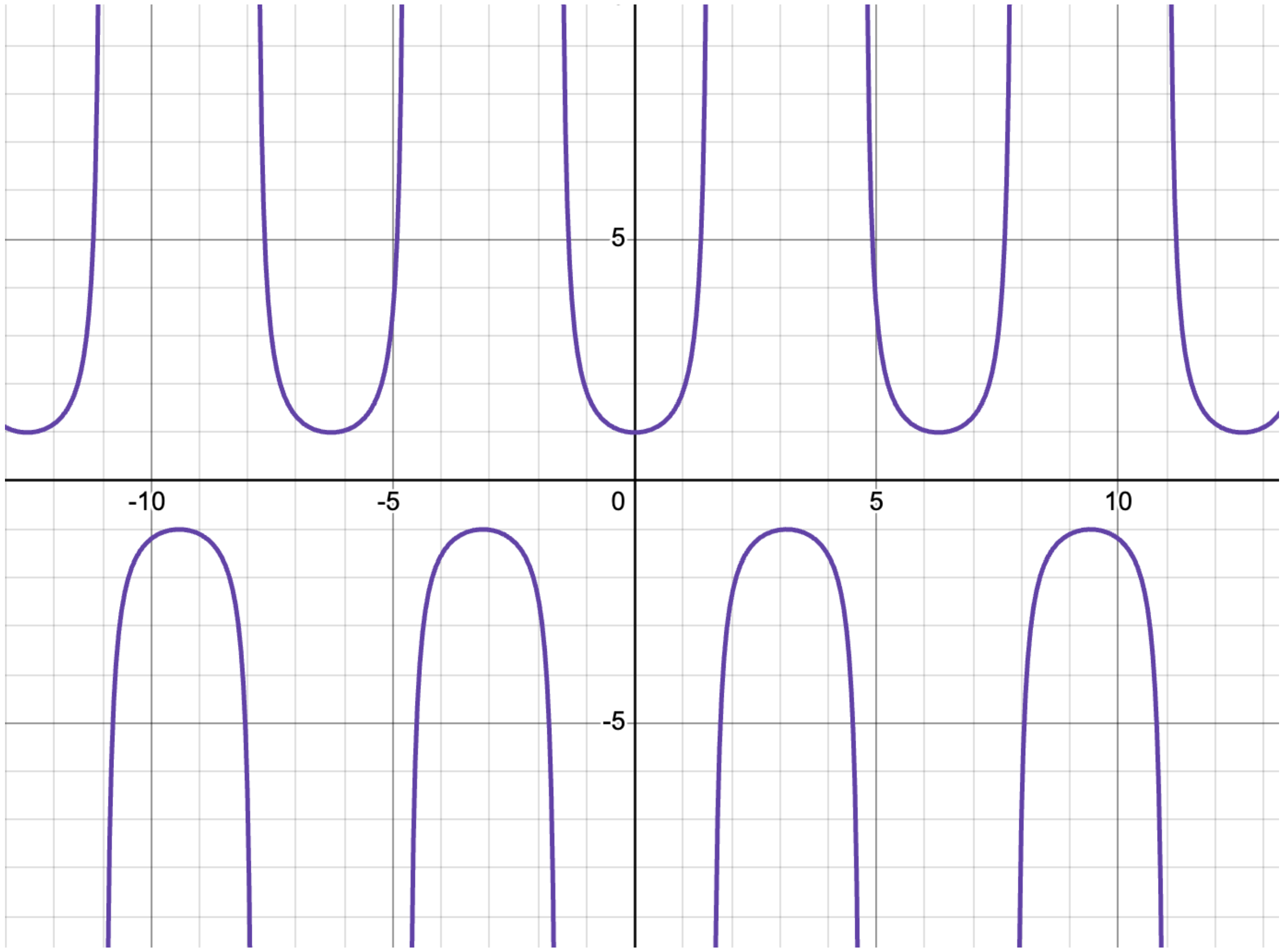
- Domain: {x| x≠(2k+1)π/2}
- Range: all real numbers
- V.A: x=(2k+1)π/2
- Period: π
- Frequency: 1/π
- zero/ point of symmetry: (kπ,0)
- Increasing: (π/2+kπ, π/2+(k+1)π)
- Odd Function
- Concavity
    - concave up: (kπ, π/2+kπ)
    - concave down: (π/2+kπ, (k+1)π)

