IDX G9 Math H
Study Guide Issue S1 Final
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Content

7.1 Ratios and Proportions

- **Terms:**
  - Ratio: comparison of two quantities
  - Proportion: equation stating two ratios are equal

- Means: the middle terms in a proportion
- Extremes: the outer terms in a proportion
- Scale: the ratio of a measurement in model comparing to real-world

- **Properties of proportions (the equivalent to a/b=c/d)**
  - Product of extremes = product of means (ad=bc)
  - Inverts still equal (b/a=d/c)
  - Componendo ((a+b)/b=(c+d)/d)
  - This happens because b/b and d/d is 1
  - Dividendo ((a-b)/b=(c-d)/d)

7.2 Similar Polygons

- **Terms:**
  - Similar: two shapes have the same shape
  - similarity ratio: constant ratio of corresponding sides
  - golden rectangle: rectangle which ratio of length to width equals the golden ratio (1.618)
  - golden ratio: an irrational number form dividing line into 2 parts (1.618)

7.3 Triangles Similar

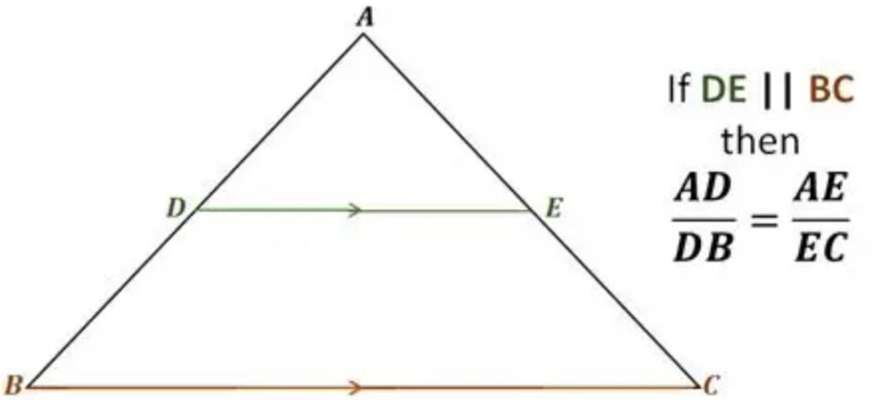
- **AA postulate**
  - If two angles of a triangle are congruent to the two angles of another, then the two triangles are similar
- **SAS theorem**
  - if one angle of a triangle is congruent to one angle of another, and sides including the two angles are proportional, then the two triangles are similar
- **SSS theorem**

- If the corresponding sides of two triangles are proportional, then the two triangles are similar

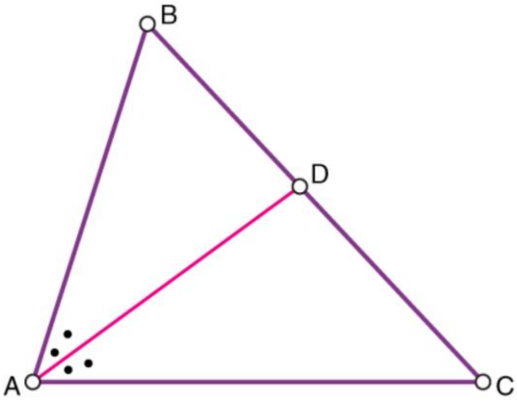
7.4 Similarities in Right Triangles

- **Methods to prove similar triangles**
- **Proportional properties**
- **Means and extremes**

