## IDX G9 Math H
## Study Guide Issue S1 Monthly 2
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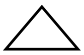
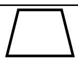
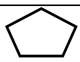
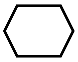
**Contents:**



### 3.5 The Polygon Angle-Sum Theorem

- **Convex polygon (default polygon)**
  - A polygon with no diagonal with points outside the polygon
- Concave polygon
  - A polygon with at least one diagonal with points outside the polygon
- **Equilateral polygon**
  - A polygon with congruent sides
- **Equiangular polygon**
  - A polygon with congruent angles
- **Regular polygon**
  - A polygon that is both equiangular and regular

- A triangle only needs to be (equilateral or equiangular) to be regular while others need to be both
- 
- In this case it is equilateral but not equiangular

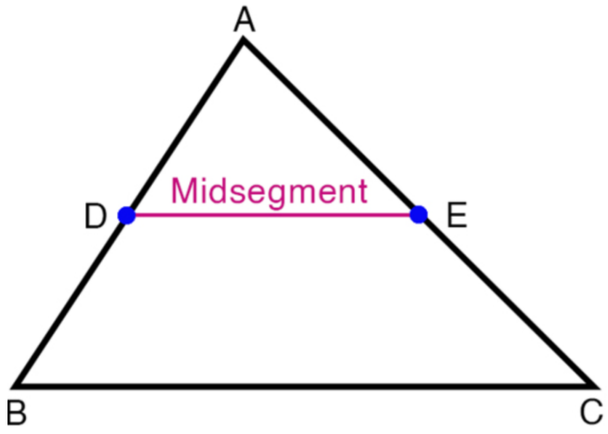
| Polygon | Number of Sides | Number of Triangles formed | Sum of the interior angle measures |
|---|---|---|---|
|  | 3 | 1 | 180 |
|  | 4 | 2 | 360 |
|  | 5 | 3 | 540 |
|  | 6 | 4 | 720 |
| | n | n-2 | 180(n-2) |

- **Polygon Angle-Sum Theorem**
  - The sum of the measures of the interior angles of an n-gon is 180(n-2).
- **Polygon Exterior Angle-Sum Theorem**
  - The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.
  - **This is only applicable in concave shapes**
  - 
  - Prove: From the Polygon Angle-Sum Theorem, we know that the sum of the measures of the interior angles of an n-gon is 180(n-2), which is simplified to 180n-360. Taking the example of a regular n-gon, the measure of each angle of the n-gon is 180-360/n. Since the sum of an angle and its exterior angle is 180, the measure of each exterior angle of the n-gon is 180-(180-360/n) = 360/n, so the sum of the measures of the exterior angles is (360/n) * n = 360. Therefore, the sum of the measures of the exterior angles of a n-gon is always 360.

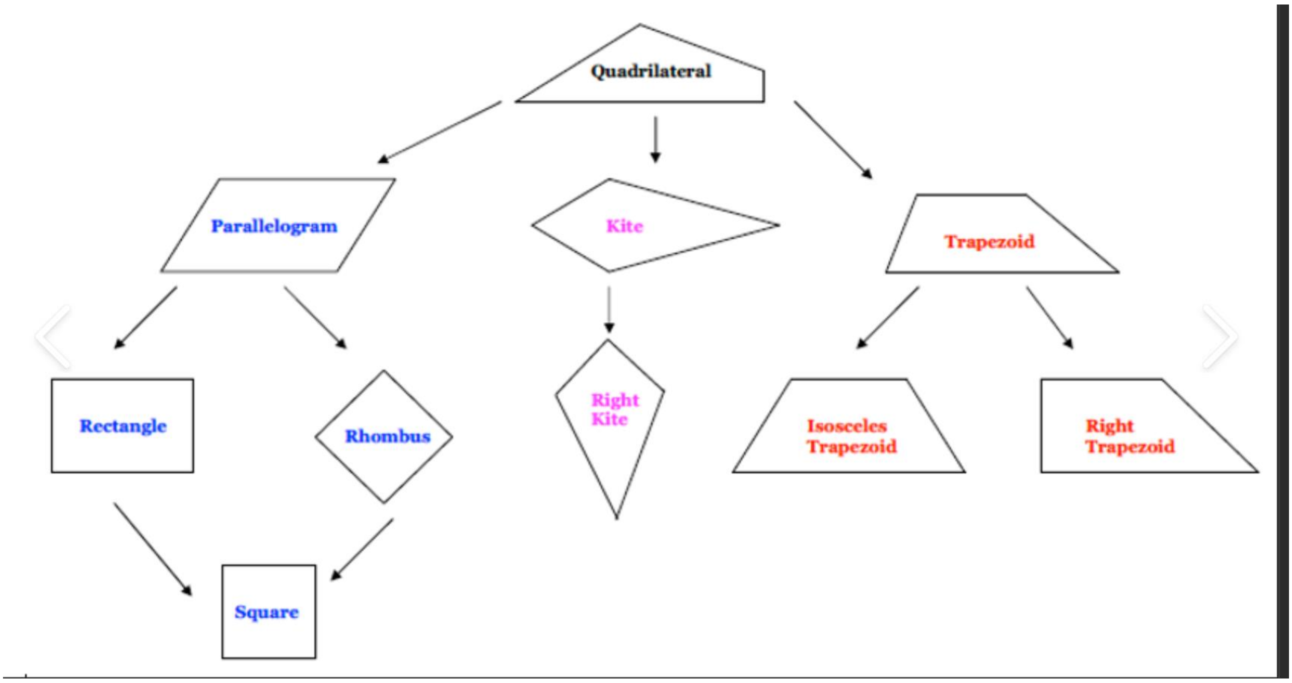
| Number of Sides | 3 | 4 | 5 | 6 | 7 | 8 | 9 | n |
|---|---|---|---|---|---|---|---|---|
| Number of Diagonals | 0 | 2 | 5 | 9 | 14 | 20 | 27 | n(n-3)/2 |

