IDX G9 Math S
Study Guide S1 Midterm
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Contents:

Sets and Venn diagrams

- Important number sets: Natural numbers (N), Integers (Z), Positive Integers (Z+ or N*), Rational numbers (Q) and Real numbers (R).

Relationship between sets include:

- Subsets
  - Given sets A and B, when x is an element of A, x is also a element of B, then we call A is the subset of B
  - **A smaller or equal to B**
- Proper Subset
  - Given sets A and B, x is an element of B but x is not an element of A, then we call A is the proper subset of B
  - **A smaller than B**

Operation in sets:

- Intersection
  - The intersection of two sets A and B is the set of elements that are both in set A **and** set B

- Union
    - The union of two sets A and B is the set of elements that are in either set A **or** set B

Types of Notation:

- Interval Notation
    - {variable is an element of…| set of numbers that the variable lies into}
- Bracket Notation
    - () meaning not including the number, [] meaning including the number

Chapter 1: Tools of Geometry & Reasoning

1-1 Patterns and Inductive Reasoning

- Definition: A method of reasoning that uses observed patterns to make generalizations (conjectures).
- Process: Observe a pattern → Make a conjecture → Test the conjecture.
    - Example: Observing that the sum of two odd numbers is always even.
- **Inductive reasoning is powerful but not foolproof; a single counterexample can disprove a conjecture.**

1-3 Points, Lines, and Planes

- Point: A location with no size. Named by a capital letter
    - Example: Point A
    - Collinear points: Points that lies on the same line
- **If three points can form a triangle, they are not collinear**
- Line: A series of points that extends in two opposite directions without end. Named by any two points on it
    - Example: Line AB
- Plane: A flat surface that extends infinitely in all directions. Named by a capital cursive letter or by three non-collinear points
    - Example: Plane P or Plane ABC
- Coplanar: Pointe and lines that lie in the same plane
- Postulates: Basic rules accepted without proof
    - Example: "Through any two points there is exactly one line"

1-4 Segments, Rays, Parallel Lines and Planes

- Segment: A part of a line with two endpoints.
    - Notation: AB (with a bar on top).
- Ray: A part of a line with one endpoint that extends infinitely in one direction.
    - Notation: →AB (endpoint first).
- Parallel Lines: Coplanar lines that do not intersect.
- Skew Lines: Non-coplanar lines that are not parallel and do not intersect.
- Parallel Planes: Planes that do not intersect.

1-5 Measuring Segments

- Ruler Postulate: The points on a line can be matched one-to-one with real numbers, allowing us to measure the distance between two points.
- **Segment Addition Postulate**: If point B is between points A and C, then AB + BC = AC.
- Congruent Segments (≅): Segments that have the same length.

1-6 Measuring Angles

- Angle: Formed by two rays with a common endpoint (the vertex).
    - Types: Acute (< 90°), Right (90°), Obtuse (> 90° & < 180°), Straight (180°).
- **Angle Addition Postulate**: If a point lies in the interior of an angle, then the sum of the measures of the two smaller angles is equal to the measure of the larger angle.
- Angle Pairs:
    - Complementary: Two angles whose sum is 90°.
    - Supplementary: Two angles whose sum is 180°.
    - Adjacent: Angles that share a vertex and a side.
    - Vertical: Opposite angles formed by two intersecting lines. They are always congruent.

1-7 Basic Constructions

- Using only a compass and a straightedge.
- Key constructions: congruent segments, congruent angles, perpendicular bisectors, angle bisectors.

1-8 The Coordinate Plane

- Distance Formula: square root of (x1-x2) squared + (y1-y2) squared
- Midpoint Formula: (x1+x2/2 , y1+y2/2)

Chapter 2: Geometric Reasoning & Proof

2-1 Conditional Statements

- Form: "If p, then q" (p → q), where p is the hypothesis and q is the conclusion.
- Related Conditionals:
    - Converse: "If q, then p" (q → p).
    - Inverse: "If not p, then not q" (~p → ~q).
    - Contrapositive: "If not q, then not p" (~q → ~p). Logically equivalent to the original statement.

