University of Crete
*Department of Physics*

Bachelor Thesis

# Perfect Transmission in Non-Hermitian Scattering Media

*Author:*
Alexandra Zisimopoulou

*Supervisor:*
Prof. Konstantinos Makris

Heraklion
July 28, 2018

# Contents

# Chapter 1

# Introduction

In 1977 the Nobel prize was given to the American physicist P. W. Anderson, who was the first to suggest that electron localization is possible in a lattice potential, provided that the degree of randomness (disorder) in the lattice is sufficiently large. The phenomenon on which, the diffusion of waves in a disordered medium is absent is called Anderson localization, named after the physicist P. W. Anderson.

The most appropriate context for the experimental implementation of random systems, proved to be that of optical physics. Many ground breaking experiments were demonstrated and as a result led to the new field of disordered photonics. With the exception of random lasers, most of the disordered systems examined were hermitian. Therefore, there is a lot of interest in exploring non-Hermitian random structures.

In this context, the physical motivation of this thesis is the recent publication by K. G. Makris, A. Brandstötter, P. Ambichl, Z.H Musslimani and S. Rotter [3], demonstrating that in one-dimensional disordered scattering problems there is only one certain class of non-Hermitian refractive index distributions which leads to constant intensity waves in disordered media. These refractive index distributions feature a system on which on average the gain of the system is equal to the loss. They showed that in such non-Hermitian refractive index distributions the backscattering is absent and that they feature perfect transmission through the disordered medium.

A reasonable question arising from these statements is how the backscattering and perfect transmission behaves through the disordered dielectric medium when the intensity inside the scattering region is not constant. In this thesis, we show that they exist certain class of non-Hermitian dielectric distributions, which correspond to a system on which on average gain is higher than loss or gain is lower than loss and display perfect transmission without backscattering through a scattering medium.

## 1.1 Maxwell's equations

In this paragraph, we are going to review some basic properties of electromagnetism that are essential and useful for our thesis. To begin with, in order to answer our question we need to map Ampere's circuital law and Faraday's law of induction to the frequency domain [2]. We present the differential form of Maxwell equations in the time domain.

$$Ampere's\ circuital\ law : \nabla \times \boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{J}(\boldsymbol{r}, t) + \frac{\partial \boldsymbol{D}(\boldsymbol{r}, t)}{\partial t} \tag{1.1}$$

$$Faraday's\ law\ of\ induction : \nabla \times \boldsymbol{E}(\boldsymbol{r}, t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} \tag{1.2}$$

$$Gauss's\ law : \nabla \cdot \boldsymbol{D}(\boldsymbol{r}, t) = \rho(\boldsymbol{r}, t) \tag{1.3}$$

$$Gauss's\ law\ for\ magnetism : \nabla \cdot \boldsymbol{B}(\boldsymbol{r}, t) = 0 \tag{1.4}$$

where $\boldsymbol{E}(\boldsymbol{r}, t)$ is the electric field, $\boldsymbol{B}(\boldsymbol{r}, t)$ is the magnetic field vector, $\boldsymbol{D}(\boldsymbol{r}, t)$ is the electric displacement field, $\rho(\boldsymbol{r}, t)$ is the charge inside an enclosed volume, $\boldsymbol{J}(\boldsymbol{r}, t)$ is the current density and $\boldsymbol{H}(\boldsymbol{r}, t)$ is the magnetic field . The differential form of Maxwell's equations can be put into an equivalent format by mapping time variables to the frequency domain.This is done by introducing the Fourier-transform operator as,

$$\tilde{Y}(..., \omega, ...) = \int_{-\infty}^{\infty} Y(..., t, ...) e^{j\omega t} dt \tag{1.5}$$

where variable $\tilde{Y}$ represents the Fourier transform of Y, j is the imaginary unit and $\omega$ is the associated frequency-domain variable corresponding to the

real variable, time t.
We also introduce the inverse Fourier-transform operator, defined as,

$$Y(...,t...) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{Y}(...,\omega,...)e^{-j\omega t}d\omega \tag{1.6}$$

Differentiating equation (1.6) with respect to t we get,

$$\frac{\partial Y}{\partial t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} -j\omega\tilde{Y}(...,\omega,...)e^{-j\omega t}d\omega \tag{1.7}$$

resulting in the mapping $\frac{\partial}{\partial t} \rightarrow -j\omega$ from the time domain to the frequency domain.

Applying the Fourier transform (1.6) and (1.7) to Maxwell's equations (1.1)-(1.4), we obtain the following relations,

$$\begin{aligned}\int_{-\infty}^{\infty} \nabla\times\tilde{\boldsymbol{H}}(\boldsymbol{r},\omega)e^{-j\omega t}d\omega = &\int_{-\infty}^{\infty} \nabla\times\tilde{\boldsymbol{J}}(\boldsymbol{r},\omega)e^{-j\omega t}d\omega \\ &+ \int_{-\infty}^{\infty} \frac{\partial\tilde{\boldsymbol{D}}(\boldsymbol{r},\omega)}{\partial t}e^{-j\omega t}d\omega\end{aligned} \tag{1.8}$$

$$\int_{-\infty}^{\infty} \nabla\times\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega)e^{-j\omega t}d\omega = -\int_{-\infty}^{\infty} \frac{\partial\tilde{\boldsymbol{B}}(\boldsymbol{r},\omega)}{\partial t}e^{-j\omega t}d\omega \tag{1.9}$$

$$\int_{-\infty}^{\infty} \nabla\times\tilde{\boldsymbol{D}}(\boldsymbol{r},\omega)e^{-j\omega t}d\omega = \int_{-\infty}^{\infty} \rho(\boldsymbol{r},\omega)e^{-j\omega t}d\omega \tag{1.10}$$

$$\int_{-\infty}^{\infty} \nabla\times\tilde{\boldsymbol{B}}(\boldsymbol{r},\omega)e^{-j\omega t}d\omega = 0 \tag{1.11}$$

which lead to the following frequency domain Maxwell's equations:

$$\nabla\times\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega) = j\omega\tilde{\boldsymbol{B}}(\boldsymbol{r},\omega) \tag{1.12}$$

$$\nabla\cdot\tilde{\boldsymbol{D}}(\boldsymbol{r},\omega) = \tilde{\rho}(\boldsymbol{r},\omega) \tag{1.13}$$

$$\nabla\times\tilde{\boldsymbol{H}}(\boldsymbol{r},\omega) = -j\omega\tilde{\boldsymbol{D}}(\boldsymbol{r},\omega) + \tilde{\boldsymbol{J}}(\boldsymbol{r},\omega) \tag{1.14}$$

$$\nabla\cdot\tilde{\boldsymbol{B}}(\boldsymbol{r},\omega) = 0 \tag{1.15}$$

We consider a non-conducting medium, $\sigma = 0$ which does not have any free charges, $\rho = 0$. Using the following relations for the current density, the electric displacement field and the magnetic field vector we get,

$$\begin{aligned}\tilde{\boldsymbol{J}}(\boldsymbol{r},\omega) &= \sigma\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega)\\ \tilde{\boldsymbol{D}}(\boldsymbol{r},\omega) &= \tilde{\varepsilon}(\boldsymbol{r},\omega)\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega)\\ \tilde{\boldsymbol{B}}(\boldsymbol{r},\omega) &= \mu(\boldsymbol{r},\omega)\tilde{\boldsymbol{H}}(\boldsymbol{r},\omega)\end{aligned} \tag{1.16}$$

where $\tilde{\varepsilon}(\boldsymbol{r},\omega)$and $\mu(\boldsymbol{r},\omega)$ are the dielectric permittivity and permeability, respectively. Combining equations (1.16) with (1.12) and (1.4) we derive the following expressions of Ampere's circuital law and Faraday's law of induction in the frequency domain.

$$\nabla \times \tilde{\boldsymbol{E}}(\boldsymbol{r},\omega) = j\omega\mu(\boldsymbol{r},\omega)\tilde{\boldsymbol{H}}(\boldsymbol{r},\omega) \tag{1.17}$$

$$\nabla \times \tilde{\boldsymbol{H}}(\boldsymbol{r},\omega) = -j\omega\tilde{\varepsilon}(\boldsymbol{r},\omega)\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega) \tag{1.18}$$

## 1.2 Derivation of Helmholtz Equation

### 1.2.1 Frequency-dependent Helmholtz Equation

The purpose of this work is to study whether there can be perfect transmission for a specific range of the wavelength values in non-Hermitian dielectric media. As a first step, we need to find how the electric field behaves inside a scattering medium. The electric field inside the scattering medium satisfies the one-dimensional Helmholtz Equation that describes time-independent scattering of a linearly polarised electric field $\Psi(x)$.

In the following two sections we will derive the one-dimensional Helmholtz equation. Firstly, need to take the curl on both sides in equation (1.17). Since we study non-magnetic dielectric scattering media we can assume that, $\mu(x,\omega) \approx \mu_0$ [7]. Also, using equation (1.18) we get,

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r},\omega) = \nabla \times (j\omega\boldsymbol{H}(\boldsymbol{r},\omega)) = \omega^2\mu_0\tilde{\varepsilon}(\boldsymbol{r},\omega)\boldsymbol{E} \tag{1.19}$$

where we have defined $\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega) \equiv \boldsymbol{E}(\boldsymbol{r},\omega)$ and $\tilde{\boldsymbol{H}}(\boldsymbol{r},\omega) \equiv \boldsymbol{H}(\boldsymbol{r},\omega)$.It is also true that,

$$\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2\boldsymbol{E} \tag{1.20}$$

Thus we get,

$$\nabla(\nabla \cdot \boldsymbol{E}(\boldsymbol{r}, \omega)) - \nabla^2 \boldsymbol{E}(\boldsymbol{r}, \omega) = \omega^2 \mu_0 \tilde{\varepsilon}(\boldsymbol{r}, \omega) \boldsymbol{E} \tag{1.21}$$

The following discussions concern only transverse electric polarization (TE). This means that the electric field has no component in the direction of propagation. Since we consider only TE polarized plane waves ,we have:

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}, \omega) = 0 \tag{1.22}$$

Substituting expression (1.22) to (1.21) we derive the Helmholtz equation,

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}, \omega) = -\omega^2 \mu_0 \tilde{\varepsilon}(\boldsymbol{r}, \omega) \boldsymbol{E}(\boldsymbol{r}, \omega) \tag{1.23}$$

We assume an one-dimensional scattering medium reaching from -L to L and a wave which propagates in the x direction, see Figure 1.1. The polarization of the electric field is in the y direction.

