```
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```

1.

a. To get log(1.9), -.9 must be used for x. b.

```
x = -.9;
k = 2;

%account for first iteration of sum
sum = -x;

tolerance = .00000000005;

while (abs(vpa(sum, 10) - vpa(log(1-x), 10)) > tolerance)
    sum = sum - (x.^k)/k;
    k = k+1;
end

disp(vpa(sum, 10));
disp(vpa(log(1-x), 10));
display(k);

% c. x = .3103448276
% d.
x = .3103448276;
k = 2;

%account for first iteration of sum
sum = 2*x;

tolerance = .00000000001;

while (abs(vpa(sum, 10) - vpa(log((1+x)/(1-x)), 10)) > tolerance)
    sum = sum + 2*((x.^(2*k-1))/(2*k-1));
    k = k+1;
end

disp(vpa(sum, 10));
disp(vpa(log((1+x)/(1-x)), 10));
display(k);

% e. The second method is much more efficient because the program
% iterated only 11 times as opposed to 185 times for the first method.
```

```
0.6418538861

0.6418538862


k =
```

```
   171

0.6418538862

0.6418538862


k =

    11
```

2.

a. Compute f(x) = ((sqrt(4+x)-2)/x) for decreasing values of x from 10^(-1) and 10^(-20)

```
disp(sprintf('\n\nTable:\n'));
disp(sprintf('x \t f(x)'));
for i=1:20,
  x=10^(-i);
  disp(sprintf('%g \t %0.10f',x,((sqrt(4+x)-2)/x)));
end

% b.
% Compute a reformulation of f(x) = ((sqrt(4+x)-2)/x) in the form
% Rf(x)= 1/((sqrt(4+x)+2)), for decreasing values of x from 10^(-1) and
% 10^(-20)

disp(sprintf('\n\nTable:\n'));
disp(sprintf('x \t f(x) \t \t Rf(x)'));
for i=1:20,
  x=10^(-i);
  disp(sprintf('%g \t %0.10f \t %0.10f',x,((sqrt(4+x)-2)/x),(1/((sqrt(4+x)+2)))));
end

% In the f(x) column, the values are between 0.22 and 0.25 until x is
% 1e-15, at which point the output becomes zero. This is because in the
% denominator, computing the sqrt(4 + x) as x approaches zero and then subtracting
% result in a number that is below machine epsilon. At this point, the
% number will be represented as zero.
%
% In the Rf(x) column, Rf(x) approaches 0.25 and then stays there after x
% becomes 1e-9. This is because the same thing that happened in the f(x)
% column is happening to the reformulated function, but this time sqrt(4+x)
% is in the denominator. As x approaches 0, sqrt(4 + x) will become 2 when
% the floating point value cannot be represented in the computer. At this
% point, the reformulated function will always output 1/4.


% b.
% Compute f(x) = (1-exp(1).^-x)/x)) for decreasing values of x from 10^(-1) and
% 10^(-20)
```

```
disp(sprintf('\n\nTable:\n'));
disp(sprintf('x \t f(x)'));
for i=1:20,
  x=10^(-i);
  disp(sprintf('%g \t %0.10f',x,((1-exp(1).^-x)/x)));
end

% b.
% Compute a reformulation of f(x) = (1-exp(1).^-x)/x)) in the form
% Rf(x)= (1-taylor(e^-x))/x, for decreasing values of x from 10^(-1) and
% 10^(-20)

disp(sprintf('\n\nTable:\n'));
disp(sprintf('x \t f(x) \t \t Rf(x)'));
for i=1:20,
  x=10^(-i);
  taylorX = (1 - (log(1125899906842624/3060513257434037)^5*x^5)/120 + (log(1125899
  disp(sprintf('%g \t %0.10f \t %0.10f',x,((1-exp(1).^-x)/x),taylorX));
end

