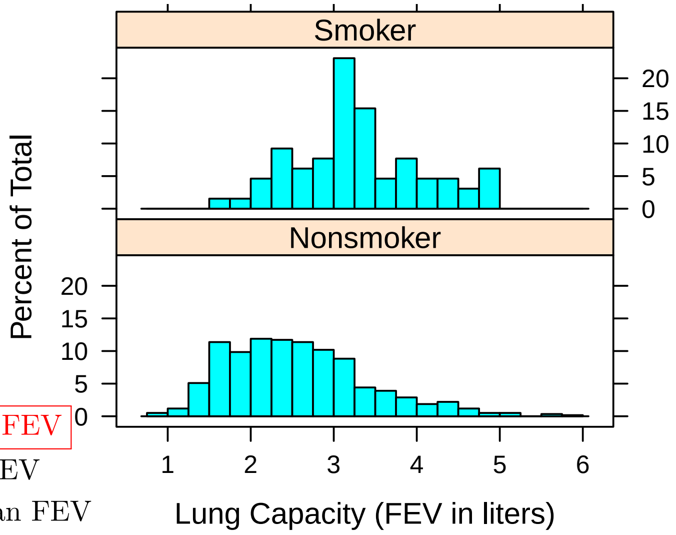
2020 Summer STAT 22000 Midterm Exam Solutions

1. We will examine data on lung capacity, as measured by a quantity called forced expiratory volume (to be abbreviated FEV), the amount of air an individual can exhale in the first second of forceful breath (in liters). The following graph displays the distributions of FEV values for 65 smokers and 589 nonsmokers who participated in a research study.



(1a) (2pts) For the 589 nonsmokers in the study,

(i) [✓ the mean FEV is higher than the median FEV]

(ii) the mean FEV is lower than the median FEV

(iii) the mean FEV is about equal to the median FEV

Select only ONE answer. ⇒ As the histogram is right-skewed, the mean $>$ the median.

(1b) (2pts) For the 65 smokers in the study, the standard deviation (SD) of the FEV values is closest to which of the following numbers **Select only ONE answer**.

(i) 0.25 [✓ (ii) 0.75] (iii) 1.5 (iv) 3

⇒ Based on the 68-95% rule, about 68% of the observations are within mean ± 1SD. The center of the histogram for smokers is around the peak 3.2. The range 3.2 ± 0.25 is too narrow to cover 68% of the area below the histogram. The range 3.2 ± 1.5 or 3 ± 3 covers almost the entire range of the histogram. So 1.5 or 3 is too big to be the SD. We see, the ranges $3.2 \pm 0.75 = (2.25, 3.75)$ and $3.2 \pm 2 \times 0.75 \approx (1.7, 4.7)$ cover about 50-70% and 90-100% of the area below the histogram, So the SD should be closest to 0.75.

(1c) (2pts) The Q3 (third quartile = 75th percentile) of the FEV of the 589 nonsmokers in the study is closest to which of the following numbers. **Select only ONE answer**.

(i) 1 (ii) 2 [✓ (iii) 3] (iv) 4.5

⇒ 75% of the area under the histogram should be on the right of Q3 and 25% on the left. The areas to the left of 1 and 2 are clearly less than 50%. Nearly all the observations are less than 4.5. So the closest one should be 3.

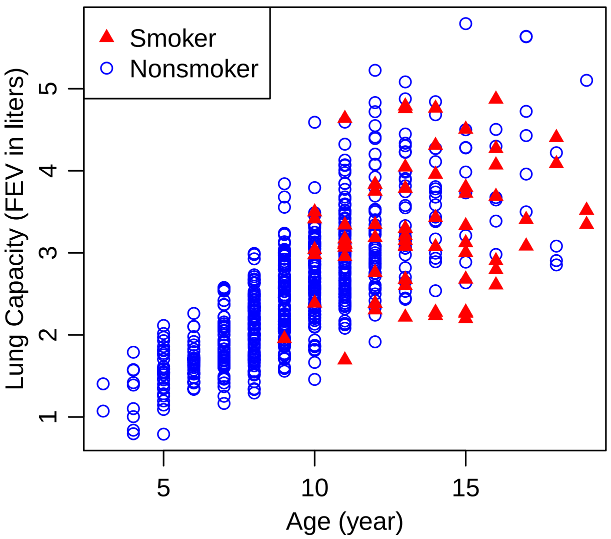
(1d) (2pts) Based on the histograms, which group — smokers or non-smokers — tends to have larger lung capacities (larger FEV)? Or are they about the same? **Select only ONE answer**.

