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PEEC Model of a Spiral Inductor Generated by Fasthenry

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Summary. A symmetric generalized state-space model of a spiral inductor is obtained by the inductance extraction software package Fasthenry.

23.1 Fasthenry

Fasthenry[KTW94] is a software program which computes the frequency-dependent resistances and inductances of complicated three-dimensional packages and interconnect, assuming operating frequencies up to the multi-gigahertz range. Specifically, it computes the complex frequency-dependent impedance matrix $Z_p(\omega) \in \mathbb{C}^{p\times p}$ of a p-terminal set of conductors, such as an electrical package or a connector, where $Z_p(\omega)$ satisfies

$$Z_p(\omega)I_p(\omega) = V_p(\omega).$$

The quantities $I_p, V_p \in \mathbb{C}^p$ are the vectors of terminal current and voltage phasors, respectively. The frequency-dependent resistance and inductance matrices $R_p(\omega)$ and $L_p(\omega)$ are related to $Z_p(\omega)$ by:

$$Z_p(\omega) = R_p(\omega) + i\omega L_p(\omega), \tag{23.1}$$

and are important physical quantities to be preserved in reduced models.

To compute $Z_p(\omega)$, Fasthenry generates an equivalent circuit for the structure to be analyzed from the magneto-quasistatic Maxwell equations via the mesh-formulated partial element equivalent circuit (PEEC) approach using multipole acceleration. To model current flow, the interior of the conductors is divided into volume *filaments*, each of which carries a constant current density along its length. In order to capture skin and proximity effects, the cross section of each conductor is divided into bundles of filaments. In fact, many thin filaments are needed near the surface of the conductors to capture the current crowding near the conductor surfaces at high frequencies (the skin effect). The interconnection of the filaments, plus the sources at the terminal pairs, generates a "circuit" whose solution gives the desired inductance and resistance matrices. For complicated structures, filaments numbering in the tens of thousands are not uncommon.

To derive a system of equations for the circuit of filaments, sinusoidal steady-state is assumed, and following the partial inductance approach in [Rue72], the filament current phasors can be related to the filament voltage phasors by

$$ZI_b = V_b, \tag{23.2}$$

where $V_b,\ I_b \in \mathbb{C}^b$, b is the number of filaments (number of branches in the circuit), and $Z \in \mathbb{C}^{b\times b}$ is the complex impedance matrix given by

$$Z = R + i\omega L, \tag{23.3}$$

where ω is the excitation frequency. The entries of the diagonal matrix $R \in \mathbb{R}^{b\times b}$ represent the DC resistance of each current filament, and $L \in \mathbb{R}^{b\times b}$ is the *dense* matrix of partial inductances. The partial inductance matrix is dense since every filament is magnetically coupled to every other filament in the problem.

To apply the circuit analysis technique known as Mesh Analysis, Kirchhoff's voltage law is explicitly enforced, which implies that the sum of the branch voltages around each mesh in the network is zero (a mesh is any loop of branches in the graph which does not enclose any other branches). This relation is represented by

$$MV_b = V_s \qquad M^T I_m = I_b, \tag{23.4}$$

where $V_s \in \mathbb{C}^m$ is the mostly zero vector of source branch voltages, $I_m \in \mathbb{C}^m$ is the vector of mesh currents, $M \in \mathbb{R}^{m\times b}$ is the mesh matrix. Here, m is the number of meshes, which is typically somewhat less than b, the number of filaments. The terminal source currents and voltages of the p-conductor system I_p and V_p are related to the mesh quantities by $I_p = N^T I_m, V_s = NV_p$, where $N \in \mathbb{R}^{m\times p}$ is a terminal incidence matrix determined by the mesh formulation.

Combining (23.4) and (23.2) yields

$$MZM^T I_m = V_s,$$

from which we obtain

$$I_p = N^T (MZM^T)^{-1} NV_p,$$

which gives the desired complex impedance matrix

$$Z_p(\omega) = (N^T(\tilde{R} + i\omega\tilde{L})^{-1}N)^{-1},$$

where $\tilde{L} = MLM^T \in \mathbb{R}^{m\times m}$ is the *dense* mesh inductance matrix, $\tilde{R} = MRM^T \in \mathbb{R}^{m\times m}$ is the sparse mesh resistance matrix.