In addition, our results are associated with an incidence direction of the plane wave from left to the right. An incidence direction to the left with the same wavenumber k would not change our results in terms of the transmission coefficient but it would change the reflection coefficient as well as the intensity variations inside the medium.

It is also known that the speed of light is equal to, $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, where $\varepsilon_0$ is the permittivity of the vacuum and $\mu_0$ is the vacuum permeability and $c = \frac{\omega}{k}$, where $\omega$ and $k = \frac{2\pi}{\lambda}$ are the frequency and the wavenumber of the wave, respectively. Thus equation (1.23) becomes,

$$\frac{\partial^2}{\partial x^2} \Psi(x, \omega) = -k^2 \varepsilon(x, \omega) \Psi(x, \omega) \tag{1.24}$$

where $\Psi(x)$ is the perpendicular component of the electric field. Also we have defined the relative dielectric permittivity as, $\varepsilon(\boldsymbol{r}, \omega) \equiv \varepsilon_r(\boldsymbol{r}, \omega) = \frac{\tilde{\varepsilon}(\boldsymbol{r}, \omega)}{\varepsilon_0}$

Hence equation (1.24) becomes,

$$\frac{\partial^2}{\partial x^2} \Psi(x, \omega) = -k^2 \varepsilon(x, \omega) \Psi(x, \omega) \tag{1.25}$$

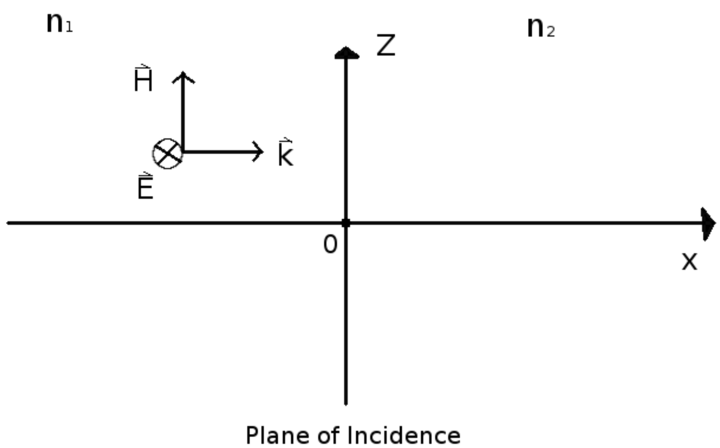



Figure 1.1: TE polarization

### 1.2.2 Frequency-independent Helmholtz Equation

At this point, we define a non-Hermitian dielectric permittivity function as following,

$$\varepsilon(x) = \varepsilon_R(x) + i\varepsilon_I(x) \tag{1.26}$$

where $i$ represents the imaginary unit, $\varepsilon_R(x)$ and $\varepsilon_I(x)$ the real and the imaginary part of the dielectric function respectively.

The wave is being enhanced (gain) if the imaginary part of the dielectric function has a negative value and is being absorbed (loss) when it has a positive value [3].

It is likely to wonder why we are able to use the Helmholtz equation (1.25) since the propagation of light in a medium which contains gain is in general a quantum problem. We will prove that for a gain system we can use Helmholtz Equation under certain conditions, which means also that for a system exhibiting gain and loss we will also be able to use Helmholtz Equation.

The originally quantum problem can be reduced to a semiclassical problem and the electromagnetic field can be described by a system of equations, called the Maxwell-Bloch equations. We can derive the nonlinear Helmholtz equation using Maxwell-Bloch equations [2].

$$\frac{\partial^2 \Psi(x,\omega)}{\partial x^2} + k^2 \tilde{\varepsilon}_{gain}(x,\omega)\Psi(x,\omega) \tag{1.27}$$

The gain medium is described by the effective dielectric constant,

$$\tilde{\varepsilon}_g(x) = \frac{g^2}{\hbar\varepsilon_0(\omega - \Omega_\alpha + i\gamma_\perp)} \frac{D_0(x)}{1 + \frac{4g^2}{\gamma_\parallel\gamma_\perp\hbar^2}\sum_\mu \Gamma(\omega_\mu) \mid \Psi_\mu \mid^2} \tag{1.28}$$

where $g$ is the dipole moment of two-level emitters, $\Gamma(\omega)$ is the Lorentzian gain curve, $\gamma_\parallel, \gamma_\perp$ are the damping constants, $D_0(x)$ is the spatially inhomogeneous pump profile and $\Omega_\alpha$ is difference in frequency of the energy states of the the level atom.
If the field strength is small,

$$\mid \Psi(x) \mid^2 << \frac{\gamma_\parallel\gamma_\perp\hbar^2}{4g^2} \tag{1.29}$$

then the propagation of the wave in the medium can be described using a field-independent dielectric distribution:

$$\varepsilon_g(x,\omega) \approx \frac{\gamma_\perp \bar{D}_0(x)}{\omega - \Omega_\alpha + i\gamma_\perp} \tag{1.30}$$

where $\varepsilon_g(x,\omega) \equiv \frac{\tilde{\varepsilon}_g(x,\omega)}{\varepsilon_0}$ is the dielectric permittivity distribution and $\bar{D}_0(x) \equiv \frac{g^2 D_0(x)}{\hbar}$.

For the purpose of our study, we can assume without any loss of generality, that for a range of the wavelength values up to $\pm(20-30)\%$ of an original value, the dielectric permittivity distribution does not depend considerably on the frequency, $\omega$. This means that we do not consider dispersion effects in this thesis. We can derive this conclusion from expression (1.30), since in our case $\omega \approx \Omega_\alpha$. Thus,

$$\varepsilon_g(x) = -i\bar{D}_0(x) \tag{1.31}$$

As we have stated before this derivation concerns a gain medium without loss. Comparing (1.31) and (1.26) we conclude that,

$$\varepsilon_I(x) = -\bar{D}(x) \tag{1.32}$$

Now, starting from equations (1.25) and (1.27) we can write the frequency-independent Helmholtz Equation,

$$\frac{\partial^2}{\partial x^2}\Psi(x) = -k^2\varepsilon(x)\Psi(x) \tag{1.33}$$

Moreover, we note that the gain values that we consider in this thesis are below the lasing thresholds and thus, we are free to use the Helmholtz equation (1.33) for our calculations.

## 1.3 Time-averaged Poynting Vector

Furthermore, we will study the behaviour of flux of the wave in a dielectric medium. The Poynting vector in one-dimensional space is defined as follows,

$$\boldsymbol{S}(t) = \boldsymbol{E}_R(x,t) \times \boldsymbol{H}_R(x,t) \tag{1.34}$$

We consider a sinusoidal wave with a frequency $\omega$ and period $T = \frac{2\pi}{\omega}$. In addition, as we have mentioned before, for our purposes we assume an one-dimensional scattering medium and a wave propagating along x direction while the polarization of the electric field is in the y direction. Thus, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ are,

$$\boldsymbol{E}(x,t) = \tilde{E}(x)e^{-j\omega t}\hat{y} \tag{1.35}$$

$$\boldsymbol{H}(x,t) = \frac{1}{c\mu_0}\tilde{H}(x)e^{-j\omega t}\hat{z} \tag{1.36}$$

where $\tilde{\boldsymbol{E}}(x)$ and $\tilde{\boldsymbol{H}}(x)$ are arbitrary complex functions. Thus,

$$\boldsymbol{E}_R(x,t) = Re(\tilde{\boldsymbol{E}}(x)e^{-j\omega t}) \tag{1.37}$$

$$\boldsymbol{H}_R(x,t) = Re(\tilde{\boldsymbol{H}}(x)e^{-j\omega t}) \tag{1.38}$$

where $\tilde{\boldsymbol{E}}(x) = \tilde{E}(x)\hat{y}$ and $\tilde{\boldsymbol{H}}(x) = \tilde{H}(x)\hat{z}$. It is also true that,

$$Re(\tilde{\boldsymbol{E}}(x)e^{-i\omega t}) = \frac{\tilde{\boldsymbol{E}}(x)e^{j\omega t} + \tilde{\boldsymbol{E}}^*(x)e^{-j\omega t}}{2} \tag{1.39}$$

$$Re(\tilde{\boldsymbol{H}}(x)e^{-i\omega t}) = \frac{\tilde{\boldsymbol{H}}(x)e^{j\omega t} + \tilde{\boldsymbol{H}}^*(x)e^{-j\omega t}}{2} \tag{1.40}$$

Thus the Poynting vector can be written as,

$$\boldsymbol{S}(t) = \frac{Re(\tilde{\boldsymbol{E}}(x) \times \tilde{\boldsymbol{H}}^*(x))}{2} + \frac{Re(\tilde{\boldsymbol{E}}(x) \times \tilde{\boldsymbol{H}}(x)e^{2j\omega t})}{2} \tag{1.41}$$

The average of the Poynting vector over time is given by,

$$\langle \boldsymbol{S} \rangle = \frac{1}{T}\int_0^T \boldsymbol{S}(t)dt \tag{1.42}$$

Substituting equation (1.41) to (1.42) we get,

$$\langle \boldsymbol{S} \rangle = \frac{1}{T}\int_0^T [\frac{Re(\tilde{\boldsymbol{E}}(x) \times \tilde{\boldsymbol{H}}^*(x))}{2} + \frac{Re(\tilde{\boldsymbol{E}}(x) \times \tilde{\boldsymbol{H}}(x)e^{2j\omega t})}{2}]dt \tag{1.43}$$

We know that $e^{j\omega t} = cos(2\omega t) + jsin(2\omega t)$. The second term of the integral is equal to,

$$\begin{aligned}\int_0^T [\frac{Re(\tilde{\boldsymbol{E}}(x) \times \tilde{\boldsymbol{H}}(x)e^{2j\omega t})}{2}]dt = \int_0^T & [\tilde{\boldsymbol{E}}_{Re}(x) \times \tilde{\boldsymbol{H}}_{Re}(x)cos(2\omega t) \\ & - \tilde{\boldsymbol{E}}_{Re}(x) \times \tilde{\boldsymbol{H}}_{Im}(x)sin(2\omega t) \\ & - \tilde{\boldsymbol{E}}_{Im}(x) \times \tilde{\boldsymbol{H}}_{Re}(x)sin(2\omega t) \\ & - \tilde{\boldsymbol{E}}_{Im}(x) \times \tilde{\boldsymbol{H}}_{Im}(x)cos(2\omega t)]dt\end{aligned} \tag{1.44}$$

Knowing that $T = \frac{2\pi}{\omega}$ we conclude that the integral (1.44) equals to zero. Thus the time-average Poynting vector is,

$$\langle \boldsymbol{S} \rangle = \frac{1}{2}Re(\tilde{\boldsymbol{E}}(x) \times \tilde{\boldsymbol{H}}^*(x)) \tag{1.45}$$

Now we will use Faraday's law of induction, see equation (1.2) in order to derive the relation of the phase of magnetic field with the phase of the electric field. Using the fact that the speed of light is equal to $c = \frac{\omega}{k}$ and the third relation of (1.16) we derive that the second part of (1.2) becomes,

$$-\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} = ik\tilde{H}(x)e^{-j\omega t}\hat{z} \tag{1.46}$$

The only non-zero component of the electric field is, $E_y(x)$. As a consequence the first part of (1.2) becomes,

$$\nabla \times \boldsymbol{E}(x,t) = \frac{\partial E(x,t)}{\partial x}\hat{z} = \Psi_x e^{-j\omega t}\hat{z} \tag{1.47}$$

Setting (1.46) equal to (1.47) we get the expression for the phase of the magnetic field,

$$\tilde{H}(x) = \frac{-j}{k}\Psi_x \tag{1.48}$$

where $\Psi_x \equiv \frac{\partial \Psi}{\partial x} \equiv \frac{\partial \tilde{E}}{\partial x}$.
Consequently, the time-averaged Poynting vector or the flux of the wave, equation (1.45), can be written equivalently as,

$$S(x) = \frac{1}{2k} Re(j\Psi(x)\Psi_x^*(x)) \tag{1.49}$$

where $S(x) \equiv \langle \boldsymbol{S} \rangle$.

