IDX G9 Physics H

Study Guide Issue 1

By Albert and Alvin, Edited by Grace

Table of Contents:

2.4 Acceleration

Acceleration

- **Definition: The rate at which an object's velocity changes**
- **Formula:** $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta \vec{t}} = \dfrac{\overline{v_f} = \overline{v_i}}{t_f - t_i} \dfrac{m}{s^2}$
- Direction of acceleration is the same as the direction of F_{net} $\left(a = \dfrac{F_{net}}{m}\right)$

Instantaneous Acceleration

- **Definition: The change in velocity at an instant of time**
- Formula: $\vec{a}(t) = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{dv}{dt}$ (optional to memorize)

2.5 Motion at constant acceleration

- **IMPORTANT FORMULAS (only use when acceleration is constant)**

1. $a = \dfrac{\Delta v}{t} = \dfrac{v_f - v_i}{t}$

2. $\Delta d = \dfrac{(v_i + v_f)}{2} t$

3. $\Delta d = v_i t + \dfrac{1}{2} a t^2$ / $\Delta d = v_f t - \dfrac{1}{2} a t^2$

4. $\Delta d = \dfrac{(v_f{}^2 - v_i{}^2)}{2a}$

2.7 Falling Objects

Free Fall

- **Definition: The motion of a falling object when air resistance is negligible and the action can be considered due to gravity alone**
- Galileo's hypothesis: at a given location on the Earth and in the absence of air resistance, all objects fall with the **same constant acceleration**
  - **gravitational acceleration: $g = 9.8 m/s^2$** toward the **center** of Earth
    - "+g" defines downward to be the positive direction
    - "-g" defines upward to be the positive direction
- Important information about free fall (not given in the question)
  - **If object is dropped: $vi = 0\ m/s, a = 9.8 m/s^2$(down is +)**

o **If object is thrown with force: $vi \neq 0 \ m/s, a = 9.8 m/s^2$ (down is +)**

Object Thrown Directly Upward
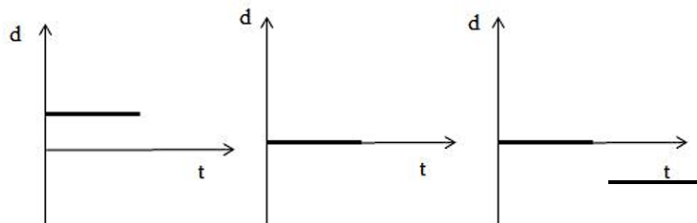
- If upward is "+":
    1. a is constant all along
    2. Initial v is +, initial Δd is 0
    3. Highest position, v=0, Δd is the largest in + direction
    4. If the object is above the initial position, Δd is +
       If the object is below the initial position, Δd is -
    5. Rising: v +, Δd +;
       Falling: v -, Δd + (above initial position) – below initial
    6. Same Δd (height), same speed.
    7. Returns to original position, v=-vi, Δd =0, $t_{rise} = t_{fall}$

2.8 Graphical Analysis of Linear Motion

- **The y-intercept** represents the **initial position**
- **Axes meaning:**
    o y-axis → position
    o x-axis → time

3 types of Motion Graph:

- Position-Time graph (d-t graph)
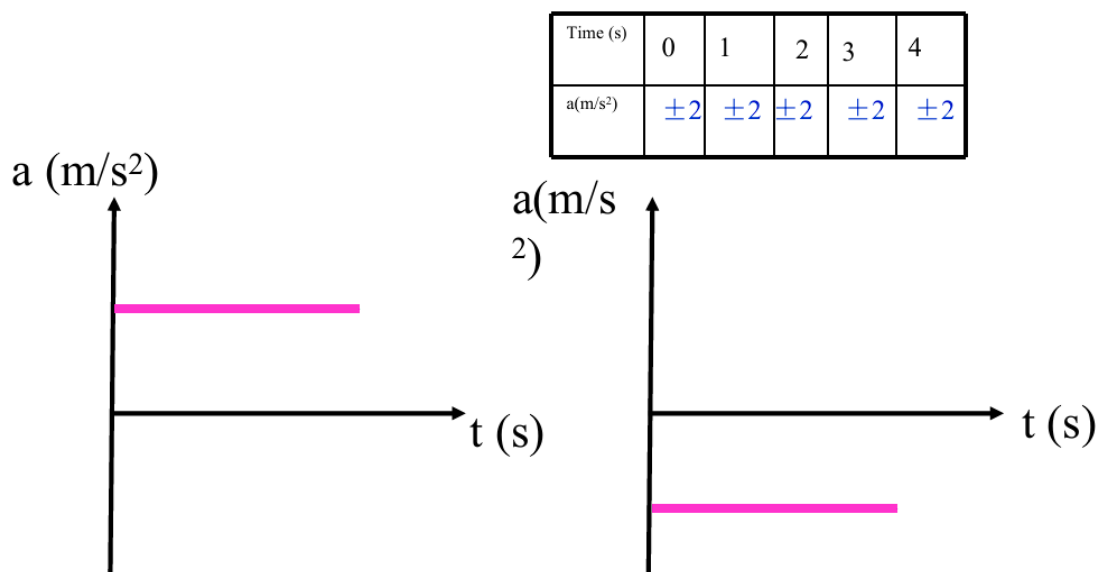   o No Motion (at rest): Horizontal straight line

