IDX G10 AP Precalculus
Study Guide Issue S1 Midterm
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Contents:

4.1 Functions

Basic Concepts

- A **function** is a correspondence or rule that assigns to every element in a set exactly one element in a another set
- the **domain** is every x value that satisfies the function; **range** is every value of the function
- a function f mapping a domain element x to a range element f(x), read "the value of f at x" or "f of x"
- y=f(x) or {(x,y)|y=f(x)}

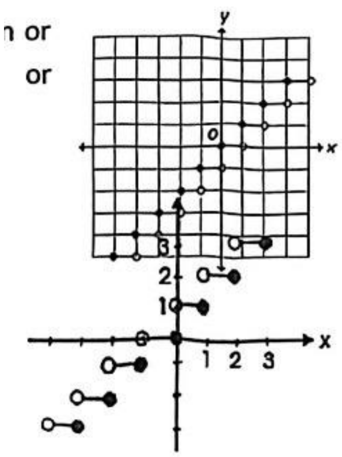
Graph

- In an xy-plane, the graph of f(x) consists of all points of (x,f(x))
    - The **domain** of y=f(x) is the set of x-coordinates of points on the graph of f. Projecting the graph of f on the x-axis gives a graph of the domain.
    - The **range** of y=f(x) is the set of the y-coordinated of points on the grph of f. Projecting the graph of f on the y-axis gives a graph of the range.

    - The **zeros** of the y=f(x) are the x-intercepts of the graph
- x is called the **independent variable**; y is called the **dependent variable**
- Functions are a subset of the more general class of correspondences called **relations**
    - $x^2y=4x-y$ is a function; $y^2=4x^2$ is a relation (can't pass the vertical line test)

The Greatest Integer Function

- Assigns to each number the greatest integer less than or equal to the number. y= $\lfloor x \rfloor$ or y=int(x)
- c(x)= $\lceil x \rceil$ is called the ceiling of x, and is the least integer greater than or equal to x.

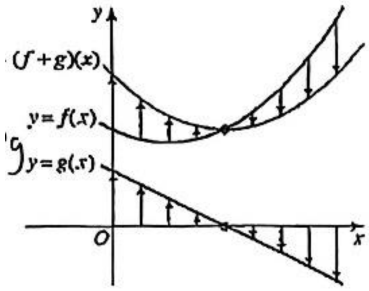


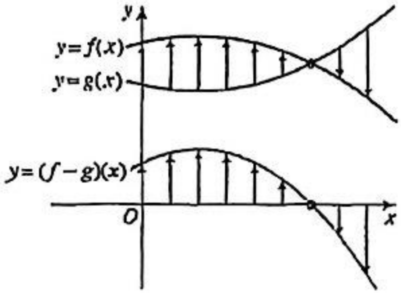
Equal Function

- Two functions f and g are equal functions if the following two conditions are satisfied:
    - The **domain** of f is equal to the domain of g
    - For all x in the domain, **f(x)=g(x)**

4.2 Operations on Functions

- Giving two functions f and g, new functions f+g, f-g, f*g, f/g can be formed by adding, subtracting, multiplying, and dividing
- The **domain** of each resulting function consists of those values of x **common** to the domain of f and g. The domain of the **quotient** function is further restricted by excluding all values that make g(x)=0
    - (f +or- g)(x)=f(x)+or-g(x) Df+or-g: Df ∩ Dg
    - (f * g)(x)=f(x) * g(x) Df * Dg: Df ∩ Dg
    - (f/g)(x)=f(x)/g(x) D(f/g): Df ∩ Dg ∩ {x|g(x)≠0}



