IDX G9 Math S+
Study Guide Issue 1
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Contents:

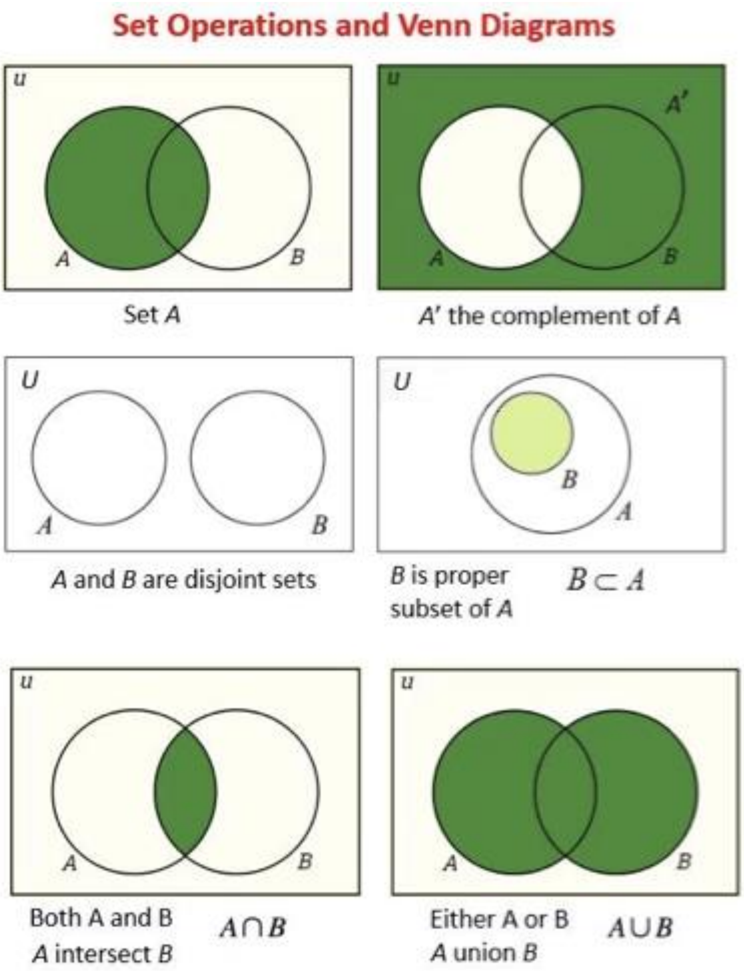
Set

- Sets and Venn Diagrams
    - A= {1,2,3} – {} represents set / 1,2,3 represents elements
    - B= {Pencils, Pens, Rulers} – Other values that are not numbers could be elements
    - n(A) = 3 (# of elements in set A)
    - A set is a collection of distinct numbers or objects. Each object is called an element or member of the set.
- Properties of Sets:
    - An element is either in the set or not in the set.
    - The elements in a set are distinct. Same element can appear only once.
    - There is no fixed order when we describe the elements in the set.

- Two sets are equal if they contain exactly the same elements

- **Important Number Set, Classification of Sets**
  - Important Number Sets:
    - N – Natural Numbers
    - Z – Integers
    - Z^+ or N* - Positive Integers
    - Z^- - Negative Integers
    - Q – Rational Numbers
    - R – Real numbers
  - Classification of Sets:
    - **-Finite** – {1,2,3} has particular defined value
    - **-Infinite** – {1,2,3…} doesn't has particular defined value
    - *Specific – {} or ∅
    - ∅, {0}, {∅}
      - n(∅) = 0
      - ∅ is a set for 1st set.
      - n({0}) = 1
      - ∅ ∈ {∅}
      - n({∅}) = 1
- Descriptive Method, Sets, Properties of Subsets
  - **Descriptive Method**
    - The descriptive method **uses words or mathematical rules to describe the members of the set** without listing them explicitly.
    - Examples
      - B = {X / X is a weekday} (Descriptive Method)
      - B = {Monday, Tuesday, Wednesday, Thursday, Friday} (Roster Method)
  - **Sets**
    - A=B Same Elements
    - A⊆B Fewer Elements – A is a subset of B
    - A⊂B – A is a proper subset of B
    - A = {1,2,3} B = {1,2,3,4…}
    - A⊂B: B must have all the elements that are in set A.
  - **Properties of Subsets**

