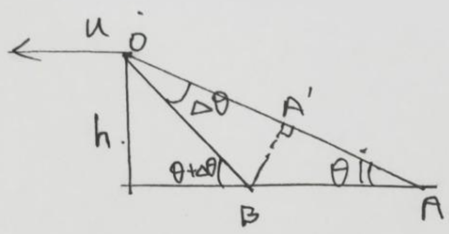
1.



① $v = \dfrac{u}{\cos\theta}$

②

$$\Delta v = \frac{u}{\cos(\theta+\Delta\theta)} - \frac{u}{\cos\theta}$$

$$\Delta v = \frac{\cos\theta - \cos\theta\cos\Delta\theta + \sin\theta\sin\Delta\theta}{\cos\theta\cos(\theta+\Delta\theta)}$$

略去二级以上小量得

$$\therefore \Delta v = \frac{u\sin\theta\,\Delta\theta}{\cos^2\theta}$$

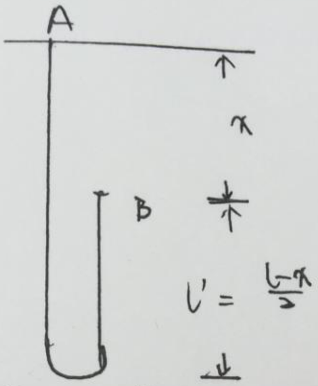
$$\therefore a = \lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}$$

$$a = \frac{u\sin\theta}{\cos^2\theta}\cdot\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$$

$$\Delta\theta \approx \frac{BA'}{OB} \approx \frac{u\Delta t\tan\theta}{h/\sin\theta}$$

$$\therefore a = \frac{u^2}{h}\tan^3\theta$$

2 .



设B下落x后右半部分速度为v.

体系动量为 $p = \frac{L-x}{2}\rho v$

dt 时间后

$$\frac{dp}{dt} = -\frac{\rho v}{2}\frac{dx}{dt} + \frac{L-x}{2}\rho\frac{dv}{dt}$$

$\frac{dx}{dt} = v$ 由于右半部分自由落体

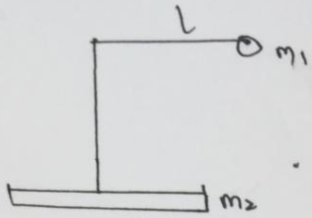
$\therefore \quad v = \sqrt{2gx}$.

而 $\frac{dv}{dt} = g$.

$\therefore \quad \rho L g - F_T = -\rho g x + \frac{L-x}{2}\rho g$

$\therefore \quad F_T = \frac{L+x}{2}\rho g + \rho x g$

3.



① 水平方向动量守恒得

$$m_1 v_1 = m_2 v_2$$

机械能守恒

$$m_1 g l = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\therefore v_2 = \sqrt{\frac{2 m_1^2 g l}{m_2^2 + m_1 m_2}}$$

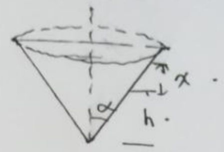
② 做功

对 m_1 球

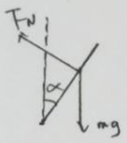
$$\begin{cases} m_1 g l + W = \frac{1}{2} m_1 v_1^2 \\ v_1 = \frac{m_2}{m_1} v_2 \end{cases}$$

$$\therefore W = \frac{-m_1^2 g l}{m_1 + m_2}$$

4.



①



$$mg\cot\alpha = m\frac{v_0^2}{R}$$

$$R = h\tan\alpha$$

$$\therefore v_0 = \sqrt{gh}$$

② $\frac{1}{2}mv_0^2 + mg(h+x) = \frac{1}{2}mv_1^2 + mgh$

（能量守恒）

$(h+x)\tan\alpha \cdot mv = h\tan\alpha \cdot mv_1$

（角动量守恒）

$$\therefore 2gx^3 + (4gh - v_1^2)x^2 - 2h(v_1^2 - gh)x = 0$$

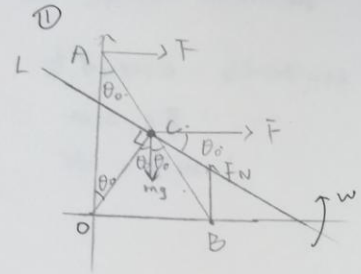
$$v_1 = 2\sqrt{gh}$$

$\therefore x = \sqrt{3}h$（负根舍去）

∴ 小球在 h 到 $(1+\sqrt{3})h$ 两高度间往返运动.

5.

(1)



在A不脱离墙面之间

质心C到O的距离

恒为 $\frac{L}{2}$

∴ C在以O为圆心 $\frac{L}{2}$ 为半径

的圆上做圆周运动.