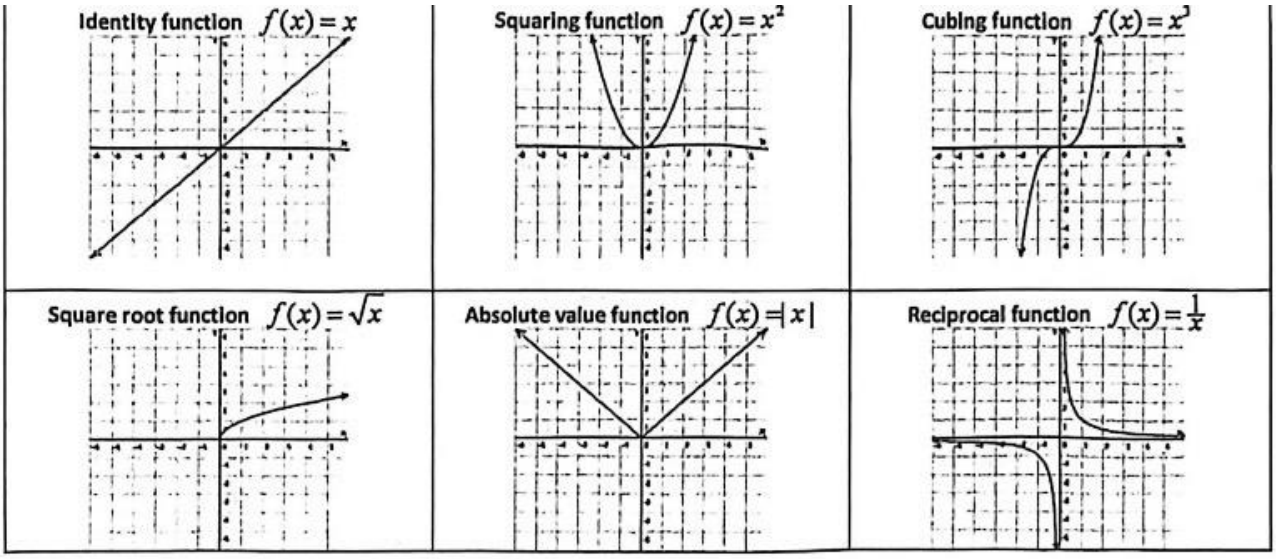


The Composite of Functions

- Another method of combining two functions is by successive application of the function in a specific order. This binary operation is called **composition of functions.**
- (f o g)(x)=f(g(x)), f circle g of x equals f of g of x
- The **domain** is the set of elements x in the domain of g such that g(x) is in the domain of f
- D(fog)={x| x Œ Dg and g(x) Œ Df}
- D(fogoh)={x| x Œ Dh and h(x) Œ Dg and g(h(x)) Œ Df}

4.3 Reflecting Graphs; Symmetry

Be familiar with these graphs



Transformation: reflections, translations and the original graphs

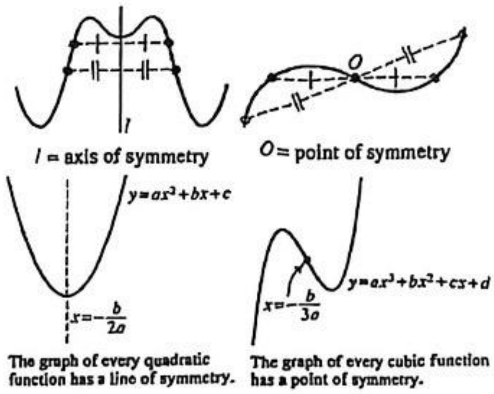
- Congruent graphs: Reflection, translations and the original graphs
- Same shapes: Dilation
- The **reflection** in the line t is the congruent graph symmetric to the original graph with respect to; the reflection is the **mirror image** of the graph where line t is the **mirror of the reflection**

- y=-f(x) reflect y=f(x) in the x-axis
- y=f(-x) reflect y=f(x) in the y-axis
- reflect about y=x, replace x by y, y by x

➢ ***Symmetry***

– A line l is called an **axis of symmetry** of a graph if it is possible to pair the points of the graph in such a way that l is the perpendicular bisector of the segment joining each pair.

– A point O is called a **point of symmetry** of a graph if it is possible to pair the points of the graph in such a way that O is the midpoint of the segment joining each pair.

