



IDX G9 Math S+
Study Guide Issue Semester 1 Final
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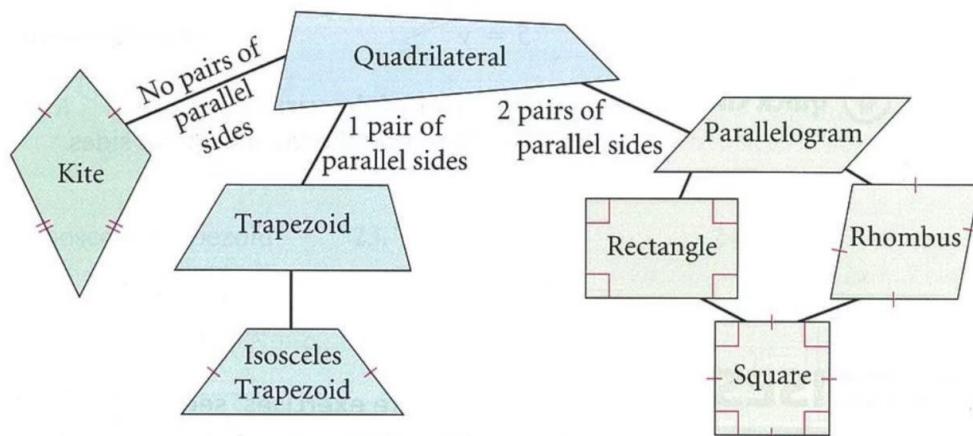
The Polygon Angle-Sum Theorems

- A polygon is a closed plane figure with at least three sides that are segments. The sides intersect only at their endpoints, and no adjacent sides are collinear.
- The Name of Polygons:
 - 3 sides: Triangle
 - 4 sides: Quadrilateral
 - 5 sides: Pentagon
 - 6 sides: Hexagon

- 7 sides: Heptagon
 - 8 sides: octagon
 - 9 sides: Nonagon
 - 10 sides: Decagon
 - 11 sides: Hendecagon
 - 12 sides: Dodecagon
 - n-sides: n-gon
- Special Polygons:
 - Equilateral polygon, equiangular polygon, regular polygon
 - Theorem 3-14: Polygon Angle-Sum Theorem
 - The sum of the measures of the interior angles of an n-gon is $180(n-2)$
 - Theorem 3-15: Polygon Exterior Angle-Sum Theorem
 - The sum of the measures of the exterior angles of a polygon, one at each vertex is 360.
 - Characteristics of Convex Polygons:
 - Number of sides: n
 - Number of diagonals from one vertex: $n-3$
 - Number of triangles formed: $n-2$
 - Total number of diagonals: $n(n-3)/2$

Classifying Quadrilaterals

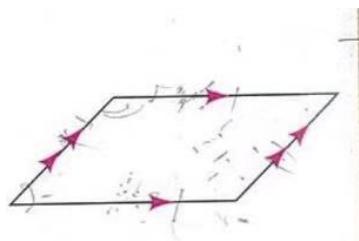
The diagram below shows the relationships among special quadrilaterals.



Definitions

Special Quadrilaterals

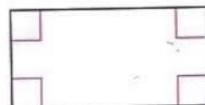
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.



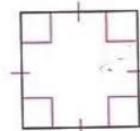
A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



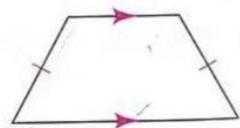
A **square** is a parallelogram with four congruent sides and four right angles.



A **kite** is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent.