Finally, we can write the mesh analysis circuit equations in the generalized state-space form:

$$E\frac{dx}{dt} = Ax + Bu, \tag{23.5}$$

$$y = B^T x, \tag{23.6}$$

where $E := \tilde{L}$ and $A := -\tilde{R}$ are both symmetric matrices, $B = N$, and u and y are the time-domain transforms of V_p and I_p, respectively. The transfer function of (23.5-23.6) evaluated on the imaginary axis gives the inverse of $Z_p(\omega)$:

$$Z_p(\omega) = (G(i\omega))^{-1}.$$

23.2 Spiral Inductor

This inductor which first appeared in [KWW00] is intended as an integrated RF passive inductor. To make it also a proximity sensor, a $0.1\mu m$ plane of copper is added $45\mu m$ above the spiral. The spiral is also copper with turns $40\mu m$ wide, $15\mu m$ thick, with a separation of $40\mu m$. The spiral is suspended $55\mu m$ over the substrate by posts at the corners and centers of the turns in order to reduce the capacitance to the substrate. (Note that neither the substrate nor the capacitance is modeled in this example.) The overall extent of the suspended turns is $1.58mm \times 1.58mm$. The spiral inductor, including part of the overhanging plane, is shown in Figure 23.1. In Figures 2(a) and 2(b), we show the resistance and impedance responses (the $R_p(\omega)$ and $L_p(\omega)$ matrices from (23.1)) of the spiral inductor corresponding to a PEEC model using 2117 filaments (state-space matrices of order 1434, single-input single-output). The frequency dependence of the resistance shows two effects, first a rise due to currents induced in the copper plane and then a much sharper rise due to the skin effect. Capturing the rise due to the skin effect while also maintaining the low frequency response is a challenge for many model reduction algorithms.

23.3 Symmetric Standard State-Space System

For certain applications one may prefer to change the generalized state-space model (23.5-23.6) to the standard state-space form. The following is a way of effecting the transformation while preserving symmetry and follows the approach used in [MSKEW96].

The mesh inductance matrix $\tilde{L}$ is symmetric and positive definite. Hence, it has a unique symmetric positive definite square root, $\tilde{L}^{\frac{1}{2}}$, satisfying $\tilde{L}^{\frac{1}{2}}\tilde{L}^{\frac{1}{2}} = \tilde{L}$. Then we use the coordinate transformation $\tilde{x} = \tilde{L}^{\frac{1}{2}}x$ to obtain the standard state-space system:

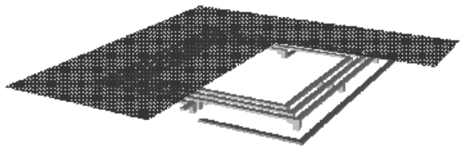
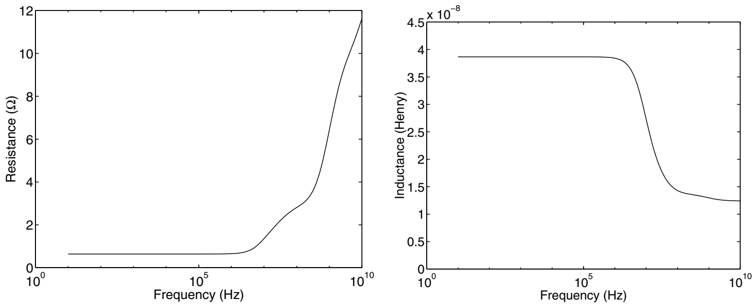


Fig. 23.1. Spiral inductor with part of overhanging copper plane



(a) Resistance (b) Inductance

Fig. 23.2. PEEC model of spiral inductor using 2117 filaments

$$\frac{d\tilde{x}}{dt} = \tilde{A}\tilde{x} + \tilde{B}u, \tag{23.7}$$

$$y = \tilde{B}^T\tilde{x}, \tag{23.8}$$

where $\tilde{A} = -\tilde{L}^{-\frac{1}{2}}\tilde{R}\tilde{L}^{-\frac{1}{2}}$ is symmetric ($\tilde{L}^{-\frac{1}{2}}$ is symmetric) and $\tilde{B} = B\tilde{L}^{-\frac{1}{2}}$.

If the original matrices are too large for testing purposes when comparing with methods requiring $O(n^3)$ work, applying the Prima[OCP98] algorithm to the generalized state-space model in (23.5-23.6) with a reduction order of around 100 will produce a smaller system with virtually the same frequency response. Indeed this is what has been done when this example was used previously in numerous papers, including [LWW99].

Note

We make a note here that the example first used in [LWW99] and subsequently in other papers comes from a finer discretization of the spiral inductor than shown here. That example started with state-spaces matrices of order 1602 (compared to order 1434 here). The order 500 system was obtained by running

Prima with a reduction order of 503. Due to the loss of orthogonality of the Arnoldi vectors the reduced matrix E_r has three zero eigenvalues. The modes corresponding to the zero eigenvalues were simply removed to give a new positive definite E_r matrix and a system of order 500. The frequency response of the resulting system is indistinguishable from the original.

Acknowledgments

Most of the description of the spiral inductor and how the model was generated by Fasthenry is paraphrased from [KTW94] and [KWW00].

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