IDX G9 Math H

Study Guide S2 M1 Issue 1

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Contents:

1.1 Sets and Venn Diagrams

Key points:

- Definition and properties of sets, subsets, elements
- Representation
- Operations

Sets and Elements

- **Set - A collection of distinct numbers or objects *(A,B,C,……)***
- **Object - each object of the set *(a,x,y,……)***
- **∈ - is an element of (), is in**
- **∈/ - is not an element of (), is not in**
    - Ex. In set A {1,2,3}, 1, 2, and 3 are all elements: 3∈A, 4∈/A

Number Set types:

- $\mathbb{Z}$-Integers

- ℕ-Natural numbers set (integer>=0)
- ℤ+ or ℕ∗-Positive integers
- ℚ-Rational numbers
- ℝ-Real numbers
- ℝ>ℚ>ℤ>ℕ>ℤ+ or ℕ∗ (from largest range to smallest)

- A set is finite when n(A)/amount of elements is a defined value.
  - Ex. A={0,1,2} n(A)=3
- Otherwise, it's an infinite set.
  - Ex. A={x∈ℕ|x} (A contains all natural numbers)
- **The empty set ∅ or { } is a set which contains no elements**
- ∅-empty set, {0}-set with an element "0", {∅}-set with an element "empty set"

Bracket representation

- [x,y] represents a closed interval $\{a|x<=a<=y\}$
- (x,y) represents an open interval $\{a|x<a<y\}$
- When an interval extends for no endpoints.
  - Ex. $\{x|x<=a\}$ we write (-∞,a]

Properties of sets

- Any element is either in or not in a set.
- The same element can only appear once in a set (Ex. {1,2,2} is not a set)
- There is no requirement for a fixed order of the elements in the set.
- Two sets are equal as long as they contain the same elements.
  - Ex. {1,2,3}={3,2,1}

Relationships between sets

- **For any $x \in A \Longrightarrow x \in B$, then we can call set A is subset of B, or we could write as "A⊆B"**
- If $A \subseteq B$ and there exists $x \in B \Longrightarrow\ x \notin A$ (or $A \neq B$), then we call set A is the proper subset of B, or we could write as "A⊂B"□

□

Operations

- **Intersection: In both sets (A∩B={x|x ∈A and x ∈B})**
  - we call A and B mutually exclusive if A∩B= ∅
- **Union: Either in the sets (A∪B={x|x ∈A or x ∈B})**
- **Complement: All elements not part of a set (A′ or $\bar{A}$= {x| $x \in U$ and $x \notin A$}, U is the universal set of all elements)**

Venn diagrams

- A venn diagram consists of a universal set U, and subsets in U.

1.2 Inductive reasoning

- We use inductive reasoning to prove what the next terms in a sequence will be based on patterns we observe
- Conjecture-conclusion you reach by inductive reasoning
- **Explicit formula-defining a only with respect to n (All terms can be found through explicit formula)**
- **Recursive formula-defining a(n) using preceding terms (a1, a2,…,an-1)**
- **Counterexample-an example for which the conjecture is incorrect**

1.3 Logic statement

- **Statement-declarative sentence which's either true or false, not both.**
  - Composed with hypothesis and conclusion (we could transfer it into a conditional statement: if a, then b"
- Conditional statement
  - If "if a, then b" is defined true, then we could say a implies b, a is sufficient for b, b is necessary for a.
- Equivalent statement
  - If p->q and while q->p, we could conclude p⇔q, which could be written as p is equivalent to q

- **Law of detachment-If the conditional is true and its hypothesis is true, then its conclusion is true (if p->q is a true statement and p is true, then q is true)**

- **Law of syllogism-If p->q and q->r are true, then p->r is a true statement.**
- For a conditional statement "if p then q ":
  - **Negation: if q then not q**
  - **Converse: if q then p**
  - **Inverse: if not p, then not q**
  - **Contrapositive: if not q, then not p (conditional and contrapositive are equivalent statements)**

1.4 Deductive Reasoning

- Deductive Reasoning: A logical reasoning process that moves from general statements to a conclusion.
  - Example questions: Given some statements with limited amount of true statements, determine which ones are correct or observe some facts.

