

CONVOLUTION AND APPROXIMATE IDENTITIES

SIMON GRÜNING

2. EXAMPLES OF TOPOLOGICAL GROUPS

DEFINITION 2.1. *Topological Group*

DEFINITION 2.2. *Locally Compact*

DEFINITION 2.3. *Haar Measure*

EXAMPLE 2.1. $\mathbb{R}^n, \mathbb{Z}^n, \mathbb{T}^n$

EXAMPLE 2.2. $dx/|x|$

EXAMPLE 2.3. *Heisenberg Group* \mathbb{H}^n

3. CONVOLUTION

DEFINITION 3.1. Let $f, g \in L^1(G)$. Define the **convolution** $f * g$ by

$$(f * g)(x) := \int_G f(y)g(y^{-1}x)d\lambda(y) \quad (1)$$

Note that on \mathbb{R}^n with an additive structure (our preferred environment for later chapters), we will simply have:

$$(f * g)(x) = \int_{\mathbb{R}^n} f(y)g(x - y)dy \quad (2)$$

THEOREM 3.1. *Convolution is define λ almost everywhere. To see this we take the L_1 norm on the definition to find it finite:*

$$\int_G \int_G f(y)g(y^{-1}x)d\lambda(y)d\lambda(x) \quad (3)$$

blablah

4. BASIC CONVOLUTION INEQUALITIES

THEOREM 4.1. *Minkowskis Inequality, triangle inequality for L_p spaces*

THEOREM 4.2. *Youngs Inequality*

THEOREM 4.3. *Youngs Inequality for Weak Type Spaces ouch proof*

(Simon Grüning) UNIVERSITY OF ZURICH, RÄMISTRASSE 71, 8006 ZURICH
E-mail address: simon.gruening@uzh.ch.

5. APPROXIMATE IDENTITIES

Approximation of dirac delta function , identity element of convolutions

DEFINITION 5.1. An **approximate identity** (as $\varepsilon \rightarrow 0$) is a family of $L^1(G)$ functions k_ε with the following three properties:

- (i) There exists a constant $c > 0$ such that $\|k_\varepsilon\|_{L^1(G)} \leq c$ for all $\varepsilon > 0$.
- (ii) $\int_G k_\varepsilon(x) d\lambda(x) = 1$ for all $\varepsilon > 0$.
- (iii) For any neighborhood V of the identity element e of the group G we have $\int_{V^c} |k_\varepsilon(x)| d\lambda(x) \rightarrow 0$ as $\varepsilon \rightarrow 0$.

EXAMPLE 5.1.

THEOREM 5.1. *approx. id. on locallz compact group G with left Haar measure*

THEOREM 5.2. *ke familz of funcns on loc compact group G with properties...*

6. REQUIRED STUFF

- (1) hausdorf topological space
- (2) counting measure
- (3) area of intersecting circles
- (4) banach algebra
- (5) hoelders inequality
- (6) fubini
- (7) chebyschevs inequality
- (8) lebesgue dominated conv. thm.
- (9) measure theoretic support

chapter 1 stuff:

- (1) L_p norms and other defs etc.
- (2) distr. functions

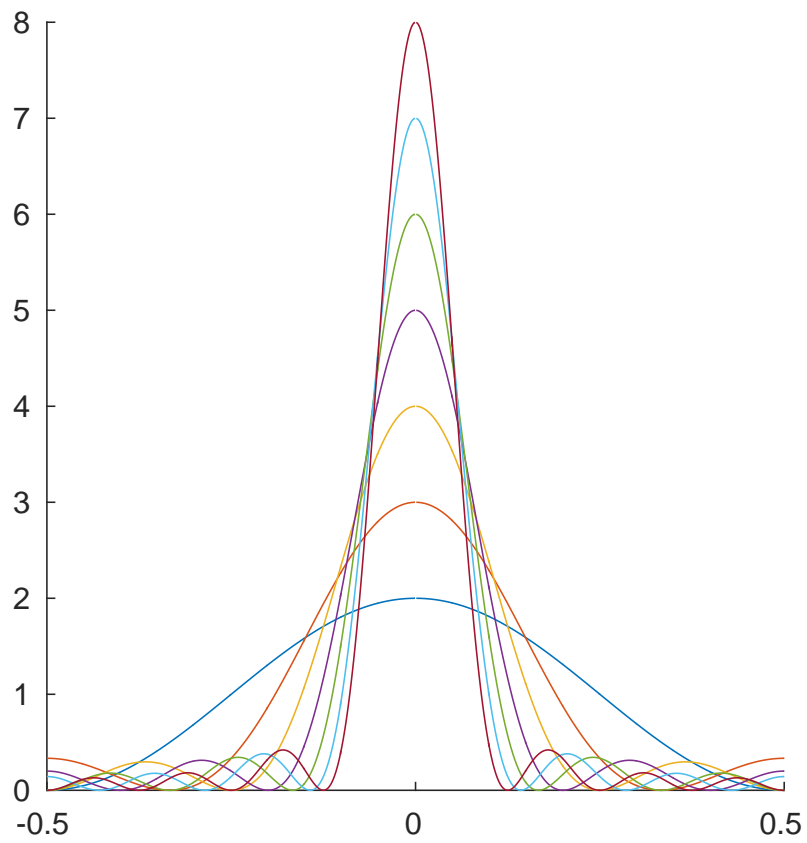


FIGURE 1. Fejer Kernel