STAT22000 Summer 2020 Homework 7

All page, section, and exercise numbers below refer to the course text (*OpenIntro Statistics*, 3rd edition, by Diez, Barr, and Cetinkaya-Rundel.).

Reading: Section 3.1 and 3.4

Problems for Self-Study : (Do Not Turn In)

- Exercise 3.3, 3.5, 3.11 on p.158-160 and Exercise 3.25, 3,27, 3.31 on p.163-164 Answers can be found at the end of the book (p.410-411).

Problems to Turn In: due **midnight of Monday, July 13**, on Canvas.

1. Exercise 3.12 on p. 160
2. A student takes a multiple-choice quiz with 5 questions, each with four possible selections for the answer. A passing grade is 60% or better (i.e., answering at least 3 of 5 questions correctly). Suppose that the student was unable to find time to study for the exam and just guesses at each question. Find the probability that the student ....
   (a) gets exactly 3 questions correct.
   (b) passes the exam.
   (c) How many questions would you expect the student to get correct?
   (d) Obtain the standard deviation of the number of questions that the student gets correct.
3. A fair 6-face die is going to be rolled some number of times.
   (a) Is it more likely that the ace (one spot) comes up 20% or more of the time in 60 rolls or 600 rolls? Explain. (Note that the probability that a fair 6-face die shows an ace in one roll is $1/6 = 16\frac{2}{3}\%$.)
   (b) Is it more likely to get 8 to 12 aces in 60 rolls or 98 to 102 aces in 600 rolls? Explain.

   Hint: Think about Law of Large Number (LLN). Getting an ace when rolling a fair 6-face die is like getting heads when tossing an unfair coin with only $1/6$ *probability to land heads.*
4. Here we will use what we learned about the Binomial distribution to check our answer for Problem 3(b).
   (a) Find out the expected value and the standard deviation of the number of aces obtained in 60 rolls of a fair 6-face die. Ditto for 600 rolls. Find out the z-scores for the count 8 and 12 aces in 60 rolls and 98 and 102 aces in 600 rolls. Use the z-scores to explain which one is more likely.
   (b) Use the normal approximation to find the probability of getting 98 to 102 aces in 600 rolls of a fair 6-face die. Be sure to check the condition required for using this approximation. Do not use continuity correction.
   (c) Repeat the previous part but using the normal approximation with continuity correction.
   (d) Find the exact probability in (b) in R using the command below.

   ```
   pbinom(102, size=600, p=1/6)-pbinom(97, size=600, p=1/6)
   ```

   Compare the exact probability with the approximate probabilities in the previous two parts.

   Remark. The probability of getting 8 to 12 aces in 60 rolls can be calculated using the normal approximation with continuity correction or using the R command

   ```
   > sum(dbinom(8:12, size=60, p=1/6))
   [1] 0.6138631
   ```

   to be around 0.61 (not required to submit). Along with the calculation in part (b-e), you should be convinced that the probability is indeed higher with 60 rolls than with 600 rolls for the observed number of aces obtained to stay within 2 from the expected number of aces obtained.