IDX G10 AP Calculus
Study Guide Issue #4
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7.2 Areas in the Plane

Area between curves

If f and g are continuous with $f(x) \geq g(x)$ for all x in $[a,b]$, then the area between the curves $y = f(x)$ and $y = g(x)$ from a to b is

$$A = \int_a^b [f(x) - g(x)]dx$$

Consider dividing the function into multiple parts if the boundaries are changing

Integrating with respect to y can be more convenient in some cases. Universally, the area from a to b is always

$$A = \int_a^b |[f(x) - g(x)]|dx$$

7.3 Volumes

The volume of a solid of known integrable function, where the cross-section area of the rotated solid is $A(x)$, from $x = a$ to $x = b$ is:

$$V = \int_a^b A(x)dx$$

For a solid rotated about the x-axis, $A(x) = \pi f(x)^2$.
Find volume with the method of slicing:

1. Sketch the solid and a typical cross-section
2. Find a formula for $A(x)$ based on the x-coordinate
3. Find the upper and lower limits of integration as a region
4. Integrate $A(x)$ over this region to find the volume

Cylindrical shells:

$$V = 2\pi \int_a^b rh\, dx$$

Here r and h are not constants, r describes the distance between the functional value and the rotational axis while h describes the distance of the x coordinate from the initial region value: a.

7.1 Integral as Net Change

Linear motion:

- Integrating velocity $v(t)$ gives displacement
- Integrating the absolute value of velocity $|v(t)|$ gives total distance traveled
- New position = initial position + displacement

$$x(t) = x(t_0) + \int_{t_0}^t v(t)dt$$

$$x(t) - x(t_0) = \int_{t_0}^t v(t)dt$$

Work
When a body moves a distance *d* along a straight line as a result of the action of a force of a constant magnitude *F* in the direction of motion, the word done by the force is $W = Fd$
Hooke's Law: $F = kx$. So over a distance, at each point x there is a small displacement dx so we can integrate over kxdx.

7.4 Lengths of Curves

Length of a smooth curve

- Requires a continuous first derivative

If a smooth curve begins at (a, c) and ends at (b, d), and $a < b$, then the length (arc length) of the curve is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

If *y* is a smooth function of *x* on $[a, b]$.

$$L = \int_c^d \sqrt{a + \left(\frac{dx}{dy}\right)^2}\, dy$$

If *x* is a smooth function of *y* on $[c, d]$.

8.4 Improper Integrals

- Integrals with infinite integration limits

1. If $f(x)$ is continuous on $[a, \infty)$, then:

$$\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$$

2. If $f(x)$ is continuous on $(-\infty, b]$, then:

$$\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$$

3. If $f(x)$ is continuous on $(-\infty, \infty)$, then:

$$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^\infty f(x)dx$$

Where *c* is any real number

If the limits are finite, the improper integral converges. If the limits DNE, the improper integral diverges and has no value.

Improper Integrals with Infinite Discontinuities

- Integrals of functions that become infinite at a point within the interval of integration

1. If $f(x)$ is continuous on $(a, b]$, then

$$\int_a^b f(x)dx = \lim_{c \to a^+} \int_c^b f(x)dx$$

2. If $f(x)$ is continuous on $[a, b)$, then

$$\int_a^b f(x)dx = \lim_{c \to b^-} \int_a^c f(x)dx$$

3. If $f(x)$ is continuous on $[a, c) \cup (c, b]$, then

$$\int_a^b f(x)dx = \lim_{q \to c^-} \int_a^q f(x)dx + \lim_{r \to c^+} \int_r^b f(x)dx$$

Theorem: Comparison Test

Let *f* and *g* be continuous on $[a, \infty)$ with $0 \leq f(x) \leq g(x)$ for all $x \geq a$. Then,

1. If $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ converges as well
2. If $\int_a^\infty f(x)dx$ diverges, then $\int_a^\infty g(x)dx$ diverges as well

10.1 Parametric Functions

Parametric Differentiation Formulas

If *x* and *y* are both differentiable functions of *t* and if $\frac{dx}{dt} \neq 0$, then

$$\frac{dy}{dx} = \frac{dy}{dt} \bullet \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

If $y' = \frac{dy}{dx}$ is also a differentiable function of *t*, then

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Length of a Parametric Curve

Let *L* be the length of a parametric curve that is traversed exactly once as *t* increases from t_1 to t_2. If $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are continuous functions of *t*, then

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Cycloids

Supposed that a wheel of radius a rolls along a horizontal line without slipping. The path traced by the point P on the wheel's edge is a cycloid

The position $P(x, y)$ is a point on the edge of the wheel when the wheel has turn t radians

Parametric equations:

$$x = at - a\sin t$$

$$y = a - a\cos t$$

Length of one arch of the cycloid is $8a$