

Ромько Валерий, #1 [Найти $15^{34} \bmod 19$]

$$\begin{aligned} 15^{34} &\equiv [15 \equiv -4 \pmod{19}] \equiv (-4)^{34} \equiv 16^{17} \equiv [16 \equiv -3 \pmod{19}] \equiv (-3)^{17} \equiv (-3)^{16} (-3)^1 \equiv (-3)^8 9^8 \equiv \\ &\equiv (-3) \cdot 81^4 \equiv [81 \equiv 5 \pmod{19}] \equiv (-3) \cdot 5^4 \equiv (-3) \cdot 25^2 \equiv [25 \equiv 6 \pmod{19}] \equiv (-3) \cdot 6^2 \equiv \\ &\equiv (-3) \cdot 36 \equiv [36 \equiv 17 \pmod{19}] \equiv (-3) \cdot 17 \equiv [17 \equiv -2 \pmod{19}] \equiv (-3)(-2) \equiv 6 \pmod{19} \end{aligned}$$

Ответ: $15^{34} \equiv 6 \pmod{19}$

#2 [найти $\varphi(4432)$]

4432	2
2216	2
1108	2
554	2
277	277

$$\begin{aligned}\varphi(2^4 \cdot 277) &= \varphi(2^4) \varphi(277) = (2^4 - 2^3)(277 - 1) = \\ &= (16 - 8) \cdot 276 = 8 \cdot 276 = 2208\end{aligned}$$

Ответ: $\varphi(4432) = 2208$

$$\#3 \text{ [Найти } 10^{-1} \pmod{57}]$$

$$\begin{array}{r|l} 57 & 3 \\ 19 & 19 \\ 1 & \end{array}$$

$$\varphi(57) = \varphi(3 \cdot 19) = \varphi(3) \varphi(19) = (3-1)(19-1) = 2 \cdot 18 = 36$$

$$\begin{aligned} 10^{-1} \pmod{57} &\equiv 10^{\varphi(57)-1} \pmod{57} \equiv 10^{35} \equiv 10 \cdot 10^{34} \equiv 10 \cdot 100^{17} \equiv [100 \equiv 43 \pmod{57}] \equiv \\ &\equiv 10 \cdot 43^{17} \equiv [43 \equiv -14 \pmod{57}] \equiv (-140)(-14)^{16} \equiv [-140 \equiv 31 \pmod{57}] \equiv 31 \cdot (-14)^{16} \equiv \\ &\equiv 31 \cdot 196^8 \equiv [196 \equiv 25 \pmod{57}] \equiv 31 \cdot 25^8 \equiv 31 \cdot 625^4 \equiv [625 \equiv 55 \pmod{57}] \equiv \\ &\equiv 31 \cdot 55^4 \equiv [55 \equiv -2 \pmod{57}] \equiv (31) \cdot (-2)^4 \equiv [31 \equiv -26 \pmod{57}] \equiv \\ &\equiv (-26) \cdot (-2)^4 = (-26) \cdot 16 = -416 \equiv [-416 \equiv 40 \pmod{57}] \equiv 40 \pmod{57} \end{aligned}$$

$$\text{Ответ: } 10^{-1} \pmod{57} = 40$$

#4 [Найти $\text{gcd}(a, b)$ и x, y , то $44x + 28y = \text{gcd}(44, 28)$]

• Будем решать рекурсивно.

① $a = 44, b = 28$

② $a = 28 \bmod 44 = 28, b = 44$

③ $a = 44 \bmod 28 = 16, b = 28$

④ $a = 28 \bmod 16 = 12, b = 16$

⑤ $a = 16 \bmod 12 = 4, b = 12$

⑥ $a = 12 \bmod 4 = 0, b = 4$

$x = 0, y = 1;$

$\text{gcd} = 4$

$x = 1 - \left\lfloor \frac{12}{4} \right\rfloor \cdot 0 = 1$

$y = 1$

$x = 0 - \left\lfloor \frac{16}{12} \right\rfloor \cdot 1 = -1$

$y = 1$

$x = 1 - \left\lfloor \frac{28}{16} \right\rfloor \cdot (-1) = 1 - (-1) = 2$

$y = -1$

$x = -1 - \left\lfloor \frac{44}{28} \right\rfloor \cdot 2 = -1 - 2 = -3$

$y = 2$

$x = 2 - \left\lfloor \frac{28}{44} \right\rfloor \cdot (-3) = 2 - 0 = 2$

$y = -3$

Ответ: $\begin{cases} x = 2 \\ y = -3 \\ \text{НОД}(44, 28) = 4 \end{cases}$

②

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