# 1.4 Transfer matrix of one layer

## 1.4.1 Boundary conditions for a single interface

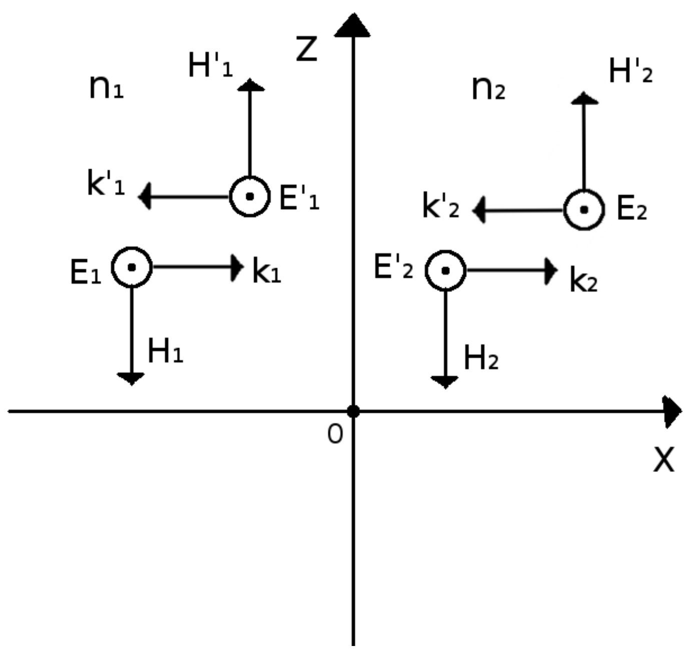



Figure 1.2: TE polarization

To begin with, we consider a plane wave travelling as shown in Figure 1.2, see [4]. Thus, using Maxwell's equations the electric field and the magnetic field can be written as follows,

$$\tilde{\boldsymbol{E}} = Ee^{ikx}\hat{y} \tag{1.50}$$

$$\boldsymbol{H} = \frac{\boldsymbol{k} \times \boldsymbol{E}}{\omega\mu_0} \tag{1.51}$$

$$\tilde{\boldsymbol{H}} = He^{ikx}\hat{z} \tag{1.52}$$

The boundary conditions for the electromagnetic field to the single interface (x=0),

$$\hat{n} \times (\tilde{\boldsymbol{E}}_2^{tot} - \boldsymbol{E}_1^{tot}) = 0 \tag{1.53}$$

$$\hat{n} \times (\tilde{\boldsymbol{H}}_2^{tot} - \boldsymbol{H}_1^{tot}) = 0 \tag{1.54}$$

are leading to the following relations,

$$E_1 + E_1' = E_2 + E_2' \tag{1.55}$$

$$H_1 + H_1' = H_2 + H_2' \tag{1.56}$$

Substituting relation (1.50) to (1.51) we get the following relation between the magnetic and the electric field,

$$H_i = \frac{E_i}{\omega\mu_0}e^{ikx}k_i \tag{1.57}$$

where i can take the values one or two, which represent the left medium and the right medium, respectively. It is also true that the two wavevectors can be written as follows,

$$k_1 = k_0 n_1 = \frac{2\pi n_1}{\lambda_0} \tag{1.58}$$

$$k_2 = k_0 n_2 = \frac{2\pi n_2}{\lambda_0} \tag{1.59}$$

Thus, substituting equation (1.57), (1.58) and (1.59) to (1.56) we get,

$$n_2(E_2 - E_2') = n_1(E_1 - E_1') \tag{1.60}$$

The boundary conditions that resulted in equations (1.60) and (1.55) can be written in matrix form as follows,

$$D_{S1} \cdot \begin{pmatrix} E_1 \\ E_1' \end{pmatrix} = D_{S2} \cdot \begin{pmatrix} E_2 \\ E_2' \end{pmatrix} \tag{1.61}$$

where $D_{Si}$ is the dynamic matrix for TE polarization,

$$D_{Si} \equiv \begin{pmatrix} 1 & 1 \\ n_i & -n_i \end{pmatrix} \tag{1.62}$$

### 1.4.2 Derivation of transfer matrix

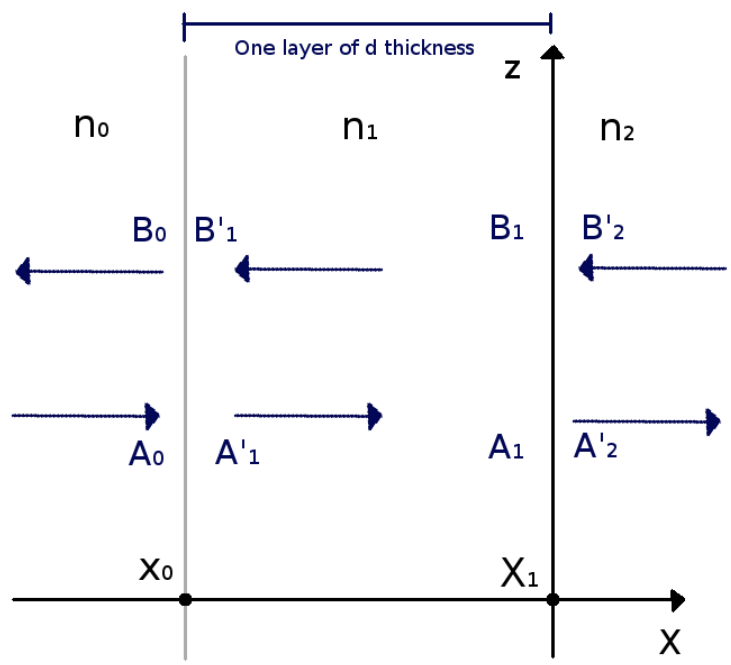



Figure 1.3: One layer scattering

At this point we consider a plane wave travelling in the $\hat{x}$ direction, as shown in Figure 1.3. We will study the transmittance and the reflectance of light through one layer for TE polarization. The layer is homogeneous inside which means that the dielectric distribution and the refractive index are constants. More specifically,

$$n(x) = \begin{cases} n_0, & \text{if } x < x_0 \\ n_1, & \text{if } x_0 < x < x_1 \\ n_2, & \text{if } x > x_1 \end{cases} \tag{1.63}$$

The electric field inside every side of the medium satisfies the Helmholtz equation (1.33), where $\varepsilon(x) = constant$. The solution of (1.33) when the dielectric distribution is constant are plane waves. The electric field can be written as a superposition of transmitted and reflected waves.

$$E(x) = \begin{cases} A_0 e^{ik_0(x-x_0)} + B_0 e^{-k_0(x-x_0)}, & \text{if } x < x_0 \\ A_1 e^{ik_1(x-x_1)} + B_1 e^{-k_1(x-x_1)}, & \text{if } x_0 < x < x_1 \\ A_2' e^{ik_2(x-x_1)} + B_2' e^{-k_2(x-x_1)}, & \text{if } x > x_1 \end{cases} \tag{1.64}$$

where $A_i$ and $B_i$ are the amplitudes of the right and left travelling waves at $x = x_i$ interface, inside the i-medium, respectively. Also $A_i'$ and $B_i'$ are the amplitudes of the right travelling wave and left travelling wave at $x = x_{i-1}$ interface, inside the i-medium, respectively. Now we will derive the relation between the amplitudes $A_1'$, $B_1'$ and the amplitudes $A_1$, $B_1$. Using the second relation in (1.64) and placing $x = x_0$ as well as $d = x_1 - x_0$ we get,

$$E(x = x_0) = A_1 e^{-ik_1 d} + B_1 e^{ik_1 d} \tag{1.65}$$

Given that $A_1'$ and $B_1'$ are the amplitudes of the right and left travelling waves at $x = x_0$ interface we conclude that,

$$A_1' = A_1 e^{-ik_1 d} \tag{1.66}$$

$$B_1' = B_1 e^{-ik_1 d} \tag{1.67}$$

These relations in matrix form can be written as follows,

$$\begin{pmatrix} A_1' \\ B_1' \end{pmatrix} = \begin{pmatrix} e^{-i\phi_1} & 0 \\ 0 & e^{i\phi_1} \end{pmatrix} \cdot \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{1.68}$$

where $\phi_1 \equiv k_1 d$. We can define the propagation matrix as following,

$$P_1 \equiv \begin{pmatrix} e^{-i\phi_1} & 0 \\ 0 & e^{i\phi_1} \end{pmatrix} \tag{1.69}$$

Applying equation (1.61) at the $x = x_0$ interface, where $E_1 = A_0$,$E_2 = A_1'$,$E_1' = B_0$ and $E_2' = B_1'$ we get,

$$D_{S0} \cdot \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_{S1} \cdot \begin{pmatrix} A_1' \\ B_1' \end{pmatrix} = D_{S1} \cdot P_1 \cdot \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{1.70}$$

and

$$D_{S1} \cdot \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = D_{S2} \cdot \begin{pmatrix} A_2' \\ B_2' \end{pmatrix} \tag{1.71}$$

Combining relations (1.70) and (1.71) we get,

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_{S0}^{-1} D_{S1} P_1 D_{S1}^{-1} D_{S2} \cdot \begin{pmatrix} A_2' \\ B_2' \end{pmatrix} \tag{1.72}$$