Cotangent Function



- Domain: $\{x|\ x \neq k\pi\}$
- Range: all real numbers
- V.A: $x=k\pi$
- Period: π
- Frequency: $1/\pi$
- zero/ point of symmetry: $((2k+1)\pi/2,0)$
- Decreasing: $(k\pi, (k+1)\pi)$
- Odd Function
- Concavity
  - concave up: $(k\pi, \pi/2+k\pi)$
  - concave down: $(\pi/2+k\pi, (k+1)\pi)$

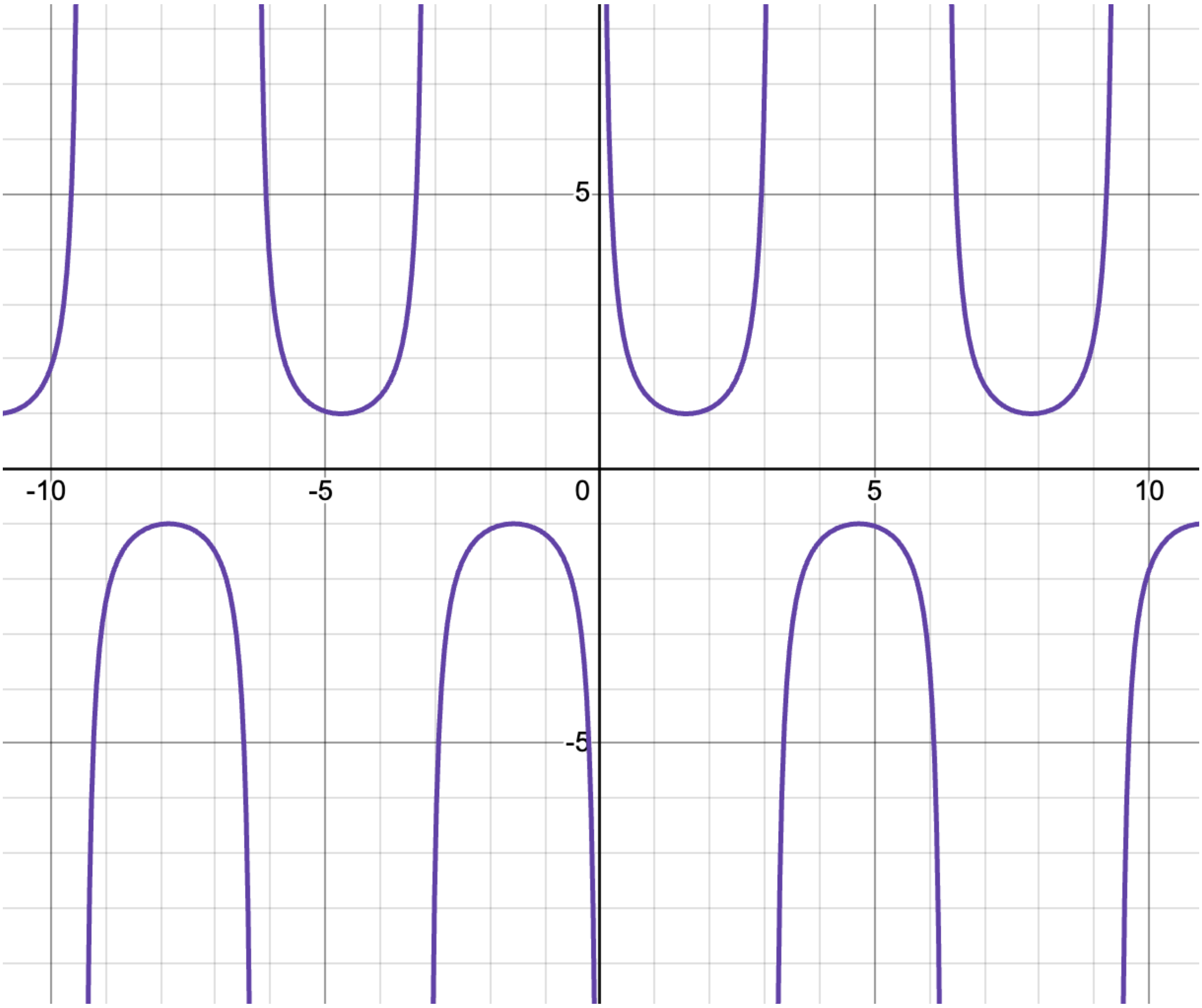
Cosecant Function



- Domain: {x| x≠(2k+1)π/2}
- Range: {y≥1 or y≤1}
- V.A: x=kπ+2/π
- Period: 2π
- Frequency: 1/(2π)
- point of symmetry: ((2k+1)π,0)
- Increasing:
  - [2kπ, 2kπ+π/2]
  - [2kπ+π/2, 2kπ+π]
- Decreasing:
  - [2kπ+π, 2kπ]
  - [2kπ+π/2, 2kπ+3π/2]
- Even Function
- Concavity

- concave up: (2kπ-π/2, 2kπ+π/2)
- concave down: (2kπ+π/2, 3π/2+2kπ)

Secant Function



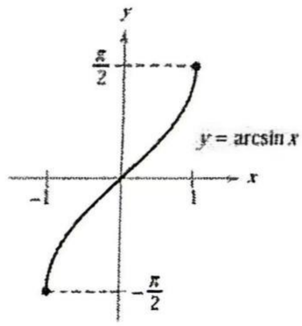
- Domain: {x| x≠kπ}
- Range: {y≥1 or y≤1}
- V.A: x=kπ
- Period: 2π
- Frequency: 1/(2π)
- point of symmetry: (kπ,0)
- line of symmetry: x=(2k+1)π/2
- Increasing:
  - [2kπ-π, 2kπ-π/2]
  - [2kπ+π/2, 2kπ+π]

- Decreasing:
  - [2kπ-π, 2kπ]
  - [2kπ, 2kπ+π/2]
- Odd Function
- Concavity
  - concave up: (2kπ, (2k+1)π)
