

PERMUTATIONS AS GENOME REARRANGEMENTS

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1. BLOCK INTERCHANGES

DEFINITION 1.1. *Block Interchange*

DEFINITION 1.2. *Block Interchange Distance* $bid(p, q)$ $bid(p) := bid(p, id)$

REMARK 1.1. $bid(p, q)$ is a metric and is left-invariant, just as we have seen for $btd(p, q)$.

DEFINITION 1.3. *Cycle Graph* $G(p)$

LEMMA 1.1. *The cycle graph $G(p)$ has a unique decomposition into edge-disjoint directed cycles in which the colors of the edges alternate.*

Proof. It is easy to see that a path of alternating colours cannot branch since by construction there is always only one edge per color directed away from every node. We begin at the 0 node and complete a cycle (this must occur since $G(p)$ is finite). Once we have done so, we remove the edges of this cycle from $G(p)$ and iterate. Notice that in removing full cycles the structure is maintained. The cycles are then edge-disjoint by construction. \square

REMARK 1.2. *The cycles are only edge-disjoint not vertex-disjoint.*

DEFINITION 1.4. $c(G(p))$, $c(\Gamma(p))$ *From now on the cycles of $G(p)$ will refer to the alternating cycles, not the traditional cycles of a permutation we have seen previously.*

EXAMPLE 1.1. $p = 123$ $p = 4213$? $p = 4312$ *draw cycle graph count cycles find minimal transpositions to id and show how it works*

Before we can find a formula for $bid(p)$ we shall examine how single block interchanges can influence the block interchange distance of a permutation in the following two lemmas.

LEMMA 1.2. 9.10 *Let $p \in S_n$ with $p \neq id$.*

LEMMA 1.3. 9.11

THEOREM 1.1. 9.9