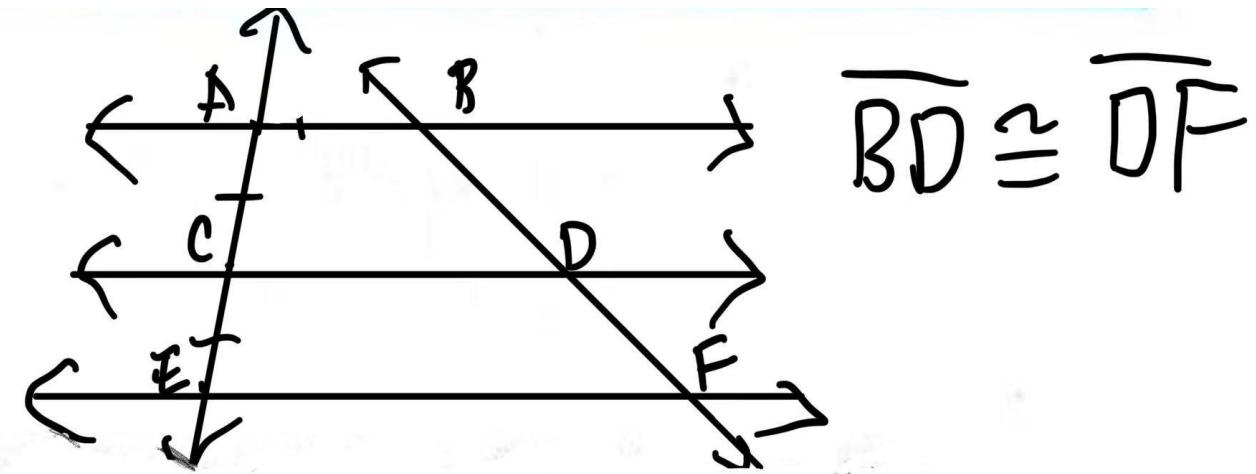


A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The **isosceles trapezoid** at the right is a trapezoid whose nonparallel opposite sides are congruent.



Properties of Parallelograms

- Theorem 6-1: Opposite sides of a parallelogram are congruent.
- Theorem 6-2: Opposite angles of a parallelogram are congruent.
- Theorem 6-3: The diagonals of a parallelogram bisect each other.
- Theorem 6-4:
 - If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments of every transversal.



Proving that a Quadrilateral is a Parallelogram

- Theorem 6-5: If both pair of opposite sides of a quadrilateral are congruent, then it is a parallelogram.
- Theorem 6-6: If both pairs of opposite angle of a quadrilateral are congruent, then it is a parallelogram.
- Theorem 6-7: If diagonals of a quadrilateral bisects each other, then it is a parallelogram.
- Theorem 6-8: If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Special Parallelograms

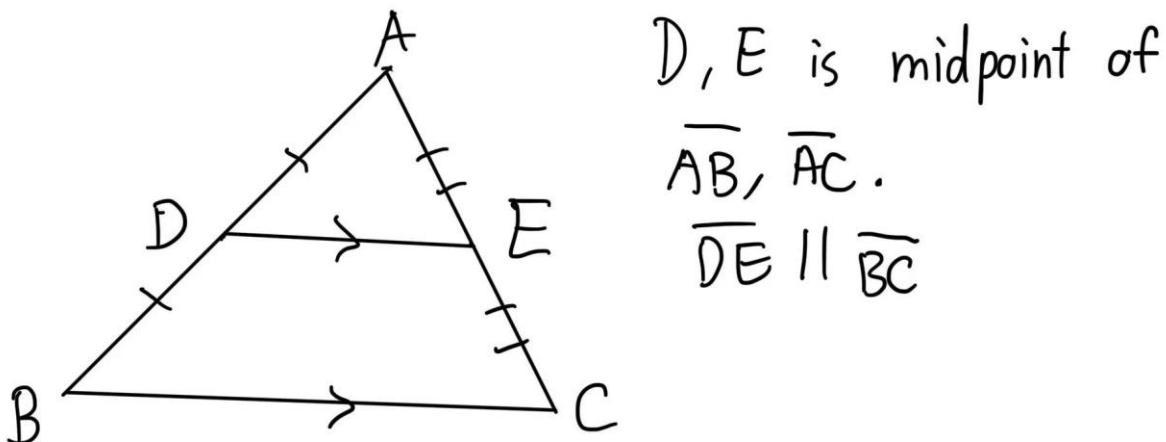
- Definition of a Rhombus: A parallelogram with 4 sides congruent.
- Definition of a Rectangle: A parallelogram with 4 angles congruent.
- Definition of a Square: A parallelogram with 4 sides and angles congruent.
- Theorem 6-9: Each diagonal of a rhombus bisects two angles of the rhombus.
- Theorem 6-10: The diagonals of a rhombus are perpendicular.
- Theorem 6-11: The diagonals of a rectangle are congruent.
- Theorem 6-12 (Converse of the Thrm 6-9): If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus.
- Theorem 6-13 (Converse of the Thrm 6-10): If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
- Theorem 6-14 (Converse of the Thrm 6-11): If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Trapezoid and Kites

