IDX G9 Math S+

Study Guide Issue S1 Monthly 2

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Contents:

Congruent Figures

- Congruent polygons have congruent corresponding parts-their matching sides and angles. Matching vertices are corresponding vertices. When you name congruent polygons, always list corresponding vertices in the same order.
    - Theorem 4-1
        - If two angles of one triangle are congruent to two angles of another triangles, then the third angles are congruent.

Triangle Congruence by SSS and SAS

- Postulate 4-1 Side-Side-Side (SSS) Postulate
    - If the three sides of one triangle are congruent to the three side of another triangle, then the two triangles are congruent.
- Postulate 4-2 Side-Angle-Side (SAS) Postulate
    - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Triangle Congruence by ASA and AAS

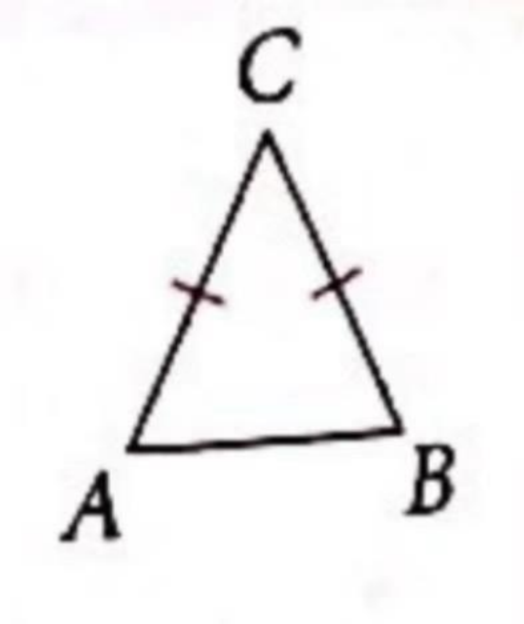
- Postulate 4-3 Angle-Side-Angle (ASA) Postulate
    - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
- Theorem 4-2 Angle-Angle-Side Theorem
    - If the angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

Using Congruent Triangles: CPCTC

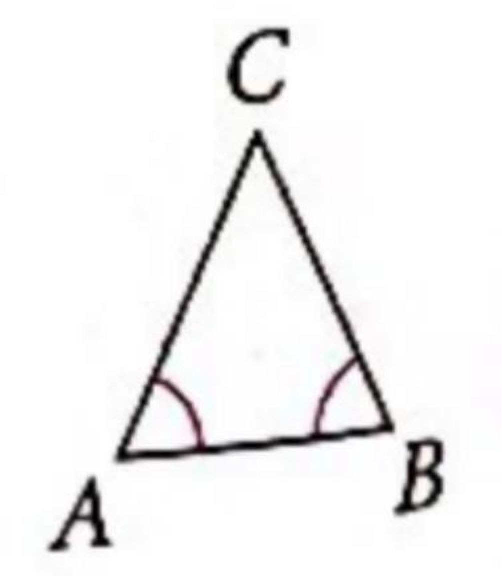
- CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
    - With SSS, SAS, ASA, and AAS, you know how to use three parts of triangles to show that the triangles are congruent. Once you have triangles congruent, you can make conclusions about their other parts because, by definition, corresponding parts of congruent triangles are congruent. You can abbreviate this as CPCTC.

Isosceles and Equilateral Triangles

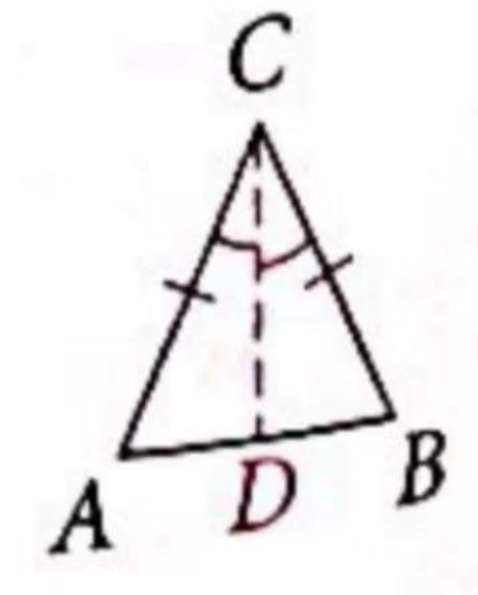
- The congruent sides of an isosceles triangle are its legs. The third side is the base. The two congruent sides form the vertex angle. The other two angles are the base angles.
- Theorem 4-3 Isosceles Triangle Theorem
    - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.



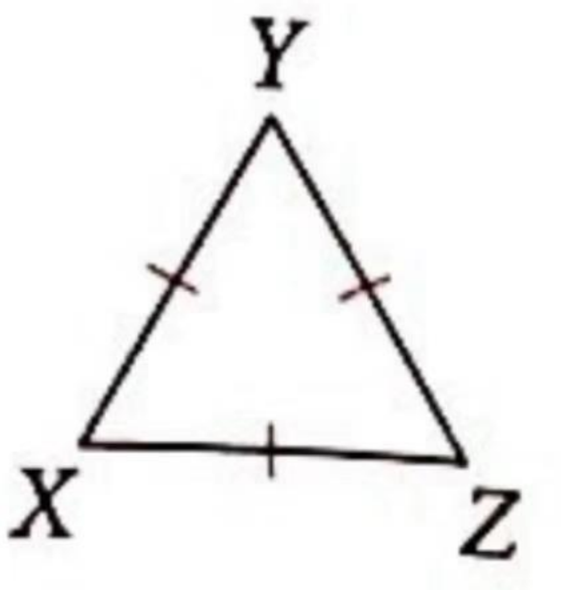
- Theorem 4-4 Converse of Isosceles Triangle Theorem
    - If two angles of a triangle are congruent, then the sides opposite the angles are congruent.



