

# Assignment3

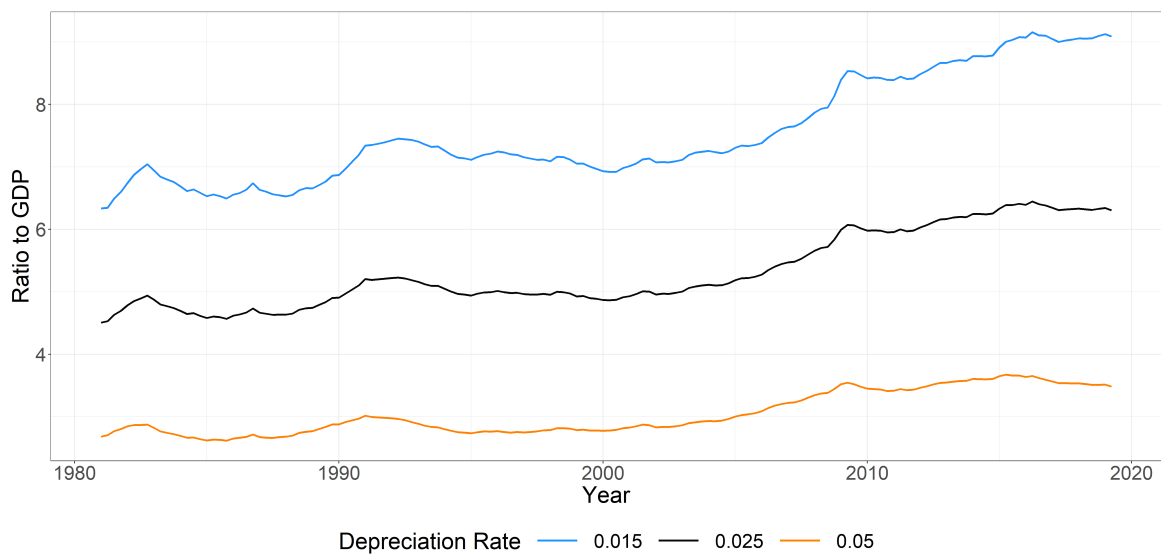
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November 2019

1.

(a)

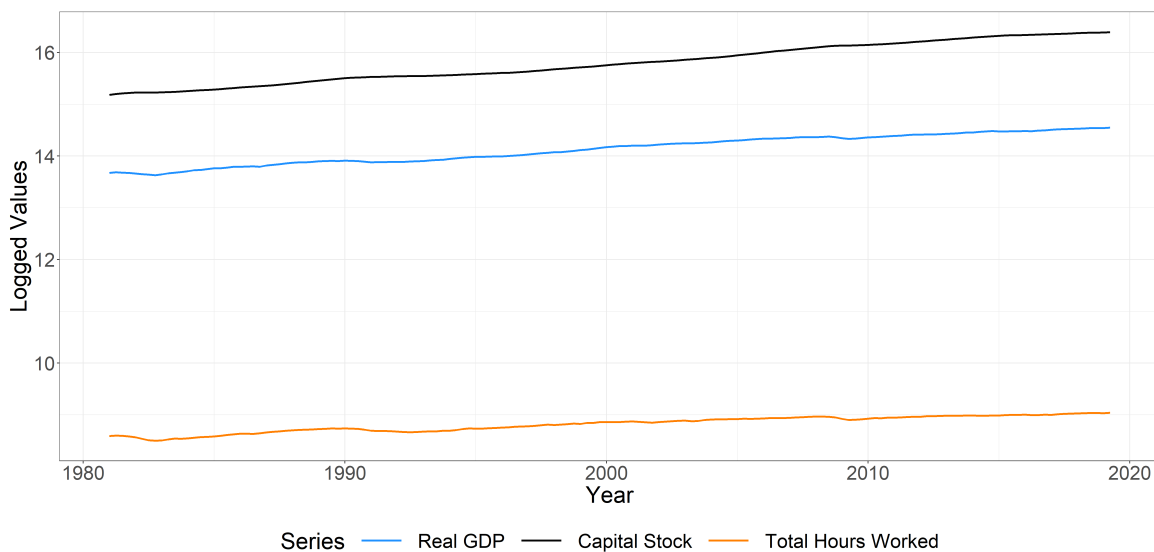
Figure 1: Imputed Capital Stock as a Ratio to Real GDP in Canada, 1981Q1 - 2019Q2



Source: Adapted from the Statistics Canada Table: 36-10-0104-01 (2012 chained prices) using real seasonally adjusted quarterly data for the components of GDP from 1961Q1-2019Q2. 1961Q1 capital stock imputed by investments divided by depreciation rate. Each following quarter was imputed by the sum of undepreciated capital stock and investment in the quarter..

(b)

Figure 2: Logged Series of Real GDP, Imputed Capital Stock and Total Hours Worked in Canada, 1981Q1 - 2019Q2



Source: Adapted from the Statistics Canada Table: 36-10-0206-01 and 36-10-0104-01 (2012 chained prices) and using real seasonally adjusted quarterly data from 1961Q1-2019Q2. 1961Q1 capital stock imputed by investments divided by depreciation rate. Each following quarter was imputed by the sum of undepreciated capital stock and investment in the quarter. Total Hours Work calculated per population and total hours worked in 2019Q2 as base, and adjusted by the index.

(c)

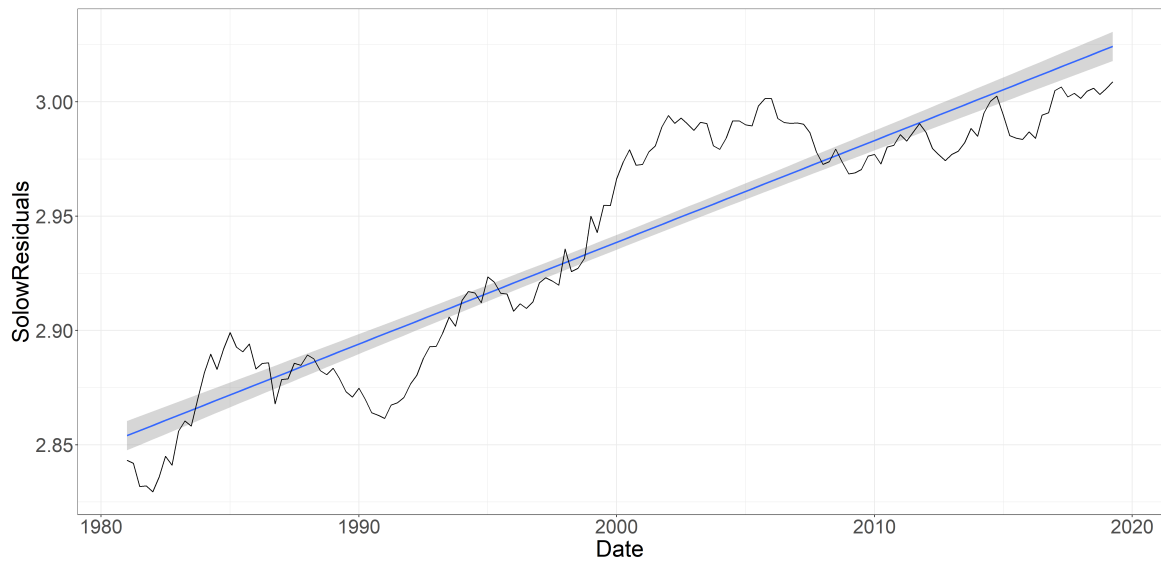
Solow Residuals ( $SR_t$ ) found by

$$\begin{aligned}\ln SR_t &= \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln N_t \\ &= \ln(e^{z_t} \mu A_0) \\ &= \ln A_0 + \ln \mu + z_t\end{aligned}$$

(d)

Estimated  $\mu = 0.012$  and  $\ln A_0 = 2.805$

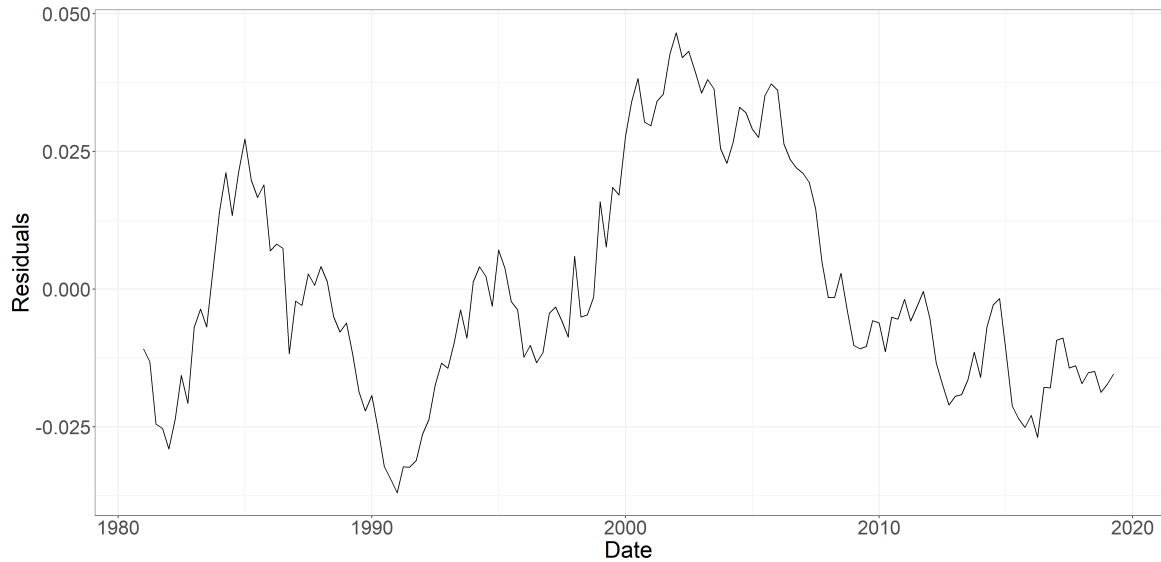
Figure 3: Solow Residual for Canada, 1981Q1 - 2019Q2



Source: Adapted from the Statistics Canada Table: 36-10-0206-01 and 36-10-0104-01 (2012 chained prices) and using real seasonally adjusted quarterly data from 1961Q1-2019Q2.