  - concave down: ((2k+1)π, (2k+2)π)

7.5 The Inverse Trigonometric Functions

➢ The Sine Function and the Inverse Sine Function

The domain of $y = \sin x$ is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$. And the corresponding range is $[-1, 1]$.

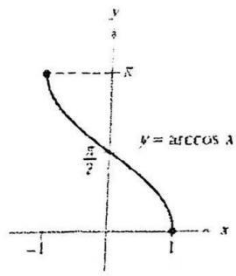


The inverse of $y = \sin x$ is called the inverse sine function, written $y = \sin^{-1}x$ or $y = \arcsin x$, and is defined as follows: $y = sin^{-1}x$ if and only if $sin\, y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

The domain of $y = \sin^{-1}x$ is $[-1, 1]$, and the range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

➢ The Cosine Function and the Inverse Cosine Function

The domain of $y = \cos x$ is restricted to $[0, \pi)$. And the corresponding range is $[-1, 1)$

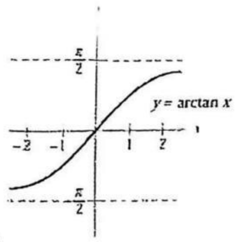


The inverse of $y = \cos x$ is called the inverse cosine function, written $y = \cos^{-1}x$ or $y = \arccos x$, and is defined as follows: $y = cos^{-1}x$ if and only if $cos\, y = x$ and $0 \le y \le \pi$

The domain of $y = \cos^{-1}x$ is $[-1, 1)$, and the range is $(0, \pi]$.