[✓ (i) smokers] (ii) nonsmokers (iii) about the same

⇒ The center of the histogram for smokers is higher than the center for non-smokers.

(1e) (5pts) Now let's consider one more variable — **age** of the participants. The scatterplot below shows the relationship between the age and lung capacity of smokers and non-smokers.

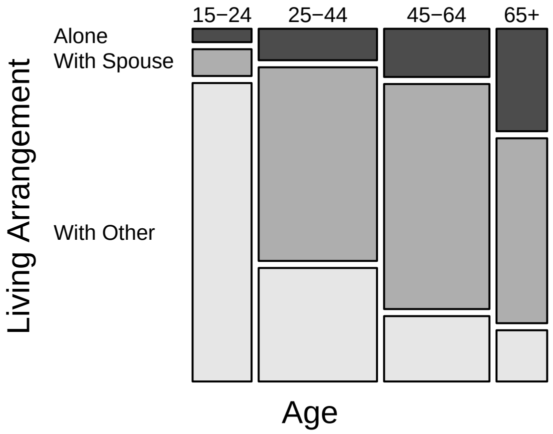
After accounting for the age of the participants, which group — smokers or non-smokers — tends to have larger lung capacities (larger FEV)? Or are they about the same? Do you get the same conclusion as in the previous part? How do you explain the seemly contradictory conclusions here and the previous part?



Answer: In the study, smokers and nonsmokers of the same age didn't differ significantly in their lung capacities (or nonsmokers had slightly lower lung capacities). This result is different from what we observed in (1d) without considering the age of the participants.

We see different results because all the participants in the study were children age 3-19. Smokers were mostly older children and older children had greater lung capacities. So it appears that smokers had greater lung capacities than nonsmokers but it's primarily because smokers in the study were older children who had greater lung capacities.

2. As reported by the U.S. Census Bureau in *American's Families and Living Arrangements*, the living arrangements by age of U.S. citizens age 15+ are as shown in the mosaic plot on the right.



(2a) (2pts) Which of the following groups contains the most number of people? **Select only ONE answer**.

(i) age 15-24 and live alone

(ii) [✓ age 25-44 and live with other]

(iii) age 45-64 and live alone

(iv) age 65+ and live with other

(2b) (2pts) Based on the mosaic plot, which of the following statement is TRUE? **Select only ONE answer**.

(i) [✓ Over half of people in the age group 45-64 live with spouse.]

(ii) The living arrangement is independent of age for U.S. citizens age 15+.

(iii) Older people are more likely to live with others than younger people.

(iv) There are more people in the age group 65+ than in the age group 45-64.

3. "*Where do you get most of your information about current news events?*" This question was asked in the 2008 General Social Survey. Possible answers included television, Internet, and newspapers, as well as other possibilities such as radio, family, and friends. The table one the right summarizes the results by age group.

Age	TV	Internet	Newspapers	Other	Total
18-29	109	92	25	36	262
30-49	272	157	88	63	580
50+	345	59	165	63	632
Total	726	308	278	162	1474

(3a) (2pts) What percentage of respondents were between 18 and 29 years old and got news primarily from the internet? **Show your calculation.**

Answer: $\frac{92}{1474} \approx 0.0624 = 6.24\%$.

(3b) (4pts) Which of the following statements implies there is an association between the respondents' age and the primary source they get news. **Select all that apply.**

[✓ (i)] There was a higher percentage of people getting news primarily from the internet among younger people than among older people. ⇒ P(Source = Internet | Age = young) ≠ P(Source = Internet | Age = old), which implies dependence.

(ii) In the 18-29 age group, there were more people getting news primarily from TV and from the internet than from newspaper and from other sources. ⇒ If the same thing is observed in other age groups, the two variables could be independent

[✓ (iii)] Those who got news primarily from the internet were generally younger than those got news primarily by reading newspaper. ⇒ P(Age | Source = Internet) ≠ P(Age | Source = Newspaper), which implies dependence.

[✓ (vi)] The percentage of people getting news primarily from the internet in the 18-29 age group was higher than the percentage getting news primarily from the internet over all age groups. ⇒ P(Source = Internet | Age = 18-29) ≠ P(Source = Internet), which implies dependence.

(v) Among those who got news primarily from the Newspaper, a majority of them were 50 or older. ⇒ If 50+ year-olds were a majority for all news source, the two variables could be independent

4. (4pts) Please find the mean, median and standard deviation (SD) for the data below.

3, 2, 8, 4 8

Show your work to receive full credit.