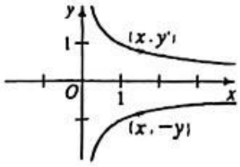


Special Tests for the Symmetry of a Graph

Symmetry in the x-axis

Replace (x,y) by $(x,-y)$

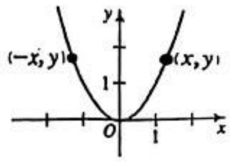
$y^2x=1$ and $(-y)^2x=1$ equivalent



Symmetry in the y-axis

Replace (x,y) by $(-x,y)$

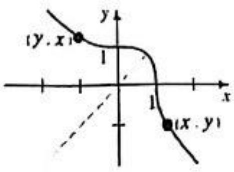
$y=x^2$ and $y=(-x)^2$ equivalent



Symmetry in the line y=x

Replace (x,y) by (y,x)

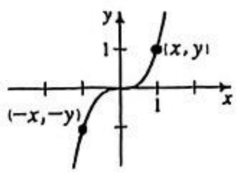
$x^3+y^3=1$ and $y^3+x^3=1$ equivalent



Symmetry in the origin

Replace (x,y) by $(-x,-y)$

$y=x^3$ and $-y=(-x)^3$ equivalent



Even and Odd function

- Even function: f(-x)=f(x)
- Odd function: f(-x)=-f(x)

Increasing and Decreasing

let f be a function defined on an interval I and let x_1 and x_2 be any two points in I

- f increases on I if $x_1 < x_2$, $f(x_1)<=f(x_2)$
- f decreases on I if $x_1 < x_2$, $f(x_1)>=f(x_2)$

- f increases strictly on I if $x_1 < x_2$,$f(x_1)<f(x_2)$
- f decreases strictly on I if $x_1 < x_2$, $f(x_1)>f(x_2)$

4.4 Periodic Functions; Stretching and Translating Graphs

Periodic Function

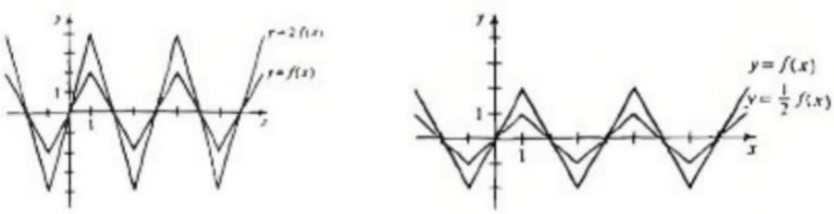
- A function f is periodic if there is a positive number p, called a **period** of f, such that $f(x+p)=f(x)$ for all x in the domain of f
- The smallest period of a periodic function is called the **fundamental** period of the function
- The definition of a periodic function implies that if f is a periodic function with period p, then $f(x)=f(x+mp)$ for all x and any integer m
- If f is a periodic function with maximum M and minimum m, the **amplitude** A of the function is **1/2(M-m)**

Stretching and Shrinking Graphs

➢ ***Stretching and Shrinking Graphs***

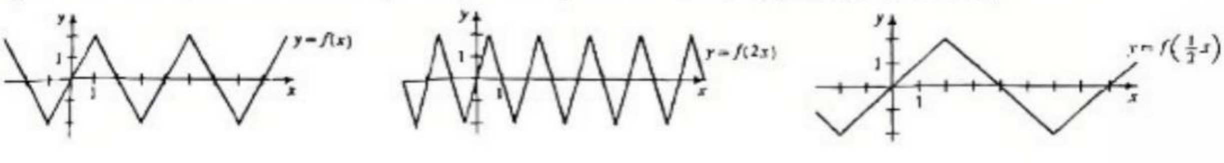
-- The graph of $y = cf(x)$ where c is positive (and not equal to 1) is obtained by ***vertically*** stretching or shrinking the graph of $y = f(x)$.

→ In the graphs below, notice that points on the x-axis for $c > 1$ (**a vertical stretch**) or toward the x-axis for $0 < c < 1$ (**a vertical shrink**).



-- The graph of $y = f(cx)$ where c is positive (and not equal to 1) is obtained by horizontally stretching or shrinking the graph of $y = f(x)$.

→ In the graphs below, notice that points on the y-axis remain fixed, while all other points move toward the y-axis for $c > 1$ (**a horizontal shrink**) or toward the y-axis for $0 < c < 1$ (**a horizontal stretch**).



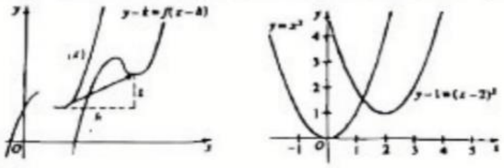
-- **Changing the Period and Amplitude of a Periodic Function**

If a periodic function f has period p and amplitude A, then:

$y = cf(x)$ has period p and amplitude cA, and $y = f(cx)$ has $\frac{p}{c}$ and amplitude A

➢ ***Translating Graphs***

The graph of $y - k = f(x - h)$ is obtained by translating the graph of $y = f(x)$ horizontally h units and vertically k units.