(e)

Figure 4: Residuals of the Solow Residual for Canada, 1981Q1 - 2019Q2



Source: Adapted from the Statistics Canada Table: 36-10-0206-01 and 36-10-0104-01 (2012 chained prices) and using real seasonally adjusted quarterly data from 1961Q1-2019Q2.

(f)

$$\rho = 0.9586, \sigma_e^2 = 0.0003367$$

(g)

$$\rho = 0.9586, \sigma_e^2 = 0.0006303$$

Using labor productivity as the solow residual does not take into account of capital shocks, unlike the model in part f.

2.

(a)

$$Y_t = C_t + I_t = zK_t^\alpha N_t^{1-\alpha}$$

$$I_t = K_{t+1} - (1 - \delta)K_t$$

$$\frac{I_t}{N_t} = \frac{K_{t+1}}{N_t}$$

$$\frac{I_t}{N_t} = \frac{K_{t+1}}{N_{t+1}} \frac{N_{t+1}}{N_t} - (1 - \delta) \frac{K_t}{N_t}$$

$$i_t = k_{t+1} - (1 - \delta)k_t$$

$$y_t = zk_t^\alpha = c_t + i_t$$

$$y_t = zk_t^\alpha = c_t + k_{t+1} - (1 - \delta)k_t$$

(b)

The social planner's problem is given by:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

s.t.

$$c_t + k_{t+1} - (1 - \delta)k_t = z_t k_t^\alpha$$

Lagrangian:

$$L = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} - \sum_{t=0}^{\infty} \lambda_t [c_t + k_{t+1} - (1 - \delta)k_t - z_t k_t^\alpha]$$

First order condition w.r.t

$$c_t: \beta^t c_t^{-\gamma} - \lambda_t = 0 \rightarrow \beta^t c_t^{-\gamma} = \lambda_t$$

$$c_{t+1}: \beta^{t+1} c_{t+1}^{-\gamma} - \lambda_{t+1} = 0 \rightarrow \beta^{t+1} c_{t+1}^{-\gamma} = \lambda_{t+1}$$

$$k_{t+1}: \lambda_t = \lambda_{t+1}(1 - \delta) + \alpha z_{t+1} k_{t+1}^{\alpha-1}$$

$$\lambda_t : c_t + k_{t+1} - (1 - \delta)k_t - z_t k_t^\alpha$$

Dynamic equations:

1) Euler

$$\frac{U_{c_t}}{U_{c_{t+1}}} = \frac{\beta^t c_t^{-\gamma}}{\beta^{t+1} c_{t+1}^{-\gamma}} = \frac{\lambda_t}{\lambda_{t+1}} \rightarrow \left( \frac{c_{t+1}}{c_t} \right)^\gamma = \beta \left( 1 - \delta + \alpha \frac{y_{t+1}}{k_{t+1}} \right)$$

2) Resource constraint per worker:

$$c_t + k_{t+1} = (1 - \delta)k_t + y_t$$

3) Output per worker:

$$y_t = z_t k_t^\alpha$$

(c)

Steady State:

1) Euler Equation

$$\begin{aligned} 1 &= \beta(1 - \delta + z\alpha k^{\alpha-1}) \\ 1 - \beta(1 - \delta) &= \beta z\alpha k^{\alpha-1} \\ \frac{\frac{1}{\beta} - 1 + \delta}{z_\alpha} &= k^{\alpha-1} k &= \left( \frac{\beta z\alpha}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

2) Steady State resource constraint per worker:

$$\begin{aligned} c &= z k^\alpha - \delta k \\ &= z \left( \frac{\beta z\alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left( \frac{\beta z\alpha}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

3) Output per worker

$$y = z \left( \frac{\beta z\alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}}$$

(d)

From output per worker,

$$y = z \left( \frac{\beta z\alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}}$$

Double check this derivative

$$\frac{\partial y}{\partial z} = \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} + \frac{z \alpha}{1 - \alpha} \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{2\alpha-1}{1-\alpha}}$$

$$\frac{\partial y}{\partial z} = \frac{1}{1 - \alpha} \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} > 0$$

If there is an increase in  $z$ , the steady state quantity of  $y$  increases.

$$\frac{\partial y}{\partial \beta} = \frac{\alpha z}{1 - \alpha} \left( \frac{\alpha \beta z}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}-1} \left( \frac{\alpha z}{1 - \beta(1 - \delta)} - \frac{\alpha(\delta - 1)\beta z}{(1 - \beta(1 - \delta))^2} \right)$$

$$\frac{\partial y}{\partial \beta} = \frac{1}{1 - \alpha} \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} \frac{1}{\beta^2} > 0$$

If there is an increase in  $\beta$ , the steady state quantity of  $y$  increases.

### 3

#### (a)

Since there's unity in the work force,  $N_t = 1$  then  $k_t = K_t/N_t = K_t$

Then for the firms problem, they maximize profit taking prices as given.

$$\max_{k_t} \Pi = z k_t^\alpha N_t^{1-\alpha} - w_t N_t - (r_t + \delta) k_t = z k_t^\alpha - w_t - (r_t + \delta) k_t$$

Where  $w_t$  is the wage at time  $t$ .

First Order condition w.r.t.  $k_t$

$$\alpha z k_t^{\alpha-1} - \delta = r_t$$

#### (b)

$$\max_{c_t, a_{t+1}} V_t = \sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

Such that

$$a_{t+1} + c_t = w_t + (1 + r_t) a_t + \pi_t$$

where  $\pi_t$  are the dividends received. However, in perfect competition dividends and profits are equal to 0.

Lagrangian:

$$L = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} - \sum_{t=0}^{\infty} \lambda_t [a_{t+1} + c_t - w_t - (1+r_t)a_t - \pi_t]$$

FOC set to 0 with respect to

$c_t$ :

$$\beta^t c_t^{-\gamma} = \lambda_t$$

$c_{t+1}$

$$\beta^{t+1} c_{t+1}^{-\gamma} = \lambda_{t+1}$$

$a_{t+1}$ :

$$\lambda_t = \lambda_{t+1}(1+r_{t+1})$$

$\lambda_t$  :

$$a_{t+1} + c_t = w_t + (1+r_t)a_t$$

Household Equations:

1) Euler

$$\left( \frac{c_{t+1}}{c_t} \right)^{\gamma} = \beta(1+r_{t+1})$$

2) Household Budget Constraint

$$a_{t+1} + c_t = w_t + (1+r_t)a_t + \pi_t$$

No labor supply decision since workforce is at unity,  $n_t = 1$ .

**(c)**

Define: A competitive equilibrium is a set of price  $(r_{t+1}, w_t)$  and allocations  $(c_t, n_t, k_{t+1}, a_{t+1})$ ,  $k_t, a_t$  are given. The optimality conditions of household and firm, and the transversality condition holding. The labor market ( $n_t^d = n_t^s$ ) and asset market ( $k_t = a_t$ ) clears. Both budget constraints holding with equality.

At competitive equilibrium  $\pi_t = \Pi_t = 0$

We have 6 endogenous variables,

$$i_t = y_t - c_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

$$r_t = \alpha z k_t^{\alpha-1} - \delta$$

$$\left(\frac{c_{t+1}}{c_t}\right)^\gamma = \beta(1 + r_{t+1})$$

$$y_t = z k_t^\alpha$$

**(d)**

The Euler equations are different in these two questions. There is an interest rate in Q3(b).

**4**

**(a)**

Since we assume unit mass of households  $H_t = H_0 = 1$ , we have  $y_t = Y_t/H_t = Y_t$

Our resource constraint in capita terms is

$$\begin{aligned} y_t &= c_t + i_t \\ &= c_t + k_{t+1} - (1 - \delta)k_t \\ z k_t^\alpha n_t^{1-\alpha} &= c_t + k_{t+1} - (1 - \delta)k_t \end{aligned}$$

**(b)**

The social planner's problem is given by

$$\max_{c_t, k_{t+1}, n_t} \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \theta \ln(l_t)\}$$

Such that

$$z k_t^\alpha n_t^{1-\alpha} = c_t + k_{t+1} - (1 - \delta)k_t$$

Lagrangian:

$$L = \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \theta \ln(l_t)\} - \sum_{t=0}^{\infty} \lambda_t [z k_t^\alpha n_t^{1-\alpha} + (1 - \delta)k_t - c_t - k_{t+1}]$$

First order conditions with respect to

$n_t$  Note ( $l_t = 1 - n_t$ ):

$$-\beta^t \frac{\theta}{1-n_t} = (1 - \alpha) \lambda_t y_t \frac{1}{n_t}$$

$c_t$ :

$$-\beta^t \frac{1}{c_t} = \lambda_t$$

$c_{t+1}$ :

$$-\beta^{t+1} \frac{1}{c_{t+1}} = \lambda_{t+1}$$



$k_{t+1}$ :

$$\lambda_t = \lambda_{t+1}(1 - \delta + a y_{t+1} \frac{1}{k_{t+1}})$$

$\lambda_t$ :

$$y_t + (1 - \delta)k_t = c_t + k_{t+1}$$

Dynamic equations:

Labor Supply

$$\begin{aligned}\beta^t \frac{\theta}{1 - n_t} &= (1 - \alpha) \lambda_t y_t \frac{1}{n_t} \\ &= (1 - \alpha) \beta^t \frac{1}{c_t} y_t \frac{1}{n_t} \\ \theta c_t n_t &= (1 - \alpha) y_t - (1 - \alpha) y_t n_t \\ n_t &= \frac{(1 - \alpha) y_t}{\theta c_t + (1 - \alpha) y_t}\end{aligned}$$

Euler equation  $\frac{U_{c_t}}{U_{c_{t+1}}}$

$$\begin{aligned}\frac{c_{t+1}}{c_t} &= \frac{\lambda_t}{\lambda_{t+1}} \beta \\ &= \beta(1 - \delta + \alpha \frac{y_{t+1}}{k_{t+1}})\end{aligned}$$

Resource Constraint per worker:

$$y_t + (1 - \delta)k_t = c_t + k_{t+1}$$

Output per worker:

$$y_t = z_t k_t^\alpha n_t^{1-\alpha}$$

(c)

Steady State Euler Equation

$$\begin{aligned}1 &= \beta(1 - \delta + \alpha \frac{y_{t+1}}{k_{t+1}}) \\ 1 - \beta(1 - \delta) &= \beta z \alpha k^{\alpha-1} n^{1-\alpha} \\ \frac{1 - \beta(1 - \delta)}{\beta z \alpha} &= \left(\frac{k}{n}\right)^{\alpha-1} \\ \left(\frac{\beta z \alpha}{1 - \beta(1 - \delta)}\right)^{\frac{1}{1-\alpha}} &= \frac{k}{n}\end{aligned}$$

Steady State for output per worker

$$\begin{aligned}
y &= zk^\alpha n^{1-\alpha} \\
\frac{y}{n} &= z \left( \frac{k}{n} \right)^\alpha \\
&= z \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Steady State for resource constraint per worker

$$\begin{aligned}
c &= y + k - \delta k - k \\
\frac{c}{n} &= \frac{y}{n} - \delta \frac{k}{n} \\
&= z \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}}
\end{aligned}$$

Steady State for n

$$\begin{aligned}
n &= \frac{(1 - \alpha)y}{\theta c + (1 - \alpha)y} \\
&= \frac{(1 - \alpha)\frac{y}{n}}{\theta \frac{c}{n} + (1 - \alpha)\frac{y}{n}} \\
&= \frac{(1 - \alpha)z \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha)z \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} + \theta \left[ z \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left( \frac{\beta z \alpha}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} \right]}
\end{aligned}$$

(d)

Assign  $Q = \frac{\beta \alpha}{1 - \beta(1 - \delta)}$

$$\begin{aligned}
n &= \frac{(1 - \alpha)z(zQ)^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha)z(zQ)^{\frac{\alpha}{1-\alpha}} + \theta[z(zQ)^{\frac{\alpha}{1-\alpha}} - \delta(zQ)^{\frac{1}{1-\alpha}}]} \quad \text{divide numerator and denominator by } (zQ)^{\frac{\alpha}{1-\alpha}} \\
&= \frac{(1 - \alpha)z}{(1 - \alpha)z + \theta(z - \delta zQ)} \\
&= \frac{(1 - \alpha)}{(1 - \alpha) + \theta - \theta \delta Q}
\end{aligned}$$

Therefore, z has no effect on the steady state quantities of n.

## 5

(a)

We define the real interest rate  $r_t$  and the rental rate of capital,  $\mu_t$  such that the relationship  $r_t = \mu_t - \delta$  is satisfied.

$$\max_{n_t, k_t} \Pi = z_t k_t^\alpha n_t^{1-\alpha} - w_t n_t - (r_t + \delta) k_t$$

FOC:

$$\begin{aligned} n_t : \alpha \frac{y_t}{k_t} - \delta &= r_t \\ k_t : (1 - \alpha) \frac{y_t}{n_t} &= w_t \end{aligned}$$

Since perfect competition  $\pi = 0$

**(b)**

$$\begin{aligned} y_t &= Y_t = C_t + I_t = z K_t^\alpha N_t^{1-\alpha} \\ I_t &= K_{t+1} - (1 - \delta) K_t \\ z_{t+1} &= z_t^\rho \epsilon_t \\ \epsilon_t &\text{ iid}(0, \delta_\epsilon^2) \end{aligned}$$

$$\max_{c_{t+s}, k_{t+s}, l_{t+s}} E_0[V_t] = E_0 \left[ \sum_{t=1}^{\infty} \beta^t \left( \frac{c_t^{1-\gamma}}{1-\gamma} + \theta \frac{l_t^{1-\eta}}{1-\eta} \right) \right]$$

s.t.

$$a_{t+1} + c_t = w_t n_t + (1 + r_t) a_t + \pi_t$$

where  $n_t + l_t = 1$

Set Lagrangian:

$$L = E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \left( \frac{c_t^{1-\gamma}}{1-\gamma} + \theta \frac{l_t^{1-\eta}}{1-\eta} \right) \right\} - E_0 \left\{ \sum_{t=1}^{\infty} \lambda_t (a_{t+1} + c_t - w_t n_t + (1 + r_t) a_t) \right\}$$

FOC:

$$\begin{aligned} c_t : \beta^t c_t^{-\gamma} &= \lambda_t \\ c_{t+1} : \beta^{t+1} E_0[c_{t+1}^{-\gamma}] &= E_0[\lambda_{t+1}] \\ a_{t+1} : E_0[\lambda_{t+1} (1 + r_{t+1})] &= \lambda_t \\ n_t : \beta^t \theta (1 - n_t)^{-\eta} &= \lambda_t w_t \\ \lambda_t : a_{t+1} + c_t &= w_t n_t + (1 + r_t) a_t \end{aligned}$$

Euler Equation:

$$E_0 \left[ \left( \frac{c_{t+1}}{c_t} \right)^\gamma | z_t \right] = \beta E_0[(1 + r_{t+1} | z_t)]$$

Labor Supply Decision:

$$\theta(1 - n_t)^{-\eta} = c^{-\gamma} w_t$$

Household Budget Constraint:

$$a_{t+1} + c_t = w_t n_t + (1 + r_t) a_t$$

**(c)**

Define: A competitive equilibrium is a set of price  $(r_{t+1}, w_t)$  and allocations  $(c_t, n_t, k_{t+1}, a_{t+1})$ ,  $k_t, a_t$  are given. The optimality conditions of household and firm, and the transversality condition holding. The labor market ( $n_t^d = n_t^s$ ) and asset market ( $k_t = a_t$ ) clears. Both budget constraints holding with equality.

Dynamic equations:

Euler Equation:

$$1 = E_0 \left[ \frac{c_t}{c_{t+1}} (\beta(1 + r_{t+1})^{\frac{1}{\gamma}}) | z_t \right]$$

Labor supply decision:

$$n_t = 1 - \left( \frac{\theta c_t^\gamma}{w_t} \right)^{\frac{1}{\eta}}$$

National account:

$$y_t = c_t + i_t$$

Interest rate:

$$r_t = \alpha \frac{y_t}{k_t} - \delta$$

Labor supply:

$$w_t = (1 - \alpha) \frac{y_t}{n_t}$$

Output:

$$y_t = z k_t^\alpha n_t^{1-\alpha}$$

Capital accumulation:

$$k_{t+1} = (1 - \delta) k_t + i_t$$

**(d)**

$$\theta = 1.7$$

Steady state values:

$$k = 11.40$$

$$z = 1.00$$

$$c = 0.83$$

$$r = 0.01$$

$$n = 0.30$$

$$w = 2.37$$

$$i = 0.28$$

$$y = 1.11$$

The process is saddle-path stable.

(e)

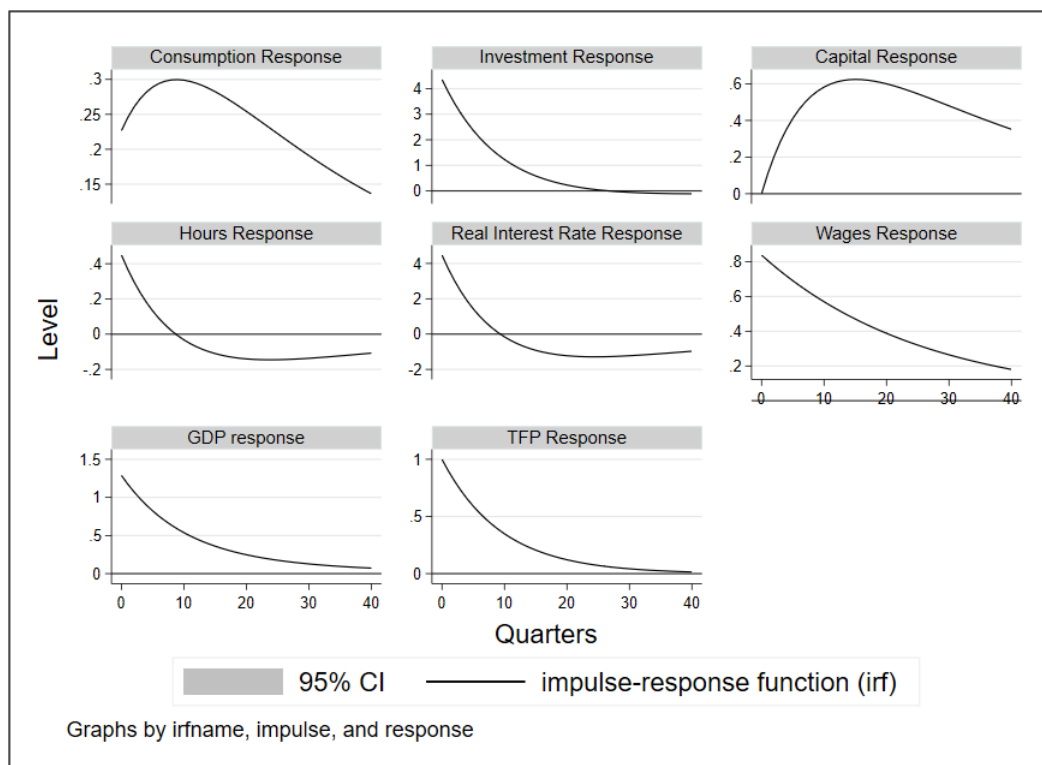
Table 1: Model and Data Moments, 1981Q1 - 2019Q2