Supposing that the wave is coming from the left to the right side of the system we notice that the matrix $\begin{pmatrix} A_2' \\ B_2' \end{pmatrix}$ represents the amplitudes of the outgoing wave while the matrix $\begin{pmatrix} A_0 \\ B_0 \end{pmatrix}$ represents the amplitudes of the incoming wave. This means that the matrix

$$T^{-1} = D_{S0}^{-1} D_{S1} P_1 D_{S1}^{-1} D_{S2} \tag{1.73}$$

represents the inverse of the transfer matrix. By using the matrix identity:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{1.74}$$

we can calculate the transfer matrix and find the following expression [4],

$$T_S = \frac{n_0}{2n_2} \begin{pmatrix} cos(\phi_1)(1 + \frac{k_2}{k_0}) + isin(\phi_1)(\frac{k_1}{k_0} + \frac{k_2}{k_1}) & cos(\phi_1)(\frac{k_2}{k_0} - 1) + isin(\phi_1)(\frac{k_1}{k_0} - \frac{k_2}{k_1}) \\ cos(\phi_1)(\frac{k_2}{k_0} - 1) - isin(\phi_1)(\frac{k_1}{k_0} - \frac{k_2}{k_1}) & cos(\phi_1)(1 + \frac{k_2}{k_0}) - isin(\phi_1)(\frac{k_1}{k_0} + \frac{k_2}{k_1}) \end{pmatrix} \tag{1.75}$$

## 1.5 Relation of scattering matrix to transfer matrix

In this section we will derive the relation between the scattering matrix and the transfer matrix. We consider one interface as shown in Figure 1.2. If we define the transfer matrix as,

$$T \equiv \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \tag{1.76}$$

then it is true that we can relate the incoming wave with the outgoing wave as following,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_1 \\ E_1^{'} \end{pmatrix} = \begin{pmatrix} E_2^{'} \\ E_2 \end{pmatrix} \tag{1.77}$$

Thus we get,

$$T_{11}E_1 + T_{12}E_1^{'} = E_2^{'} \tag{1.78}$$

$$T_{21}E_1 + T_{22}E_1^{'} = E_2 \tag{1.79}$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} E_1^{'} \\ E_2^{'} \end{pmatrix} \tag{1.80}$$

This means that,

$$S_{11}E_1 + S_{12}E_2 = E_1^{'} \tag{1.81}$$

$$S_{21}E_1 + S_{22}E_2 = E_2^{'} \tag{1.82}$$

We can solve equation (1.82) for the amplitude $E_2$ and substitute it in equation (1.81). Comparing the new expressions for $E_2$ and $E_2^{'}$ with the equations (1.78) and (1.79) we find the expressions for the elements of the scattering matrix S as following,

$$S_{12} = \frac{1}{T_{22}} \tag{1.83}$$

$$S_{11} = \frac{-T_{21}}{T_{22}} \tag{1.84}$$

$$S_{22} = \frac{T_{12}}{T_{22}} \tag{1.85}$$

$$S_{21} = \frac{T_{22}T_{11} - T_{21}T_{12}}{T_{22}} = \frac{1}{T_{22}} \tag{1.86}$$

by using the fact that $det(T) = 1$. Therefore the S matrix in terms of the elements of the T matrix is,

$$S = \begin{pmatrix} \frac{-T_{21}}{T_{22}} & \frac{1}{T_{22}} \\ \frac{1}{T_{22}} & \frac{T_{12}}{T_{22}} \end{pmatrix} \tag{1.87}$$

# Chapter 2

# Constant Intensity Waves

## 2.1 Dielectric Function

We consider a one-dimensional dielectric medium. Demanding the wave to have constant intensity inside the scattering medium, the ansatz is a wave with constant amplitude and a position dependent phase.

$$\Psi(x) = e^{ik\int_{-L}^{x} W(x')dx'} \tag{2.1}$$

where W(x) is a real arbitrary function.
Inserting expression (2.1) into the Helmholtz equation (1.33), leads to the following expression for the dielectric function,

$$\varepsilon(x) = W^2(x) - \frac{i}{k}\frac{dW}{dx} \tag{2.2}$$

Notice that the relation between the refractive index distribution and any dielectric permittivity distribution is in given by,

$$n(x) = \sqrt{\varepsilon(x)} \tag{2.3}$$

The following perfect transmission boundary conditions,

$$\frac{d\psi(\pm L)}{dx} = ik\Psi(\pm L) \tag{2.4}$$

generate the following boundary conditions for the function W(x),

$$W(L) = W(-L) = 1 \tag{2.5}$$

As a consequence of the above restrictions of the boundary conditions, we have to choose our W(x) such that is even at the end points of the medium. As a result, the following integral is zero for any arbitrary function W(x),

$$\int_{-L}^{L} Im(\varepsilon(x))dx = 0 \tag{2.6}$$

which implies that the spatial average gain-loss over the scattering region is zero.

At this point, one might wonder how the problem would be affected if the dielectric permittivity function had a different expression than (2.2) , resulting in a system which features in average gain greater that loss,

$$\int_{-L}^{L} Im(\varepsilon(x))dx < 0 \tag{2.7}$$

or loss greater that gain,

$$\int_{-L}^{L} Im(\varepsilon(x))dx > 0 \tag{2.8}$$

This was the first question we asked ourselves and inspired the work presented in this Bachelor Thesis.

In addition, there is a restriction imposed by the physical values of the experimentally accessible materials. We have to choose our W(x) such that the real part of the refractive index is $n_R(x) > 1$ and $Re(\varepsilon(x)) > 1$.

## 2.2 Numerical Results

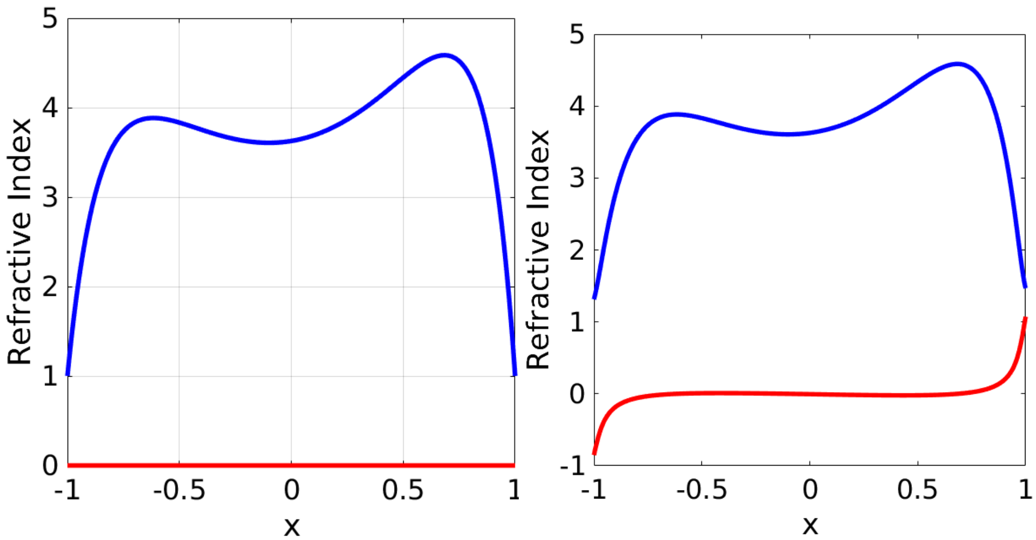
Our first results are based in the function,

$$W(x) = -2.7e^{\frac{(x+0.05)^2}{0.58}}(x^2-1) + 0.1e^{\frac{-(x-0.5)^2}{1.5}}(x^2-1) + 1 \tag{2.9}$$

We assume that the medium extends from -1 to 1 and that the wave number has the normalized value $k = \frac{2\pi}{0.55} = 11.42$

To begin with, we consider a refractive index distribution without gain and loss , $n_I(x) = 0$. Using only the real part $\varepsilon_R(x)$ of equation (2.2) and the relations (2.3) and (2.2), we can easily depict, see Appendix A, the corresponding refractive index distribution, $n(x) = n_R(x)$ and the intensity profile, see Figures 2.1(a) and 2.2(a) blue line, respectively. The transmittance and the reflectance of the light in the dielectric structure as a function of the wavelength are illustrated in Figures 2.3(a) and 2.4(a), respectively.

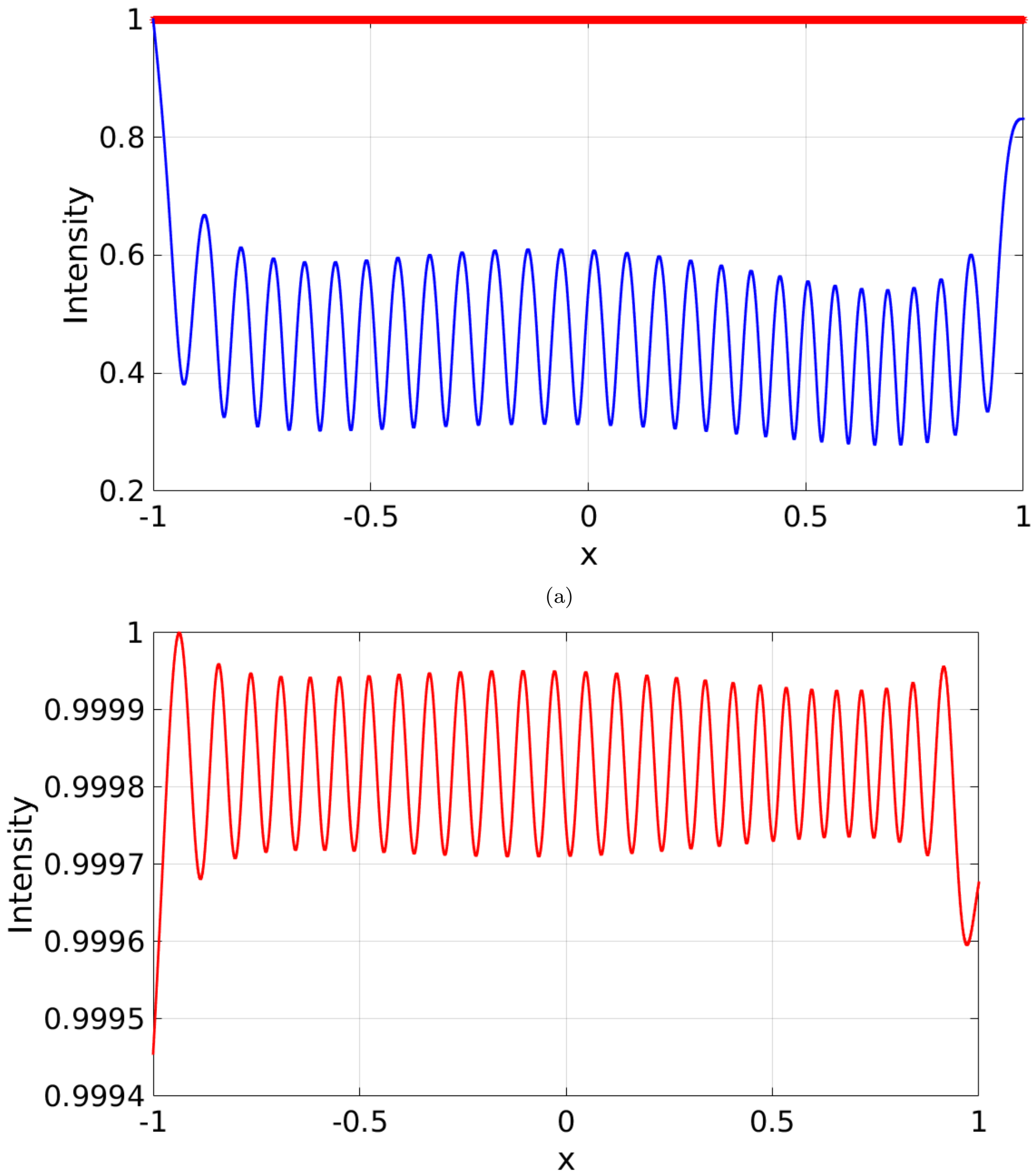
On the contrary, adding gain and loss to our system, $Im(\varepsilon(x)) \neq 0$ results in a constant intensity profile, see Figure 2.2(b). This is also illustrated in Figure 2.2(a) red line, where we observe that the intensity in this case is constant comparing to the intensity of the dielectric distribution of the hermitian medium. The refractive index distribution can be calculated numerically by inserting relations (2.2) , (2.9) to (2.3) and is illustrated in Figure 2.1(b). Moreover, the transmittance and the reflectance of the light as a function of the wavelength are displayed in Figures 2.3(b) and 2.4(b), respectively.