**5.1 Midsegments of Triangles**

- **Midsegment**
  - A segment in a triangle whose endpoints are the midpoints of two sides of the triangle. The midsegment is parallel to the third side and is half the length of the third side.
  - In Triangle OPQ, given R, S are the midpoints of OP, OQ. Prove: RS = ½ PQ and RS//PQ. Note: = will replace 'is congruent to' for clarity.
  - Proving: Draw a line parallel to OP such that when RS is extended it meets the line at F. Because OP // QF, Angle SOR =Angle SQF, and Angle ORS = QFS. Furthermore, OS = QS because S is the midpoint of OQ. From the above three congruent statements, we can prove Triangle ORS is congruent to Triangle QFS by AAS congruency. By CPCTC (corresponding parts of congruent triangles are congruent), we can then prove that OR=QF, and RS=SF. From OR=QF and OR=PR (R bisects OP), we can use the Transitive Property of Congruence to show PR=QF. We know that PR//QF and PR=QF, so we can prove PRQF is a parallelogram through the property of parallelograms (will be covered later). The property of a parallelogram then shows that RF=PQ, and since RF=RS+SF and RS=SF, we can get 2RS=PQ, so RS=1/2PQ. The property of parallelogram also shows that RF//PQ, and since RS and RF are on the same line, RS//PQ, so we have proved both properties of a midsegment.

- 


**6.1 Classifying Quadrilaterals**




- **Parallelogram**
  - A quadrilateral with both pairs of opposite sides parallel
- **Rhombus**
  - A parallelogram with 4 congruent sides
  - Classifications that also apply: Parallelogram
- **Rectangle**
  - A parallelogram with 4 right angles
  - Classifications that also apply: Parallelogram
- **Square**

- A parallelogram with 4 congruent sides and 4 right angles
- Classifications that also apply: Parallelogram, Rhombus, Rectangle

- **Kite**
  - A quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent
- **Trapezoid**
  - A quadrilateral with exactly one pair of parallel sides
- **Isosceles Trapezoid**
  - A trapezoid with exactly one pair of parallel sides

| Shape | Number of differing side lengths (at most) | Number of right angles (at least) | Number of pairs of parallel opposite sides |
|---|---|---|---|
| Square | 0 | 4 | 2 |
| Kite | 2 | 0 | 0 |
| Parallelogram | 2 | 0 | 2 |
| Trapezoid | 4 | 0 | 1 |
| Rectangle | 2 | 4 | 2 |
| Rhombus | 0 | 0 | 2 |