	Data		Model	
	SDx/SDy	Corr.	SDx/SDy	Corr.
GDP	1.00	1.00	1.00	1.00
Consumption	0.62	0.81	0.50	0.81
Investment	4.96	0.82	2.87	0.95
Wages	0.72	0.69	0.95	0.94
Hours	0.89	0.20	0.33	0.31

Notes: Summary statistics for cyclical component from the Butterworth filter using quarterly data from 1981q1 to 2019q2. Relative standard deviation, and contemporaneous pairwise correlation. Source: Statistics Canada.

(f)

Figure 5: TFP Impulse-Response Function



Graphs by IRFs, impulse, and response modeled after a real business cycle model with iso-elastic utility over consumption and leisure .

(g)

Steady State Values at  $\theta = 2.4$ :

$k = 11.46$

$z = 1.00$

$c = 0.83$

$r = 0.01$

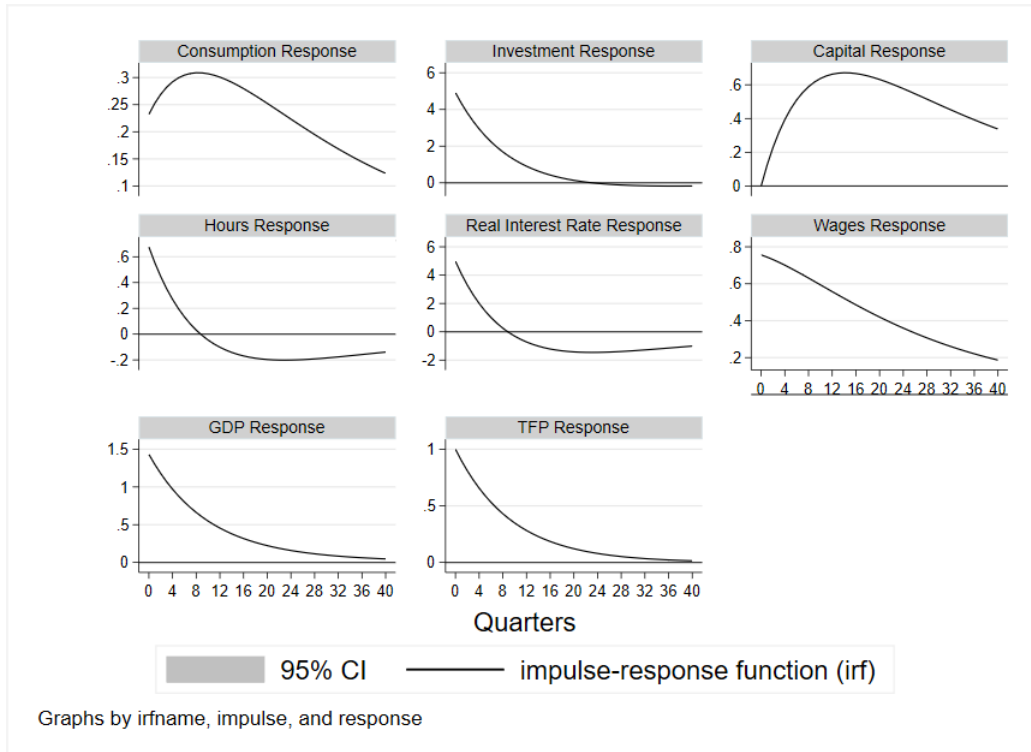
$n = 0.30$   
 $w = 2.37$   
 $i = 0.29$   
 $y = 1.12$

Table 2: Model and Data Moments, 1981Q1 - 2019Q2

	Data		Model	
	SDx/SDy	Corr.	SDx/SDy	Corr.
GDP	1.00	1.00	1.00	1.00
Consumption	0.62	0.81	0.47	0.78
Investment	4.96	0.82	2.97	0.96
Wages	0.72	0.69	0.91	0.89
Hours	0.89	0.20	0.45	0.42

Notes: Summary statistics for cyclical component from the Butterworth filter using quarterly data from 1981q1 to 2019q2. Relative standard deviation, and contemporaneous pairwise correlation. Source: Statistics Canada.

Figure 6: TFP Impulse-Response Function

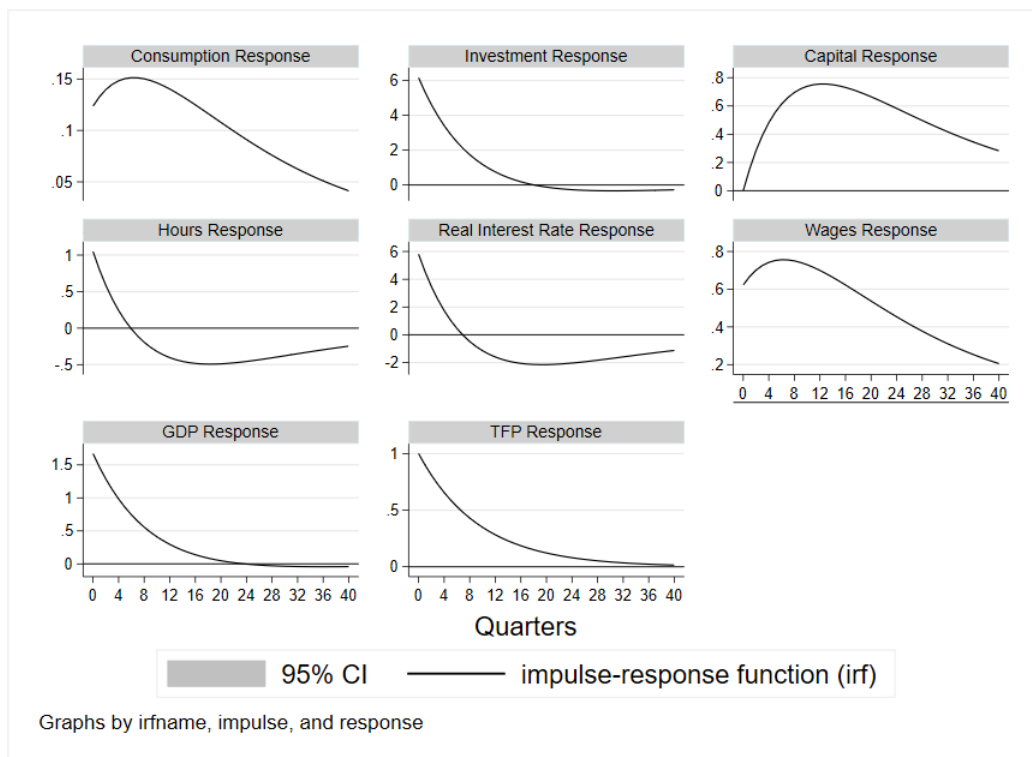


Graphs by IRFs, impulse, and response modeled after a real business cycle model with iso-elastic utility over consumption and leisure .

This did not make the model significantly closer to the moments in the data. Consumption is farther, investment is slightly closer, wages are closer, hours are ambiguous.

(h)

Figure 7: TFP Impulse-Response Function



Graphs by IRFs, impulse, and response modeled after a real business cycle model with iso-elastic utility over consumption and leisure .

Table 3: Model and Data Moments, 1981Q1 - 2019Q2

	Data		Model	
	SDx/SDy	Corr.	SDx/SDy	Corr.
GDP	1.00	1.00	1.00	1.00
Consumption	0.62	0.81	0.21	0.67
Investment	4.96	0.82	3.53	0.99
Wages	0.72	0.69	1.04	0.67
Hours	0.89	0.20	0.83	0.36

Notes: Summary statistics for cyclical component from the Butterworth filter using quarterly data from 1981q1 to 2019q2. Relative standard deviation, and contemporaneous pairwise correlation. Source: Statistics Canada.

This made consumption significantly farther, investment significantly closer, wages farther and hours significantly closer. Overall, it can be argued that the model fit has improved.