(a) (b)

Figure 2.1: (a) Hermitian refractive index (blue line) without gain and loss (red line). (b) Non-Hermitian refractive index where the blue line represents the real part of the refractive index while the red line the imaginary part.

Notice, that we can state that the wave is perfectly transmitted through




(a)



(b)

Figure 2.2: (a) Intensity of the Hermitian refractive index (blue line) and intensity of the CI wave for the corresponding non-Hermitian refractive index (red line). (b) A better illustration of the intensity of the CI wave.

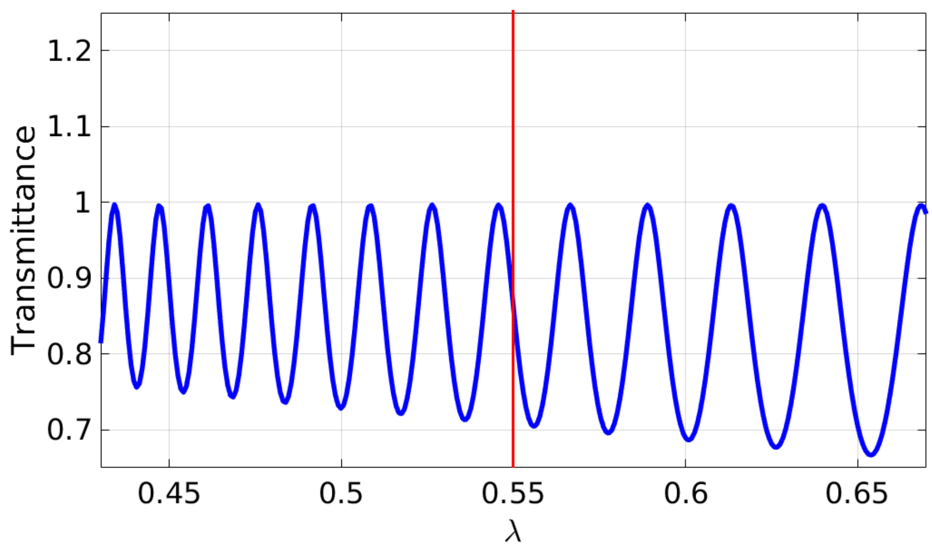
the medium when the transmittance has a value around the value 1, for a wide range of the wavelength values and when at the same time, for the same range of the wavelength values, the reflectance is near zero. Specifically the order of magnitude of the reflectance on the two sides of Figure 2.4(b) is $10^{-2}$, while in the middle it reaches the value $10^{-4}$. Consequently, we can state that the wave is transmitted perfectly with no backscattering through the medium when we add gain and loss for a range of the wavelength values from $0.43\mu m$ to $0.67\mu m$.

Comparing the figures which correspond to a dielectric structure composed of only $n_R(x)$ to the figures which correspond to a dielectric structure composed of both $n_R(x)$ and $n_{Im}(x)$, we conclude the following.

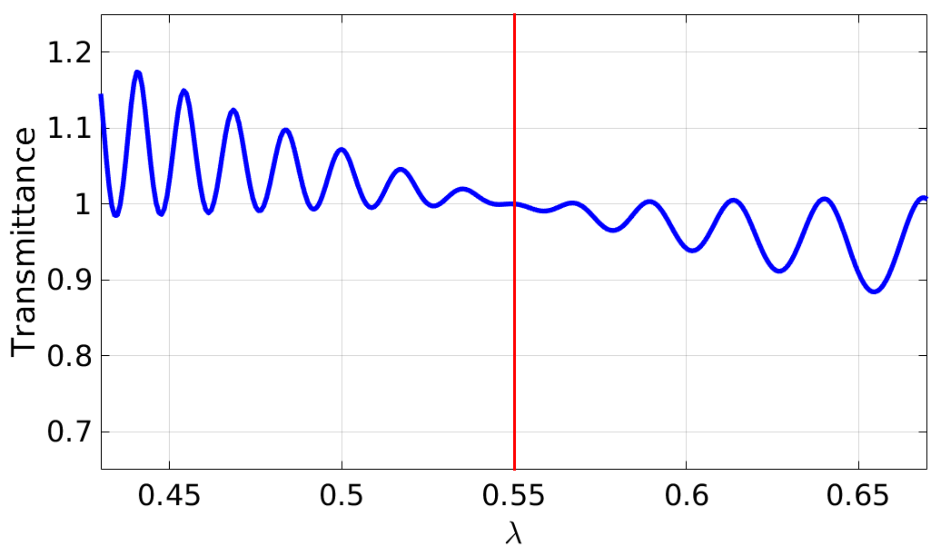
Adding gain and loss to the scattering medium results to a state which features constant intensity, see Figure 2.2(a) red line and is fully transmitted 30% around the value $\lambda = 0.55$, which corresponds to the wavelength featuring constant intensity.

In contrast, when gain and loss is absent, the wave appears to be transmitted perfectly for some certain values of the wavelength and not for the whole range, see Figures 2.3(a) and 2.4(a). More specific, the reflectance takes values from 0.32 to $10^{-3}$, and the transmittance takes values from 0.68 to 1. However adding gain and loss to the structure results in a perfect transmitted wave for every value of the wavelength from $0.43\mu m$ to $0.67\mu m$.

Moreover, we claimed that in average the gain of the system is equal to the loss of the system, see expression (2.6). This conclusion is also valid for this example and is demonstrated in Figure 2.5.

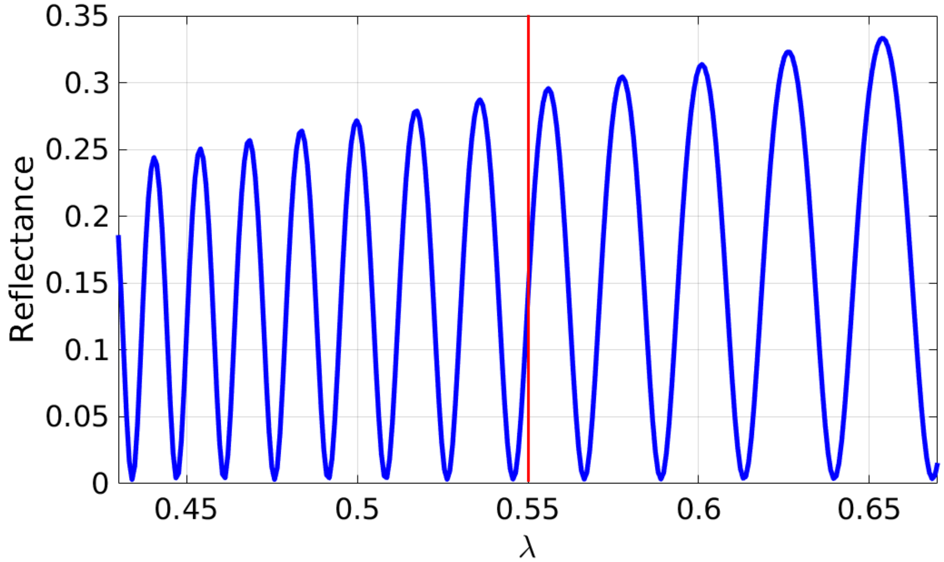



(a)

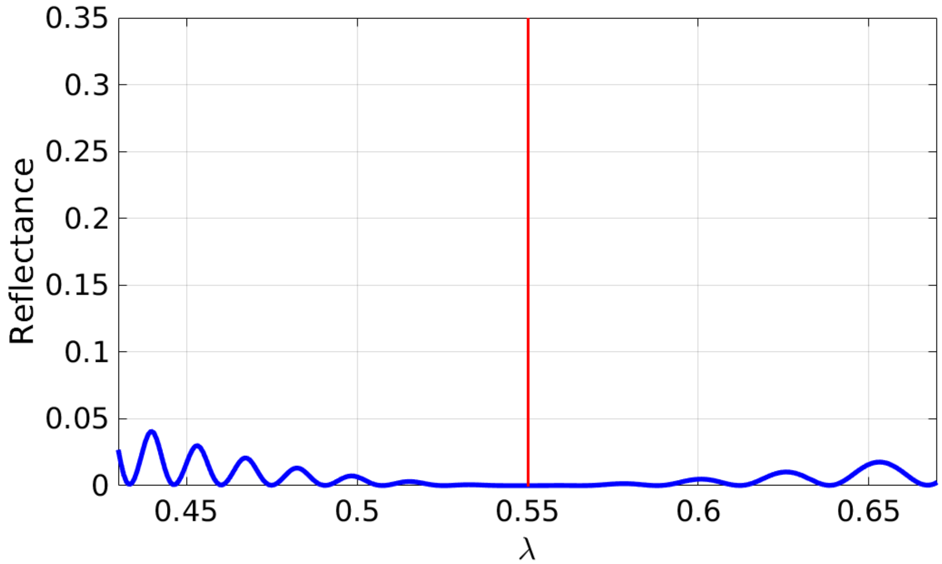



(b)

Figure 2.3: (a)Transmittance of the wave as a function of the wavelength when gain and loss is absent.(b)Transmittance of the wave as a function of the wavelength when we add gain and loss to the medium.




