IDX G10 AP Calculus
Study Guide Issue S1 Midterm
By 10-10 Emma, Edited by 10-3 Eric

Contents:

3.4 Velocity, Acceleration, Speed, and Parametric Functions

displacement	$\Delta s = x(t+\Delta t) - x(t)$
average velocity	$\frac{\Delta s}{\Delta t} = \frac{x(t+\Delta t)-x(t)}{\Delta t}$
(instantaneous) velocity	$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t)-x(t)}{\Delta t}$
speed	$\lvert v(t)\rvert = \left\lvert\frac{ds}{dt}\right\rvert$
acceleration	$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Slope of a parametric curve

A parametrized curve $(x(t), y(t))$ is differentiable at t if x and y are differentiable at t. If $\frac{dy}{dt}, \frac{dx}{dt}$, and $\frac{dy}{dx}$ exist, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$$

View the parametric function as 2D motion

Position	$\overrightarrow{r(t)} = \langle x(t), y(t) \rangle$
Velocity	$\overrightarrow{v(t)} = \langle x'(t), y'(t) \rangle$
Speed	$\left\|\overrightarrow{v(t)}\right\| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$
Acceleration	$\overrightarrow{a(t)} = \langle x''(t), y''(t) \rangle$

8.2 L'Hopital's Rule

Note:

1. L'Hopital's rule applies to one-sided limits
2. The constant a given in the theorem may be $\pm\infty$

Theorem 1: L'Hopital's Rule (first form)

If $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

Theorem 2: L'Hopital's Rule (stronger form)

If $f(a) = g(a) = 0$, and if f and g are differentiable on an open interval I containing a, and supposed that $g'(x) \neq 0$ on I if $x \neq a$. Then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Provided the latter limit exists.

Theorem 3: L'Hopital's Rule (∞/∞ form)

If $f(x)$ and $g(x)$ both approach infinity as $x \to a$, then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Provided the latter limit exists.

Intermediate Forms: $\mathbf{1^{\infty}, 0^{0}, \infty^{0}}$

$$\lim_{x\to a}\ln f(x) = L \Rightarrow \lim_{x\to a} f(x) = e^{L}$$

4.1 Extreme Values of Functions

Absolute (Global) Extreme Values

Let f be a function with domain D. Then $f(c)$ is the

a) Absolute maximum value on D if and only if $f(x) \le f(c)$ for all $x \in D$

b) Absolute minimum value on D if and only if $f(x) \ge f(c)$ for all $x \in D$

Theorem: the Extreme Value Theorem

If f if continuous on a closed interval [a,b], then f has both a maximum value and a minimum value on the interval.

Relative (Local) Extreme Values

Let c be an interior point of the domain of the function f. Then $f(c)$ is a

a) Local maximum value at c if and only if $f(x) \le f(c)$ for all x in some open interval containing c.

b) Local minimum value at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c.

A function f has a local maximum or local minimum at an endpoint c if the appropriate inequality holds for all x in some half-open domain interval containing c.

Theorem: Local Extreme Values

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c, then

$$f'(c) = 0$$

Critical Point

A point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a critical point of f.

** a critical point is a value, not a point**

4.2 Mean Value Theorem

Rolle's Theorem

If $y = f(x)$ is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), and $f(a) = f(b)$, then there is at least one point c in (a,b) at which

$$f'(c) = 0$$

Mean Value Theorem

If $y = f(x)$ is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Increasing and Decreasing Functions

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. f increases on I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
2. f increases on I if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

Monotonic Functions

A function that is always increasing on an interval or always decreasing on an interval.

Corollary 1

Let f be continuous on [a,b] and differentiable on (a,b).

1. Let $f' > 0$ at each point of (a,b), then f increases on (a,b).
2. Let $f' < 0$ at each point of (a,b), then f decreases on (a,b).

Other Consequences

1. Functions with $f' = 0$ are constant
2. Functions with the same derivative differ by a constant

Antiderivative

A function $F(x)$ is an antiderivative of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The process of finding an antiderivative is antidifferentiation.

4.3 Connecting f' and f'' with the graph of f

First derivative test for local extrema (continuous function)

Critical point	Local max: $f' > 0$ for $f(x) < c$ and $f' < 0$ for $f(x) > c$
	Local min: $f' < 0$ for $f(x) < c$ and $f' > 0$ for $f(x) > c$
	No extreme: $f' > 0$ for $f(x) < c$ and $f' > 0$ for $f(x) > c$ or

	$f' < 0$ for $f(x) < c$ and $f' < 0$ for $f(x) > c$
Left endpoint	Local max: $f < 0$ for $f(x) > a$
	Local min: $f > 0$ for $f(x) > a$
Right endpoint	Local max: $f' > 0$ for $f(x) < b$
	Local min: $f' < 0$ for $f(x) < b$

Concavity

The graph of a differentiable function $y = f(x)$ is

a) Concave up on an open interval I if y' is increasing on I

b) Concave down on an open interval I if y' is decreasing on I

Concavity Test

The graph of a twice differentiable function $y = f(x)$ is

a) Concave up on any interval where $y'' > 0$

b) Concave down on an open interval where $y'' < 0$

Points of Inflection

A point where $f'' = 0$ or $f'' DNE$ and where the concavity changes

Second Derivative Test for Local Extrema

a) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$

b) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$

4.4 Modeling and Optimization

Apply the derivative tests and concavity tests in order to find the optimal value of the function provided int he real world scenario.

4.5 Linearization and Newton's Method

Linearization and Linear Approximation

- Differentiable curves are always locally linear
- Tangent line is a good representation of the curve if close enough to the point of tangency
- Provides a good approximation of the value of the function nearby

Linearization

If f is differentiable at $x = a$, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a)$$

Defines the linearization of f at a. The approximation $f(x) \approx L(x)$ is the standard linear approximation of f at a. The point $x = a$ is the center of the approximation.

Therefore, we can deduce:

$$f(a + \Delta x) \approx L(a + \Delta x) = f'(a) \bullet \Delta x + f(a)$$

$$\because \Delta y = f(a + \Delta x) - f(a) \approx f'(a) \bullet \Delta x$$

$$\therefore \Delta y \approx f'(a) \bullet \Delta x$$

As $\Delta x \to 0$, $dy = f'(a) \bullet dx$

Differentials

Let $y = f(x)$ be a differentiable function. The differential dx is an independent variable. The differential dy (or df, the differential of f) is

$$df = dy = f'(x)dx$$

Note: the dependent variable depends on both x and dx.

Sum and product Rules for Differential

$$d(x + y) = dx + dy$$

$$d(xy) = xdx + ydy$$

Note: chain rule and quotient rule also apply for differentials

Estimate Change with Differentials

Differential estimate of change:

Let $f(x)$ be differentiable at $x = a$. The approximate change in the value of f when x changes from a to $a + dx$ is

$$df = f'(a)dx$$

4.6 Related Rates

Step 1: Find relationship of the values in the question.

Step 2: Take derivative on both sides in relation to a constant value, usually time.

Step 3: Analyze at specific points given in the question.