Problem 1

```
                                            90%      correctly flags
                    20%    contains error
                                            10%      mistakenly ignores
Tax returns
                    80%    without error    2%       mistakenly flags
                                            98%      correctly ignores
```
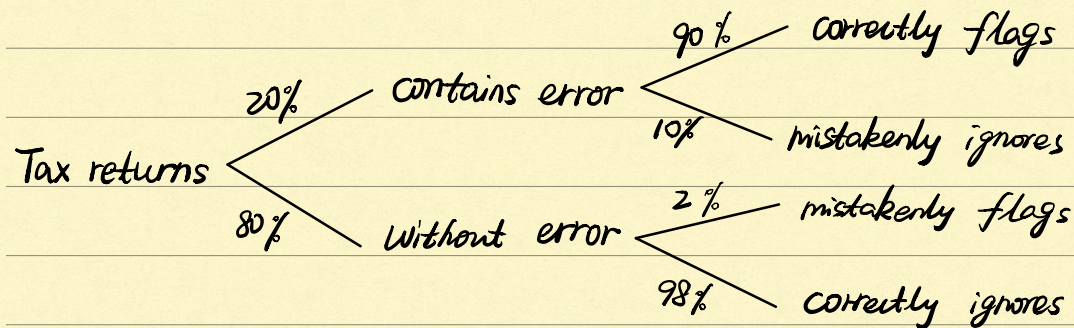
(a) $P = 20\% \times 90\% + 80\% \times 2\% = 19.6\%$

(b) $P = 20\% \times 90\% \div 19.6\% = 91.8\%$

(c) $P = 20\% \times 10\% \div (20\% \times 10\% + 80\% \times 98\%) = 2.49\%$

Problem 2

(a) $P = 1 - 0.21264 - 0.04325 - 0.00306 = 0.74105$

(b) Expected value $= 0.00306 \times 119 + 0.04325 \times 2 + 0 - 0.74105 \times 1 = -0.29041$

(c) $SD = sqrt\left[(-1+0.29041)^2 \times 0.74105 + (0+0.29041)^2 \times 0.21264 + \right.$
$$\left. (2+0.29041)^2 \times 0.04325 + (119 + 0.29041)^2 \times 0.00306\right] = 6.65$$

(d) Expected net profit $= -0.29041 \times 100 = -29.041$

(e) $SD_{tot} = \sqrt{SD^2 \times 100} = SD \times 10 = 66.5$

(f) $P = \frac{20}{80} \times \frac{19}{79} \times \frac{18}{78} \times \frac{17}{77} = 0.00306$

Problem 3

(a) $E(X) = 35 \times \frac{1}{38} - 1 \times \frac{37}{38} = -\frac{1}{19} \approx -0.0526$

$V(X) = (35 + \frac{1}{19})^2 \times \frac{1}{38} + (-1 + \frac{1}{19})^2 \times \frac{37}{38} \approx 33.2$

(b) $E(Y) = 17 \times \frac{2}{38} - 1 \times \frac{36}{38} = -\frac{1}{19} \approx -0.0526$

$V(Y) = (17 + \frac{1}{19})^2 \times \frac{2}{38} + (-1 + \frac{1}{19})^2 \times \frac{36}{38} \approx 16.2$

(c) $E(T) = 34 \times \frac{1}{38} + 16 \times \frac{2}{38} - 2 \times \frac{35}{38} = -\frac{2}{19} \approx -0.105$

$V(T) = (34 + \frac{2}{19})^2 \times \frac{1}{38} + (16 + \frac{2}{19})^2 \times \frac{2}{38} + (-2 + \frac{2}{19})^2 \times \frac{35}{38} \approx 47.57$

(d) They are not independent, because one can not win both the single bet and the split bet, there is only one correct number.

(e) $V(T) \neq V(X) + V(Y)$, because X and Y are not independent.