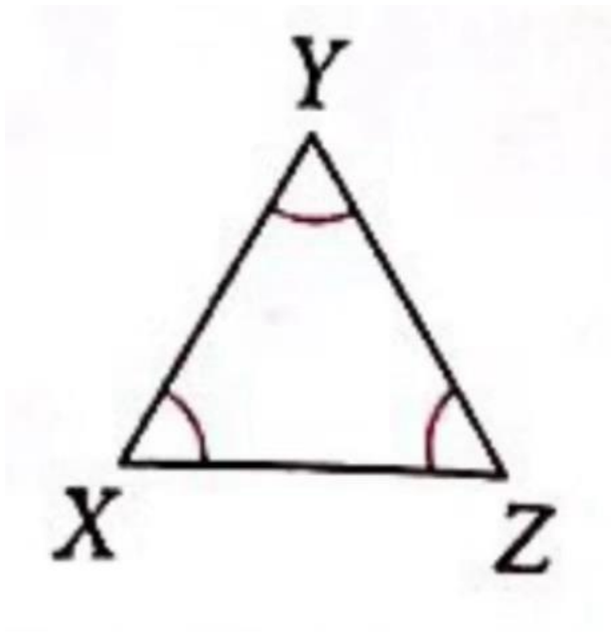
- Theorem 4-5 The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.



- A corollary is a statement that follows immediately from a theorem.
- Corollary to Theorem 4-3
    - If a triangle is equilateral, then the triangle is equiangular.

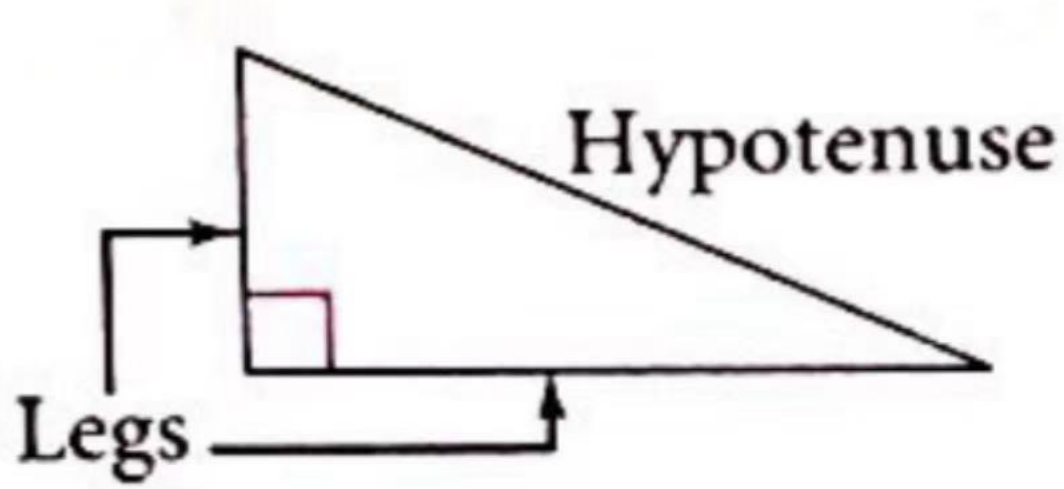


- Corollary to Theorem 4-4
    - If a triangle is equiangular, then the triangle is equilateral.



Congruence in Right Triangles

- In a right triangle, the side opposite the right angle is the longest side and is called the hypotenuse.
- The other two sides are called legs.

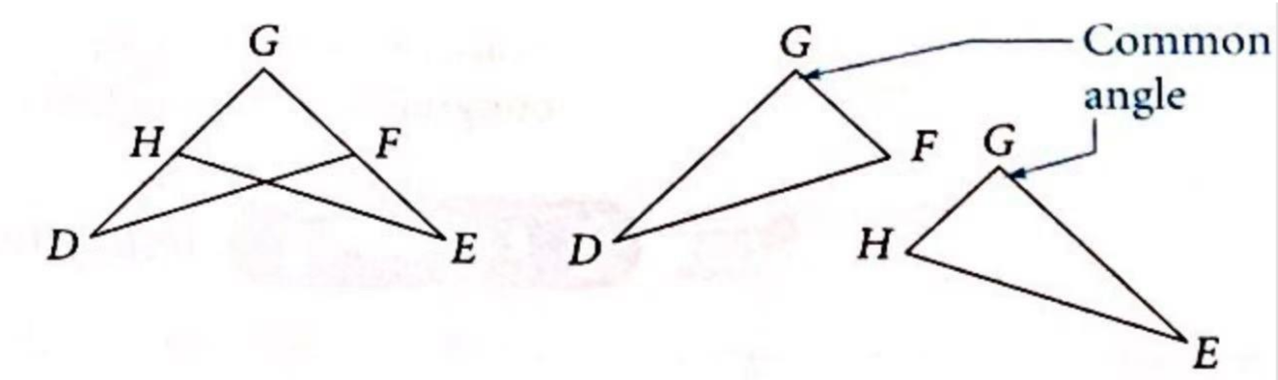


- Right triangles provide a special case for which there is an SSA congruence rule.It occurs when hypotenuses are congruent and one pair of legs are congruent.
- Theorem 4-6 Hypotenuse-Leg (HL) Theorem
  - If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Using Corresponding Parts of Congruent Triangles

