NOTE: This is an official document by Indexademics. Unless otherwise stated, this document may not be accredited to individuals or groups other than the club IDX, nor should this document be distributed, sold, or modified for personal use in any way.
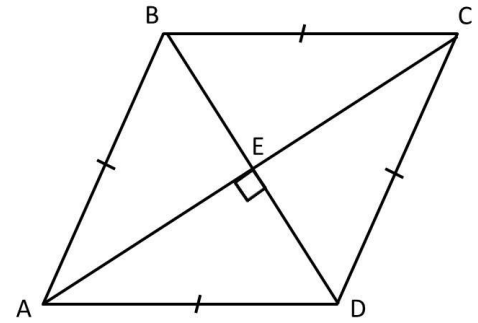
Contents:

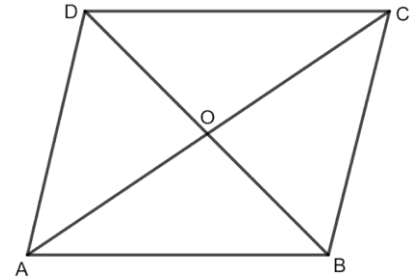3.1 Concepts and properties of polygons

- **Polygon**: a closed plane figure with at least 3 sides that are line segments
    - Equilateral polygon: polygon with all sides congruent
    - Equiangular polygon: polygon with all angles congruent
    - Regular polygon: polygon that is both equilateral and equiangular
    - A polygon is either concave or convex:
        - Concave polygon: polygon with at least one diagonal outside of it.
        - Convex polygon: polygon with no diagonals outside of it.
- **Vertices**: the intersection of endpoints in a polygon (number of vertices=number of sides, used to classify a polygon, ex. A polygon with n sides is a n-gon)
- **Diagonals**: the segments that connect two nonconsecutive vertices (for a n-gon: the number of diagonals is n(n-3)/2)
- **Interior angle sum theorem**: The sum of interior angles of an n-gon is $(n-2) \cdot 180°$

- **Exterior angle sum theorem**: The sum of exterior angles of an n-gon is 360°. (Derived by $n \cdot 180° - (n-2) \cdot 180° = 360°$)

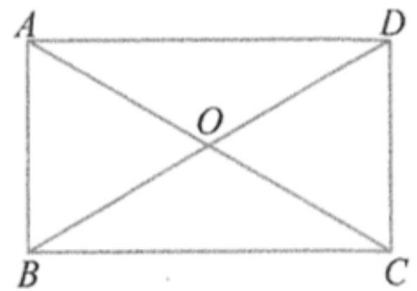## 3.2 Parallelograms

- Parallelogram: a quadrilateral with both pairs of opposite sides parallel.

- Properties of parallelogram:
    - AB ∥ CD, AD ∥ BC
    - $AB = CD$, $AD = BC$ (proved by AAS)
    - $\angle A = \angle C$, $\angle B = \angle D$ (use alternate interior angle theorem to prove)
    - AC, BD bisect each other (Prove △AOD≅△COB)

- Determination of parallelogram (these conditions can prove that ABCD is a parallelogram):
    - AB ∥ CD, AD ∥ BC
    - $AB = CD$, $AD = BC$
    - $AB \parallel CD$, $AB = CD$ (one pair of congruent, parallel side)
    - $\angle A = \angle C$, $\angle B = \angle D$
    - AC, BD bisect each other
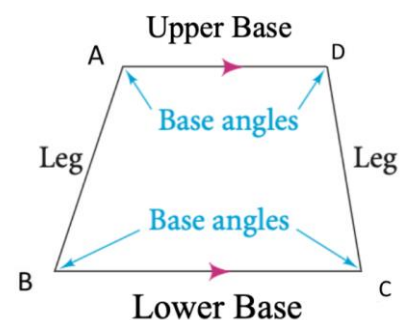
3.3 Special parallelogram

- Rectangle: a parallelogram with four right angles.
    - Properties of rectangle:
        - $\angle A = \angle B = \angle C = \angle D = 90°$
        - $AC = BD$, $AO = BO = CO = DO$
    - Determinations of rectangle:
        - $\angle A = \angle B = \angle C = 90°$ (three right angles)
        - If ABCD is a parallelogram: $AC = BD$
- Rhombus: a parallelogram with four congruent sides.
    - Properties of rhombus:
        - $AB = BC = CD = DA$

O

- $AC \perp BD$ (so $S_{ABCD} = \frac{1}{2}AC \cdot BD$)

- $\angle BAO = \angle DAO, \angle ABO = \angle CBO, \angle BCO = \angle DCO, \angle CDO = \angle ADO.$
  (diagonals are angle bisectors)

  o Determinations of rhombus:

  - $AB = BC = CD = DA$

  - A parallelogram with perpendicular diagonals

- Square: a parallelogram with four congruent sides and four right angles.

  o Properties of square:

  - $\angle A = \angle B = \angle C = \angle D = 90°$

  - $AB = BC = CD = DA$

  - Square has the properties of both rhombus and rectangle

  o Determinations of square:

