# CONVOLUTION AND APPROXIMATE IDENTITIES

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## 2. Examples of Topological Groups

DEFINITION 2.1. *Topological Group*

DEFINITION 2.2. *Locally Compact*

DEFINITION 2.3. *Haar Measure*

EXAMPLE 2.1. $\mathbb{R}^n, \mathbb{Z}^n, \mathbb{T}^n$

EXAMPLE 2.2. $dx/|x|$

EXAMPLE 2.3. *Heisenberg Group* $\mathbb{H}^n$

## 3. Convolution

DEFINITION 3.1. *Let* $f, g \in L^1(G)$. *Define the* ***convolution*** $f * g$ *by*

$$(f * g)(x) = \int_G f(y)g(y^{-1}x)d\lambda(y) \tag{1}$$

## 4. Basic Convolution Inequalities

THEOREM 4.1. *Minkowskis Inequality, triangle inequality for Lp spaces*

THEOREM 4.2. *Youngs Inequality*

THEOREM 4.3. *Youngs Inequality for Weak Type Spaces ouch proof*

## 5. Approximate Identities

Approximation of dirac delta function , identity element of convolutions

DEFINITION 5.1. *An* ***approximate identity*** *(as* $\varepsilon \to 0$*) is a family of* $L^1(G)$ *functions* $k_\varepsilon$ *with the following three properties:*

*(i) There exists a constant* $c > 0$ *such that* $\|k_\varepsilon\|_{L^1(G)} \leqslant c$ *for all* $\varepsilon > 0$ .
*(ii)* $\int_G k_\varepsilon(x)d\lambda(x) = 1$ *for all* $\varepsilon > 0$.
*(iii) For any neighborhood* $V$ *of the identity element* $e$ *of the group* $G$ *we have* $\int_{V^c} |k_\varepsilon(x)|d\lambda(x) \to 0$ *as* $\varepsilon \to 0$ .

EXAMPLE 5.1.

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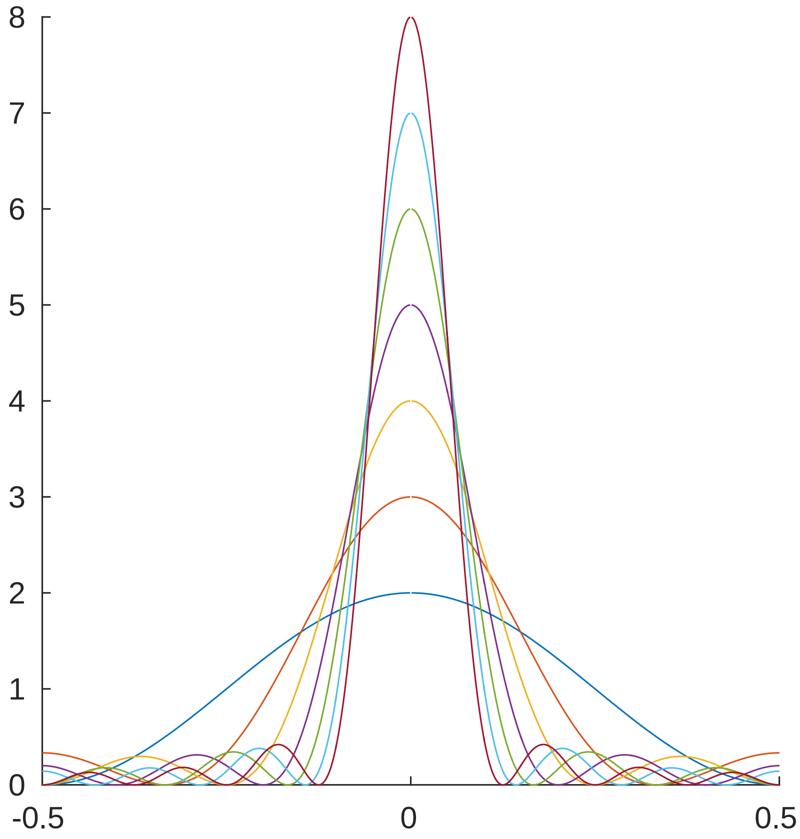


FIGURE 1. Fejer Kernel

THEOREM 5.1. *approx. id. on locallz compact group $G$ with left Haar measure*

THEOREM 5.2. *ke familz of funcs on loc compact group $G$ with properties...*

## 6. Required Stuff

(1) hausdorf topological space
(2) counting measure

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(3) area of intersecting circles
(4) banach algebra
(5) hoelders inequality
(6) fubini
(7) chebyschevs inequality
(8) lebesgue dominated conv. thm.
(9) measure theoretic support

chapter 1 stuff:

(1) Lp norms and other defs etc.
(2) distr. functions