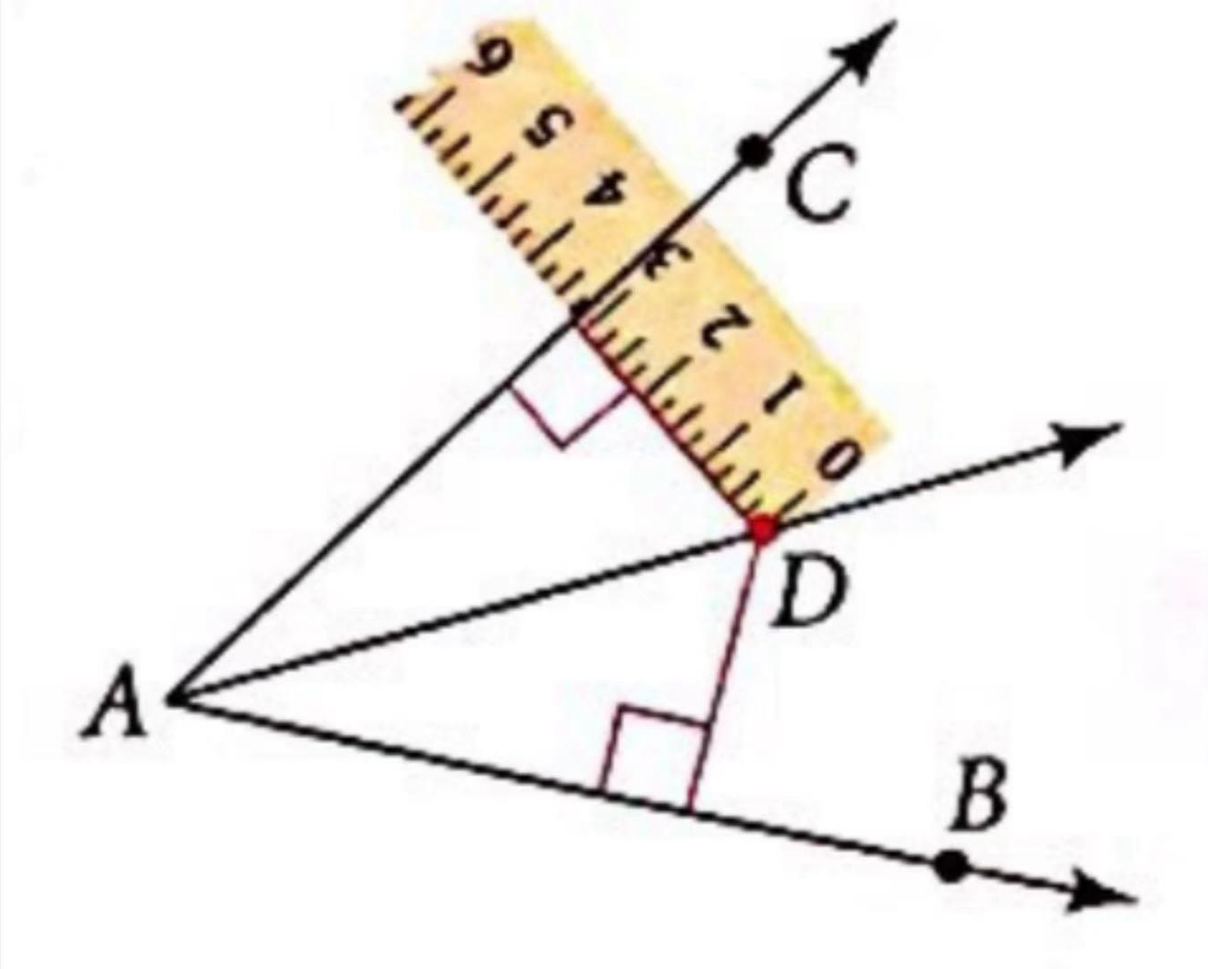
- Some triangle relationships are difficult to see because the triangles overlap.
- Overlapping triangles may have a common side or angle. You can simplify your work with overlapping triangles by separating and redrawing the triangles.

- Identifying Common Parts



Bisectors in Triangles

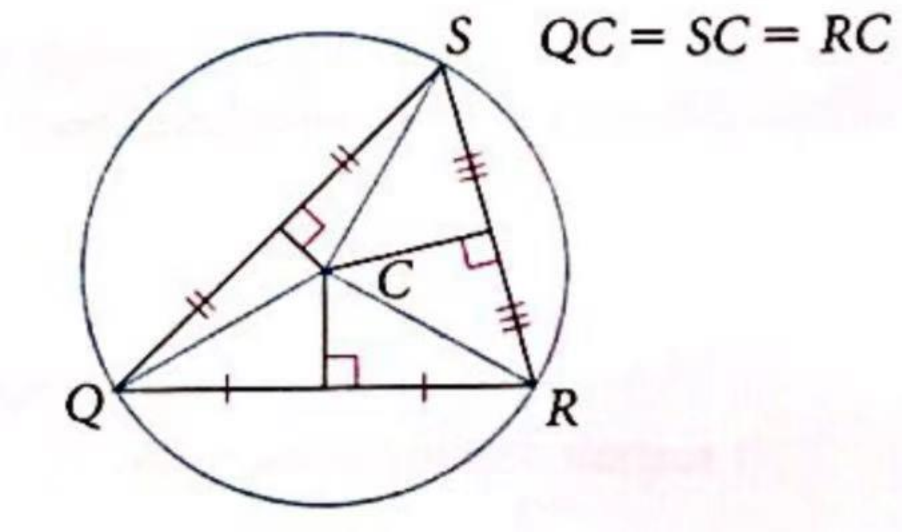
- Theorem 5-2 Perpendicular Bisector Theorem
  - If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.
- Theorem 5-3 Converse of the Perpendicular Bisector Theorem
  - If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
- Theorem 5-4 Angle Bisector Theorem
  - If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.
- Theorem 5-5 Converse of the Angle Bisector Theorem
  - If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.
  - The distance from a point to a line is the length of the perpendicular segment from the point to the line. In the diagram, AD is the bisector of angle CAB. If you measure the lengths of the perpendicular segments from D to the two sides of the angle, you will find that the lengths are equal so D is equidistant from the sides.



