P8131_HW7

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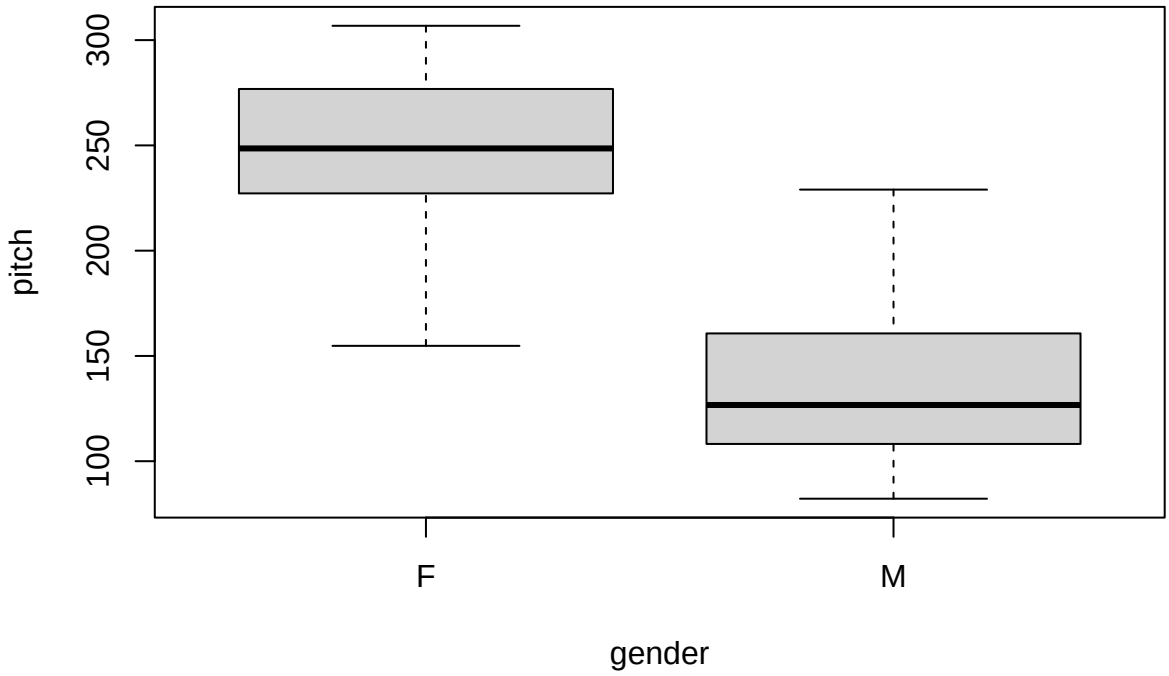
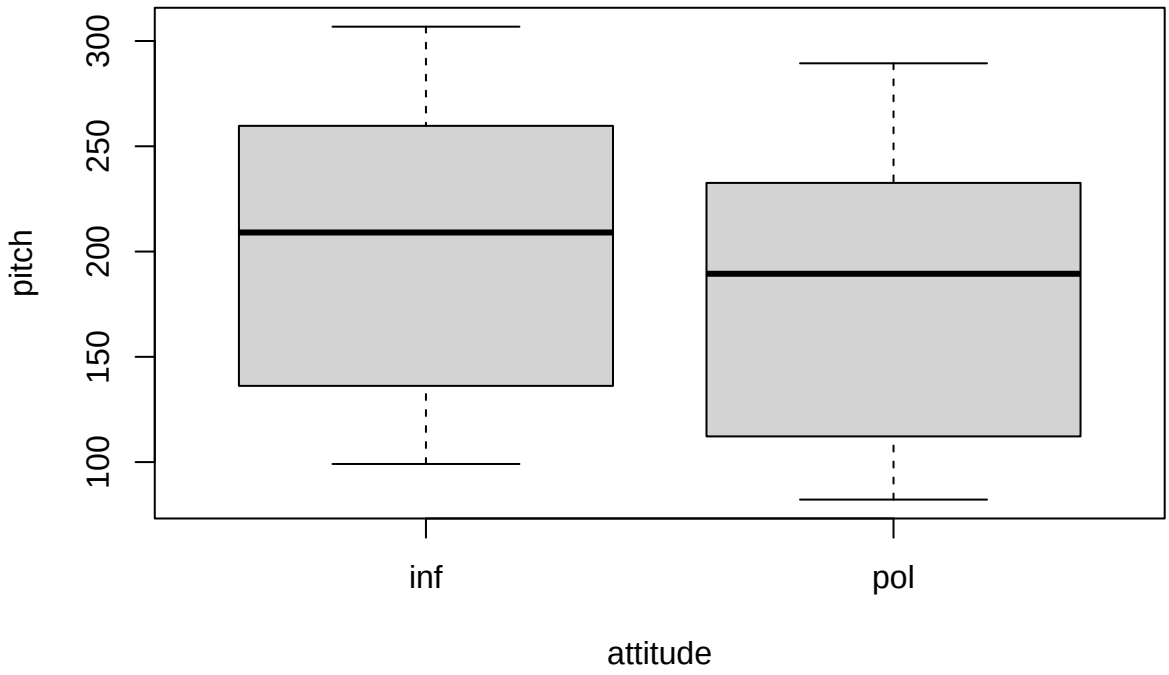
```
library(readr)
library(tidyverse)
library(dplyr)
library(nlme)
library(lme4)
```

