

STAT 22000 Lecture Slides

Overview of Confidence Intervals

Yibi Huang
Department of Statistics
University of Chicago

This set of slides covers section 4.2 in the text

- Overview of Confidence Intervals

Confidence intervals

- A plausible range of values for the population parameter is called a *confidence interval*.
- Using only a sample statistic to estimate a parameter is like fishing in a murky lake with a spear, and using a confidence interval is like fishing with a net.

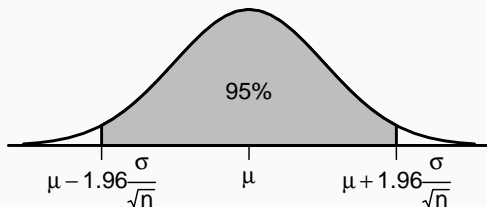


We can throw a spear where we saw a fish but we will probably miss. If we toss a net in that area, we have a good chance of catching the fish.



- If we report a point estimate, we probably won't hit the exact population parameter. If we report a range of plausible values we have a good shot at capturing the parameter.

Recall that CLT says, for large n , $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$. For a normal curve, 95% of its area is within 1.96 SDs from the center. That means, **for 95% of the time, \bar{X} will be within $1.96 \frac{\sigma}{\sqrt{n}}$ from μ .**



Alternatively, we can also say, **for 95% of the time, μ will be within $1.96 \frac{\sigma}{\sqrt{n}}$ from \bar{X} .**

Hence, we call the interval

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}} = \left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}} \right)$$

a **95% confidence interval for μ .**

Procedures to Construct a 95% Confidence Interval for μ

1. Take a simple random sample (or i.i.d. sample) of size n and find the sample mean \bar{X} .
2. If n is large, the 95% confidence interval for μ is given by

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

But σ is usually unknown ...

But σ is usually unknown ...

When the population SD σ is unknown, we replace it with our best guess — the sample SD s . So an approximate 95% confidence interval for μ is

$$\bar{X} \pm 1.96 s / \sqrt{n}$$

- However, this replacement is hazardous because
 - s is a poor estimate of σ if the sample size n is small and
 - s is very sensitive to outliers
- So we require $n \geq 30$ and sample shouldn't have any outlier nor be too skewed \Rightarrow Need to check histogram of the data
- We will discuss working with samples where $n < 30$ in the next chapter

Other Conditions Required to Use a Confidence Interval

Independence: Observations in the sample must be independent

- If the observations are from a simple random sample and consist of $< 10\%$ of the population, then they are nearly independent.
- Subjects in an experiment are considered independent if they undergo random assignment to the treatment groups.
- If a sample is from a seemingly random process, e.g. the lifetimes of wrenches used in a particular manufacturing process, checking independence is more difficult. In this case, use your best judgement.

Example: Average number of exclusive relationships

A random sample of 50 college students were asked how many exclusive relationships they have been in so far. This sample yielded a mean of 3.2 and a standard deviation of 1.74. Estimate the true average number of exclusive relationships using this sample.

$$\bar{x} = 3.2 \quad s = 1.74$$

The approximate 95% confidence interval is about

$$\begin{aligned}\bar{x} \pm 1.96 \times \text{SE} &= \bar{x} \pm 1.96 \times \frac{s}{\sqrt{n}} \\ &= 3.2 \pm 1.96 \times \frac{1.74}{\sqrt{50}} \\ &\approx 3.2 \pm 0.5 = (2.7, 3.7)\end{aligned}$$

True or False

True or False and explain: We are 95% confident that the average number of exclusive relationships college students in this sample have been in is between 2.7 and 3.7.

False. The confidence interval $\bar{X} \pm 1.96SE$ definitely (100%) contains the sample mean \bar{X} , not just with probability 95%.

True or False and explain: 95% of college students have been in 2.7 to 3.7 exclusive relationships.

False. The confidence interval is for covering the population mean μ , not for covering 95% of the entire population. If 95% of college students have been in 2.7 to 3.7 exclusive relationships, the SD won't be as large as 1.74.

True or False

True or False and explain: There is 0.95 probability that the true mean number of exclusive relationships of college students falls in the interval (2.7, 3.7)

True or False and explain: The interval (2.7, 3.7) has probability of 0.95 of enclosing the true mean number of exclusive relationships of college students.

Both are False. The population mean μ is a fixed number, not random. It is either in the interval (2.7, 3.7), or not in the interval. There is no uncertainty involved.