(i)

Steady state values at  $\theta = 20, \eta = 0.0075, \gamma = 12$

$k = 11.54$

$z = 1.00$

$c = 0.84$

$r = 0.01$   
 $n = 0.30$   
 $w = 2.37$   
 $i = 0.29$   
 $y = 1.13$

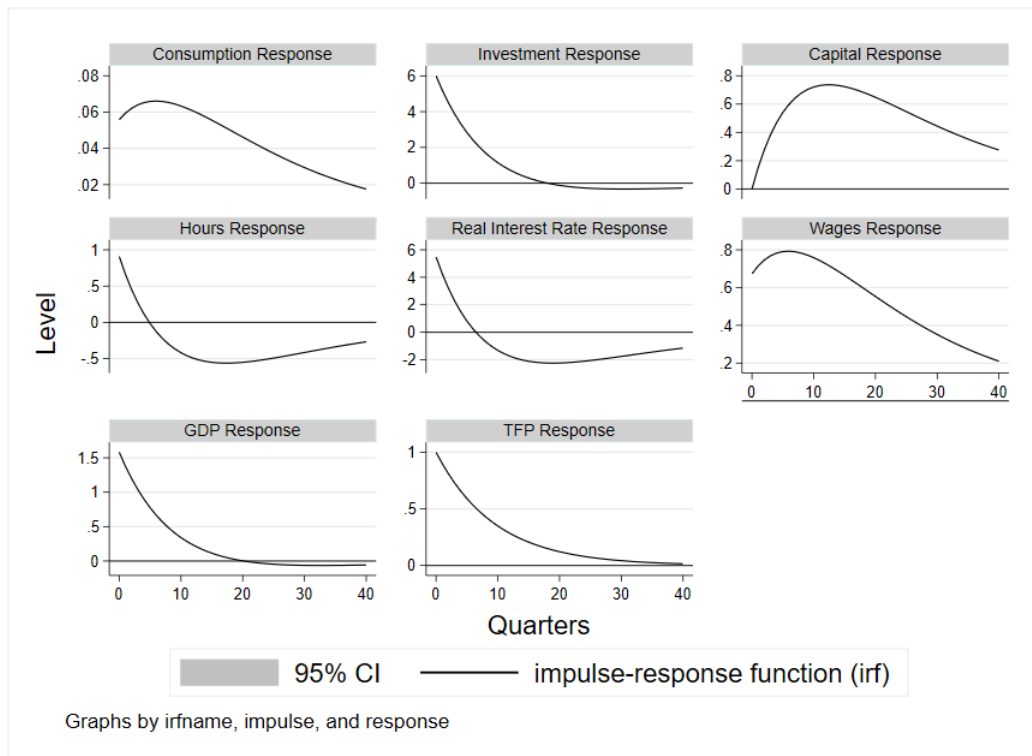
Table 4: Model and Data Moments, 1981Q1 - 2019Q2

	Data		Model	
	SDx/SDy	Corr.	SDx/SDy	Corr.
GDP	1.00	1.00	1.00	1.00
Consumption	0.62	0.81	0.10	0.63
Investment	4.96	0.82	3.73	1.00
Wages	0.72	0.69	1.18	0.63
Hours	0.89	0.20	0.95	0.28

Notes: Summary statistics for cyclical component from the Butterworth filter using quarterly data from 1981q1 to 2019q2. Relative standard deviation, and contemporaneous pairwise correlation. Source: Statistics Canada.

From inspection, this made consumption and wages standard deviations, in addition to to consumption and investment contemporaneous pairwise correlation farther, but all other entries have gotten closer. Overall the model has fit the data better. This model trade offs the consumption accuracy with households predominately focusing on their utility of leisure when giving a higher weighted  $\gamma$  and lower weighted  $\eta$ .

Figure 8: TFP Impulse-Response Function



Graphs by IRFs, impulse, and response modeled after a real business cycle model with iso-elastic utility over consumption and leisure .



(j)

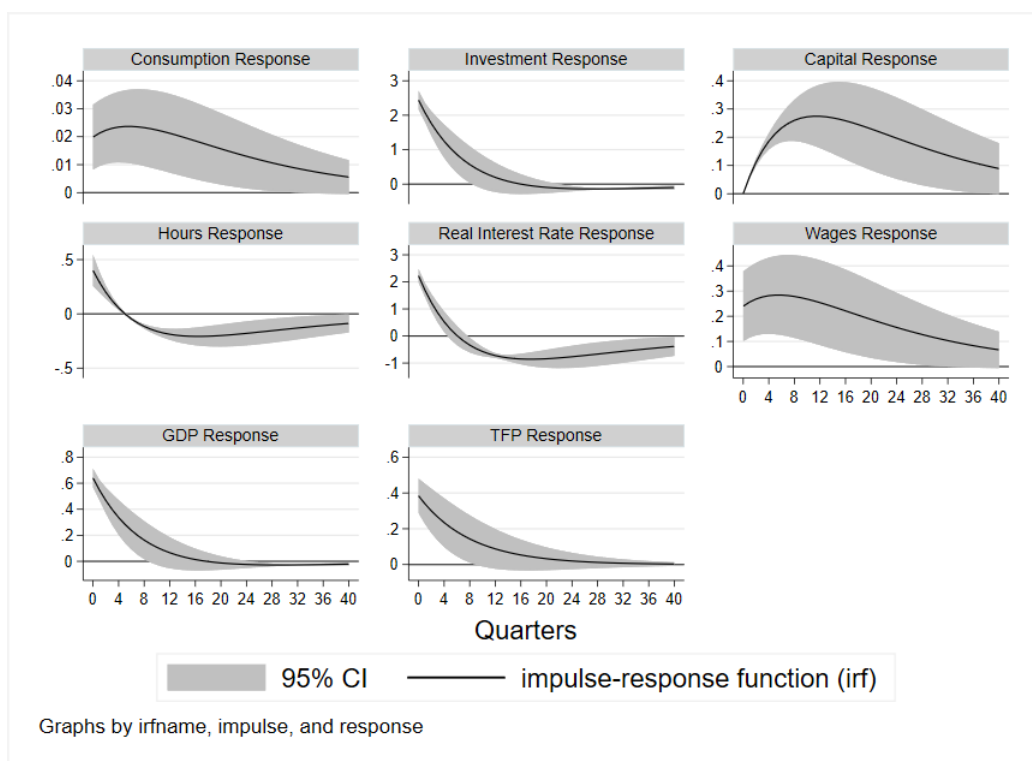
Log likelihood of -150.44

Table 5: Model and Data Moments, 1981Q1 - 2019Q2

	Data		Model	
	SDx/SDy	Corr.	SDx/SDy	Corr.
GDP	1.00	1.00	1.00	1.00
Consumption	0.62	0.81	0.9	0.60
Investment	4.96	0.82	3.75	1.00
Wages	0.72	0.69	1.06	0.60
Hours	0.89	0.20	0.93	0.39

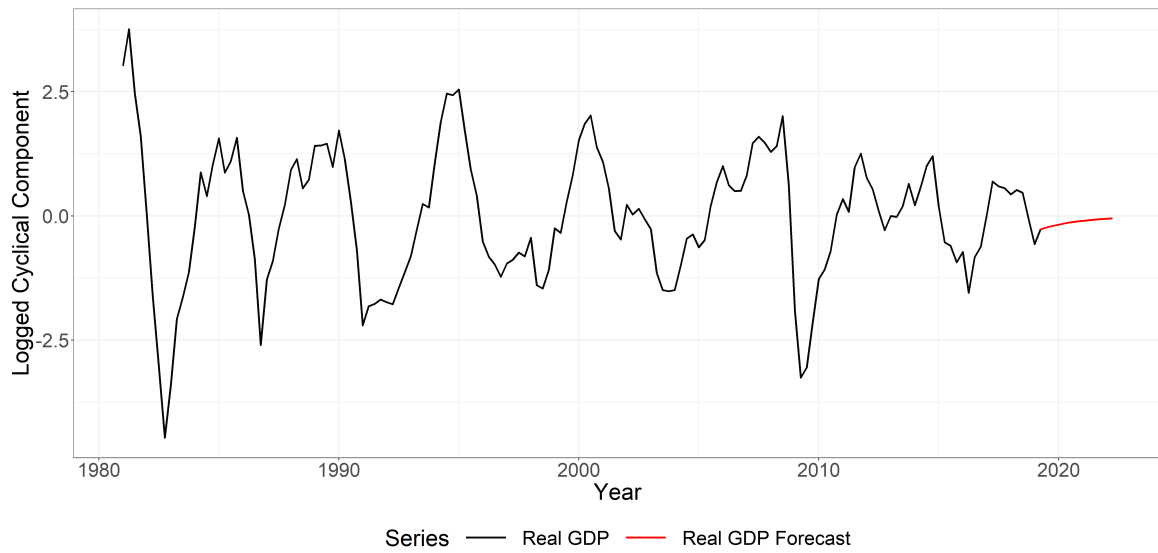
Notes: Summary statistics for cyclical component from the Butterworth filter using quarterly data from 1981q1 to 2019q2. Relative standard deviation, and contemporaneous pairwise correlation. Source: Statistics Canada.

Figure 9: TFP Impulse-Response Function



Graphs by IRFs, impulse, and response modeled after a real business cycle model with iso-elastic utility over consumption and leisure .

Figure 10: Canadian Real GDP 3-Year Forecast 1981Q1-2022Q2



Source: Statistic Canada. Forecasted on cyclical component of logged real GDP from 1981Q1-2019Q2. Forecast is from 2019Q2-2022Q2.