➢ The Tangent Function and Inverse Tangent Function

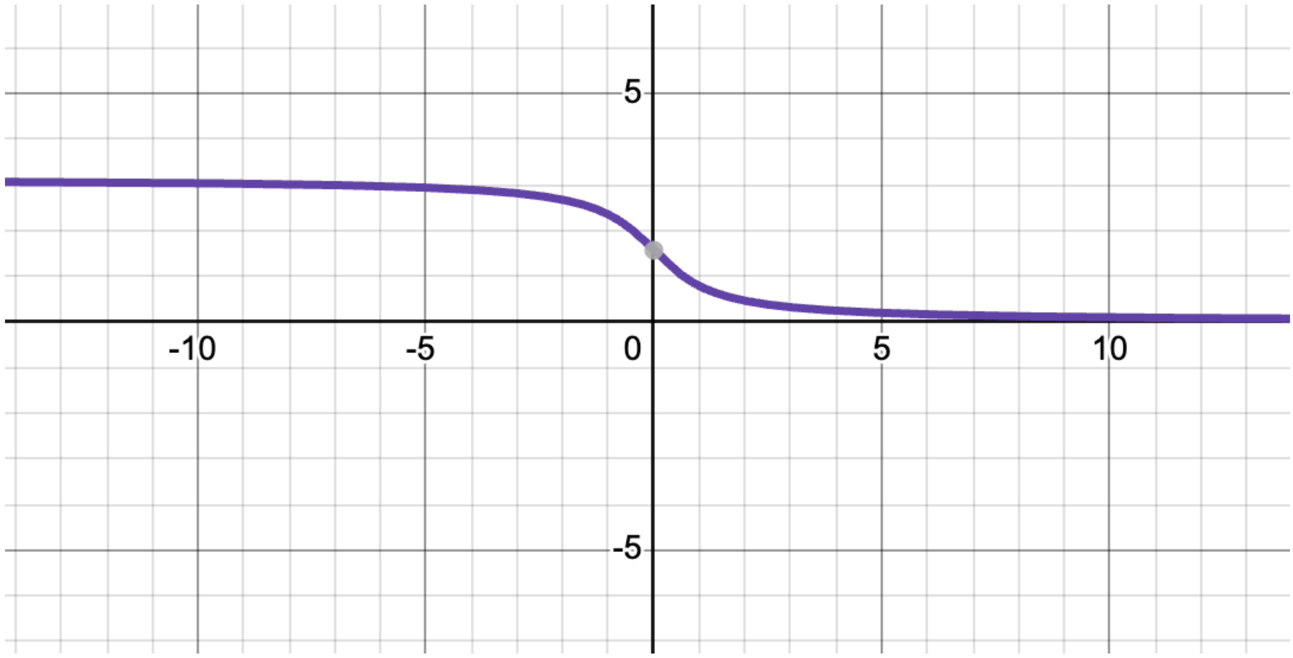
The domain of $y = \tan x$ is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2})$, and the corresponding range is $\mathbb{R}$.



The inverse of $y = \tan x$ is written $y = \tan^{-1} x$ or $y = \arctan x$, and is defined as: $y = tan^{-1}x$ if and only if $tan\, y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

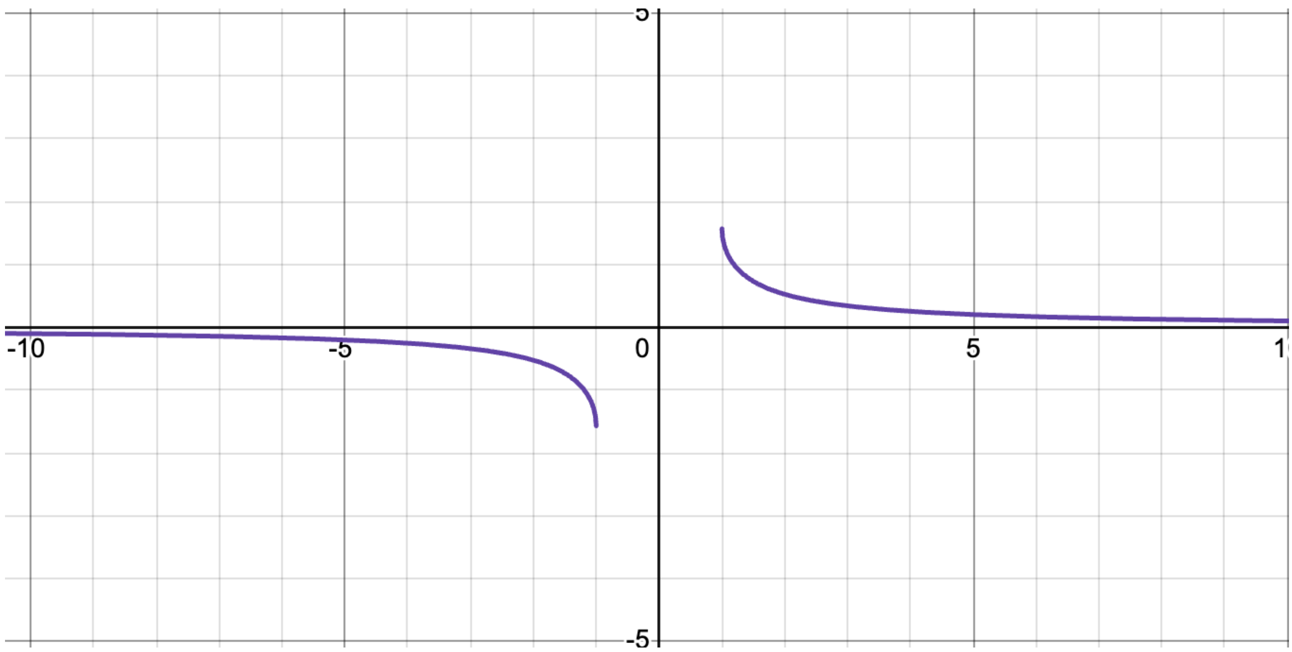
The domain of $y = \tan^{-1}x$ is $\mathbb{R}$, and the range is $[-\frac{\pi}{2}, \frac{\pi}{2})$