2-2 Biconditionals and Definitions

- Biconditional: A statement that contains "if and only if" (p ↔ q). **It is true only when both the conditional and its converse are true.**
- Good Definitions: Can be written as a true biconditional statement.

2-3 Deductive Reasoning

- Definition: The process of using logic to draw conclusions from given facts, definitions, and properties.
- **Law of Detachment**: If $p \rightarrow q$ is true and p is true, then q is true.
- **Law of Syllogism**: If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true.

2-4 Reasoning in Algebra

- Applying algebraic properties to geometric figures and measures.
- Key Properties: **Addition, Subtraction, Multiplication, Division, Substitution, Reflexive, Symmetric, Transitive, and Distributive Properties.**
    - **POE (Properties of Equality) = and POC (Properties of Congruence) ≅**

2-5 Proving Angles Congruent

- **Vertical Angles Theorem**: Vertical angles are congruent.
- **Congruent Supplements Theorem**: If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.
- **Congruent Complements Theorem**: If two angles are complementary to the same angle (or to congruent angles), then they are congruent.
- **Right Angle Theorem**: All right angles are congruent.

Chapter 3: Parallel & Perpendicular Lines

3-1 Properties of Parallel Lines

- When a transversal (a line that intersects two or more lines) crosses parallel lines, special angle pairs are formed:
    - **Corresponding Angles Postulate**: Congruent.
    - **Alternate Interior Angles Theorem**: Congruent.
    - **Alternate Exterior Angles Theorem**: Congruent.
    - **Same-Side Interior Angles Theorem**(Consecutive Interior): Supplementary
    - **Same-Side Exterior Angles Theorem:** Supplementary

3-2 Proving Lines Parallel

- The converses of the postulates and theorems from 3-1 are used to prove lines parallel.
    - Example: If corresponding angles are congruent, then the lines are parallel.

In a plane, two lines perpendicular to the same line are parallel.

3*-4 Parallel Lines and the Triangle Angle-Sum Theorem*

- **Triangle Angle-Sum Theorem**: The sum of the measures of the angles of a triangle is 180°.
- **Exterior Angle Theorem**: The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

3-5 The Polygon Angle-Sum Theorems

- **Polygon Angle-Sum Theorem**: The sum of the measures of the interior angles of an n-gon is $(n-2)180°$.

- **Polygon Exterior Angle Sum Theorem**: The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

3-6 Lines in the Coordinate Plane

- Slope-Intercept Form: $y = mx + b$ (m = slope, b = y-intercept)
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

3-7 Slopes of Parallel and Perpendicular Lines

- Parallel Lines: Have the same slope. ($m_1 = m_2$)
- Perpendicular Lines: Have slopes that are negative reciprocals. ($m_1 \quad m_2 = -1$)

3-8 Constructing Parallel and Perpendicular Lines

- Techniques for constructing a line through a given point that is parallel or perpendicular to a given line, using a compass and straightedge.

Chapter 4: Congruent Triangles

4-1 Congruent Figures

- Definition: Two figures are congruent if they have the same shape and size.
- Corresponding Parts: In congruent polygons, all corresponding sides and angles are congruent.
    - Notation: $\triangle ABC \cong \triangle DEF$

4-2 Triangle Congruence by SSS and SAS

- **SSS Postulate** (Side-Side-Side): If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
- **SAS Postulate** (Side-Angle-Side): If two sides and the included angle (the angle between the two sides) of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

4-3 Triangle Congruence by ASA and AAS

- **ASA Postulate** (Angle-Side-Angle): If two angles and the included side (the side between the two angles) of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
- **AAS Theorem** (Angle-Angle-Side): If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.

4-4 Using Congruent Triangles: CPCTC

- **CPCTC**: Corresponding Parts of Congruent Triangles are Congruent.
    - Application: Once you have proven that two triangles are congruent using SSS, SAS, ASA, or AAS, you can then use CPCTC to prove that any of

their corresponding parts (sides or angles) are also congruent. This is often the final step in a proof.