Question (a)

```
# load data
pl = read.csv("./HW7-politeness_data.csv")
attach(pl)
# boxplot
boxplot(frequency~gender, xlab="gender", ylab="pitch")
```




```
boxplot(frequency~attitude, xlab="attitude", ylab="pitch")
```



Question (b)

```
# fit LMM with random intercept
lmm1 = lme(frequency ~ gender+attitude, random=~1 | subject, data=pl, method="REML" )
summary(lmm1)
```

```
## Linear mixed-effects model fit by REML
##   Data: pl
##        AIC      BIC    logLik
##   806.0805 818.0527 -398.0402
##
## Random effects:
##  Formula: ~1 | subject
##         (Intercept) Residual
## StdDev:    24.45803 29.11537
##
## Fixed effects:  frequency ~ gender + attitude
##                  Value Std.Error DF   t-value p-value
## (Intercept)  256.98690 15.154986 77 16.957251  0.0000
## genderM     -108.79762 20.956235  4 -5.191659  0.0066
## attitudepol  -20.00238  6.353495 77 -3.148248  0.0023
##  Correlation:
##             (Intr) gendrM
## genderM     -0.691
## attitudepol -0.210  0.000
##
## Standardized Within-Group Residuals:
##        Min         Q1        Med         Q3        Max
## -2.3564422 -0.5658319 -0.2011979  0.4617895  3.2997610
##
```

```
## Number of Observations: 84
## Number of Groups: 6
```

The covariance matrix for a subject Y_i is:

$$cov(Y_i) = \begin{bmatrix} \sigma^2 + \sigma_b^2 & \sigma_b^2 & ... & \sigma_b^2 \\ \sigma_b^2 & \sigma^2 + \sigma_b^2 & ... & \sigma_b^2 \\ ... & & & \\ \sigma_b^2 & \sigma_b^2 & ... & \sigma^2 + \sigma_b^2 \end{bmatrix}$$