Inverse Function of Cotangent



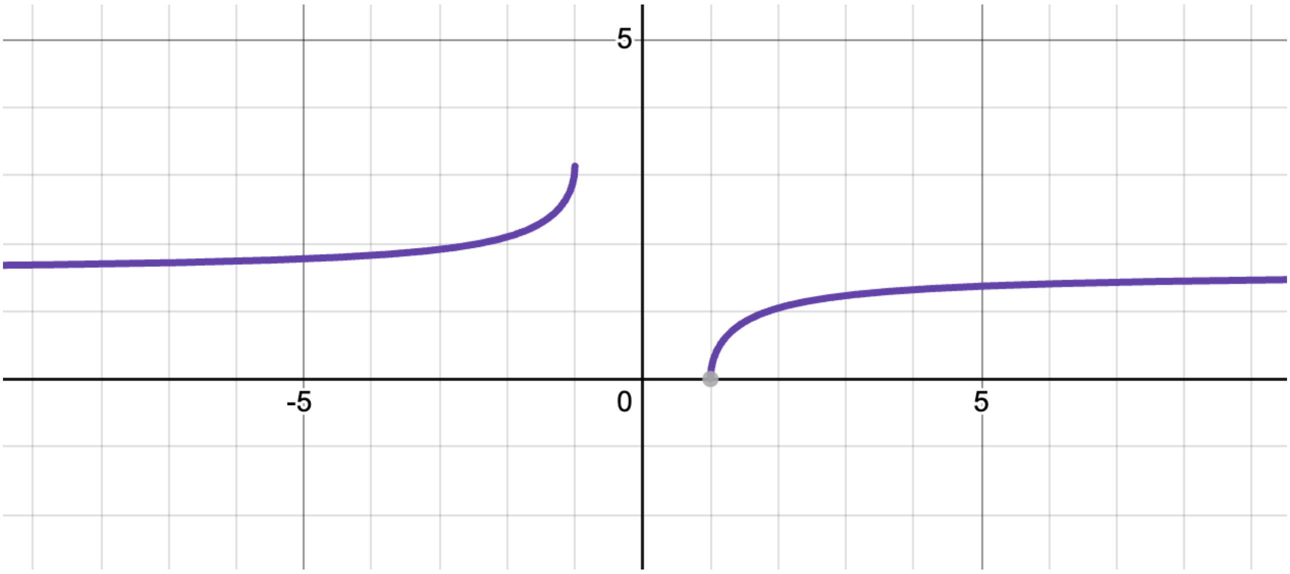
- y=cot(x)
  - Domain: [0,π]
  - Range: all real numbers
- y=Cot^{-1}(x)
  - Domain: all real numbers
  - Range: [0,π]

Inverse Function of Cosecant Function



- y=csc(x)
  - Domain: [-π/2,0) U (0, π/2]
  - Range: (-∞,-1] U [1,∞)
- y=Csc^{-1}(x)
  - Domain: (-∞,-1] U [1,∞)
  - Range: [-π/2,0) U (0, π/2]

Inverse Function of Secant Function



- y=Sec(x)
  - Domain: [0,π/2) U (π/2, π]
  - Range: (-∞,-1] U [1,∞)
- y=sec^{-1}(x)
  - Domain: (-∞,-1] U [1,∞)
  - Range: [0,π/2) U (π/2, π]