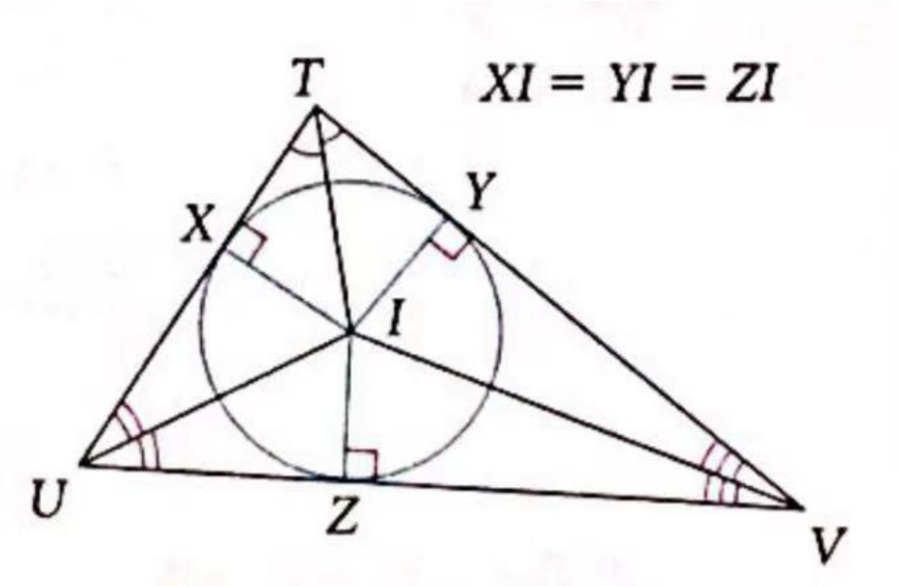
Concurrent Lines, Medians, and Altitudes

- When three or more lines intersect in one point, they are concurrent. The point at which they intersect is the point of concurrency. For any triangle, four different sets of lines are concurrent. Theorems 5-6 and 5-7 tell you about two of them.
- Theorem 5-6
    - The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.
- Theorem 5-7
    - The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.
- This figure shows triangle QRS with the perpendicular bisectors of its sides concurrent at C. The point of concurrency of the perpendicular bisectors of a triangle is called the circumcenter of the triangle.

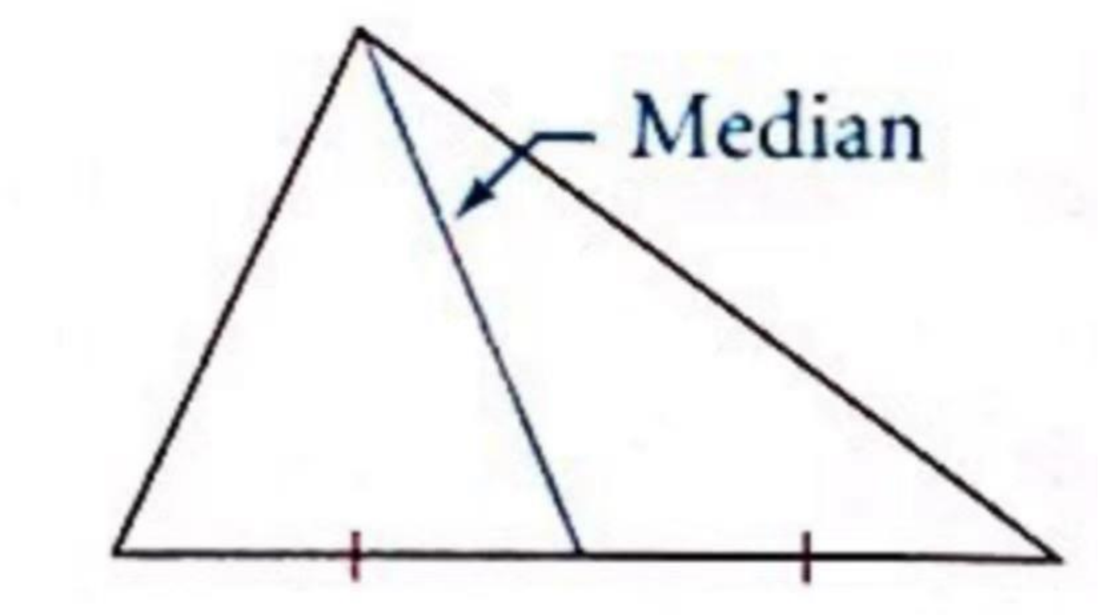
- Points Q, R, and S are equidistant from C, the circumcenter. The circle is circumscribed about the triangle.