    - A⊆A
    - A = B ó A⊆B, B⊆A
    - ∅ is a subset of every sets.
    - ∈ - Belongs to, within. (A∈ B : A is an element of B)
- Set Operations & Venn Diagram
  - **Transversal Property of Subset**
    - A⊆B, B⊆C => A⊆C
  - **Reflexive Property of Subset**
    - A⊆A



- Intersection and Union Properties
  - ∩ **Properties**
    - A∩B = B∩A
    - A∩A = A
    - A∩∅ = ∅
    - A∩B ⊆A, A∩B ⊆B
    - A∩B = A if and only if A⊆B

        - $(A \cap B) \cap C = A \cap (B \cap C)$
    - **∪ Properties**
        - $A \cup B = B \cup A$
        - $A \cup A = A$
        - $A \cup \emptyset = \emptyset$
        - $A \cup B \subseteq A$, $A \cup B \subseteq B$
        - $A \cup B = A$ if and only if $A \subseteq B$
        - $(A \cup B) \cup C = A \cup (B \cup C)$
- Bracket Notations
    - [a, b] → closed
    -] a, b [→ open
    - [a, b [→ closed at a, open at b
    -] a, b] → open at a, closed at b

1-1 Patterns and Inductive Reasoning

- **Inductive Reasoning**
    - Reasoning that is based on patterns you observe. If you observe a pattern in a sequence, you can use inductive reasoning to tell what the next terms in the sequence will be.
- A conclusion you reach using inductive reasoning is called a **conjecture**.
- Not all conjectures are true, you can prove that the conjecture is false by finding a counterexample. A c**ounterexample** of a conjecture is an example for which the conjecture is incorrect.

1-3 Points, Lines, and Planes

- In geometry, words such as *point, line,* and *plane* are undefined. In order to define these words, it is necessary to use words that need further defining. It is important to have general descriptions of their meanings.
- A **point** has no size. It is represented by a small dot and is named by a capital letter. A geometric figure is a set of points, and **space** is defined as the set of all points.
- A **line** is a series of points that extend in two opposite directions without end. A name of a line could be represented by any two points on the line, such as line AB. Another way to

name a line is with a single lowercase letter, such as line l. Points that lie on the same line are **collinear points.**

- A **plane** is a flat surface that has no thickness. A plane contains many lines and extends without end in the directions of all its lines. You can name a plane by either a single capital letter or by at least three of its noncollinear points. Points and lines in the same plane are **coplanar** (all collinear points are coplanar).
- A **postulate** or **axiom** is an accepted statement of fact.
  - Postulate 1-1: Through any two points there is exactly one line.
  - Postulate 1-2: If two lines intersect, then they intersect in exactly one point.
  - Postulate 1-3: If two planes intersect, then they intersect in exactly one line.
  - Postulate 1-4: Through any three noncollinear points there is exactly one plane.

1-4 Segments, Rays, Parallel Lines and Planes

- A **segment** is the part of a line consisting of two endpoints and all points between them.
- A **ray** is the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.
- **Opposite rays** are two collinear rays with the same endpoint. Opposite rays always form a line.
- Lines that do not intersect may or may not be coplanar.
- **Parallel lines** are coplanar lines that do not intersect. **Skew lines** are noncoplanar: therefore, they are not parallel and do not intersect.
- Segments or rays are parallel if they lie in parallel lines. They are skew if they lie in skew lines.