7.5 Proportions in Triangles

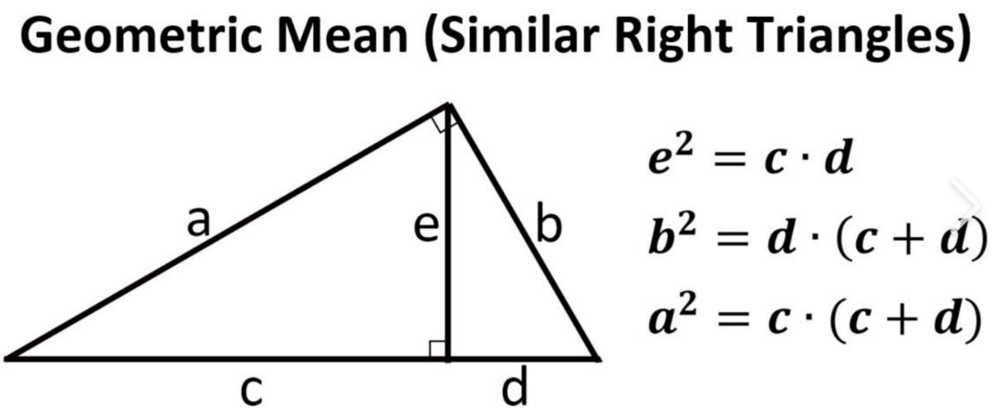
- **Side-splitter theorem**
  - If a line parallel to one side of a triangle and intersects the other two sides, then the sides are divided proportionally
  - Corollary: if three parallel lines intersect two transversals, then segments on the transversals are proportional
  - 

- **Angle bisector theorem**
  - If a ray bisects an angle, then it divided the opposite side into two segments proportional to the other two sides in the triangle

- 

8.1 The Pythagorean theorem and Converse

- **Pythagorean theorem**
  - In a right triangle, sum of squares of lengths of legs is equal to square of length of the hypotenuse ($a^2+b^2=c^2$)
- **Converse**
  - If the square of the length of one side of a triangle is equal to the sum of the square of the lengths of other two sides, then it's a right triangle ($c^2=a^2+b^2$)
- **Obtuse inequality**
  - If the square of the length of one side of a triangle is greater than the sum of the square of the length of the other two sides, then the triangle is obtuse ($c^2>a^2+b^2$)
- **Acute inequality**
  - if the square of the length of one side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle ($c^2<a^2+b^2$)
- **Pythagorean triples**
  - set of nonzero whole numbers a,b,c that satisfy $a^2+b^2=c^2$ (3,4,5; 5,12,13)
- **Geometric Mean**

- 

8.2 Special Right Triangles

- **45-45-90**
  - In 45-45-90 triangle, both legs are congruent and length of hypotenuse is root 2 times the length of a leg
- **30-60-90**
  - In a 30-60-90 triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is root 3 times the length of the shorter leg

8.3 tangent ratios

- **Terms**
  - Tangent ratio: a trigonometric ratio for right triangles, ratio of length of opposite side to adjacent side
- **Conclusion**
  - Tangent ratio depends on the size of angle but not the size of triangle

8.4 Sine and Cosine ratios

- **Terms**
  - Sine and cosine ratios: core trigonometric ratios for right triangles (sin=opposite/hypotenuse, cos=adjacent/hypotenuse)

- $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$; $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \frac{\sqrt{3}}{1}$; $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

9.2 The Area of a Triangle

- **The area K of Triangle ABC is given by: K= ½ x (one side) x (another side) x (sine of included angle)**
  - Proof: The area of a triangle ABC is 1/2bh. Here, when we take side a and angle C, h can be written as: a sin C, as sin C = h/a. Therefore, by replacing h with a sin C, we get the area of the triangle is ½ ab sin C, where a b are two sides and angle C is the angle between a and b.
- The area of a parallelogram with side lengths a and b and acute angle θ is ab sin θ, as a parallelogram is made of two triangles with area ½ ab sin C. We simply multiply by 2 to get ab sin C.
- Segment: The region bounded by an arc on the circle and the chord connecting the endpoints of the arc.
- Another more familar formula A=1.2bh

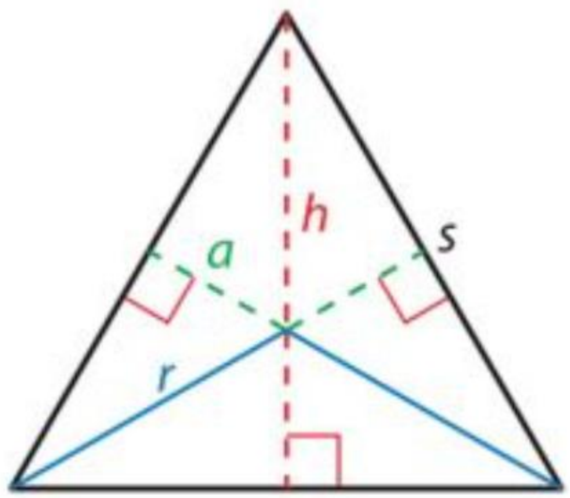
10.1 Areas of Parallelograms and Triangles