  - A rectangle with equal adjacent sides,

  - A rhombus with an right angle

3.4 Trapezoid

- Trapezoid: a quadrilateral with exactly one pair of parallel sides. Lower Base

- Isosceles trapezoid: A trapezoid with two congruent legs

  o Properties:

  - $AB = CD$

  - $\angle B = \angle C, \angle A = \angle D$

  - $AC = DB$

  - These above can also be the determination of trapezoid

- Right trapezoid: trapezoid with a right angle

- Mid-segment of trapezoid: E,F are the midpoint of AB and DC respectively

  o Properties:

  o $EF \parallel AD \parallel BC$

- $EF = \frac{1}{2}(AD + BC)$ (apply triangle mid-segment theorem to prove)
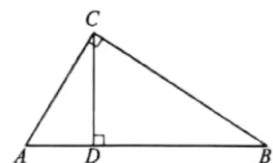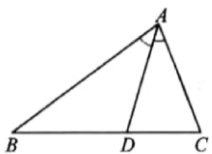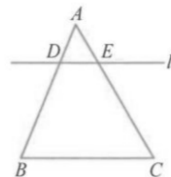
4.1 Ratio and Proportions

- Ratio: comparison between two quantities: a to b ($a: b$ or $\frac{a}{b}$)

- Proportion: a statement that two ratios are equal: a, b, c, d are proportional ($\frac{a}{b} = \frac{c}{d}$ or $a: b = c: d$)
  - A and d are extremes, b and c are means
  - If $\frac{a}{b} = \frac{b}{c}$, then b is the geometric mean of a and c

- Properties of proportion (given $\frac{a}{b} = \frac{c}{d}$):
  - $ad = bc$
  - $\frac{a+b}{b} = \frac{c+d}{d}$
  - $\frac{a+c}{a-c} = \frac{b+d}{b-d}$
  - $\frac{a+c}{b+d} = \frac{a}{b} = \frac{c}{d}$
  - $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n} = k$, $b1 + b2 + \cdots + bn \neq 0$, then $\frac{\sum_{i=1}^{n} k_i a_i}{\sum_{i=1}^{n} k_i b_i} = \frac{a_m}{b_m} = k$

- Point of golden ratio: P is on AB, AP>AB, such that AP is the geometric mean of AB and PB

4.2 Similar Triangles

- Similar triangle: corresponding angle congruent, corresponding sides proportional (this ratio is the similar ratio)

- Theorems:

  - Line splitter theorem: $l \parallel$ AB, it divides AB and AC proportionally
  - Euclidean theorem: $CD^2 = AD \cdot BD$, $AC^2 = AD \cdot AB$, $BC^2 = BD \cdot AB$
  - Triangle interior angle bisector theorem: If AD bisects $\angle BAC$, then $\frac{AB}{AC} = \frac{BD}{DC}$ (converse also true)
  - Triangle exterior angle bisector theorem: If AD bisects the exterior angle of $\angle BAC$ and $AD \cap BC = D$, then $\frac{AB}{AC} = \frac{BD}{DC}$ (converse also true)

- Determination of similar triangles ($\triangle ABC \sim \triangle A'B'C'$):

  - $\angle A = \angle A'$, $\angle B = \angle B'$ (AA~)

  - $\angle A = \angle A'$, $\dfrac{AB}{A'B'} = \dfrac{AC}{A'C'}$ (SAS)

  - $\dfrac{AB}{A'B'} = \dfrac{AC}{A'C'} = \dfrac{BC}{B'C'}$ (SSS)

  - $\triangle ABC$ and $\triangle A'B'C'$ are both right triangles, $\angle C = \angle C' = 90°$: $\dfrac{AB}{A'B'} = \dfrac{AC}{A'C'}$ (HL)

- Properties of similar triangle:

  - The ratio between corresponding altitudes, medians, angle bisectors of two similar triangles are equal to the similarity ratio

  - The ratio between perimeters of two similar triangles is equal to the similarity ratio.

  - The ratio between areas of two similar triangles is equal to the square of similarity ratio.

- Triangle interior angle bisector theorem

  - In $\triangle ABC$, if AD bisects $\angle BAC$, $AD \cap BC = D$, then $\dfrac{AB}{AC} = \dfrac{BD}{DC}$

- Converse of triangle interior angle bisector theorem

  - In $\triangle ABC$, $AD \cap BC = D$, if $\dfrac{AB}{AC} = \dfrac{BD}{DC}$, then AD bisects $\angle BAC$.

- Triangle exterior angle bisector theorem

  - In $\triangle ABC$, if AD bisects the exterior angle of $\angle BAC$, $AD \cap BC = D$, then $\dfrac{AB}{AC} = \dfrac{BD}{DC}$

- Converse of triangle exterior angle bisector theorem

  - In $\triangle ABC$, $AD \cap BC = D$, if $\dfrac{AB}{AC} = \dfrac{BD}{DC}$, then AD bisects the exterior angle of $\angle BAC$.