project 4

Yun Siyu 17307110448

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Problem 1: Warmup

1a. Question

Suppose we have a sensor reading for the second timestep, $D_2 = 0$. Compute the posterior distribution $P(C_2 = 1 \mid D_2 = 0)$.

1a. Answer

According to Bayesian network

$$
\begin{aligned}
P(C_2 = 1 \mid D_2 = 0) &= \frac{P(C_2 = 1, D_2 = 0)}{P(D_2 = 0)} \\
&= \frac{P(C_2 = 1) * P(D_2 = 0 \mid C_2 = 1)}{P(D_2 = 0)} \\
&= \frac{P(C_2 = 1) * P(D_2 = 0 \mid C_2 = 1)}{\sum_{C_2} P(D_2 = 0, C_2)} \\
&= \frac{\sum_{C_1} P(C_2 = 1 \mid C_1) * P(C_1) * P(D_2 = 0 \mid C_2 = 1)}{\sum_{C_2} P(D_2 = 0, C_2)} \\
&= \frac{\sum_{C_1} P(C_2 = 1 \mid C_1) * P(C_1) * P(D_2 = 0 \mid C_2 = 1)}{\sum_{C_2} P(D_2 = 0 \mid C_2) * P(C_2)}
\end{aligned}
$$

so we're going to compute the numerator and the denominator separately

$$(P(C_2 = 1 \mid C_1 = 0) * P(C_1 = 0) + P(C_2 = 1 \mid C_1 = 1) * P(C_1 = 1)) * P(D_2 = 0 \mid C_2 = 1) = \eta * \big[\epsilon * 0.5 + (1 - \epsilon) * 0.5\big]$$

for the denominator

$$
\begin{aligned}
P(C_2 = 1) &= P(C_2 = 1 \mid C_1 = 0) * P(C_1 = 0) + P(C_2 = 1 \mid C_1 = 1) * P(C_1 = 1) \\
&= +(1 - \epsilon) * 0.5 + \epsilon * 0.5
\end{aligned}
$$

$$
\begin{aligned}
P(C_2 = 0) &= P(C_2 = 0 \mid C_1 = 0) * P(C_1 = 0) + P(C_2 = 0 \mid C_1 = 1) * P(C_1 = 1) \\
&= \epsilon * 0.5 + (1 - \epsilon) * 0.5
\end{aligned}
$$

so the denominator is

$$\sum_{C_2} P(D_2 = 0 \mid C_2) * P(C_2) = \epsilon * 0.5 + (1 - \epsilon) * 0.5$$

Therefore,

$$
\begin{aligned}
P(C_2 = 1 \mid D_2 = 0) &= \frac{\eta * \big[\epsilon * 0.5 + (1 - \epsilon) * 0.5\big]}{\epsilon * 0.5 + (1 - \epsilon) * 0.5} \\
&= \eta
\end{aligned}
$$

1b. Question

Suppose a time step has elapsed and we got another sensor reading $D_3 = 1$, but we are still interested in C_2. Compute the posterior distribution $P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$.

1b. Answer

it is the same as 1a, so the answer is

$$P(C_2 = 1 \mid D_2 = 0, D_3 = 1) = \frac{P(C_2 = 1, D_2 = 0, D_3 = 1)}{P(D_2 = 0, D_3 = 1)}$$

For the numerator

$$\begin{aligned} P(C_2 = 1, D_2 = 0, D_3 = 1) &= \sum_{C_1, C_3} P(C_1)P(C_2 = 1 \mid C_1)P(D_2 = 0 \mid C_2 = 1)P(C_3 \mid C_2 = 1)P(D_3 = 1 \mid C_3) \\ &= \eta\big[(1-\epsilon)(1-\eta) + \epsilon\eta\big] * \big[\epsilon * 0.5 + (1-\epsilon) * 0.5\big] \end{aligned}$$

and for the denominator

$$\begin{aligned} P(C_2 = 0, D_2 = 0, D_3 = 1) &= \sum_{C_1, C_3} P(C_1)P(C_2 = 0 \mid C_1)P(D_2 = 0 \mid C_2 = 0)P(C_3 \mid C_2 = 0)P(D_3 = 1 \mid C_3) \\ &= (1-\eta) * \big[(1-\epsilon)\eta + \epsilon(1-\eta)\big] * \big[\epsilon * 0.5 + (1-\epsilon) * 0.5\big] \end{aligned}$$

$$\begin{aligned} P(D_2 = 0, D_3 = 1) &= P(C_2 = 0, D_2 = 0, D_3 = 1) + P(C_2 = 1, D_2 = 0, D_3 = 1) \\ &= \Big(\eta\big[(1-\epsilon)(1-\eta) + \epsilon\eta\big] + (1-\eta)\big[(1-\epsilon)\eta + \epsilon(1-\eta)\big]\Big) * \big[\epsilon * 0.5 + (1-\epsilon) * 0.5\big] \end{aligned}$$

so the answer is

$$\begin{aligned} P(C_2 = 1 \mid D_2 = 0, D_3 = 1) &= \frac{P(C_2 = 1, D_2 = 0, D_3 = 1)}{P(D_2 = 0, D_3 = 1)} \\ &= \frac{\eta\big[(1-\epsilon)(1-\eta) + \epsilon\eta\big]}{\eta\big[(1-\epsilon)(1-\eta) + \epsilon\eta\big] + (1-\eta)\big[(1-\epsilon)\eta + \epsilon(1-\eta)\big]} \end{aligned}$$

1c. Answer

i.

Compute and compare the probabilities $P(C_2 = 1 \mid D_2 = 0)$ and $P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$. Give numbers, round your answer to 4 significant digits.

Answer:

$$P(C_2 = 1 \mid D_2 = 0) = \eta = 0.2000$$

$$P(C_2 = 1 \mid D_2 = 0, D_3 = 1) = 0.4157$$

ii.

How did adding the second sensor reading $D_3 = 1$ change the result? Explain your intuition in terms of the car positions with respect to the observations.

Answer:
From i, we can see that $P(C_2 = 1 \mid D_2 = 0) < P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$. Hence, we can come to a conclusion that adding the second sensor $D_3 = 1$ increased the probability of $P(C_2) = 1$. The car's position observed at t+1-th step is related to the car's position at t-th step. So the observation of $D_3 = 1$ increased the probability of $C_2 = 1$.

iii.

What would you have to set ϵ while keeping $\eta = 0$ so that $P(C_2 = 1 \mid D_2 = 0) = P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$? Explain your intuition in terms of the car positions with respect to the observations.

Answer:
from the 1a and 1b, we know that

$$P(C_2 = 1 \mid D_2 = 0) = \eta$$

$$P(C_2 = 1 \mid D_2 = 0, D_3 = 1) = \frac{\eta\big[(1-\epsilon)(1-\eta) + \epsilon\eta\big]}{\eta\big[(1-\epsilon)(1-\eta) + \epsilon\eta\big] + (1-\eta)\big[(1-\epsilon)\eta + \epsilon(1-\eta)\big]}$$

so,if $P(C_2 = 1 \mid D_2 = 0) = P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$, we can solve the equation that

$$\epsilon = 0.5$$

Set $\epsilon = 0.5$. Hence $P(C_t \mid C_{t-1}) = 0.5, in\ any\ cndition$, while. Thus the effect of C_{t-1} on C_t can be eliminated, and we don't need to consider the transition probability of $P(C_3 \mid C_2)$.

Problem 2: Emission probabilities

2a. The code is in submission.py.

Problem 3: Transition probabilities

3a. The code is in submission.py.

Problem 4: Particle filtering

4a. The code is in submission.py.