IDX G9 Math H+

Study Guide S1 Finals

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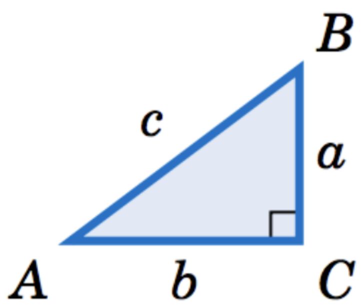
Contents:

4.3 Trigonometric ratios in acute angles

- Trigonometric ratios
    - $sin\angle A = \frac{a}{c}$
    - $cos\angle A = \frac{b}{c}$
    - $tan\angle A = \frac{a}{b}$
    - $cot\angle A = \frac{b}{a}$



- Properties:

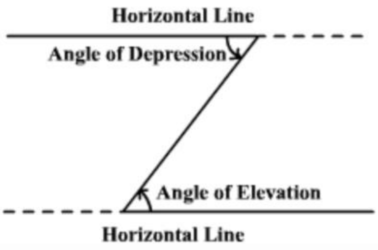
- $tan\angle A = \frac{1}{cot\angle A}$
- $tan\angle A = \frac{sin\angle A}{cos\angle A}$
- $sin^2\angle A + cos^2\angle A = 1$ (Pythagorus theorem)
- $sin\angle A = cos\angle B, tan\angle A = cot\angle B$

- Area of a triangle in terms of trigonometric ratio:

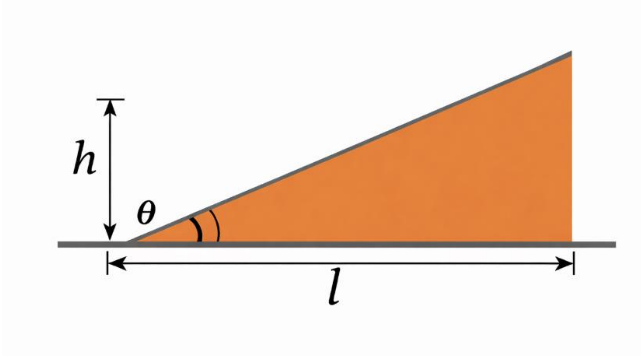
- $S_{\triangle ABC} = \frac{1}{2}bcsinA = \frac{1}{2}acsinB = \frac{1}{2}absinC$ ($csinA$ is equal to a, same for $csinB$, and $sinC =$ 1)

- Law of sine:
  - $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$ (use the above formula to prove)

<u>4.4 Application in trigonometry</u>



- Angle of elevation and angle of depression
- Course of a ship: measured in clockwise direction
- Compass bering: the acute angle from the north/south direction to the east/west direction. Ex: N50˚E
- Gradient: $i = \frac{h}{l} = tan\theta$

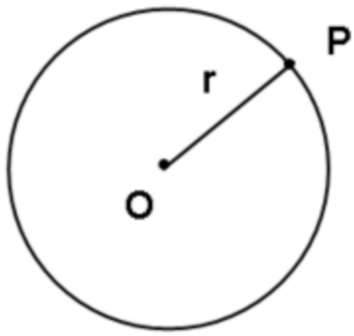


<u>Transformations</u>

- Concepts:
  - Transformation: a change in a geometric figure's position, size or shape
  - Pre-image: Original figure
  - Image: resulting figure
  - Isometry: pre-image and image are the same (includes reflection, translation, and rotation)
  - Transformation notation: used to describe the mapping process: ex. (x, y) → (x+h, y+k)
  - Point notation: used to identify image points: ex. A → A'
- Transformations:
  - Translation(slide): an isometry that maps all points of a figure the same **distance** in the same **direction**.
  - Composition of transformations: a combination of two or more transformations
  - Reflection (flip): an isometry in which a figure and its image have **opposite orientation**.

Line of reflection r: If a point B is not on r then it is the perpendicular bisector of the segment joining B and its image B'. (used to identify image)

- Rotation (turning): an isometry in which each point is turned by the same angle about a fixed point (center of rotation)



described as number of degree clockwise/counterclockwise

- Symmetry (used to describe a figure, not a transformation): A figure has symmetry if there is an isometry that maps the figure onto itself
- Types of symmetry:

  - Line symmetry/ reflectional symmetry: the isometry is reflection. The line of reflection is the axis of symmetry of the figure.
  - Rotational symmetry: a image has is its own image for at least one rotation of less than 360°. (Point symmetry: 180° rotational symmetry)

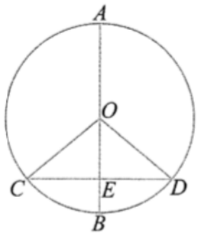
- Dilation: is a transformation whose pre-image and image are **similar**. In general, it is **not** an isometry.