Answer: The sorted list is: 2, 3, 4, 8, 8

$$\text{(1pt) Median} = 4, \text{the middle number}$$

$$\text{(1pt) Mean} = \frac{2+3+4+8+8}{5} = \frac{25}{5} = 5$$

$$\text{(2pts) SD} = \sqrt{\frac{(2-5)^2+(3-5)^2+(4-5)^2+(8-5)^2+(8-5)^2}{5-1}}$$

$$= \sqrt{\frac{9+4+1+9+9}{4}} = \sqrt{\frac{32}{3}} = \sqrt{8} \approx 2.828$$

[Grading: 1pt off if divided by $n = 5$ rather than by $n - 1 = 5 - 1 = 4$, which gives SD $= \sqrt{32/5} = \sqrt{6.4} \approx 2.53$.]

5. Below are the midterm exam scores of 23 introductory statistics students, sorted from low to high:

$$46, 47, 61, 74, 75, 77, 83, 84, 85, 86, 88, 88, 90, 90, 90, 93, 94, 94, 95, 95, 96, 96, 98$$

The five-number summary is: Min = 46, Q1 = 80, Median = 88, Q3 = 94, Max = 98.

(5a) (2pts) What value(s) would be identified as outliers based on the 1.5 IQR rule? **Select only ONE answer**.

(i) none (ii) 46 (iii) 46, 98 (vi) 98 [✓ (v) 46, 47] (v) 46, 47, 98

(5b) (1pt) Where does the lower whisker of the boxplot for the 23 scores above extend to? **Select only ONE answer**.

(i) 46 (ii) 47 (iii) 59 [✓ (iv) 61] (v) 67 (vi) 74

⇒ There are two outliers: 46 and 47, as they fall below $Q1 - 1.5 \times \text{IQR} = 80 - 1.5 \times 14 = 59$. The lower whisker of a boxplot extends to the smallest observation that is not an outlier, which is 61.

6. For a period of 5 years, physicians at McGill University Health Center followed about 5000 adults over the age of 50. The researchers were investigating whether people taking a certain class of antidepressants (SSRIs) might be at greater risk of bone fractures. Their observations are summarized in the table on the right.

Experienced	Taking SSRI?		
Fracture?	Yes	No	Total
Yes	14	244	258
No	123	4627	4750
Total	137	4871	5008

(6a) (2pts) Which pair of numbers should we examine if we want to know whether people taking SSRI might be at greater risk of bone fractures? **Select only ONE answer**.

(i) 14 vs 244 (ii) 14/5008 vs 244/5008 [✓ (iii) 14/137 vs 244/4871]
(iv) 14/258 vs 244/258 (v) 258/5008 vs 4750/5008

(6b) (2pts) Can we conclude that taking SSRI increases the risk of bone fractures? **Select only ONE answer**.

(i) No, because the study was not blinded
(ii) No, because the 5008 subjects were not randomly sampled from the population of interest
[✓ (iii) No, because the subjects were not randomized to take SSRI]
(iv) Yes, because the sample size 5008 was large
(v) Yes, because the study is a comparative experiment

7. (2pts) Suppose that 35% of the registered voters in a state are registered as Republicans, 40% as Democrats, and 25% as Independents. A newspaper wants to select a sample of 1000 registered voters to predict the outcome of the next election. If they randomly select 350 Republicans, randomly select 400 Democrats, and randomly select 250 Independents, what kind of sampling method is used here?

Select only ONE answer.

(i) simple random sampling [✓ (ii) stratified sampling]
(iii) clustered sampling (iv) multistage sampling
(v) convenience sampling (vi) voluntary response sampling

8. (4pts) After menopause, some women take supplemental estrogen. There is some concern that if these women also drink alcohol, their estrogen levels will rise too high. Nineteen (19) volunteers who were receiving supplemental estrogen were randomly divided into 2 groups, as were 20 other volunteers not on estrogen. In each case, one group drank an alcoholic beverage, the other a nonalcoholic beverage. An hour later, everyone's estrogen level was checked. Only those on supplemental estrogen who drank alcohol showed a marked increase. For each of the following statements about the study, determine whether it is true or false. No explanation is required.

(8a) T or [✓ F]: This study blocked on whether subjects drank alcohol ⇒ The study blocked on whether the subjects take supplemental estrogen.

(8b) [✓ T] or F: We cannot make a causal conclusion on the effect of supplemental estrogen on women's estrogen levels as the women were not randomized to take supplemental estrogen

(8c) T or [✓ F]: We can generalize the conclusion of the study as the subjects were randomized to drink an alcoholic or a nonalcoholic beverage ⇒ We cannot generalize the conclusion to a bigger population since the subjects were not randomly sampled from the population.