    o Uniform Motion:
        ▪ **The intersection of graph lines** shows the point where **two objects meet**

- Slanted Straight line
- Slope = **average velocity**
  - Rising slope → "+" (positive velocity); Descending slope → "–" (negative velocity)
  - Steeper slope → greater speed (larger ($|v|$))
- **Instantaneous Velocity**
  - Only when velocity is **constant**: instantaneous velocity = average velocity
- **Equation of Motion (for constant velocity)**

$$d_f = vt + d_i$$

- Accelerated Motion:
  - graph lines curved
  - average velocity = slope of the connection of the two dots on the line
  - instantaneous velocity = slope of the tangent to the curve
- Velocity-Time graph (V-T graph)
  - Uniform Motion (a = 0)
    - Horizontal line in v-t: v=constant, a=0
    - Velocity sign:
      - v > 0: moving in a positive direction
      - v < 0: moving in a negative direction
    - **Displacement = total area** between the graph line and (t)-axis
      - Above (t)-axis → ($\Delta d > 0$)
      - Below (t)-axis → ($\Delta d < 0$)
    - Meeting point: when displacement / area is equal
    - slope = Average acceleration = Instantaneous acceleration = 0
  - Uniformly Accelerated Motion
    - Slanted straight line in v-t graph
    - Slope = Average acceleration = Instantaneous acceleration
    - In V-t graph:

- Rising line → a > 0
- Descending line → a < 0
  - Steeper slope → |a| larger
  - o Motion with Changing Acceleration
    - Instantaneous acceleration = slope of the tangent to the curve at a particular moment
- Acceleration-Time graph (a-t graph)
  - o Uniformly Accelerated Motion:
    - Horizontal straight line parallel to t-axis

Time (s)	0	1	2	3	4
a(m/s²)	±2	±2	±2	±2	±2

a (m/s²)

t (s)

a(m/s²)

t (s)

Chapter 3 Kinematics in Two Dimensions; Vectors

3.1 Vectors and Scalars

- **Vector**: magnitude and direction
- **Scalar**: only magnitude
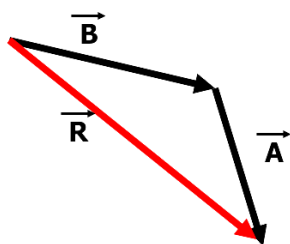- Representing Vectors:

A line and an arrow

Symbol: $\vec{A}$, $\vec{v}$ (Velocity), and $\vec{a}$ (Acceleration)
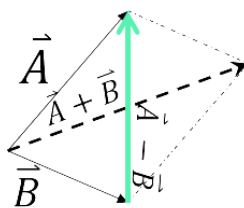
magnitude
head (tip)
direction
tail

- o **Draw to Scale**: ex. 1cm=10m/s (every vector must be at least two segments long)
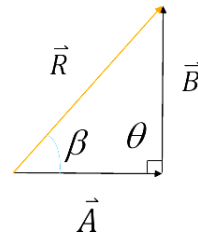
3.2 Addition of Vectors – Graphical Methods

- **Resultant/net** is sum of ≥ 2 vectors

- If $\vec{A} + \vec{B} = \vec{R}$, then $|A - B| \leq R \leq A + B$

- Tail-to-Tip method:
  - o Move the tail of a vector to the tip of a vector (without changing direction and magnitude)
  - o The resultant points from the tail of the first vector to the tip of the last vector

- Parallelogram method (only suitable for two vectors)
  - o Move the tail of two vectors together, forming an angle
  - o Construct a parallelogram using the two vectors as sides of the figure
  - o The diagonal pointing from the tail of both vectors is the resultant

- Calculation Method (Pythagorean theorem & Trigonometric ratios)
  - o Construct a triangle using the two vectors as the sides and the resultant force as the hypotenuse
  - o Calculate the angle of the resultant force using trigonometric ratios

(Tail-to-Tip Method)　　　(Parallelogram Method)　　　(Calculation Method)

3.3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

- Subtraction:

- o Vectors can be directly subtracted if they are in opposite directions
- o Otherwise, turn subtracting vector to opposite direction
- Multiplication
  - o Vectors can be multiplied by scalars
    - If scalar > 0 , the magnitude is the product & same direction as the vector
    - If scalar < 0, the magnitude is still the product & has opposite direction of the vector

3.4 Adding Vectors by Components

- know magnitude and direction → break vector into x and y components
  - o **Vector Resolution:** $\vec{A} = \vec{A_x} + \vec{A_y}$
  - o $A_x = A cos\theta$; $A_y = A sin\theta$

3.5 Projectile Motion

- **Projectile**: An object shot through the air
- **Trajectory**: The curved flight path that is followed by a moving object
- Projectile motion is a combination of two independent motions:
  - o **Horizontal motion component**: motion with constant velocity when there is no air resistance (Usually resulted from a toss/launch force)
  - o **Vertical motion component**: motion with constant acceleration (Usually resulted from a toss/launch force + gravity)
- Projectile launched horizontally:
  - o $v_y = gt$
  - o v_x is constant, $v_x = v_i$

$$t = \sqrt{\frac{2d_y}{g}} \qquad d_y = \frac{gt^2}{2} \qquad d_x = v_x t$$

- Projectile launched at an angle:

o v_y is the y component of v_i at a constant acceleration of 9.8m/s^2 downwards

o v_x is constant

$$t_{highest} = \frac{v_{iy}}{g} \; ; \; t_{flight} = 2 \times \frac{v_y}{g}$$

$$h_{max} = \frac{v_{iy}^2}{2g}$$

$$d_x = v_{ix}t_{flight}$$

3.6 Solving Problems Involving Projectile Motion

1) Some physics students stand on a cliff that is **80 meters high**. They throw a rock perfectly horizontally off the edge with an **initial speed of 15 m/s**. Assume air resistance is negligible, and the acceleration due to gravity is $g = 9.8 \text{m/s2}$. **How long** does it take for the rock to hit the ground below? What is the **final vertical velocity** of the rock just before it hits? What is the **magnitude** of the rock's **final overall velocity**? **How far** from the base of the cliff does the rock land?

Solution:

Time: Use the formula below to calculate

$$t = \sqrt{\frac{2d_y}{g}}$$

$$t = \sqrt{\frac{160}{9.8}}$$

$$t = 4.04$$

Vertical Velocity: Use the formula below to calculate

$$V_y = gt$$

$$V_y = 9.8 \times 4.04$$

$$V_y = 39.59 m/s$$

Magnitude of Final Overall Velocity: Calculate by using the Pythagorean theorem

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{(15)^2 + (39.6)^2}$$

$$v = 42.3 m/s$$

Distance from the Base of the Cliff: use formula below

$$d_X = v_x t$$

$$d_x = 15 \times 4.04$$

$$d_x = 60.6m$$

2) A cannonball is fired from ground level with an **initial velocity of 50 m/s** at an angle of **30° above the horizontal**. What is the **maximum height** reached by the cannonball? How much **time** does it spend **in the air** (total time of flight)? What is its **range** (the horizontal distance it travels before hitting the ground)? What is the **velocity** (magnitude and direction) of the cannonball **2.0 seconds after** it is fired?

Solution:

Break the Initial Velocity into x and y components:

$$v_{iy} = 50 \sin 30° = 25 m/s$$

$$v_{ix} = 50 \cos 30° = 43.3 m/s$$

Maximum Height: Calculate by using the formula below

$$h_{max} = \frac{v_{iy}^2}{2g}$$

$$h_{max} = \frac{25^2}{19.6} = 31.9m$$

Time of Flight:

$$t_{flight} = 2 \times \frac{v_y}{g}$$

$$t_{flight} = 2 \times \frac{25}{9.8} = 5.1s$$

Horizontal Distance:

$$d_x = v_{ix}t_{flight}$$

$$d_x = 43.3 \times 5.1 = 220.83m$$

Velocity as t=2s: use Pythagorean theorem and trigonometric ratios

$$v_y = v_{iy} + a_y t$$

$$v_y = 25 + (-9.8)(2) = 5.4m/s$$

$$V = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{(43.3)^2 + (5.4)^2} = 43.64m/s$$

$$\theta = \tan^{-1}(\frac{v_y}{v_x})$$

$$\theta = \tan^{-1}(\frac{5.4}{43.64}) = 7.05°$$

So, the velocity is 43.6 m/s at 7.1° below the horizontal.

3.8 Relative Velocity

- A person on the train sees a tree move at a speed of 100km/h, but a person on the ground might state that the tree is at rest; motion is relative
- **Relative velocity**: The velocity of one body relative to another
- Formula:

- $\vec{v}_{AB} + \vec{v}_{BC} = \vec{v}_{AC}$
- $\vec{v}_{AB} = -\vec{v}_{BA}$