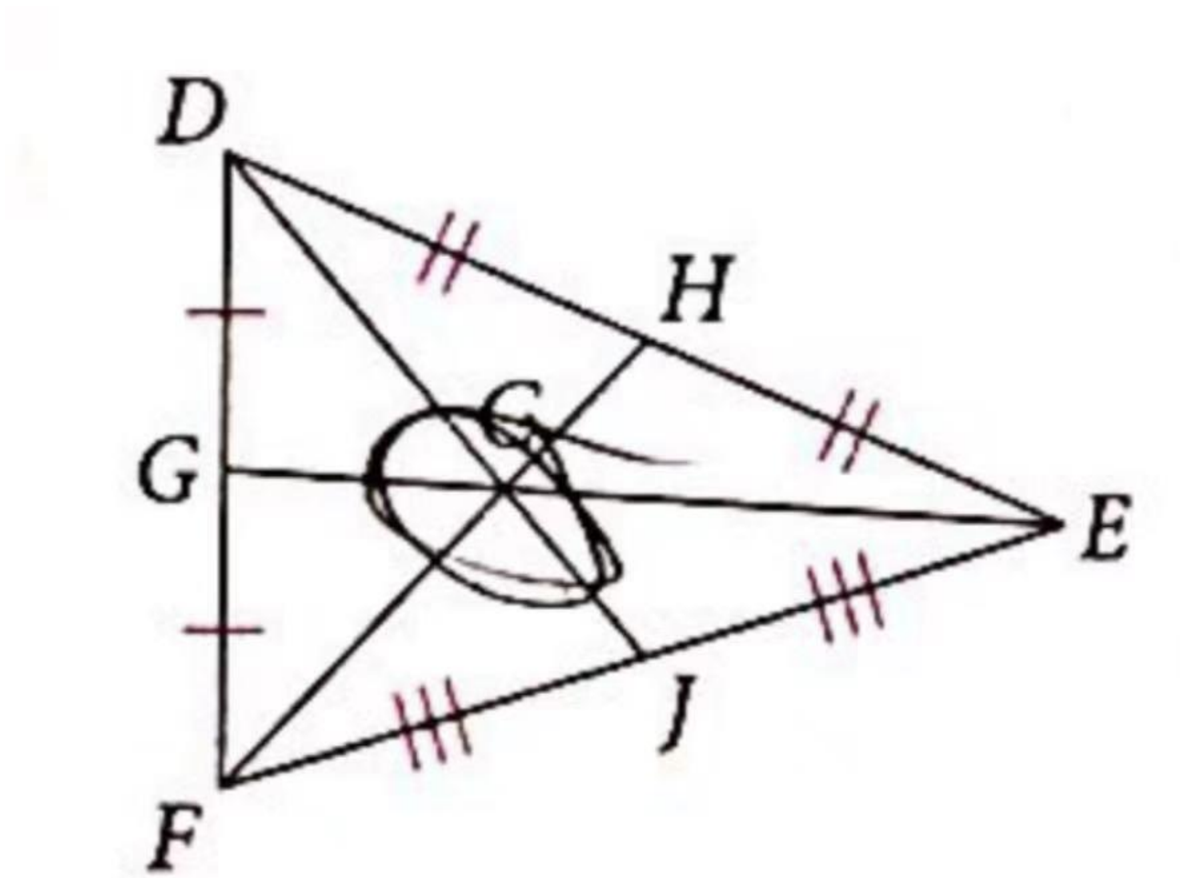
- This figure shows triangle UTV with the bisectors of its angles concurrent at I.
- The point of concurrency of the angle bisectors of a triangle is called the incenter of the triangle.
- Points X, Y, and Z are equidistant from 1, the incenter. The circle is inscribed in the triangle.



- A median of a triangle is a segment whose endpoints are a vertex and the midpoint of the opposite side.

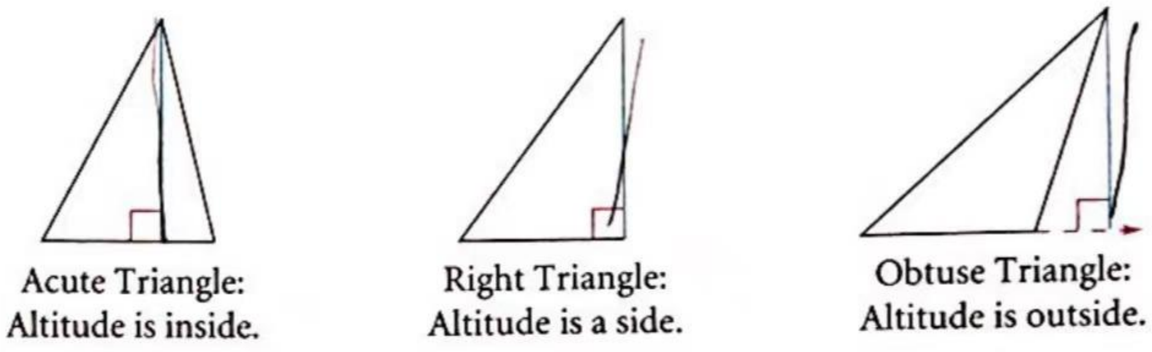


- Theorem 5-8
  - The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.
  - DC = 2/3 DJ EC = 2/3 EG FC = 2/3 FH



- In a triangle, the point of concurrency of the medians is the centroid. The point is also called the center of gravity of a triangle because it is the point where a triangular shape will balance.