**6**

**(a)**

$$\max \pi = z_t k_t^\alpha n_t^{1-\alpha} - w_t n_t - (r_t + \delta) k_t$$

FOCs:

$$k_t : \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha} - r_t - \delta = 0$$

$$r_t = \alpha \frac{y_t}{k_t} - \delta$$

$$n_t : (1 - \alpha) z_t k_t^\alpha n_t^{\alpha} - w_t = 0$$

$$w_t = (1 - \alpha) \frac{y_t}{n_t}$$

**(b)**

$$a_{t+1} + c_t = w_t n_t + \pi_t + (1 + r_t) a_t$$
$$L = \sum_{s=0}^{\infty} \beta^s [l n c_t + \theta l n (1 - n_t)] - \sum_{s=0}^{\infty} \lambda_t [a_{t+1} + c_t - w_t n_t - (1 - r_t) a_t]$$

FOCs:

$$c_t : \beta^t c_t^{-1} = \lambda_t$$
$$c_{t+1} : \beta c_{t+1}^{-1} = \lambda_{t+1}$$
$$a_{t+1} : \lambda_t = \lambda_{t+1} (1 + r_{t+1})$$
$$\lambda_t : a_{t+1} + c_t = w_t n_t + \pi_t + (1 + r_t) a_t$$
$$n_t : \beta^t \theta (1 - n_t)^{-1} = \lambda_t w_t$$

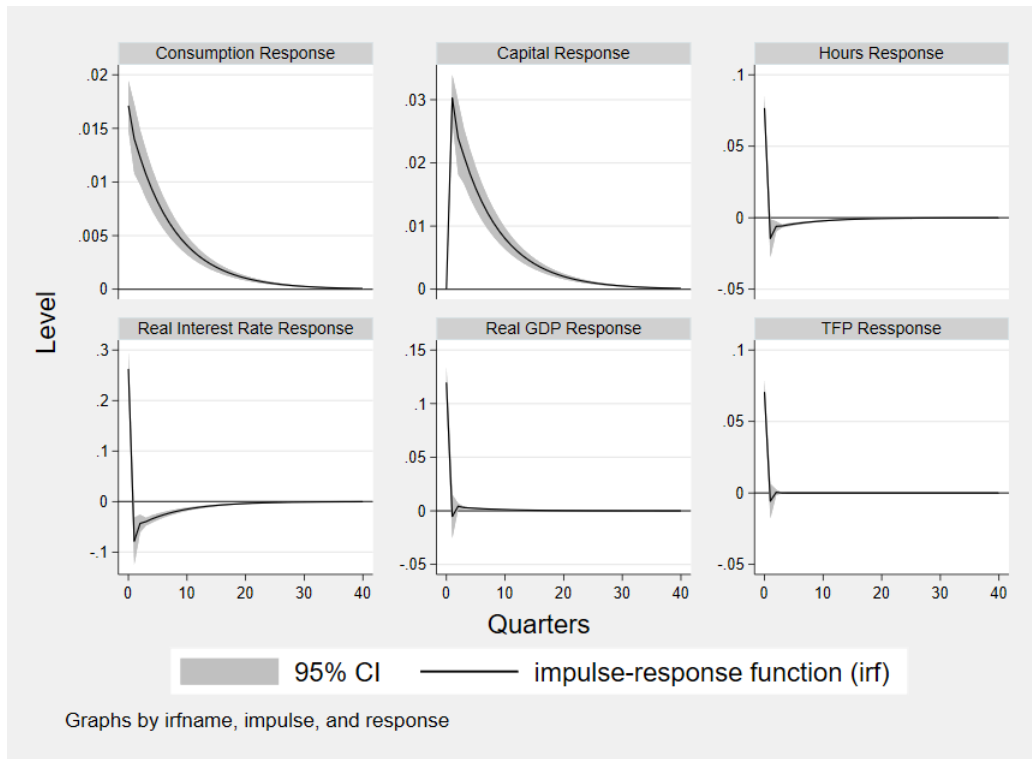
**(c)**

$$c_t : \frac{U'(c_t)}{U'(c_{t+1})} = \frac{c_t^{-1}}{c_{t+1}^{-1}} = \beta (1 + r_{t+1})$$
$$n_t : -\beta^t \theta \frac{1}{1 - n_t} + \beta^t \frac{1}{c_t} W_t = 0$$
$$n_t = 1 - \frac{c_t \theta}{w_t}$$
$$r_t = \alpha \frac{y_t}{k_t} - \delta$$
$$w_t = (1 - \alpha) \frac{y_t}{n_t}$$
$$i_t = y_t - c_t$$
$$k_{t+1} = (1 - \delta) k_t + i_t$$
$$y_t = z_t k_t^\alpha n_t^{1-\alpha}$$

**(d)**

$$\rho = -0.08051$$
$$\sigma_\epsilon = 0.006480411$$

Figure 11: TFP Impulse-Response Function



Graphs by IRFs, impulse, and response modeled on a decentralized model with log utility consumption and leisure. Real GDP taken from World Bank's Global Economy Monitor quarterly seasonally adjusted 1987Q1-2019Q2 and detrended with a butterworth filter. .

(e)

$$\max \pi = z_t k_t^{\alpha_1} n_t^{\alpha_2} O_t^{1-\alpha_1-\alpha_2} - w_t n_t - (r_t + \delta) k_t - q_t O_t$$

FOCs:

$$k_t : z_t \alpha_1 k_t^{\alpha_1-1} n_t^{\alpha_2} O_t^{1-\alpha_1-\alpha_2} - r_t - \delta = 0$$

$$r_t = \alpha_1 \frac{y_t^I}{k_t} - \delta$$

$$n_t : z_t \alpha_2 k_t^{\alpha_1} n_t^{\alpha_2-1} O_t^{1-\alpha_1-\alpha_2} - w_t = 0$$

$$w_t = \alpha_2 \frac{y_t^I}{n_t}$$

$$O_t : (1 - \alpha_1 - \alpha_2) z_t k_t^{\alpha_1} n_t^{\alpha_2} O_t^{1-\alpha_1-\alpha_2-1} - q_t = 0$$

$$q_t = (1 - \alpha_1 - \alpha_2) \frac{y_t^I}{O_t}$$

$$L = \sum_{s=0}^{\infty} \beta^s [\ln c_t + \theta \ln(1 - n_t)] - \sum_{s=0}^{\infty} \lambda_t [a_{t+1} + c_t - w_t n_t - (1 - r_t) a_t]$$

FOCs:

$$c_t : \beta^t c_t^{-1} = \lambda_t$$

$$c_{t+1} : \beta c_{t+1}^{-1} = \lambda_{t+1}$$

$$a_{t+1} : \lambda_t = \lambda_{t+1} (1 + r_{t+1})$$

$$\lambda_t : a_{t+1} + c_t = w_t n_t + \pi_t + (1 + r_t) a_t$$

$$n_t : \beta^t \theta (1 - n_t)^{-1} = \lambda_t w_t$$

$$c_t : \frac{U'(c_t)}{U'(c_{t+1})} = \frac{c_t^{-1}}{c_{t+1}^{-1}} = \beta(1 + r_{t+1})$$

$$n_t : -\beta^t \theta \frac{1}{1 - n_t} + \beta^t \frac{1}{c_t} W_t = 0$$

$$n_t = 1 - \frac{c_t \theta}{w_t}$$

$$r_t = \alpha_1 \frac{y_t^I}{k_t} - \delta$$

$$w_t = \alpha_2 \frac{y_t^I}{n_t}$$

$$i_t = y_t^I - c_t$$

$$k_{t+1} = (1 - \delta)k_t + i_t$$

$$y_t^I = z_t k_t^{\alpha_1} n_t^{\alpha_2} O_t^{1-\alpha_1-\alpha_2}$$

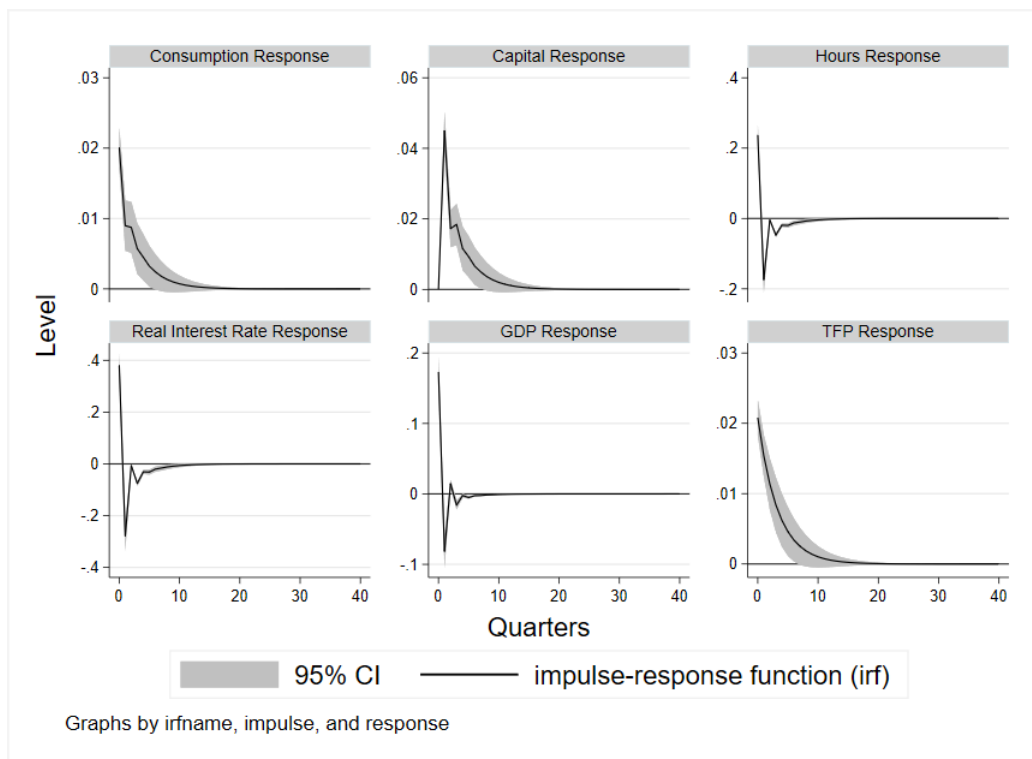
$$y_t^O = \frac{Y_t^O}{H_t} = \frac{a_t \bar{M}}{H_o} = a_t \bar{M}$$

$$q_t = (1 - \alpha_1 - \alpha_2) \frac{y_t^I}{O_t}$$

$$O_t = (1 - \alpha_1 - \alpha_2) \frac{y_t^I}{q_t}$$

(f)

Figure 12: TFP Impulse-Response Function



Graphs by IRFs, impulse, and response modeled on a decentralized model with log utility consumption and leisure. Real GDP taken from World Bank's Global Economy Monitor quarterly seasonally adjusted 1987Q1-2019Q2 and detrended with a butterworth filter. Quarterly Real Crude Oil Prices from the same time is taken from US Energy Administration Information and detrended using the Butterworth filter.

$$\rho_a = 0.6471$$

$$\rho_z = 0.7394$$

$$\sigma_e^2 = 0.1828$$

$$\sigma_\epsilon^2 = 0.0004$$

$\sigma_\epsilon^2$  decreased and  $\rho_z$  increased.