(a)




(b)

Figure 2.4: (a) Reflectance of the wave as a function of the wavelength when gain and loss is absent.(b) Reflectance of the wave as a function of the wavelength when we add gain and loss to the medium.

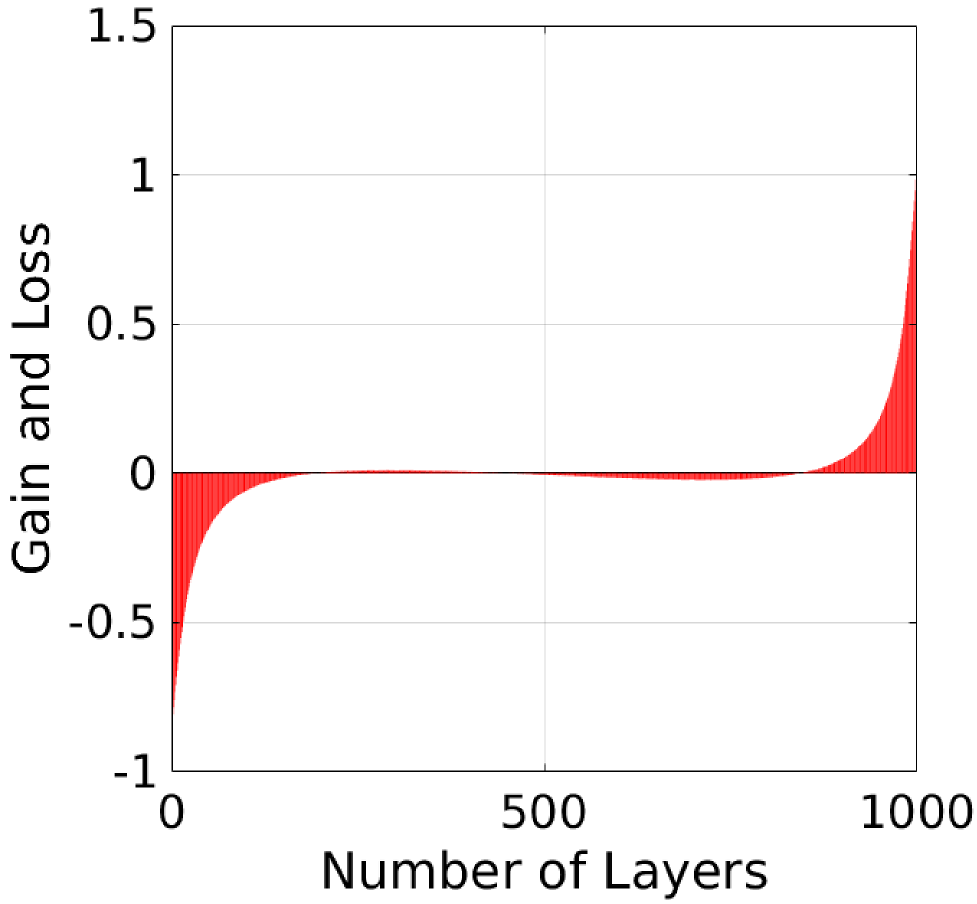



Figure 2.5: Gain and loss in every layer

## 2.3 Flux of the dielectric function

We begin taking the derivative of (2.1),

$$\Psi_x = ikW(x)e^{ik\int_{-1}^{x} W(x')dx'} \tag{2.10}$$

where the function W(x) is given by (2.9).
Now we can calculate the flux of the wave using equation (1.49). Thus substituting (2.10) and (2.1) to (1.49) we get,

$$S(x) = \frac{1}{2}W(x) \tag{2.11}$$

We conclude that the physical meaning of the arbitrary function W(x) that we defined earlier is that it equals to the flux of the wave inside the scattering medium. This conclusion is indicated in Figure 2.6.

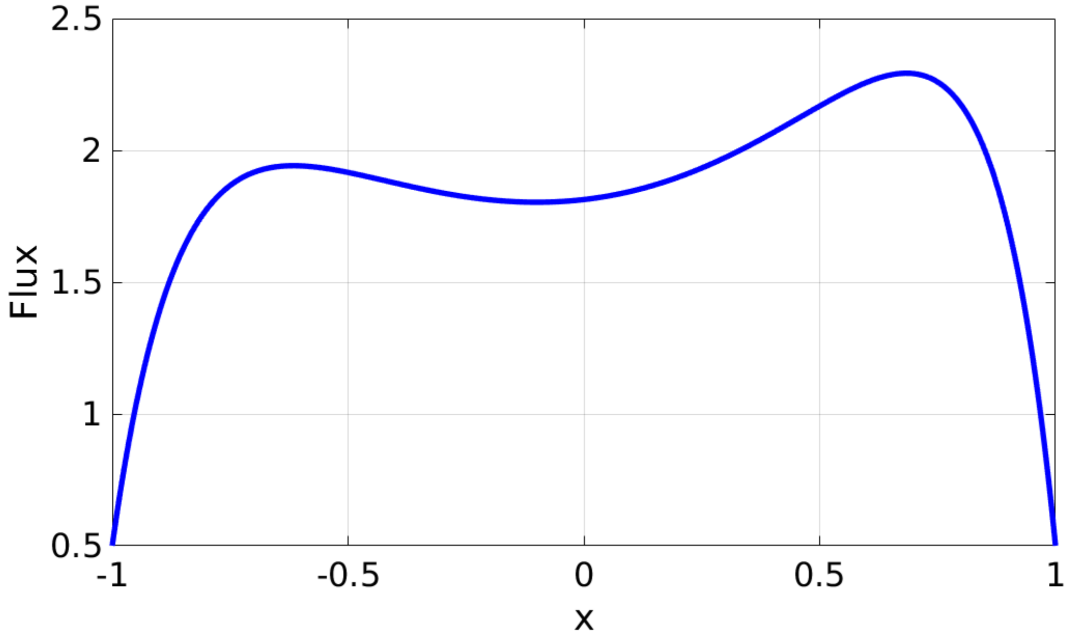



Figure 2.6: Flux

# Chapter 3

# Non Constant Intensity Waves

The primary inquiry of this Thesis is to study whether there can be perfect transmission through a dielectric medium for a wide range of the wavelength values, when the intensity is not a constant. In the following sections we will study systems which do not exhibit constant intensity waves, but their dielectric permittivity distribution is such that the transmission of the wave through the medium is perfect. The imbalance between the gain and loss of the system, which is indicated by the imaginary part of the dielectric permittivity function, can lead to wave propagation without backscattering. This imbalance is expressed through the expressions (2.7) and (2.8).

As a starting point, the general idea was to use different expressions of the wave function comparing to (2.1), in order to construct different dielectric permittivity functions. We are allowed to do that since we can relate equivalently the wave function $\Psi(x)$ with the dielectric function $\varepsilon(x)$ through the Helmholtz Equation (1.33). In the following chapter we construct one such wave function, which leads to a changing intensity over the spatial coordinate x and is realized by multiplying a real arbitrary function f(x) to the wave function $\Psi(x)$, see (2.1). The arbitrary function f(x) is such that the final wave function is not equivalent to the wave function of the constant intensity waves (2.1).

## 3.1 Dielectric function

We start by multiplying the wave function (2.1) with a real arbitrary function f(x), where $f(x) \in [-L, L]$ and we choose $f(x) \neq 0,\ \forall x \in [-L, L]$. Thus, the wave function inside the medium and its first and second derivative are,

$$\Psi(x) = e^{ik\int_{-L}^{x} W(x')dx'} f(x) \tag{3.1}$$

$$\frac{\partial \Psi(x)}{\partial x} = \Psi'(x) = f'(x) e^{ik\int_{-L}^{x} W(x')dx'} + ikW(x)f(x)e^{ik\int_{-L}^{x} W(x')dx'} \tag{3.2}$$

$$\begin{aligned}\frac{\partial^2 \Psi(x)}{\partial x} &= f''(x)e^{ik\int_{-L}^{x} W(x')dx'} + ikW(x)f'(x)e^{ik\int_{-L}^{x} W(x')dx'} \\ &+ W'(x)ikf(x)e^{ik\int_{-L}^{x} W(x')dx'} + f'(x)ikW(x)e^{ik\int_{-L}^{x} W(x')dx'} \\ &- k^2W^2(x)f(x)e^{ik\int_{-L}^{x} W(x')dx'}\end{aligned} \tag{3.3}$$

Substituting equations (3.1) and (3.3) to the Helmholtz equation (1.33) we get the first expression for the dielectric function:

$$\varepsilon(x) = W^2(x) = \frac{1}{k^2}\frac{f''(x)}{f(x)} - \frac{i}{k}(W'(x) + 2\frac{f'(x)}{f(x)}W(x)) \tag{3.4}$$

Since $f(x) \neq 0,\ \forall x \in [-L, L]$ and if $W(x) \neq 0\ \forall x \epsilon [-L, -L]$ we define

$$g(x) \equiv \frac{f'(x)}{f(x)} W(x) \tag{3.5}$$

Thus,

$$\frac{f''(x)}{f(x)} = (\frac{g(x)}{W(x)})^2 + \frac{g'(x)W(x) - W'(x)g(x)}{W^2(x)} \tag{3.6}$$

Substituting equations (3.5) and (3.6) to (3.4) we take equivalently the second expression for the dielectric function.

$$\begin{aligned}\varepsilon_2 =& W^2(x) - \frac{1}{k^2}(\frac{g(x)}{W(x)})^2 - \frac{g'(x)W(x) - W'(x)g(x)}{W^2(x)} \\ &- \frac{i}{k}(W'(x) + 2g(x))\end{aligned} \tag{3.7}$$

The perfect transmission boundary conditions (2.4), generate the following ansatz,

$$g(\pm L) = ikW^2(\pm L) = ikW(\pm L) \tag{3.8}$$

The functions W(x) and g(x) are real, so we have to demand the following boundary conditions for the function W(x),

$$W(\pm L) = W^2(\pm L) = 1 \tag{3.9}$$

or

$$W(\pm L) = W^2(\pm L) = 0 \tag{3.10}$$

However using relation (3.5) and the fact that we have chosen $f(x) \neq 0$, $\forall x \in [-L, L]$, we conclude that $W(\pm L) = 1$. Consequently the boundary conditions for the function g(x) are ,

$$g(\pm L) = 0 \tag{3.11}$$

## 3.2 Numerical results

As a first example, we define W(x) and g(x) as following,

$$W(x) = -1.05x^2 + 2.05cosh(x^2 - 1) + 5e^{x^2-3.45}cos(\frac{\pi x}{2}) \tag{3.12}$$

$$g(x) = 13sin(5\pi x) + 0.0001cos(\frac{\pi x}{2}) \tag{3.13}$$

We also asume that the medium extends from -1 to 1 and that the wave number has the value $k = \frac{2\pi}{0.4} = 15.71$. Notice that $W(x) \neq 0$, $\forall x \in [-1, 1]$, $W(\pm 1) = 1$ and $g(\pm 1) = 0$.
The corresponding refractive index distribution is shown in Figure 3.1(b). Computing the average gain and loss over the medium leads us to:

$$\int_{-1}^{1} Im[\varepsilon(x)] = -3 \cdot 10^{-4} \tag{3.14}$$

The value of the integral is negative, which means that on average gain is higher than loss.Figure 3.1(b) shows the values of gain and loss in every layer.