% In the f(x) column, the values of f(x) are near 1 until x = 1e-17, then
% they drop to zero. While the limit of the function as x approaches zero
% is 1, the numerator drops to zero because of the way the
% computer stores exponents. In IEEE representation, there are fewer bits to store
% As x becomes increasingly small, the bits to store the exponent of e in
% the numerator round down, causing e^-x to drop to one and the numerator, 1-e^-x,
%
% In the Rf(x) column, the values of Rf(x) are increasing. This is because
% the limit of the denominator of the Taylor approximation approaches 1 as
% x approaches zero, while the numerator approaches 0.
```

```
Table:

x    f(x)
0.1    0.2484567313
0.01    0.2498439450
0.001    0.2499843770
0.0001    0.2499984375
1e-05    0.2499998438
1e-06    0.2499999843
1e-07    0.2499999985
1e-08    0.2499999763
1e-09    0.2500000207
1e-10    0.2500000207
1e-11    0.2499778162
1e-12    0.2500222251
1e-13    0.2486899575
1e-14    0.2220446049
1e-15    0.0000000000
1e-16    0.0000000000
1e-17    0.0000000000
1e-18    0.0000000000
```

1e-19	0.0000000000
1e-20	0.0000000000

Table:

x	f(x)	Rf(x)
0.1	0.2484567313	0.2484567313
0.01	0.2498439450	0.2498439450
0.001	0.2499843770	0.2499843770
0.0001	0.2499984375	0.2499984375
1e-05	0.2499998438	0.2499998438
1e-06	0.2499999843	0.2499999844
1e-07	0.2499999985	0.2499999984
1e-08	0.2499999763	0.2499999998
1e-09	0.2500000207	0.2500000000
1e-10	0.2500000207	0.2500000000
1e-11	0.2499778162	0.2500000000
1e-12	0.2500222251	0.2500000000
1e-13	0.2486899575	0.2500000000
1e-14	0.2220446049	0.2500000000
1e-15	0.0000000000	0.2500000000
1e-16	0.0000000000	0.2500000000
1e-17	0.0000000000	0.2500000000
1e-18	0.0000000000	0.2500000000
1e-19	0.0000000000	0.2500000000
1e-20	0.0000000000	0.2500000000

Table:

x	f(x)
0.1	0.9516258196
0.01	0.9950166251
0.001	0.9995001666
0.0001	0.9999500017
1e-05	0.9999950000
1e-06	0.9999995000
1e-07	0.9999999495
1e-08	0.9999999939
1e-09	0.9999999717
1e-10	1.0000000827
1e-11	1.0000000827
1e-12	0.9999778783
1e-13	1.0003109452
1e-14	0.9992007222
1e-15	0.9992007222
1e-16	1.1102230246
1e-17	0.0000000000
1e-18	0.0000000000
1e-19	0.0000000000
1e-20	0.0000000000

Table:

x	f(x)	Rf(x)
0.1	0.9516258196	19.0483758333
0.01	0.9950166251	199.0049833751
0.001	0.9995001666	1999.0004998334
0.0001	0.9999500017	19999.0000499983
1e-05	0.9999950000	199999.0000050000
1e-06	0.9999995000	1999999.0000004999
1e-07	0.9999999495	19999999.0000000522
1e-08	0.9999999939	199999999.0000000000
1e-09	0.9999999717	1999999998.9999997616
1e-10	1.0000000827	19999999999.0000000000
1e-11	1.0000000827	199999999998.9999694824
1e-12	0.9999778783	1999999999999.0000000000
1e-13	1.0003109452	19999999999999.0000000000
1e-14	0.9992007222	199999999999999.0000000000
1e-15	0.9992007222	1999999999999999.0000000000
1e-16	1.1102230246	20000000000000000.0000000000
1e-17	0.0000000000	200000000000000032.0000000000
1e-18	0.0000000000	1999999999999999744.0000000000
1e-19	0.0000000000	20000000000000000000.0000000000
1e-20	0.0000000000	199999999999999967232.0000000000

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