Some Practice Questions, tell to graph these functions using translating, stretching, and shrinking

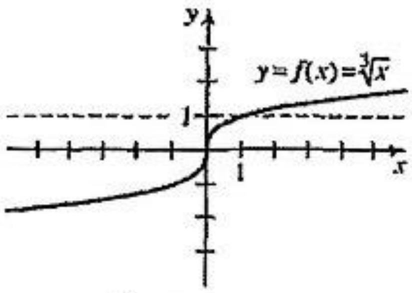
y=-2x+4

x=(1/3)y+2

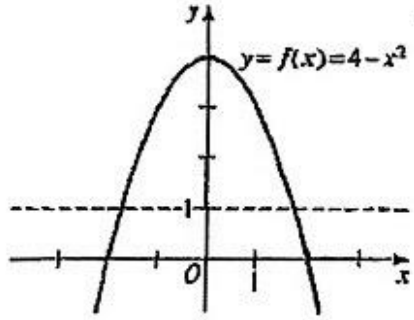
-3y=-x+7

4.5 Inverse Functions

- The inverse of a relation is a relation; but the inverse of a function **MAY NOT** be a function; But usually, we take the "inverse "as the "inverse function"
- A function f has an **inverse function** g if and only if
  - f(g(x))=x for all x in the domain of g(x)
  - g(f(x))=x for all x in the domain of f(x)
  - If two functions are inverse of each other, their composition give the identity function h(x)=x
- The inverse function of a function f is denoted f^{-1} or $f^{-1}(x)$. $(f \circ f^{-1})(x)=(f^{-1} \circ f)(x)=x$
- A function y=f(x) that gas an inverse is called a **one-to-one function,** because not only does each x-value correspond to exactly and its inverse are **symmetric** with respect to the line y=x
- The Horizontal-Line Test: If the graph of the function y=f(x) is such that no horizontal line intersects the graph in more than point, then f is one-to-one and has an inverse.



f is one-to-one and has an inverse.



f is not one-to-one and has no inverse.

4.7 Forming Functions from verbal descriptions

Finish and understand every question on the packet in enough

4.8 Radical Functions

After solving the equation, check whether the solutions are real roots or gaining roots

4.8+ Partial Functions

- To decompose numerical fractions: $\frac{3}{5}+\frac{1}{7}=\frac{3\cdot 7+5\cdot 1}{5\cdot 7}=\frac{26}{35}$; $\frac{26}{35}=\frac{A}{5}+\frac{B}{7}$ (where $7A+5B=26$).
- What is partial fraction?

$$\frac{2}{x-3}+\frac{5}{x+1}=\frac{2(x+1)+5(x-3)}{(x-3)(x+1)}=\frac{7x-13}{x^2-2x-3}$$

Partial fractions   ← *Decomposition*   A single fraction

- If a fraction with denominators that can be factored into ***linear factors*** and the degree of the numerator is ***less than*** the degree of the denominator, *how to decompose the fraction?*

1. To factor the denominator	$x^2-2x-3=(x-3)(x+1)$
2. Express as two fractions using A and B for the numerators.	$\frac{7x-13}{x^2-2x-3}=\frac{A}{x-3}+\frac{B}{x+1}$
3. Multiply by the least common denominator.	$7x-13=A(x+1)+B(x-3)=(A+B)x+(A-3B)$
4. Solve for A and B.	① $\begin{cases} 7=A+B \\ -13=A-3B \end{cases}$; ② $\begin{cases} x=3: 7\cdot 3-13=A(3+1)+B\cdot 0 \Rightarrow A=2 \\ x=-1: 7\cdot(-1)-13=A\cdot 0+B\cdot(-1-3) \Rightarrow B=5 \end{cases}$
5. Express with partial fractions	$\frac{7x-13}{x^2-2x-3}=\frac{2}{x-3}+\frac{5}{x+1}$

- The denominator must be factored into three non-repeating linear factors
- In partial fraction decomposition, a linear factor (x-a) is repeated n times. If $(x-a)^n$ is a factor of the denominator, then the partial fraction decomposition must include.
- Synthetic division can be used to identify the zeros of the denominator