过C做 L ⊥ OC.

释放瞬间 $v_c=0$

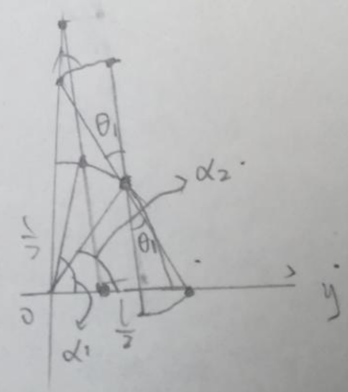
∴ $a_n=0$.

∴ $(mg-F_N)\cos\theta_0 = F\sin\theta_0$ ①

设棒转动角速度为 w.

我们尝试从几何关系求出

v_c 和 w 的关系.



$$\begin{cases} \Delta t v_c = \frac{L}{2}(\alpha_1-\alpha_2). \\ w = \frac{\theta_1}{\Delta t}. \\ \theta_1 = \alpha_1-\alpha_2 \end{cases}$$

∴ $v_c = \frac{L}{2}w$.

∴ $a_c = \frac{L}{2}\beta$. (β 角加速度). ②

∴ $a_c = \frac{F\cos\theta_0 + (mg-F_N)\sin\theta_0}{m}$ ③

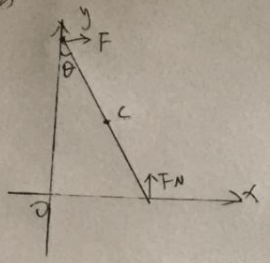
$\frac{L}{2}F_N\sin\theta_0 - \frac{L}{2}F\cos\theta_0 = M$. ④

$M = I\beta$.

∴ 由 ① ② ③ ④ 知

$$\begin{cases} F = \frac{3}{4}mg\sin\theta_0\cos\theta_0. \\ F_N = mg(1-\frac{3}{4}\sin^2\theta_0). \end{cases}$$

5.(2)



建立直角坐标系

$$m\ddot{x}_c = F$$

$$m\ddot{y}_c = F_N - mg$$

$$J_c\ddot{\theta} = F_N\frac{l}{2}\sin\theta - F\frac{l}{2}\cos\theta$$

由几何关系

$$x_c = \frac{l}{2}\sin\theta \qquad y_c = \frac{l}{2}\cos\theta$$

$$\dot{x}_c = \frac{l}{2}\cos\theta\cdot\dot{\theta},\quad \dot{y}_c = -\frac{l}{2}\sin\theta\dot{\theta}$$

$$\ddot{x}_c = \frac{l}{2}\cos\theta\ddot{\theta} - \frac{l}{2}\sin\theta(\dot{\theta})^2$$

$$\ddot{y}_c = -\frac{l}{2}\sin\theta\ddot{\theta} - \frac{l}{2}\cos\theta\dot{\theta}^2$$

下滑过程中 $\dot{\theta}\neq 0$

但机械能守恒

$$mg\frac{l}{2}\cos\theta_0 = mg\frac{l}{2}\cos\theta + \frac{1}{2}m(\dot{x}_c + \dot{y}_c) + \frac{1}{2}J_c\dot{\theta}^2$$

$J_c = \frac{1}{12}ml^2$ 代入

即可解得

$$\ddot{\theta} = \frac{3g\sin\theta}{2l}$$

$$\ddot{x}_c = \frac{3}{4}g\sin\theta(3\cos\theta - 2\cos\theta_0)$$

$$\ddot{y}_c = \frac{1}{4}g(9\cos^2\theta - 3 - 6\cos\theta\cos\theta_0)$$

$$\therefore F = \frac{3}{4}mg\sin\theta(3\cos\theta - 2\cos\theta_0)$$

$$F_N = \frac{1}{4}mg(1 + 9\cos^2\theta - 6\cos\theta\cos\theta_0)$$

当 $\theta = \theta_0$ 时结果与(1)相同

∴可见此方法也可用于(1)求解.

$$F_N = \frac{1}{4}mg[(3\cos\theta - \cos\theta_0)^2 + 1 - \cos^2\theta_0]$$

θ 从 θ_0 增加到 $\frac{\pi}{2}$　F_N 恒大于 0

而 θ 从 θ_0 增大到 $\frac{\pi}{2}$ 过程中

F 随着 $\cos\theta$ 减小，将从正变为负　∴ $F=0$ 时的 θ 即为 θ_m

$$\therefore 3\cos\theta_m = 2\cos\theta_0$$

$$\therefore \theta_m = \arccos\left(\frac{2}{3}\cos\theta_0\right)$$

▲ 此题也可用(1)方法求解.

这里给出一种新的方法。