- **Theorem 10-1: Area of a Rectangle: A= bh**
- **Theorem 10-2: Area of a Parallelogram: A = bh**
- **Theorem 10-3: Area of a Triangle: A = ½ bh**

10.2 Areas of Trapezoids, Rhombuses, and Kites

- **Theorem 10-4: Area of a Trapezoid: ½ (b1+b2) h**
- **Theorem 10-5: Area of a Rhombus or Kite: ½ (diagonal 1 + diagonal 2)**

10.3 Areas of Regular Polygons

- **Apothem (in a regular polygon): The perpendicular distance from the center of the circumscribed circle of the polygon to the side of the polygon.**
- 

- **Area of a regular n-gon with a known side length and apothem: A = ½ ans = ½ ap.**
  - Proof: A regular n-gon can be divided into n triangles with the base as the side length of the polygon and height as the apothem. Therefore, the area of one triangle is ½ as. Since there is n of these triangles, we multiply to get ½ ans. Since ns is also the perimeter of the polygon, we can also write the area as ½ ap.
- **Finding the area of a regular n-gon with known radius**
  - First, divide the n-gon into n isosceles triangles. Draw the perpendicular bisector of one of the triangles, giving you angle $\theta = 360/2n$. We can then find ½ of the base of the isosceles triangle as $r \sin \theta$, so the entire base if $2r \sin \theta$. Furthermore, the apothem of the n-gon, also the height of the triangle, is $r \cos \theta$. The area of each isosceles triangle is bh: ½ $(2r \sin \theta)(r \cos \theta) = r^2 \sin \theta \cos \theta$. Finally, since there is n of these triangles in the polygon, the total area is $nr^2 \sin \theta \cos \theta$.

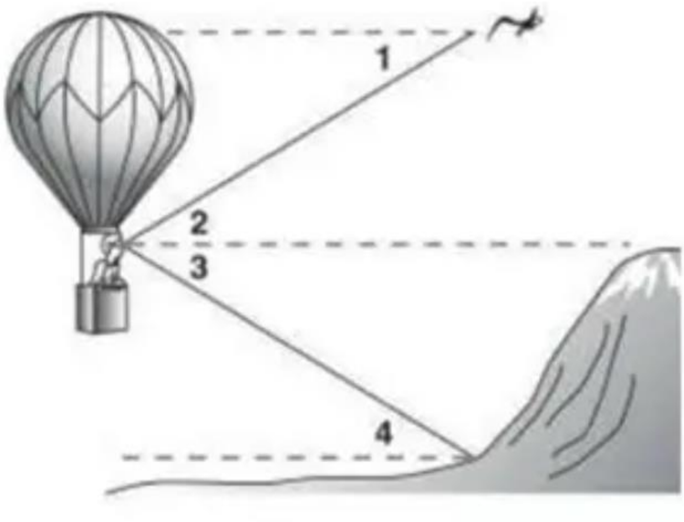
10.4 Perimeters and Areas of Similar Figures

- **Theorem 10-7: If the similarity ratio of two figures is a/b, then the ratio of their perimeters is a/b, of their areas is a^2/b^2.**

8.5 Angles of Elevation and Depression

- Angle of elevation: The angle between the horizontal line and the line of sight towards an object which is above the horizontal line.

- Angle of depression: The angle between the horizontal line and the line of sight towards an object which is below the horizontal line.



9.5 Application of Trigonometry to Navigation and Surveying

- Course: The course of a ship or plane is the angle, measured in degrees in clockwise, from the north direction to the direction of the ship or plane.

- Compass bearing: Measured in the same way as the course of a ship, and where the direction is the line towards another object.

- In surveying, a compass reading is given as an acute angle from the north-south line toward the east or west.

    - Ex. N20° E, S30° W