where

$$\sigma^2 = 847.7049, \sigma_b^2 = 598.1953$$

```
# covariance matrix for estimates of fixed effects
vcov(lmm1)
```

```
##             (Intercept)       genderM   attitudepol
## (Intercept)   229.67362 -2.195819e+02 -2.018345e+01
## genderM      -219.58189  4.391638e+02  7.288702e-15
## attitudepol   -20.18345  7.288702e-15  4.036690e+01
```

```
# BLUPs for subject-specific intercepts
random.effects(lmm1)
```

```
##    (Intercept)
## F1  -13.575831
## F2   10.170522
## F3    3.405309
## M3   27.960288
## M4    4.739325
## M7  -32.699613
```

```
# residuals
pl$frequency-fitted(lmm1)
```

```
##          F1          F1          F1          F1          F1          F1
## -10.1086926 -38.9110735  61.6913074  16.2889265 -19.5086926  43.4889265
##          F1          F1          F1          F1          F1          F1
##  27.3913074  33.3889265   8.4913074   8.9889265 -42.2086926 -12.7110735
##          F1          F1          F3          F3          F3          F3
## -26.9110735 -68.6086926 -10.6898326 -23.0922136  -3.5898326  -9.3922136
##          F3          F3          F3          F3          F3          F3
##  26.6101674   5.6077864  35.0101674  46.4077864  -7.7898326  -7.8922136
##          F3          F3          F3          F3          M4          M4
## -13.8898326  18.4077864   4.0077864 -54.8898326 -22.2262298 -29.3286108
##          M4          M4          M4          M4          M4          M4
##  96.0737702 -38.0286108 -20.7262298  60.6713892  60.4737702   9.9713892
##          M4          M4          M4          M4          M4          M4
## -31.1262298 -26.0286108 -22.9262298 -16.7286108  -6.9286108  -6.4262298
##          M7          M7          M7          M7          M7          M7
##  -9.3872916 -16.3896725 -13.2872916 -11.1896725  -9.5872916  -5.2896725
##          M7          M7          M7          M7          M7          M7
```

```
##   1.6127084   4.5103275  -1.7872916 -12.5896725  13.3127084  -7.2896725
##          M7          M7          F2          F2          F2          F2
##   8.9103275  12.1127084 -14.4550462 -35.8574271  -0.8550462  -7.4574271
##          F2          F2          F2          F2          F2          F2
##  42.2449538  34.6425729  -3.9550462  29.0425729  30.5449538  27.0425729
##          F2          F2          F2          F2          M3          M3
## -39.1550462 -41.2574271  13.8425729 -19.9550462  -2.3471929  12.6504261
##          M3          M3          M3          M3          M3          M3
## -13.7471929  23.5504261   4.0528071   9.9504261  51.3528071  14.7504261
##          M3          M3          M3          M3          M3          M3
##   4.5528071 -19.6495739  -9.4471929 -18.1495739 -15.0495739  -2.8471929
## attr(,"label")
## [1] "Fitted values"
```

Question (c)

```
lmm2.1 = lme(frequency ~ gender+attitude, random=~1|subject, data=pl, method="ML" )
lmm2.2 = lme(frequency ~ gender+attitude+gender*attitude, random=~1|subject, data=pl, method="ML" )
anova(lmm2.1, lmm2.2)
```

```
##        Model df      AIC      BIC    logLik   Test  L.Ratio p-value
## lmm2.1     1  5 825.6363 837.7904 -407.8182
## lmm2.2     2  6 826.2508 840.8357 -407.1254 1 vs 2 1.385523  0.2392
```

The P-value is greater than 0.05, thus we fail to reject the null and use the smaller model without the interaction term.

Question (d)

```
# fit LMM with random intercepts for subject and scenario
lmm3 = lmer(frequency ~ gender + attitude + (1|subject) + (1|scenario), data=pl)
summary(lmm3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: frequency ~ gender + attitude + (1 | subject) + (1 | scenario)
##    Data: pl
##
## REML criterion at convergence: 784.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.2690 -0.6331 -0.0878  0.5204  3.5326
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  scenario (Intercept) 224.5    14.98
##  subject  (Intercept) 613.2    24.76
##  Residual             637.8    25.25
## Number of obs: 84, groups:  scenario, 7; subject, 6
```

```
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  256.987     16.101  15.961
## genderM     -108.798     20.956  -5.192
## attitudepol  -20.002      5.511  -3.630
##
## Correlation of Fixed Effects:
##             (Intr) gendrM
## genderM     -0.651
## attitudepol -0.171  0.000
```

```
VarCorr(lmm3)
```

```
##  Groups   Name        Std.Dev.
##  scenario (Intercept) 14.983
##  subject  (Intercept) 24.763
##  Residual             25.254
```

The covariance matrix for a subject Y_i is:

$$cov(Y_i) = \begin{bmatrix} \sigma^2 + \sigma^2_{b_1} + \sigma^2_{b_2} & \sigma^2_{b_1} + \sigma^2_{b_2} & ... & \sigma^2_{b_1} + \sigma^2_{b_2} \\ \sigma^2_{b_1} + \sigma^2_{b_2} & \sigma^2 + \sigma^2_{b_1} + \sigma^2_{b_2} & ... & \sigma^2_{b_1} + \sigma^2_{b_2} \\ & ... & & \\ \sigma^2_{b_1} + \sigma^2_{b_2} & \sigma^2_{b_1} + \sigma^2_{b_2} & ... & \sigma^2 + \sigma^2_{b_1} + \sigma^2_{b_2} \end{bmatrix}$$

where

$$\sigma^2 = 637.78, \sigma^2_{b_1} = 613.19, , \sigma^2_{b_2} = 224.5$$