1.5 Basic Geometric Elements

- **Points: A location; no size; represented by a small dot. Notation: capital letters like A, B,C, …**
- **Lines: A set of points that extend in two different directions without end.**
  - Notation: lowercase letter like a, b, c, l, … ($A \in a$)
- **Segments: Part of a line consisting of two endpoints and all the points between them.**
  - Notation: a bar on two capital letters representing each endpoint of the segment
- **Ray: The part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.**
  - Notation: an arrow on letter A representing the one endpoint and B representing the other point on the segment
- **Opposite rays: two collinear rays with the same end point with opposite direction that always form a line.**
- **Collinear points: points that are on the same line.**
- **Planes: a flat surface that has no thickness and extends without end in the directions of all the lines on it. It contains infinitely many lines.**
  - Notation: lowercase Greek letter like α, β, γ…

- Acute angles - angles with degree between 0 and 90
- Right angles - angles with degree of 90
- Obtuse angles - angles with degree between 90 and 180
- Straight angles - angles with degree of 180
- Vertical angles - two angles with opposite rays as sides
- Adjacent angles - coplanar angles with a common side, a common vertex with no shared interior points
- Complementary angles - two angles whose measures have the sum 90
- Supplementary angles - two angles whose measures have the sum 180
- **Postulates: An accepted statement of fact (don't need to prove)**
    - 1. Only one straight line can be drawn through two given points.
    - 2. Three non-collinear points can determine a plane.
    - 3. If two lines intersect, they intersect in exactly one point.
    - 4. If two distinct points on a line are on a plane, then the line is on the plane.
    - 5. If two planes intersect, they intersect in exactly one line.
    - 6. If two distinct planes have one point in common, they have only one line in common which contains this common point.
    - 7. If A, B, C are collinear, and B is between A and C, then AB+BC=AC
    - 8. If B is an interior point of AOC, then $m\angle AOC = m\angle AOB + m\angle BOC$
- **Theorem: statement that needs proof**
    - **Vertical angles theorem: vertical angles are congruent.**
    - **Congruent complement/supplement theorem: if two angles are the complement supplement of the same angle, they are congruent.**
- Reasonings for proof
    - **Properties of Equality**
        - Addition P.O.E: If $a=b$, then $a + c = b + c$.
        - Subtraction P.O.E: If $a=b$, then $a - c = b-c$.
        - Multiplication P.O.E: If $a=b$, then $a \cdot c = b \cdot c$.
        - Division P.O.E: If $a=b$ and $c\neq0$, then $a/c=b/c$
        - Reflexive P.O.E: $a=a$
        - Symmetric P.O.E: If $a=b$, then $b=a$.
        - Transitive P.O.E: If $a=b$ and $b=c$, then $a=c$.

        - Substitution P.O.E: If a = b, then b can replace a in any expression.
        - Distributive P.O.E: a(b + c) = ab + ac

    - **Properties of Congruence**
        - Reflexive P.O.C: AB = AB; ∠A =∠A
        - Symmetric P.O.C: If AB = CD, then CD = AB; If ∠A = ∠B, then∠ B = ∠A.
        - Transitive P.O.C: If AB = CD and CD = EF, then AB = EF; If ∠A = ∠B and ∠B = ∠C, then ∠A = ∠C.

1.6 Intersecting and Parallel Lines – Concepts:

- **Transversal: A line that intersects two different lines at two different points**
- **Corresponding angles: The pair of angles on the same side of the transversal, and on the same side of the two lines**
- **Alternate interior angle: The pair of angles on the opposite sides of the transversal and are outside of the two lines**
- **Alternate exterior angle: The pair of angles on the opposite sides of the transversal and between the two lines**
- **Same-side interior angle: The pair of angles on the same side of the transversal and between the two lines**
- **Same-side exterior angle: The pair of angles on the same sides of the transversal and are outside of the two lines –**
- **Corresponding angle postulate: If a transversal intersects two parallel lines, the corresponding angles formed are congruent (Converse is also correct)**
- Theorems (If a transversal intersects two parallel lines):
    - **Alternate Interior Angles Theorem: Alternate interior angles are congruent**
    - **Same-Side Interior Angles Theorem: Same-side interior angles are supplementary**
    - **Alternate Exterior Angles Theorem: Alternate exterior angles are congruent**

- **Same-Side Exterior Angles Theorem: Same-side exterior angles are supplementary**
- Converse of the above four theorems are all correct

- Properties of parallel lines
  - If two lines are parallel to the same line, then they are parallel.
  - For two parallel lines, the distance from any point on one line to the other is a constant, and it's defined as the distance between the two parallel lines.