  - Every dilation has a **center** and a **scale factor** n, where n>0.
    - If n > 1, the dilation is an enlargement.
    - If 0 < n < 1, the dilation is a reduction.
    - For any point R, R' is on ray CR, and CR′ = n·CR

6.1 Concepts and properties of circles

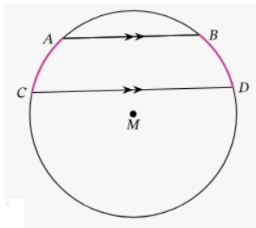
- Circle: The set of points equidistant from a fixed point (Center of the circle)
  - Radius: distance from any point on the circle to the center
  - Circle with center O is denoted as $\odot\ O$
- Properties of circle:
  - Congruent circles have congruent radii.
- Relationship between point and circle:
  - If P is the interior of $\odot\ O$, then $|PO| < r$
  - If P is on $\odot\ O$, then $|PO| = r$
  - If P is the interior of $\odot\ O$, then $|PO| > r$

- Arc: part of a circle
  - Minor arc: smaller than a semicircle
  - Major arc: greater than a semicircle
- Chord: A segment with end points on a circle (chord through the center is the diameter)
- Central angle: An angle whose vertex is the center of the circle (The measure of the arc intercepted by a central angle is the measure of this central angle, ex. $m\angle AOB = m\widehat{AB}$)
  - Central angle theorem: If two central angles in the same circle are congruent, then their intercepted arcs, intercepted chords, and their apothems are respectively congruent.

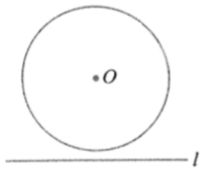


6.2 Chords and arcs

- Perpendicular diameter theorem: If a diameter is perpendicular to a chord, then it bisects the chord and its corresponding arcs (Proved by HL)

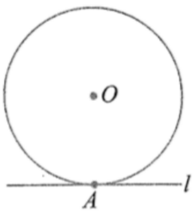


  - Corollary: The arcs included (intercepted) by two parallel chords are congruent. ($\widehat{AB} = \widehat{CD}$)

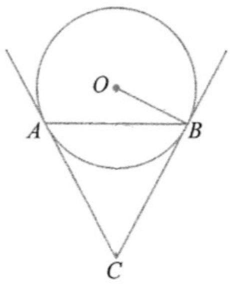


6.3 Relationship between lines and circles

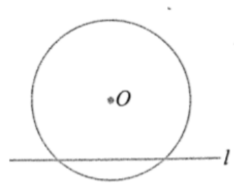
- No intersection: the line is exterior of the circle
- Exactly one point of intersection: the line is tangent to the circle. (The line: tangent line, the intersection point: point of tangency).



  - A line through the center is perpendicular to the tangent line must pass through the point of tangency.
  - A line through the point of tangency that is perpendicular to the tangent line must pass through the center.

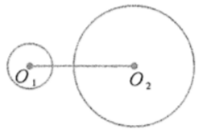


  - Through an exterior point of a circle, there are two tangent lines from this point to the point of tangency are congruent. The line through the center and this point bisects the angle formed by the two tangent lines.

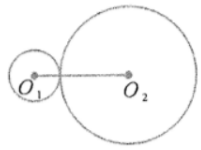


- Two points of intersection: The line intersects the circle. (The line: secant line)
- Relationship between two circles:

o External

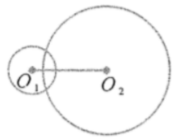


o Externally tangent



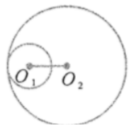
▪ The line connecting two centers of tangent circles passes through the point of tangency

o Intersect

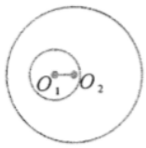


▪ The line connecting two centers of intersecting circles is the perpendicular bisector of their common chord.

o Internally tangent



o Interior



o Concentric

- Common tangent: a line is tangent to two circles

o External: two circles are on the same side of the tangent line

o Internal: two circles are on the different sides of the tangent line