Yes they do align.

## Appendix

```
library(dplyr)
library(ggplot2)
library(latex2exp)

setwd('D:\\Users\\Ziqiu\\OneDrive\\Documents\\Masters Courses\\EC640 Macroeconomics\\A3')
df <- read.csv('D:\\Users\\Ziqiu\\OneDrive\\Documents\\Masters Courses\\EC640 Macroeconomics\\A3\\A3.csv', header = TRUE)
df[, -1] <- as.data.frame(sapply(df[, -1], as.numeric))

df[, 1] <- as.Date(as.character(df[, 1]), format = "%m/%d/%Y")

png("1A.png", width = 465, height = 225, units='mm', res = 300)
ggplot(data = df, aes(x = Date)) +
  geom_line(aes(y = K1/Y, color = 'dodgerblue1'), size = 1.1) +
  geom_line(aes(y = K2/Y, color = 'grey2'), size = 1.1) +
  geom_line(aes(y = K3/Y, color = 'darkorange1'), size = 1.1) +
```

```

theme_bw() +
scale_color_identity(guide = 'legend',
                     breaks = c('dodgerblue1','grey2','darkorange1'),
                     labels = c('0.015', '0.025', '0.05')) +
theme(legend.position="bottom",
      legend.text = element_text(size=22),
      axis.text = element_text(size=22),
      axis.title = element_text(size=26),
      plot.title=element_text(size = 26),
      legend.title=element_text(size= 26),
      # Change legend key size and key width
      legend.key.size = unit(1.5, "cm"),
      legend.key.width = unit(2.0,"cm")) +
labs(x = 'Year', y = 'Ratio to GDP', color = "Depreciation Rate")
dev.off()

png("1B.png", width = 465, height = 225, units='mm', res = 300)
ggplot(data = df, aes(x = Date)) +
  geom_line(aes(y = log(Y), color = 'dodgerblue1'), size = 1.1)+
  geom_line(aes(y = log(K2), color = I('grey2')), size = 1.1)+
  geom_line(aes(y = log(H), color = 'darkorange1'), size = 1.1)+
  theme_bw() +
  scale_color_identity(guide = 'legend',
                       breaks = c('dodgerblue1','grey2','darkorange1'),
                       labels = c('Real GDP', 'Capital Stock',
                                   'Total Hours Worked')) +
  theme(legend.position="bottom",
        legend.text = element_text(size=22),
        axis.text = element_text(size=22),
        axis.title = element_text(size=26),
        plot.title=element_text(size = 26),
        legend.title=element_text(size= 26),
        # Change legend key size and key width
        legend.key.size = unit(1.5, "cm"),
        legend.key.width = unit(2.0,"cm")) +
  labs(x = 'Year', y = 'Logged Values', color = "Series")
dev.off()

###1c

df <- mutate(df, SolowResiduals = log(Y) - 0.34*log(K1) - (1-0.34)*log(H))

qlc <- lm(SolowResiduals ~ Date, data = df)

summary(qlc)
plot(df[['Date']], df[['SolowResiduals']])
abline(lm(SolowResiduals ~ Date, data = df))

# Graph of the solowresiduals
png("1D.png", width = 465, height = 225, units='mm', res = 300)
ggplot(data = df, aes(x=Date)) +
  geom_smooth(aes(y = SolowResiduals), method = 'lm', formula = y ~ x) +
  geom_line(aes(y = SolowResiduals)) +
  theme_bw() +
  theme(axis.text = element_text(size=22),
        axis.title = element_text(size=26),
        plot.title=element_text(size = 26))
dev.off()

# graphing the residuals of the linear trend in solow residuals
qlr <- data.frame("Residuals" = qlc$residuals, "Date" = df[['Date']])

png("1E.png", width = 465, height = 225, units='mm', res = 300)
ggplot(data = qlr, aes(x = Date)) +
  geom_line(aes(y = Residuals)) +
  theme_bw() +
  theme(axis.text = element_text(size=22),
        axis.title = element_text(size=26),
        plot.title=element_text(size = 26))
dev.off()

# Simple AR(1) no constant
qlf <- lm(qlr[['Residuals']][1]~qlr[['Residuals']][2:length(qlr[['Residuals'])])-1])
summary(qlf)
var(qlf$residuals)

qlg <- lm(log(Labor_prod) ~ Date, data = df)

AR1Res <- lm(qlg$residuals[1]~qlg$residuals[2:length(qlg$residuals)] - 1)
summary(AR1Res)

var(AR1Res$residuals)

clear all
capture drop _all
//Log file
capture log close
log using output.log, replace

// Options
set linesize 255

```



```

set more off
set maxiter 100
// Timer
timer on 1

//*****
// Load Data
//*****

// Set time
import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A2\CDataQ.csv", encoding(ISO-8859-9)
generate time=tq(1961q1)+_n-1
format %tq time
tsset time
gen y=0
drop if time<tq(1981q1)

// Model

matrix param = (2, 0.36, 0.025, 0.99, 0.9, 2, 1.7)
matrix colnames param = gamma alpha delta beta rho eta theta
dsgenl ( F.c = c*({beta}*(1+F.r))^(1/{gamma}) ) ///
      ( y = z*k^{alpha})*n^{1-{alpha}} ) ///
      ( r = {alpha}*y/k -{delta} ) ///
      ( n = 1-({theta}*c^{gamma}/w)^{1/{eta}} ) ///
      ( w = (1-{alpha})*y/n ) ///
      ( F.k = (1-{delta})*k-c+y ) ///
      ( ln(F.z) = {rho}*ln(z) ) ///
( i = y - c ) ///
      ,observed(y) unobserved(c r n w i) endstate(k) exostate(z) solve noidencheck from(param)

//5DF
// Check on Model

// Eigenvalue stability
estat stable

// Steady state
estat steady, compact

//Approximate state transition matrix.
estat transition

// Policy matrix
estat policy, compact

// Display estimated covariances of model variables
estat covariance, nocovariance
estat covariance

*****
// IRFS

irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r w i) _yline(0) byopts(yrescale) ///
legend( nobox region(lstyle(none)) ) xtitle("Quarters") ytitle("Level") graphregion(color(white))

// 5E
clear all

import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A3\Q5E.csv"

drop year
generate time = tq(1961q1) +_n-1
format %tq time
tsset time
drop if time<tq(1981q1)

drop quarter

gen lgdp = ln(y)
gen lc = ln(c)
gen li = ln(i)
gen lwage = ln(wage)
gen lhour = ln(hours)

tsfilter bw bwy = lgdp, maxperiod(32) order(8)
tsfilter bw bwc = lc, maxperiod(32) order(8)
tsfilter bw bwi = li, maxperiod(32) order(8)
tsfilter bw bwhours = lhour, maxperiod(32) order(8)
tsfilter bw bwage = lwage, maxperiod(32) order(8)

sum bwy bwc bwi bwhours bwage if time >= tq(1981q1)
pwcrr bwy bwc if time >= tq(1981q1), star(0.01)
pwcrr bwy bwi if time >= tq(1981q1), star(0.01)
pwcrr bwy bwhours if time >= tq(1981q1), star(0.01)
pwcrr bwy bwage if time >= tq(1981q1), star(0.01)

```

```

//5G

clear all
capture drop _all
//Log file
capture log close
log using output.log, replace

// Options
set linesize 255
set more off
set maxiter 100
// Timer
timer on 1

//*****
// Load Data
//*****

// Set time
import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A2\CDataQ.csv", encoding(ISO-8859-9)
generate time=tq(1961q1)+_n-1
format %tq time
tsset time
gen y=0
drop if time<tq(1981q1)

// Model

matrix param = (2, 0.36, 0.025, 0.99, 0.9, 1, 2.4)
matrix colnames param = gamma alpha delta beta rho eta theta
dsngenl ( F.c = c*({beta}*(1+F.r))^(1/{gamma}) ) ///
( y = z*k^{alpha}*n^{1-(alpha)} ) ///
( r = {alpha}*y/k -{delta} ) ///
( n = 1-({theta}*c^{gamma}/w)^{1/{eta}} ) ///
( w = (1-{alpha})*y/n ) ///
( F.k = (1-{delta})*k-c+y ) ///
( ln(F.z) = {rho}*ln(z) ) ///
( i = y - c ) ///
,observed(y) unobserved(c r n w i) endstate(k) exostate(z) solve noidencheck from(param)

//*****
// Check on Model

// Eigenvalue stability
estat stable

// Steady state
estat steady, compact

//Approximate state transition matrix.
estat transition

// Policy matrix
estat policy, compact

// Display estimated covariances of model variables
estat covariance, nocovariance
estat covariance

*****
// IRFS

irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r w i) yline(0) xlabel(0(4)40) byopts(yrescale) legend( nobox region(lstyle(none)) ) xtitle(

// 5H

clear all
capture drop _all
//Log file
capture log close
log using output.log, replace

// Options
set linesize 255
set more off
set maxiter 100
// Timer
timer on 1

//*****
// Load Data
//*****

// Set time
import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A2\CDataQ.csv", encoding(ISO-8859-9)
generate time=tq(1961q1)+_n-1
format %tq time

