Resilient Consensus Through Dynamic Event-Triggered Mechanism

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1 Introduction

The increasing concern for cyber security in recent years has been accompanied by the attention paid to solving the consensus problem for multi-agent systems (MASs) against malicious attacks. Open communications via shared networks are vulnerable to potential attacks, thereby causing irreparable losses. Another concern is the heavy communication burden, which is caused by frequent interactions between agents. Motivated by these observations, this paper aims to design a dynamic event-triggered (DE) controller for the MAS to achieve exact resilient consensus with reduced communication overheads. Despite the influence of noncooperative agents in the network, the states of all cooperative agents will converge to the same consensus value eventually.

2 Problem Formulation and Algorithm Design

Consider a single-integrator MAS described by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. We denote $x_i[t] \in \mathbb{R}$ as the state for agent i at time step $t \in \mathbb{Z}_{\geq 0}$. The state update for each agent $i \in \mathcal{V}$ is based on a prescribed rule, which is expressed as

$$x_i[t+1] = x_i[t] + u_i[t], \tag{1}$$

where $u_i[t]$ represents the control input. To achieve resilient consensus, the event-triggered protocol [1] is introduced as

$$u_i[t] = -\sum_{j \in \mathcal{V}_i^{\text{in}}} a_{ij}[t]\left(x_i[t] - \hat{x}_j[t]\right), \tag{2}$$

where $\hat{x}_j[t] \in \mathbb{R}$ denotes the last broadcast state value of agent j, which is defined as $\hat{x}_j[t] = x_j[t_m^j]$, $t \in [t_m^j, t_{m+1}^j)$, with $\{t_0^j, t_1^j, \ldots \in \mathbb{Z}_{>0}\}$ being the sequence of triggering times. For the dynamic triggering case, a dynamic variable $\xi_i[t] \in \mathbb{R}$ is introduced and its state update adheres to

$$\xi_i[t+1] = (1-\psi_i)\xi_i[t] + \phi_i(\vartheta[t] - |e_i[t]|), \tag{3}$$

where ψ_i and ϕ_i are parameters to be designed, $\vartheta[t]$ is a positive threshold which converges exponentially to zero. With $\xi_i[t]$, agent i determines its triggering times $\{t_m^i\}_{m=1}^{\infty}$ by the following dynamic event-triggered function (DETF):

$$t_{m+1}^i = \min\left\{t > t_m^i : \zeta_i\left(|e_i[t]| - \vartheta[t]\right) > \xi_i[t]\right\}, \tag{4}$$

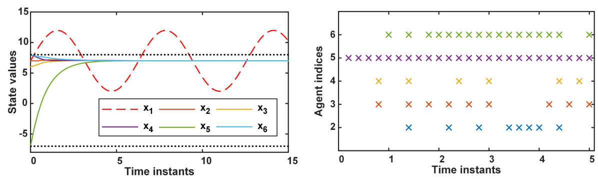
where ζ_i is also a parameter to be designed. Based on the designed DETF, we further develop a *Dynamic Event-triggered Mean-Subsequence-Reduced* (DE-MSR) algorithm. Taking the state update of cooperative agent $i \in \mathcal{C}$ as an example, the main steps of the DE-MSR algorithm are shown in Algorithm 1.

Algorithm 1 DE-MSR ALGORITHM

Initialize $x_i[0]$, $\hat{x}_i[0]$, $\xi_i[0] > 0$, ψ_i, ζ_i, and ϕ_i;
for $t = 0, 1, \ldots$, the cooperative agent $i \in \mathcal{C}$ **do**
    Implement the W-MSR algorithm [2];
    Obtain $\mathcal{R}_i^{\text{in}}[t]$ as the set of retained in-neighbors;
    Update the state according to

$$x_i[t+1] = x_i[t] - \sum_{j \in \mathcal{R}_i^{\text{in}}[t]} a_{ij}[t]\left(x_i[t] - \hat{x}_j[t]\right); \tag{5}$$

    if DETF (4) triggers **then**
        Update $\hat{x}_i[t+1]$ as $\hat{x}_i[t+1] = x_i[t+1]$;
        Send $\hat{x}_i[t+1]$ to the agents in $\mathcal{V}_i^{\text{out}}[t]$;
    else
        Set $\hat{x}_i[t+1]$ as $\hat{x}_i[t+1] = \hat{x}_i[t]$.
    end if
end for



(a) Trajectories. (b) Triggering behavior.

Fig. 1. Illustrative examples using DETF (4).

3 Illustrative Examples

The illustrative examples of the DE-MSR algorithm are depicted in Figs. 1(a) and (b). Fig. 1(a) shows that the state values of cooperative agents reach an agreement. Furthermore, it is observed from Fig. 1(b) that the communication times between agents are significantly reduced. Compared to the existing event-based resilient algorithm [1], the proposed method is superior in mitigating communication overheads while maintaining the comparable convergence rate.

References

[1] Y. Wang, H. Ishii, Resilient consensus through event-based communication, IEEE Trans. Control Netw. Syst. 7 (1) (2020) 471–482.

[2] H. J. LeBlanc, H. Zhang, X. Koutsoukos, S. Sundaram, Resilient asymptotic consensus in robust networks, IEEE J. Sel. Areas Commun. 31 (4) (2013) 766–781.