1-5 Measuring Segments

- Postulate 1-5 **Ruler Postulate**
  - The points of a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers.
- Congruent segments are equal in length and similar in shape.
- Postulate 1-6 **Segment Addition Postulate**
  - If three points A, B, and C are collinear and B is between A and C, then AB + BC = AC.

- A **midpoint** of a segment is a point that divides the segment into two congruent segments. A midpoint, or any line, ray, or other segment through a midpoint, is said to bisect the segment.

1.6 Measuring Angles

- Postulate 1-7 **Protractor Postulate**
    - Let ray OA and ray OB be opposite rays in a plane. Ray OA, ray OB, and all the rays with endpoint O that can be drawn on one side of line AB can be paired with the real numbers from 0 to 180 so that
        - Ray OA is paired with 0 and ray OB is paired with 180.
        - If ray OC is paired with x and ray OD is paired with y, then angle COD = |x - y |
- Acute Angle: $0<X<90$
- Right Angle: $X=90$
- Obtuse Angle: $90<X<180$
- Straight Angle: $X=180$
- Congruent Angle: Angles with same measure.
- Postulate 1-8 **Angle Addition Postulate:**
    - If point B is in the interior of angle AOC, then angle AOB + angle BOC = angle AOC. If angle AOC is a straight angle, then angle AOB + angle BOC = 180.

2.1 Conditional Statements

- You have heard *if-then* statements such as this one:
    - If you are not completely satisfied, then your money will be refunded.
- Another name for an if-then statement is a **conditional**. Every conditional has two parts. The part following if is the **hypothesis**, and the part following then is the **conclusion**.
    - A conditional can have a **truth value** of *true or false*. To show that a conditional is true, show that every time the hypothesis is true, the conclusion is also true. To show that a conditional is false, you need to find only one counterexample for which the hypothesis is true, and the conclusion is false.
    - The **converse** of a statement switches the hypothesis and conclusion.

2.2 Biconditional and Definitions

- If both conditional and its converse are true, you can combine them as a true **biconditional**. This is the statement you get by connecting the conditional and its converse with the word *and*. You can write a biconditional more concisely, however, by joining the two parts of each conditional with the phrase *if and only if*.
- A biconditional contains q=>p and p=>q as q<=>p.
- A good definition…
    - Is a statement that can help you identify or classify an object.
    - Has several important components.
    - Uses clearly understood terms. The terms should be commonly understood or already defined.
    - Is precise. Good definitions avoid words such as *large, sort of,* and *almost*.
    - Is reversible. That means that you can write a good definition as a true biconditional.

2.3 Deductive Reasoning

- **Deductive reasoning** (or logical reasoning) is the process of reasoning logically from given statements to conclusion. If the given statements are true, deductive reasoning produces a true conclusion.
- **Law of Detachment:** If a conditional is true and its hypothesis is true, then its conclusion is true. (If p->q is a true statement and p is true, then q is true.)
- **Law of Syllogism:** If p->q and q->r are true statements, then p->r is a true statement.

5-4 Inverses, Contrapositives, and Indirect Reasoning

- The negation of a statement has the opposite truth value. Represented in ~, for example, ~P.
- The **inverse** of a conditional statement negates both the hypothesis and the conclusion. The **contrapositive** of a conditional switches the hypothesis and the conclusion and negates both.
- Equivalent statements have the same truth value.
- Conditional and contrapositive statements share truth value and inverse and converse statements shares same truth value.
- This type of reasoning is called **indirect reasoning**. In indirect reasoning, all possibilities are considered and then all but one are proved false. The remaining possibility must be true.
- A proof involving indirect reasoning is an **indirect proof**. In an indirect proof, a statement and its negation often are the only possibilities.

- Writing an Indirect Proof
  - Step 1. State as an assumption the opposite of what you want to prove.
  - Step 2. Show that this assumption leads to a contradiction.
  - Step 3. Conclude that the assumption must be false and that you want to prove must be true.