

Lloyd's always converges, but not always at the optimal solution

- centers might be too close

↳ solve by k-means++

pick random centers, but each point has pr of being selected  
proportional to dist from center (outliers more likely)

$$L Pr(x \text{ picked}) = \frac{0(x)^2}{\sum_{x \in X} 0(x)^2} = \text{each point gets } (\text{dist to center})^2 \text{ \# 'entries'}$$

how to choose k:

1. iterate through k to find point of diminishing return

- means gotta do many attempts (not always possible)

\* sometimes, lower cost  $\neq$  best use

clustering wants:

- similar dps in same
- dissimilar dps in different

k-means only focuses on



= small inner cluster width/distance (a), larger distance between clusters (b)

$$= \underbrace{(b-a) / \max(a,b)}_{\text{want}} : \text{close to 1 when } a \ll \text{ or } b \gg$$

Silhouette Score

for each dp i:  $a_i$  = avg dist to neighbor in cluster

$b_i$  = smallest mean dist to every point in another cluster

tags = [string<sub>1</sub>, string<sub>2</sub>, string<sub>3</sub>, ...]

model = SentenceTransformer(model) ← LLM

feature\_vectors = model.encode(tags) ← for each tag, converts into array where each index is similarity score to a word in dict

clusters = Kmeans(n\_clusters=3, n\_init=20).fit(feature\_vectors).predict(feature\_vectors)

runs Kmeans with 3 centers, 20 times

equal to .fit\_predict(feature\_vectors)

for KMeans: the only 'randomness' comes from initial center selection

↳ same centers = same result

must use mean distance func to correctly cluster

↳ diff func will do something, but not what's intended

Hierarchical Clustering: clustering in tiers/thresholds  
 Outputs dendrogram



agglomerative - start w/ each point is own cluster

- compute dist between clusters
- merge 2 closest clusters
- repeat until  $\forall$  in the same cluster

divisive - start with  $\forall$  in 1 cluster

- at each step, split
- repeat until each point is its own cluster

recursively apply  
 kmeans with  $k=2$

threshold distance isn't very practical

↳ hard to know what it means

more-so to view data that can't be plotted  
 bc dp's in very high dimensions

\*  $d(x, y)$  - distance between points  
 $D(a, b)$  - distance between clusters  
 $\mu_n$  = mean / avg dp pos in  $C_n$

how to:

1. make matrix of all points vs themselves
2. compute distance between all points
3. group together 2 closest points
4. redefine matrix, include combined points as clusters
5. redefine dist to  $\forall$  other points
6. add closest dp to cluster
7. repeat 4-7 until  $\forall$  in 1 cluster

ways to calc  $D(C_1, C_2)$

1. single-length dist: min dist between any points in  $C_1, C_2$ 
  - ↳ sensitive to noise in between dps
  - ↳ creates elongated clustering

2. complete length dist: max dist between any points in  $C_1, C_2$ 
  - ↳ opposite to SL: handles noise well
  - ↳ sensitive to outliers

3. average-link dist:  $\frac{1}{|C_1| \cdot |C_2|} \sum d(p_1, p_2)$

4. centroid dist: similar to kmeans
  - ↳ pos of  $C$  is average position of  $\forall p \in C$

5. Ward's Dist:  $\sum_{p \in C_{12}} d(p, \mu_{12}) - \sum_{p \in C_1} d(p, \mu_1) - \sum_{p \in C_2} d(p, \mu_2)$ 
  - ↳ merging  $C_1$  and  $C_2$

(spread around mean in merged) - (spread around mean in  $C_1$ ) - (spread around mean in  $C_2$ )

## Density Based

density: I take radius of length  $\epsilon$  around point

↳ if # points within region  $>$  threshold: dense ✓

core point:  $\epsilon$ -neighborhood contains  $\geq \langle \text{min\_points} \rangle$

border point: in  $\epsilon$ -neighborhood of core point

noise point: neither core or border

DFS method: 1. iterate through dataset

2. if  $dp$  is core:

- iterate through neighborhood

- if neighbor is core: repeat ↗

↳ else: neighbor is border, add to cluster

- continue iterating through neighborhood

- once no more undiscovered nodes in neighborhood,

repeat step 1 with next undiscovered  $dp$  in dataset

since \*density definition\* is defined, won't pick up on differently dense areas = mark as outliers  
↳ thus makes clusters of the same density