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CLASSICAL FOURIER ANALYSIS: CONVOLUTION AND APPROXIMATE IDENTITIES

1. Examples of Topological Groups

Definition 1.1. *Topological Group*

Definition 1.2. *Locally Compact*

Definition 1.3. *Haar Measure*

Example 1.1. $\mathbb{R}^n, \mathbb{Z}^n, \mathbb{T}^n$

Example 1.2. $dx/|x|$

Example 1.3. *Heisenberg Group* $\mathbb{H}^n$

2. Convolution

Definition 2.1. *Let $f, g \in L^1(G)$. Define the **convolution** $f * g$ by*

$$(f * g)(x) = \int_G f(y)g(y^{-1}x)d\lambda(y) \tag{1}$$

3. Basic Convolution Inequalities

Theorem 3.1. *Minkowskis Inequality, triangle inequality for Lp spaces*

Theorem 3.2. *Youngs Inequality*

Theorem 3.3. *Youngs Inequality for Weak Type Spaces ouch proof*

4. Approximate Identities

Approximation of dirac delta function , identity element of convolutions

Definition 4.1. *An **approximate identity** (as $\varepsilon \to 0$) is a family of $L^1(G)$ functions k_ε with the following three properties:*

(i) There exists a constant $c > 0$ such that $\|k_\varepsilon\|_{L^1(G)} \leqslant c$ for all $\varepsilon > 0$.
(ii) $\int_G k_\varepsilon(x)d\lambda(x) = 1$ for all $\varepsilon > 0$.
(iii) For any neighborhood V of the identity element e of the group G we have $\int_{V^c} |k_\varepsilon(x)|d\lambda(x) \to 0$ as $\varepsilon \to 0$.

Example 4.1.

Theorem 4.1. *approx. id. on locallz compact group G with left Haar measure*

Theorem 4.2. *ke familz of funcs on loc compact group G with properties...*

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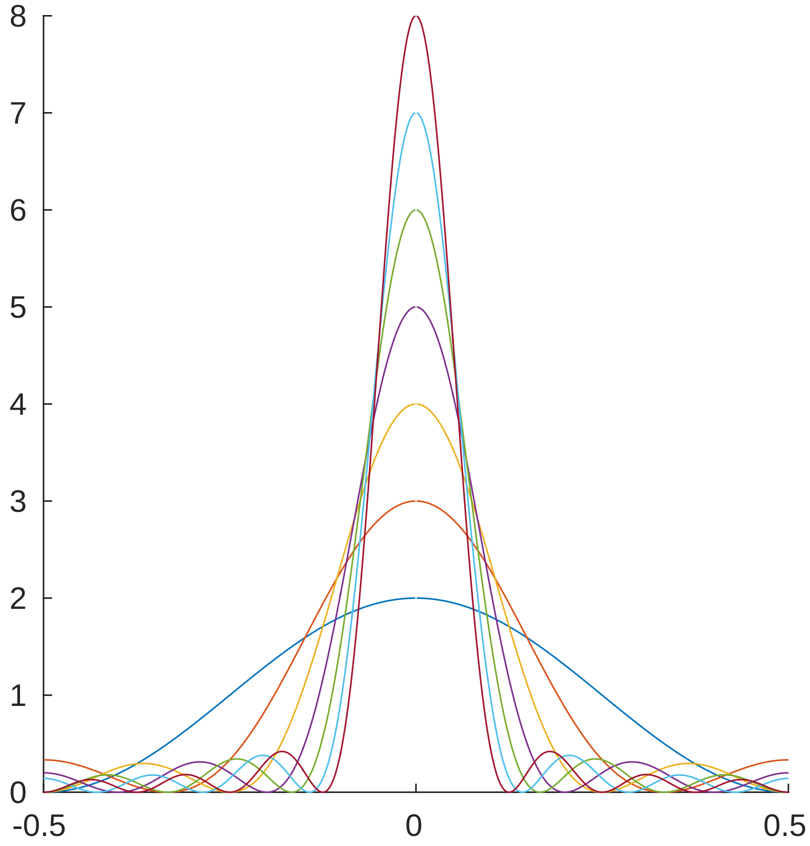

Figure 1. Fejer Kernel

5. Required Stuff

(1) hausdorf topological space
(2) counting measure
(3) area of intersecting circles
(4) banach algebra
(5) hoelders inequality
(6) fubini
(7) chebyschevs inequality
(8) lebesgue dominated conv. thm.
(9) measure theoretic support

chapter 1 stuff:

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(1) Lp norms and other defs etc.
(2) distr. functions