- Theorem 6-15: The base angles of an isosceles trapezoid are congruent.
- Theorem 6-16: The diagonals of an isosceles trapezoid are congruent.
- Theorem 6-17: The diagonals of a kite are perpendicular.

Midsegments of Triangles

- Theorem 5-1: Triangle Midsegment Theorem
 - The midsegment is parallel to the third side and is 1/2 its length.



Ratios and Proportions

- A ratio is a comparison of two quantities. You can write the ratio of a to b or a : b as the quotient a/b when b is not equal to 0. Unless otherwise stated, the terms and expressions appearing in ratios in this book are assumed to be nonzero.
- A proportion is a statement that two ratios are equal. You can write a proportion in these forms:
 - $a/b = c/d$ and $a:b = c:d$
- When three or more ratios are equal, you can write an extended proportion.

Property

$\frac{a}{b} = \frac{c}{d}$ is equivalent to

Properties of Proportions

$$(1) ad = bc$$

$$(2) \frac{b}{a} = \frac{d}{c}$$

$$(3) \frac{a}{c} = \frac{b}{d}$$

$$(4) \frac{a+b}{b} = \frac{c+d}{d}$$

- Multiplying both sides of $a/b = c/d$ by bd results in the first property, called the Cross-Product Property. You may state this property as "The product of the extremes is equal to the product of the means."

means
 ↓ ↓
 $a : b = c : d$
 ↑ Extremes ↑
 $\frac{a}{b} = \frac{c}{d}$
 $ad = bc$

- In a scale drawing, the scale compares each length in the drawing to the actual length. The lengths used in a scale can be in different units. A scale might be written as 1 in. to 100 mi, 1 in. = 12 ft, or 1 mm : 1 m. You can use proportions to find the actual dimensions represented in a scale drawing.

Similar Polygons

- Two figures that have the same shape but not necessarily the same size are similar (\sim).
- Two polygons are similar if ...
 - Corresponding angles are congruent.
 - Corresponding sides are proportional.
- The ratio of the lengths of corresponding sides is the similarity ratio.
- A golden rectangle is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle. A pattern of repeated golden rectangles is shown at the right. Each golden rectangle that is formed is copied and divided again. Each golden rectangle is similar to the original rectangle.
- In any golden rectangle, the length and width are in the golden ratio which is about 1.618: 1.

Proving Triangles Similar

- Postulate 7-1: Angle-Angle Similarity (AA \sim) Postulate
 - If two angles of a triangle are congruent to two angles of another triangle, then the two triangles are similar.
- Theorem 7-1: Side-Angle-Side Similarity (SAS \sim) Theorem
 - If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar.
- Theorem 7-2: Side-Side-Side Similarity (SSS \sim) Theorem
 - If the corresponding sides of two triangles are proportional, then the triangles are similar.

Similarity in Right Triangles

- Theorem 7-3:
 - The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.
- Corollary 1 to Theorem 7-3:
 - The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.
- Corollary 2 to Theorem 7-3:
 - The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse.

Proportions in Triangles

- Theorem 7-4: Side-Splitter Theorem
 - If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.
- Corollary to Theorem 7-4
 - If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.
 - $a/b = c/d$