**6.2 Properties of Parallelograms**

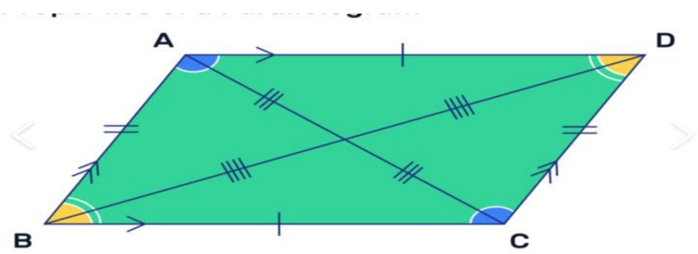
- **Theorem 6-1**
  - **Opposite sides of a parallelogram are congruent.**
  - Prove: Assume there is a parallelogram ABCD. Draw a diagonal BD. From the Alternate Interior Angles Theorem, we can get Angle ABD = Angle CDB, and Angle ADB = Angle CBD. Furthermore, we have BD=DB by the Reflexive Property of Congruence. Therefore, we can prove Triangle ABD is congruent to triangle CDB by ASA congruency. By CPCTC (corresponding parts of congruent triangles are congruent), we have AB is congruent to CD.
- **Theorem 6-2**
  - **Opposite angles of a parallelogram are congruent.**
  - Prove: Assume there is a parallelogram ABCD. Draw a diagonal BD. From the Alternate Interior Angles Theorem, we can get Angle ABD = Angle CDB, and Angle ADB = Angle

CBD. Furthermore, we have BD=DB by the Reflexive Property of Congruence. Therefore, we can prove Triangle ABD is congruent to Triangle CDB by ASA congruency. By CPCTC (corresponding parts of congruent triangles are congruent), we have Angle A is congruent to Angle C.

- **Theorem 6-3**
  - **The diagonals of a parallelogram bisect each other.**
  - Prove: Assume there is a parallelogram ABCD. The diagonals BD and AC intersect at point O. From the Alternate Interior Angles Theorem, we can prove Angle ADO = Angle CBO, and Angle DAO = Angle BCO. Furthermore, because opposite sides of a parallelogram are congruent, AD = CB. We can then prove Triangle ADO is congruent to CBO by ASA congruency. By CPCTC (corresponding parts of congruent triangles are congruent), we can show AO=CO and DO=BO, so the two diagonals AC and BD bisect each other.
- **Theorem 6-4**
  - If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments of every transversal.

The adjacent angles of parallelogram are supplementary

**6.3 Proving quadrilateral is parallelogram**




- **Definition and properties of parallelogram**
  - Opposite sides are parallel and equal
  - Opposite angles are equal
  - Diagonals bisect each other

- **Theorems decides that the quadrilateral is parallelogram**

- If both pairs of opposite sides of a quadrilateral are parallel then the quadrilateral is a parallelogram

  -could be proven by the definitions of parallelogram

- If both pairs of opposite sides of a quadrilateral are equal in length then the quadrilateral is a parallelogram

  -could be proven by congruent triangles after connecting diagonals.

- If one pair of opposite sides of a quadrilateral is both parallel and equal then the quadrilateral is a parallelogram. (only in the same pair of sides)

  -could be proven by congruent triangles after connecting diagonals

- If the diagonals od a quadrilateral bisect each other then the quadrilateral is a parallelogram

  -could be proven by congruent triangles

**6.4 special quadrilaterals**

- **Properties**
  - Rhombus

    -Each diagonal bisects two angles

    -All sides are congruent

    converse: if one of the diagonals in a parallelogram bisect two angles, it's a rhombus

    -diagonals are perpendicular to each other

    Converse: if diagonals in a parallelogram are perpendicular to each other, it's a rhombus.

  - Rectangle

    -Diagonals are equal in length

    Converse: if diagonals of a parallelogram are equal, it is a rectangle.

**6.5 trapezoids and kites**

| | Normal trapezoid | Isosceles trapezoid | kite |
|---|---|---|---|
| side | One pair parallel, other pair neither parallel nor equal | One pair parallel, other pair equal in length | Two adjacent sides congruent, opposite sides not congruent |
| angle | Same side angles supplementary | Same side angles supplementary, opposite angles supplementary | Only one pair of opposite angles congruent |
| diagonal | Neither equal nor bisect | Equal in length but not bisected | Perpendicular bisect each other, bisects two opposite angles |

- **Trapezoids**
    - The base angles of an isosceles trapezoids are equal

      -Can be proved through constructing parallel lines of the sides which are not parallel to form a parallelogram and a triangle which is isosceles.
    - The diagonals of an isosceles trapezoids are equal

      -Can be proved through congruent triangles

- **Kite**
    - Diagonals perpendicular bisect each other