```

```

tsset time
gen y=0
drop if time<1981q1

// Model

matrix param = (5, 0.36, 0.025, 0.99, 0.9, 0.01, 6)
matrix colnames param = gamma alpha delta beta rho eta theta
dsge1 ( F.c = c*({beta}*(1+F.r))^(1/{gamma}) ) ///
      ( y = z*k^({alpha})*n^(1-({alpha})) ) ///
      ( r = {alpha}*y/k -{delta} ) ///
      ( n = 1-({theta}*c^{gamma}/w)^(1/{eta}) ) ///
      ( w = (1-({alpha})*y/n ) ///
      ( F.k = (1-({delta})*k-c+y ) ///
      ( ln(F.z) = {rho}*ln(z) ) ///
      ( i = y - c ) ///
      ,observed(y) unobserved(c r n w i) endstate(k) exostate(z) solve noidencheck from(param)

//*****
// Check on Model

// Eigenvalue stability
estat stable

// Steady state
estat steady, compact

//Approximate state transition matrix.
estat transition

// Policy matrix
estat policy, compact

// Display estimated covariances of model variables
estat covariance, nocovariance
estat covariance

*****
// IRFS

irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r w i) yline(0) xlabel(0(4)40) byopts(yrescale) legend( nobox region(lstyle(none)) ) xtitle=

// 5I

clear all

import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A2\CDataQ.csv", encoding(ISO-8859-9)
generate time=tq(1961q1)+_n-1
format %tq time
tsset time
gen y=0
drop if time<1981q1

// Model

matrix param = (12, 0.36, 0.025, 0.99, 0.9, 0.0075, 20)
matrix colnames param = gamma alpha delta beta rho eta theta
dsge1 ( F.c = c*({beta}*(1+F.r))^(1/{gamma}) ) ///
      ( y = z*k^({alpha})*n^(1-({alpha})) ) ///
      ( r = {alpha}*y/k -{delta} ) ///
      ( n = 1-({theta}*c^{gamma}/w)^(1/{eta}) ) ///
      ( w = (1-({alpha})*y/n ) ///
      ( F.k = (1-({delta})*k-c+y ) ///
      ( ln(F.z) = {rho}*ln(z) ) ///
      ( i = y - c ) ///
      ,observed(y) unobserved(c r n w i) endstate(k) exostate(z) ///
      solve noidencheck from(param)

//*****
// Check on Model

// Eigenvalue stability
estat stable

// Steady state
estat steady, compact

//Approximate state transition matrix.
estat transition

// Policy matrix
estat policy, compact

// Display estimated covariances of model variables
estat covariance, nocovariance
estat covariance

*****

```

```

// IRFS

irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r w i) yline(0) ///
byopts(yrescale) legend( nobox region(lstyle(none)) ) xtitle("Quarters") ///
ytitle("Level") scheme(s2mono)

clear all
import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A3\CDataQA3.csv", encoding(ISO-8859-9)
generate time=tq(1961q1)+_n-1
format %tq time
tsset time
//Create GDP Data
generate lgdp = 100*ln(gdp)
tsfilter bw y_bw = lgdp, maxperiod(32) order(8) trend(trend_bw)
gen y= y_bw
drop if time<tq(1981q1)

//*****
// Model 5J

constraint 1 _b[beta] = 0.99
constraint 2 _b[gamma] = 12
constraint 3 _b[alpha] = 0.36
constraint 4 _b[delta] = 0.025
constraint 5 _b[theta] = 20
constraint 6 _b[eta] = 0.0075
constraint 7 _b[rho] = 0.9

dsngenl ( F.c = c*({beta}*(1+F.r))^(1/{gamma})) ) //
( y = z*k^{alpha})*n^{1-(alpha)} ) ///
( r = {alpha}*y/k -{delta} ) ///
( n = 1-({theta}*c^{gamma}/w)^(1/{eta}) ) ///
( w = (1-{alpha})*y/n ) ///
( i = y-c ) ///
( F.k = (1-{delta})*k+i ) ///
( ln(F.z) = {rho}*ln(z) ) ///
, constraint(1/6) observed(y) unobserved(c r n w i) endstate(k) exostate(z) noidencheck
// solve from(param)

//*****
// Check on Model

estimates store dsge_est
tsappend, add(12)
forecast create dsgemodel
forecast estimates dsge_est
forecast solve, prefix(y_f) begin(tq(2019q3))
replace y_f =. if time <= tq(2019q1)
replace y_f =y if time == tq(2019q2)
tsline y_f y

//*****
// Check on Model

// Eigenvalue stability
estat stable

// Steady state
estat steady, compact

//Approximate state transition matrix.
estat transition

// Policy matrix
estat policy, compact

// Display estimated covariances of model variables
estat covariance, nocovariance
estat covariance

*****
// IRFS

irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r w i) yline(0) xlabel(0(4)40) ///
byopts(yrescale) legend( nobox region(lstyle(none)) ) xtitle("Quarters") scheme(s2mono)

### Q5K

df2 <- read.csv('D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A3\A35K.csv',
header = TRUE)
df2[,-1] <- as.data.frame(sapply(df2[,-1], as.numeric))
df2[,1] <- as.Date(as.character(df2[,1]), format = "%m/%d/%Y")

png("Q5K.png", width = 465, height = 225, units='mm', res = 300)
ggplot(data = df2, aes(x=Date)) +
geom_line(aes(y = y, color = 'grey2'), size = 1.1)+

```

```

geom_line(aes(y = y_fy, color = 'red'), size = 1.1) +
theme_bw() +
scale_color_identity(guide = 'legend',
                     breaks = c('grey2', 'red'),
                     labels = c('Real GDP', 'Real GDP Forecast')) +
theme(legend.position="bottom",
      legend.text = element_text(size=22),
      axis.text = element_text(size=22),
      axis.title = element_text(size=26),
      plot.title=element_text(size = 26),
      legend.title=element_text(size= 26),
      # Change legend key size and key width
      legend.key.size = unit(1.5, "cm"),
      legend.key.width = unit(2.0,"cm")) +
labs(x = 'Year', y = 'Logged Cyclical Component', color = "Series")
dev.off()

** 6D

clear all
import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A3\Q6D.csv", encoding(ISO-8859-9)
generate time=tq(1987q1)+_n-1
format %tq time
tsset time
gen lgdp = ln(gdp)

tsfilter bw y = lgdp, maxperiod(32) order(8)

constraint 1 _b[beta] = 0.96
constraint 2 _b[alphaa] = 0.32
constraint 3 _b[alphab] = 0.64
constraint 4 _b[delta] = 0.05
constraint 5 _b[theta] = 2.3
dsge1 ( F.c = c*{beta}*(1+F.r) ) ///
( y = z*k^({alphaa})*n^({alphab}) ) ///
( r = {alphaa}*y/k -{delta} ) ///
( n = 1-({theta}*c/w) ) ///
( w = {alphab}*y/n ) ///
( F.k = (1-({delta}))*k-c+y ) ///
( ln(F.z) = {rho}*ln(z) ) ///
,constraint(1/5) observed(y) unobserved(c r n w) endstate(k) exostate(z) noidencheck

//*****
// Check on Model
// Eigenvalue stability
estat stable
// Steady state
estat steady, compact
//Approximate state transition matrix.
estat transition
// Policy matrix
estat policy, compact
// Display estimated covariances of model variables
estat covariance, nocovariance
estat covariance

//*****
// IRFS
irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r) yline(0) ///
byopts(yrescale) legend( nobox region(lstyle(none)) ) xtitle("Quarters") ///
ytitle("Level") scheme(s2mono)

// 6F

clear all
import delimited "D:\Users\Ziqiu\OneDrive\Documents\Masters Courses\EC640 Macroeconomics\A3\Q6F.csv", encoding(ISO-8859-9)
generate time=tq(1987q1)+_n-1
format %tq time
tsset time
gen loil = ln(oil)
gen lgdp = ln(gdp)

tsfilter bw q = loil, maxperiod(32) order(8)
tsfilter bw y = lgdp, maxperiod(32) order(8)

constraint 1 _b[beta] = 0.96
constraint 2 _b[alphaa] = 0.32
constraint 3 _b[alphab] = 0.64
constraint 4 _b[alphac] = 0.04
constraint 5 _b[delta] = 0.05
dsge1 ( F.c = c*{beta}*(1+F.r) ) ///
( y = z*k^({alphaa})*n^({alphab})*o^({alphac}) ) ///
( o=a*1 ) ///
( q=1-({alphaa}-{alphab}*y/o) ) ///
( i=y-c ) ///
( r = {alphaa}*y/k -{delta} ) ///
( n = {alphab}*y/k-{delta} ) ///
( w = {alphab}*y/n ) ///

```

```

( F.k = (1-{delta})*k-c+y ) ///
( ln(F.a)={rho_a}*ln(a) ) ///
( ln(F.z) = {rho}*ln(z) ) ///
,constraint(1/5) observed(y q) unobserved(c r n o w i) endstate(k) exostate(z a) noidencheck

irf set solowirf
irf create imp_res, replace step(40)
irf graph irf, irf(imp_res) impulse(z) response(z y k c n r) yline(0) ///
byopts(yrescale) legend( nobox region(lstyle(none)) ) xtitle("Quarters") ///
ytitle("Level") scheme(s2mono)

```