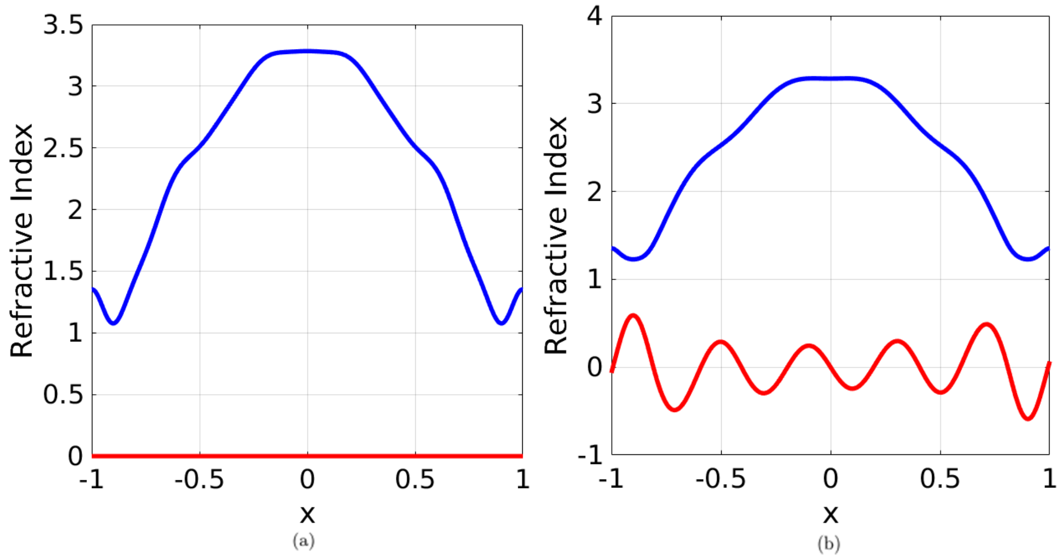



Figure 3.1: (a) Hermitian refractive index (blue line) without gain and loss (red line).(b) Non-Hermitian refractive index, where blue line represents the real part of the refractive index and red line represents the imaginary part of the refractive index.

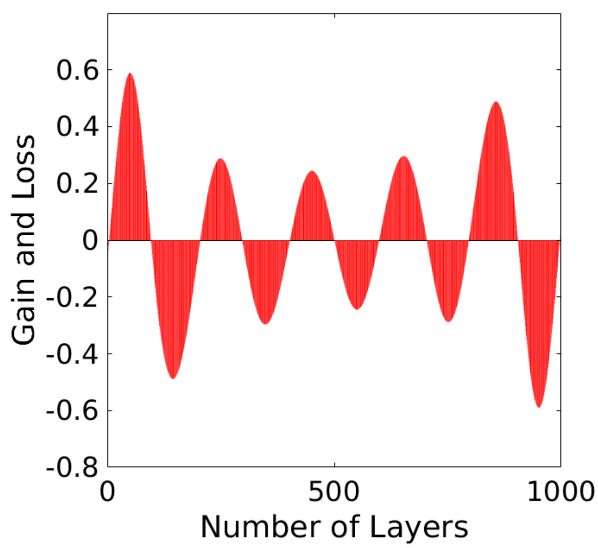



Figure 3.2: Gain and loss in every layer

The transmittance and reflectance of the light for this particular refractive index distribution is shown in pictures 3.4(b) and 3.5(b), respectively. We conclude that the wave is transmitted perfectly with no backscattering through the medium when we add gain and loss for a range of the normalized wavelength from 0.32 to 0.48, since all the values of the transmittance are near the value 1 and all the values of the reflection coefficient are near the value zero. The order of magnitude of the reflectance on the two sides of Figure 3.5(b) is $10^{-3}$, while in the middle it reaches the value $10^{-6}$.

On the other hand, when gain and loss is absent, the wave appears to be transmitted perfectly for some certain values of the wavelength and not for the whole range, see Figures 3.4(a) and 3.5(a). Specifically, the reflectance takes values from 0.28 to $10^{-3}$, and the transmittance takes values from 0.77 to 1. However adding gain and loss to the structure results in a perfect transmitted wave for every value of the wavelength from 0.32 to 0.48.

Moreover as we have expected the intensity is no longer constant, see Figure 3.3. We used both a numerical method (blue dots, see Appendix A) and an analytical expression (red dots) to calculate the intensity of the wave inside the scattering region. As we can observe from Figure 3.3 the two methods are in perfect agreement.

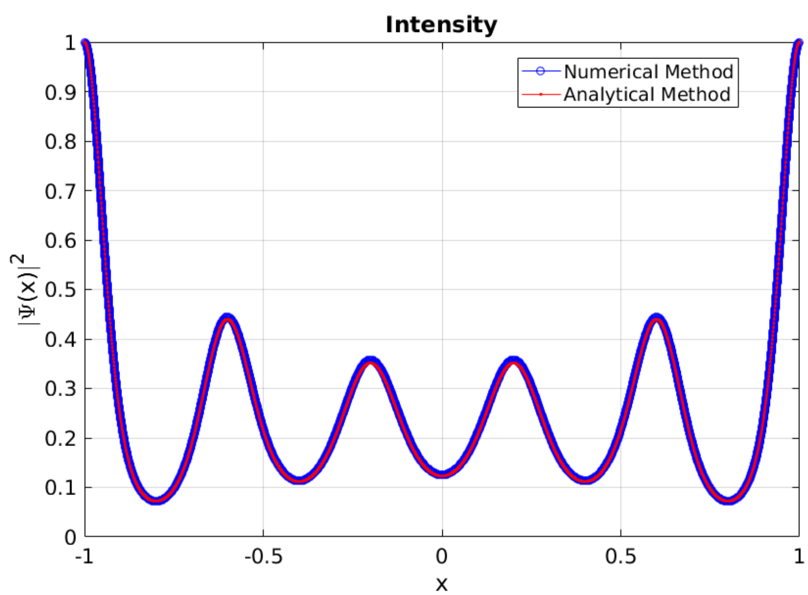



Figure 3.3

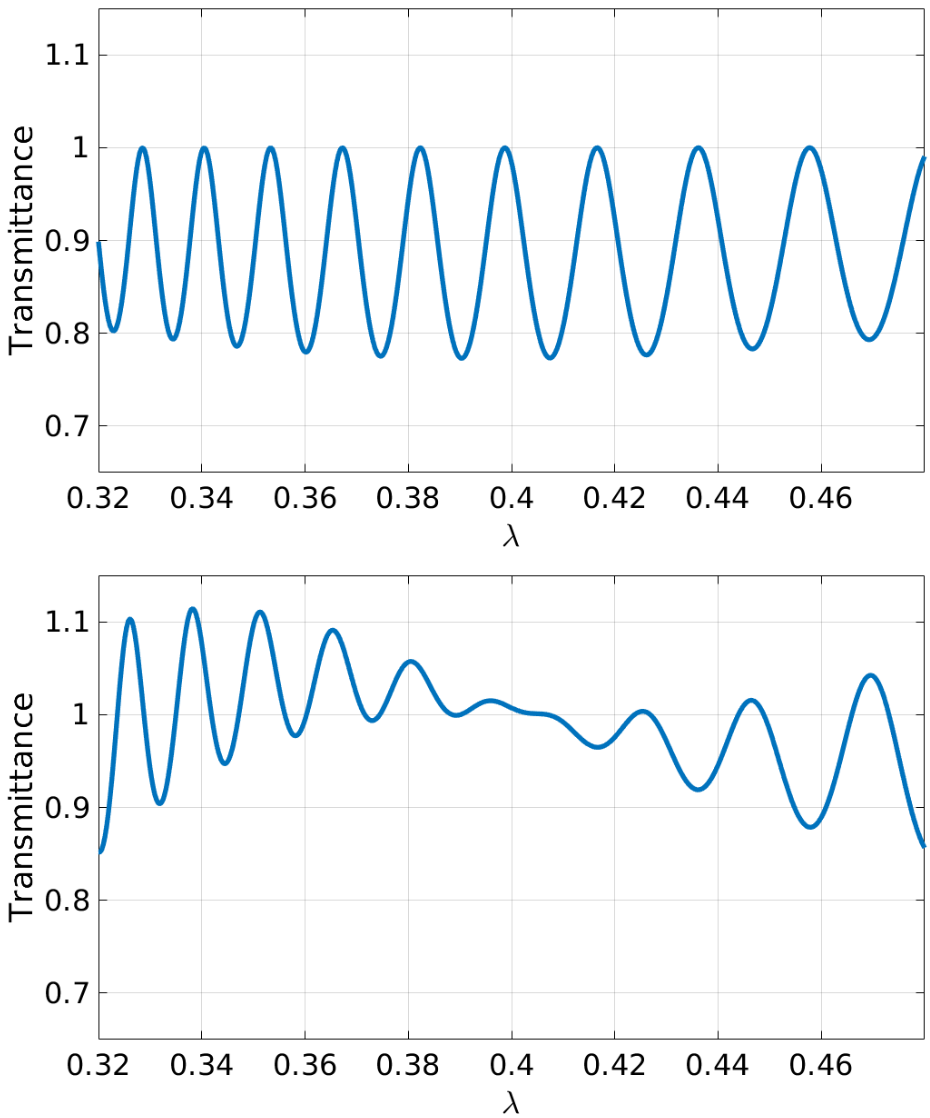



Figure 3.4: (a) Transmittance of the wave as a function of the wavelength when gain and loss is absent. (b) Transmittance of the wave as a function of the wavelength when we add gain and loss to the medium.