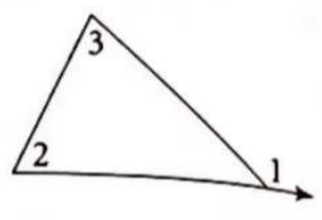
- An altitude of a triangle is the perpendicular segment from a vertex to the line containing the opposite side. Unlike angle bisectors and medians, an altitude of a triangle can be a side of a triangle or it may lie outside the triangle.



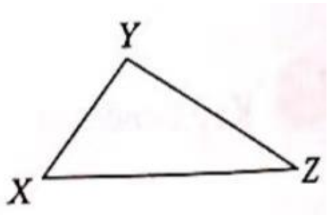
- The lines containing the altitudes of a triangle are concurrent at the orthocenter of the triangle.
- Theorem 5-9
    - The lines that contain the altitudes of a triangle are concurrent.

Inequalities in Triangle

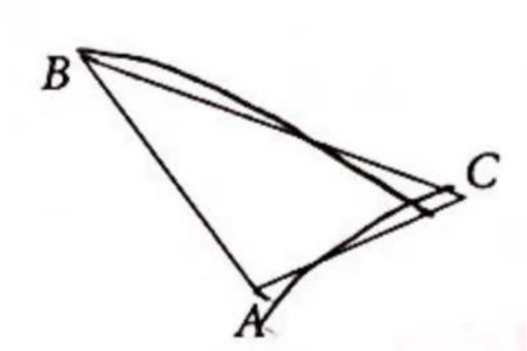
- Property
    - Comparison Property of Inequality
        - If a = b + c and c > 0, then a > b.
- Corollary
    - Corollary to the Triangle Exterior Angle Theorem
        - The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.
        - Angle 1>Angle 2, Angle 1>Angle 3



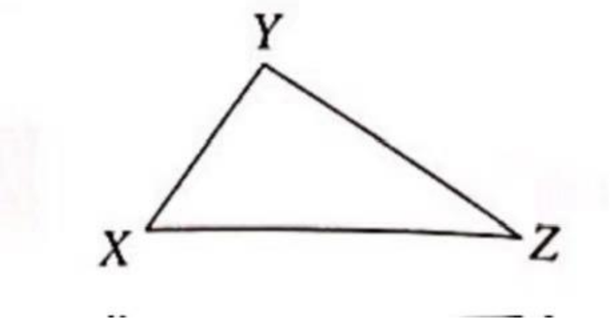
- Theorem 5-10
  - If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.
  - If XZ > XY, then Angle Y>Angle Z



- Theorem 5-11
  - If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.
  - If Angle A > Angle B, then BC > AC.



- Theorem 5-12 Triangle Inequality Theorem
  - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
    - XY + YZ > XZ
    - YZ + ZX > YX
    - ZX + XY > ZY



Basic Constructions

- In a construction you use a straightedge and a compass to draw a geometric figure. A straightedge is a ruler with no markings on it. A compass is a geometric tool used to draw circles and parts of circles called arcs.
- Four basic constructions involve constructing congruent segments, congruent angles, and bisectors of segments and angles.

1 EXAMPLE Constructing Congruent Segments

Construct a segment congruent to a given segment.

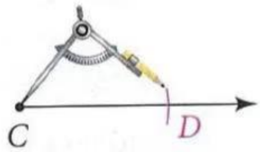
Given: $\overline{AB}$

Construct: $\overline{CD}$ so that $\overline{CD} \cong \overline{AB}$

Step 1
Draw a ray with endpoint C.

Step 2
Open the compass to the length of $\overline{AB}$.

Step 3
With the same compass setting, put the compass point on point C. Draw an arc that intersects the ray. Label the point of intersection D.



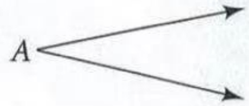
$\overline{CD} \cong \overline{AB}$

2 EXAMPLE Constructing Congruent Angles

Construct an angle congruent to a given angle.

Given: $\angle A$

Construct: $\angle S$ so that $\angle S \cong \angle A$

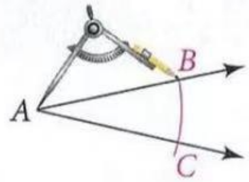


Step 1

Draw a ray with endpoint S.

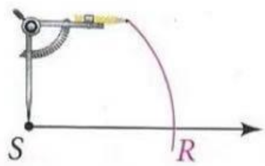
Step 2

With the compass point on point A, draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.



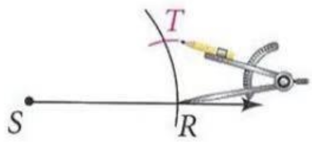
Step 3

With the same compass setting, put the compass point on point S. Draw an arc and label its point of intersection with the ray as R.



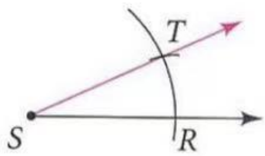
Step 4

Open the compass to the length BC. Keeping the same compass setting, put the compass point on R. Draw an arc to locate point T.



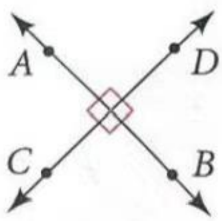
Step 5

Draw $\overrightarrow{ST}$.



$\angle S \cong \angle A$

- Perpendicular lines are two lines that intersect to form right angles. The symbol 1 means "is perpendicular to."
- A perpendicular bisector of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint, thereby bisecting the segment into two congruent segments.