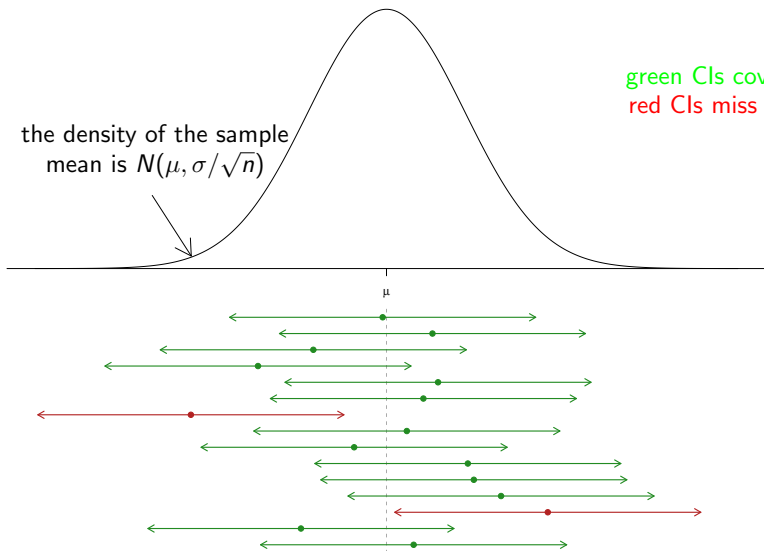
What does “95% confidence” mean?

What is the thing that has a 95% chance to happen?

- It is the **procedure to construct the 95% interval**.
- About 95% of the intervals constructed following the procedure (taking a SRS and then calculating $\bar{X} \pm 1.96 s / \sqrt{n}$) will cover the true population mean μ .
- After taking the sample and an interval is constructed, the constructed interval either covers μ or it doesn't. We don't know. Only God knows.
- Just like lottery, before you pick the numbers and buy a lottery ticket, you have some chance to win the prize. After you get the ticket, you either win or lose.

green CIs cover μ
red CIs miss μ

the density of the sample
mean is $N(\mu, \sigma/\sqrt{n})$



True or False

True or False and explain: If a new random sample of size 50 is taken, we are 95% confident that the new sample mean will be between 2.7 and 3.7.

False. The confidence interval is for covering the population mean μ , not for covering the mean of another sample. The SE σ / \sqrt{n} or s / \sqrt{n} is a typical distance between the sample mean and population mean, not a typical distance between two sample means.

True or False

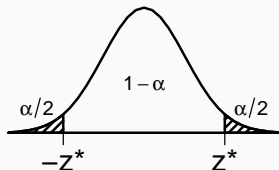
True or False and explain: This confidence interval $\bar{X} \pm 1.96 s / \sqrt{n}$ is not valid since the number of exclusive relationships is integer-valued. Neither the population nor sample is normally distributed.

False. The construction of the CI $\bar{X} \pm 1.96 s / \sqrt{n}$ only uses the normality of the sampling distribution of the sample mean. Neither the population nor the sample is required to be normally distributed. By the central limit theorem, with a large enough sample size we can assume that the sampling distribution is nearly normal and calculate a confidence interval.

Confidence Intervals at Other Confidence Levels

For a given confidence level $(1 - \alpha)$, we want to find a z^* such that

$$P(-z^* < Z < z^*) = 1 - \alpha \quad \text{or}$$



In general, a confidence intervals at confidence level $(1 - \alpha)$ is

sample mean $\pm z^* SE$

- $z^* \times SE$ is called the *margin of error*

Commonly used confidence levels:

- 90% C.I.: $\alpha = 0.1$, $z^* = 1.645$
- 95% C.I.: $\alpha = 0.05$, $z^* = 1.96$
- 99% C.I.: $\alpha = 0.01$, $z^* = 2.58$

MP-commissioned poll finds 12 per cent of British Columbians would engage in civil disobedience

There's 'a deep, deep frustration with the Trudeau government' over pipeline, Kennedy Stewart says.

By [Ainslie Cruickshank](#) StarMetro Vancouver

Sat., April 28, 2018

VANCOUVER — Twelve per cent of British Columbians are willing to engage in civil disobedience to oppose the Trans Mountain expansion project, a new poll has found, underscoring what a Burnaby MP says is a "deep frustration" with the federal government.

The online poll, conducted this month by Insights West and commissioned by NDP MP Kennedy Stewart, asked 1,021 people in B.C. between April 16-18 if they would consider civil disobedience to stop or disrupt the pipeline's construction. It found men and women were equally likely to consider civil disobedience, a release said.

. (several lines omitted).

The poll had a margin of error of plus or minus three percentage points 19 times out of 20.

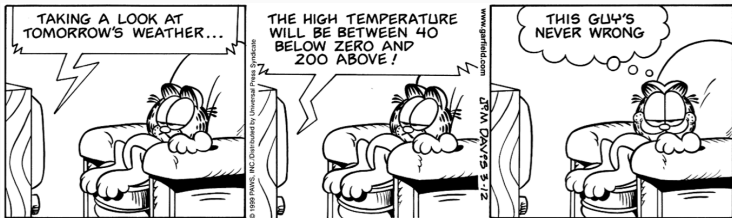
95% CI for the percentage of British Columbians that are willing to engage in civil disobedience to oppose the pipeline's construction is $12\% \pm 3\%$.

Width of an Interval

If we want to be more certain that we capture the population parameter, i.e. increase our confidence level, should we use a wider interval or a smaller interval?

A wider interval.

Can you see any drawbacks to using a wider interval?



If the interval is too wide it may not be very informative.