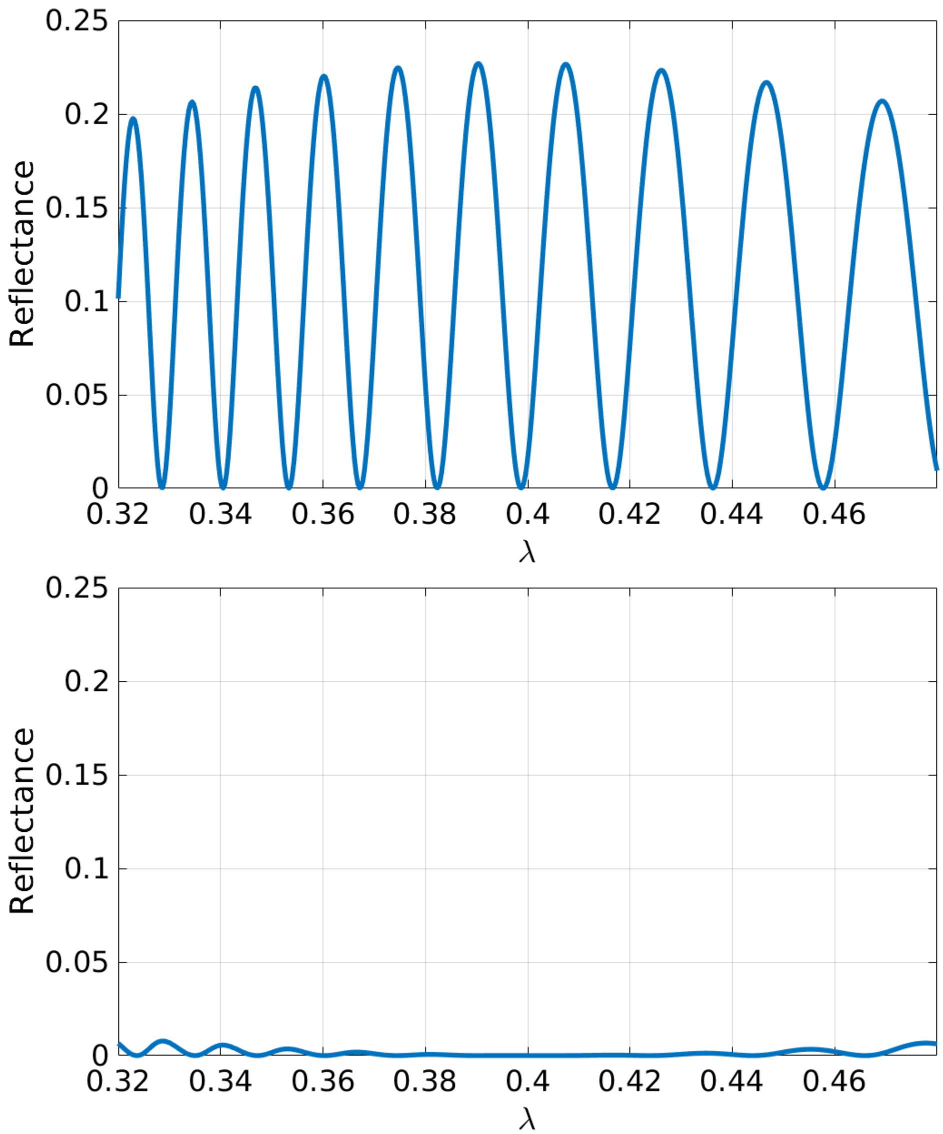



Figure 3.5: (a) Reflectance of the wave as a function of the wavelength when gain and loss is absent. (b) Reflectance of the wave as a function of the wavelength when we add gain and loss to the medium.

## 3.3 Flux of the dielectric function

We begin by taking the derivative of (3.1),

$$\Psi_x = \frac{\partial f}{\partial x} e^{ik\int_{-1}^{x} W(x')dx'} + ikW(x)e^{ik\int_{-1}^{x} W(x')dx'} \tag{3.15}$$

where the function W(x) is given by (3.12).
Now we can calculate the flux of the wave using equation (1.49). Thus substituting (3.15) and (3.1) to (1.49) we get,

$$S(x) = \frac{1}{2}Re(f(x)W(x) + \frac{i}{k}f(x)\frac{\partial f}{\partial x}) \tag{3.16}$$

Thus,

$$S(x) = \frac{1}{2}f(x)W(x) \tag{3.17}$$

We conclude that the product of the arbitrary real function W(x) with the non-zero arbitrary real function f(x) is the flux of the wave inside the medium. This conclusion is presented in Figure 3.6.

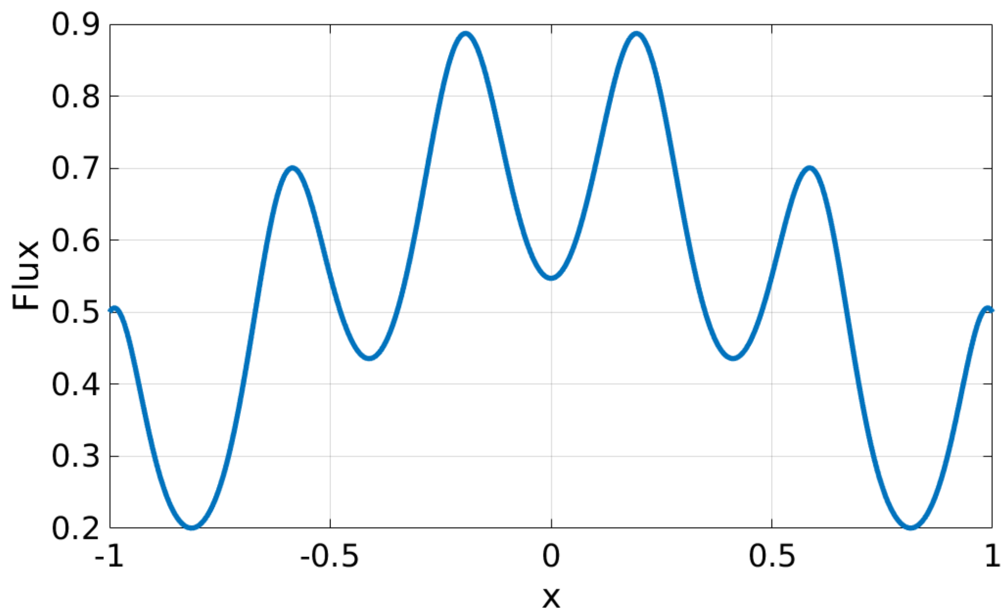



Figure 3.6: Flux

# Chapter 4

# Summary and Conclusions

In conclusion, we believe that we have examined a non-Hermitian dielectric permittivity distribution corresponding to a system on which on average gain is not equal with loss and which can exhibit perfect transmission through the medium, for a wide range of the wavelength values. It is important to notice that the perfect transmission is a sharp resonant effect when gain and loss is absent from the medium.

It is also interesting to investigate if we can achieve perfect transmission in two-dimensional or three-dimensional systems which have the characteristic that their dielectric permittivity distribution has an imaginary part with an imbalanced relevant to gain and loss. Moreover, our study might have technological applications where we want to achieve perfect transmission in complex media with no backscattering.

# Appendix A

# Computational Methods

We begin by dividing the x-region into N layers. If the number of the layers N is sufficiently large we can say that the discrete x-region is a good approximation of the continuous x-region. In our study we chose N=1000. We assume that the dielectric distribution is constant in every layer. Thus, for the n-th layer, the Helmholtz Equation, (1.33) can be written as follows,

$$\frac{\partial^2}{\partial x^2}\Psi_n(x) = -k^2\varepsilon_n\Psi_n(x) \tag{A.1}$$

As a result, the refractive index is also constant in each layer. Consider one layer of the x-region, see Figure 1.2. The value of the refractive index in this layer is the average of the values it has in $x_0$ and $x_1$. This way, using equation (2.3) we can calculate numerically the refractive index of every layer and plot it as a function of x.

After we have calculated numerically the refractive index for every layer, we use the same numerical method to plot the transmittance as well as the reflectance of the wave as a function of the wavelength. Given an incoming wave of wavelength $\lambda$, we start from the first layer and calculate consecutively the transmission coefficient $T$, expression (1.75), in every layer. The transmittance of the wave is defined as, $\mid T \mid^2$. As we change the wavelength of the incoming wave we can plot the transmittance of light as a function of the wavelength. The same method was used to plot the reflectance of light, $\mid R \mid^2$, as a function of the wavelength. Notice that while we were changing the wavelength of the incident wave the wavenumber in equation (1.33) had a constant value, since this characterizes the refractive index or in other words it describes the medium and the medium remains the same and does not change with wavelength.

Finally, we calculated numerically the wave function in every layer, using the same method. As we have stated before the electric field inside every layer satisfies Helmholtz equation (A.1). Therefore the electric field inside every layer is a superposition of reflected and transmitted plane waves. After we calculate numerically the refractive index we can also plot the intensity, $\mid \Psi(x) \mid^2$, as a function of the spatial coordinate x.

# Bibliography


[1] M. Premaratne, and G. P. Agrawal. *Light Propagation in Gain Media:Optical Amplifiers.* Cambridge University Press, 2011.

[2] A.V. Dorofeenko, A.A. Zyablovsky, A.A. Pukhov, A.A. Lisyansky, and A.P. Vinogradov.*Light propagation in composite materials with gain layers.* Physics-Uspekhi, **55**(11)1080-1097, 2012.

[3] K. G. Makris, A. Brandstötter, P. Ambichl, Z. H. Musslimani, and S. Rotter. *Wave propagation through disordered media without backscattering and intensity variations.* Light: Science & Applications(2017) **6**, e17035.

[4] P. Yeh, A. Yariv, and C.-S. Hong. *Electromagnetic propagation in periodic stratified media. i. general theory.* Journal of Optical Society of America, vol. 67, pp. 423–438, 1977.

[5] S. Yu, X. Piao, and N. Park. *Bohmian Photonics for Independent Control of the Phase and Amplitude of Waves.* Phys. Rev. Lett., vol. 120, 193902, 2018.

[6] A. Lagendijk, B. Tiggelen, and D. S. Wiersma. *Fifty years of Anderson localization.* Physics Today, vol. 62, 9, 14, 2009.

[7] L.D. Landau, and E.M. Lifshitz. *Electrodynamics of Continuous Media.* Pergamon Press, 1963.