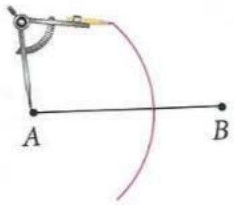


3 EXAMPLE Constructing the Perpendicular Bisector

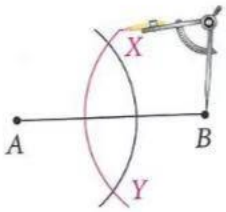
Construct the perpendicular bisector of a segment.

Given: $\overline{AB}$
Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY} \perp \overline{AB}$ at the midpoint M of $\overline{AB}$.

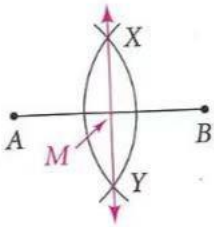
Step 1
Put the compass point on point A and draw a long arc as shown. Be sure the opening is greater than $\frac{1}{2}AB$.



Step 2
With the same compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs intersect as X and Y.

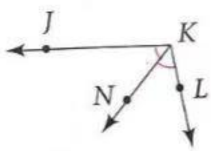


Step 3
Draw $\overleftrightarrow{XY}$. The point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ is M, the midpoint of $\overline{AB}$.



$\overleftrightarrow{XY} \perp \overline{AB}$ at the midpoint of $\overline{AB}$, so $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

4 EXAMPLE Finding Angle Measures



Algebra $\overrightarrow{KN}$ bisects $\angle JKL$ so that $m\angle JKN = 5x - 25$ and $m\angle NKL = 3x + 5$. Solve for x and find $m\angle JKN$.

$m\angle JKN = m\angle NKL$	**Definition of angle bisector**
$5x - 25 = 3x + 5$	**Substitute.**
$5x = 3x + 30$	**Add 25 to each side.**
$2x = 30$	**Subtract 3*x* from each side.**
$x = 15$	**Divide each side by 2.**
$m\angle JKN = 5x - 25 = 5(15) - 25 = 50$	**Substitute 15 for *x*.**

$m\angle JKN = 50$

- An angle bisector is a ray that divides an angle into two congruent coplanar angles.

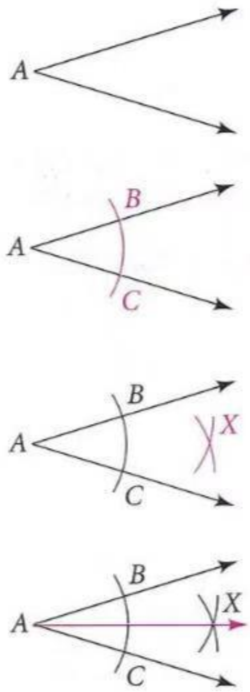
- Its endpoint is at the angle vertex. Within the ray, a segment with the same endpoint is also an angle bisector. You may say that the ray or segment bisects the angle.

5 EXAMPLE Constructing the Angle Bisector

Construct the bisector of an angle.

Given: $\angle A$

Construct: $\overrightarrow{AX}$, the bisector of $\angle A$



Step 1
Put the compass point on vertex A. Draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.

Step 2
Put the compass point on point C and draw an arc. With the same compass setting, draw an arc using point B. Be sure the arcs intersect. Label the point where the two arcs intersect as X.

Step 3
Draw $\overrightarrow{AX}$.

$\overrightarrow{AX}$ is the bisector of $\angle CAB$.

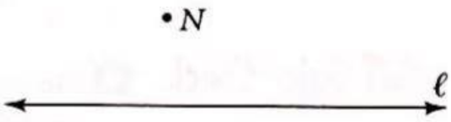
Constructing Parallel and Perpendicular Lines

1 EXAMPLE Constructing $\ell \parallel m$

Construct the line parallel to a given line and through a given point that is not on the line.

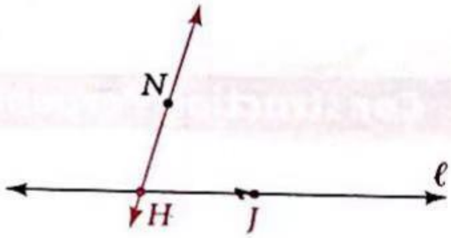
Given: line ℓ and point N not on ℓ

Construct: line m through N with $m \parallel \ell$



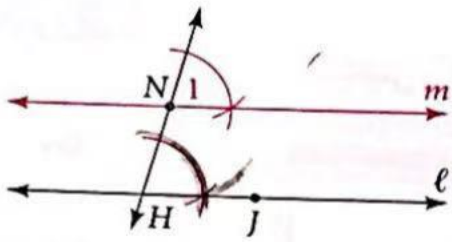
Step 1

Label two points H and J on ℓ.
Draw $\overleftrightarrow{HN}$.



Step 2

Construct $\angle 1$ with vertex at N so that $\angle 1 \cong \angle NHJ$ and the two angles are corresponding angles. Label the line you just constructed m.



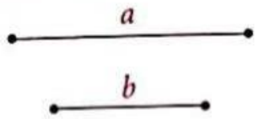
$m \parallel \ell$

2 EXAMPLE Constructing a Special Quadrilateral

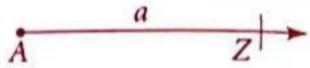
Construct a quadrilateral with one pair of parallel sides of lengths a and b.

Given: segments of lengths a and b

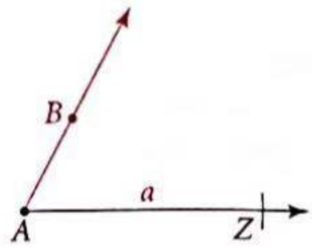
Construct: quadrilateral $ABYZ$ with $AZ = a, BY = b$, and $\overline{AZ} \parallel \overline{BY}$



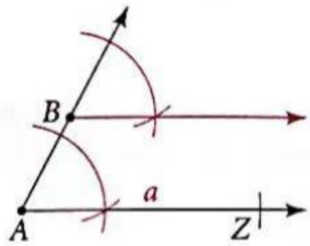
Step 1
Construct $\overline{AZ}$ with length a.



Step 2
Draw a point B not on $\overleftrightarrow{AZ}$. Then draw $\overrightarrow{AB}$.

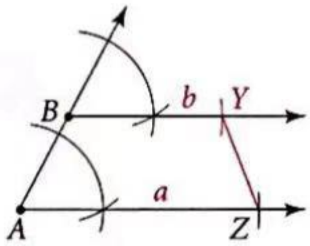


Step 3
Construct a ray parallel to $\overleftrightarrow{AZ}$ through B.



Step 4
Construct Y so that $BY = b$. Then draw $\overline{YZ}$.

Quadrilateral $ABYZ$ has $AZ = a, BY = b$, and $\overline{AZ} \parallel \overline{BY}$.

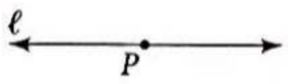


3 EXAMPLE Perpendicular at a Point on a Line

Construct the perpendicular to a given line at a given point on the line.

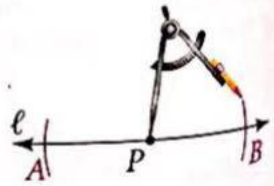
Given: point P on line ℓ

Construct: $\overleftrightarrow{CP}$ with $\overleftrightarrow{CP} \perp \ell$



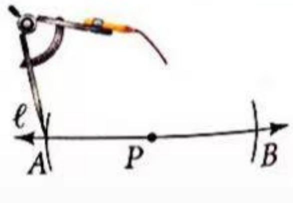
Step 1

Put the compass point on point P. Draw arcs intersecting ℓ in two points. Label the points A and B.



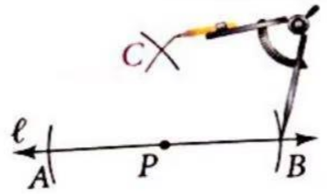
Step 2

Open the compass wider. With the compass tip on A, draw an arc above point P.



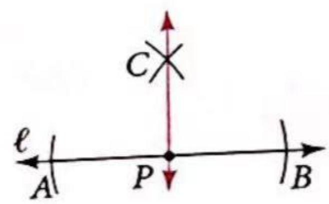
Step 3

Without changing the compass setting, place the compass point on point B. Draw an arc that intersects the arc from Step 2. Label the point of intersection C.



Step 4

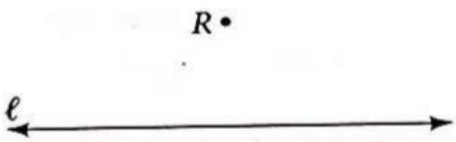
Draw $\overleftrightarrow{CP}$.



$\overleftrightarrow{CP} \perp \ell$

EXAMPLE Perpendicular From a Point to a Line

Construct the perpendicular to a given line through a given point not on the line.

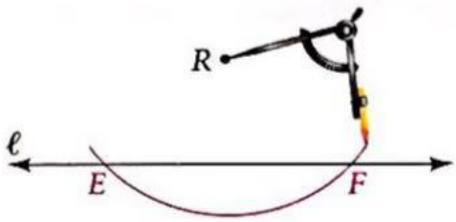


Given: line ℓ and point R not on ℓ

Construct: $\overleftrightarrow{RG}$ with $\overleftrightarrow{RG} \perp \ell$

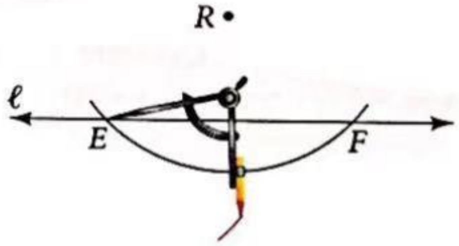
Step 1

Open your compass to a size greater than the distance from R to ℓ. With the compass point on point R, draw an arc that intersects ℓ at two points. Label the points E and F.



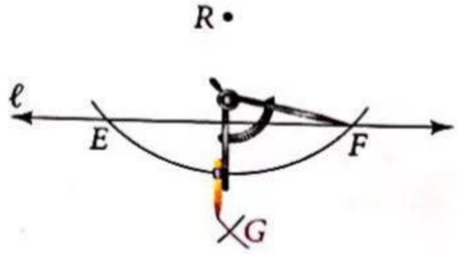
Step 2

Place the compass point on E and make an arc.



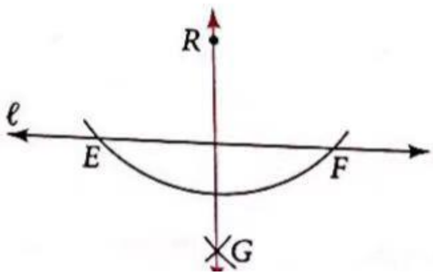
Step 3

Keep the same compass setting. With the compass tip on F, draw an arc that intersects the arc from Step 2. Label the point of intersection G.



Step 4

Draw $\overleftrightarrow{RG}$.



$\overleftrightarrow{RG} \perp \ell$