(8d) T or [✓ F]: This study used stratified sampling

⇒ The 3 strata are Republicans, Democrats, and Independents. A sample is selected from each of the strata. Simple random sampling does not guarantee the breakup of 3 party identifications in the sample to match the breakup in the population.

9. (4pts) In some jurisdictions, there are "pretrial conferences," where the judge confers with the opposing lawyers to settle the case or at least to define the issues before trial. Observational data suggests that pretrial conferences promote settlements and speed up trials, but there were doubts.

In New Jersey courts, pretrial conferences were mandatory. However, an experiment was done in 7 counties. During a 6-month period, 2954 personal injury cases (mainly automobile accidents) were assigned at random to treatment or control. For the 1495 control cases (group A), pretrial conferences remained mandatory. For the 1459 treatment cases, the conferences were made optional—either lawyer could request one. Among the treatment cases, 701 opted for a pretrial conference (group C), and 758 did not (group B).

To check whether pretrial conferences promote settlements and speed up trials, which two groups should the investigator compare? **Explain your choice**.

(i) A v.s. B (ii) B v.s. C [✓ (iii) A v.s. (B+C)] (iv) (A+C) v.s. B

(v) Any of the above works because this is a randomized controlled experiment

⇒ To make a causal conclusion, the two groups compared must be divided by randomization. In this study, the division between Group A and Group B+C is made by randomization. The division between Group B and C is self-selected. Cases that opt for a pretrial conference are probably different from cases that don't — so there will be lots of confounding. Comparing A v.s. B or (A+C) v.s. B also suffers from similar problems because one group only includes cases that chose not to have a pretrial conference and the other group is a mixture of both.

10. (4pts) A local news agency conducted a survey about unemployment by randomly dialing phone numbers until they had gathered responses from 1000 adults in their state. In the survey, 19% of those who responded said they were not currently employed. In reality, only 6% of the adults in the state were not currently employed. How do you explain the difference in the two percentages?

Answer: The difference is due to non-response bias. Adults who are employed are less likely to be available for the survey than adults who are unemployed.

Grading Remark: Many students confused "voluntary response sample" with "non-response bias." In this survey, the news agency first sampled phone numbers at random. Only those got called had chance to participate in the survey. This is not a voluntary response sample. In a voluntary response sample, the investigator don't select a sample first. All the subjects can participate in the survey if they want. As no sample is selected beforehand, it is not possible to calculate the response rate in a voluntary response sample because we have no idea how many didn't respond. This sample in this survey is not a voluntary response sample. **1pt off if mentioned this is a voluntary response sample.**

11. (4pts) Traffic checks on a certain section of highway suggest that 40% of drivers are speeding there. Two vehicles in a row are selected at random at that section of highway. For each of the following statements, determine whether it is TRUE or FALSE and **explain briefly**.

(11a) The probability that at least one of the two vehicles in a row is speeding is $0.4 + 0.4$

Answer: Let A be the event that the first vehicle is speeding and B be the event that the second vehicle is speeding. We know $P(A) = P(B) = 0.4$ since 40% of the drivers are speeding. $P(A \text{ or } B) = P(A) + P(B) = 0.4 + 0.4$ is true only if A and B are disjoint, which is not true as the two consecutive vehicles can be both speeding.

(11b) The probability that the two vehicles in a row are both speeding is 0.4×0.4

Answer: The speed of two consecutive vehicles are usually pretty close. A and B are not independent. $P(A \text{ and } B) = P(A) \times P(B) = 0.4 \times 0.4$ is true only if A and B are independent.

12. The 2018 General Social Survey (GSS) interviewed a national sample of American adults and found that 47% of survey respondents have a pet dog, 25% have a pet cat, and 14% of survey respondents had both a dog and a cat. For all the parts below, please **show your calculation**.

(12a) (2pts) What percentage of survey respondents have either a pet dog or a pet cat or both?

Answer:

$$\begin{aligned} P(\text{dog owner or cat owner}) &= P(\text{dog owner}) + P(\text{cat owner}) - P(\text{dog and cat owner}) \\ &= 47\% + 25\% - 14\% = 58\% \end{aligned}$$

(12b) (2pts) What percentage of survey respondents have no pet dog nor pet cat?

Answer:

$$P(\text{neither dog owner nor cat owner}) = 1 - P(\text{dog owner or cat owner}) = 100\% - 58\% = 42\%$$

(12c) (3pts) Among the respondents that have no pet cat, what percentage of them have a pet dog?

Answer:

$$P(\text{dog owner but not cat owner}) = P(\text{dog owner}) - P(\text{dog and cat owner}) = 47\% - 14\% = 33\%$$

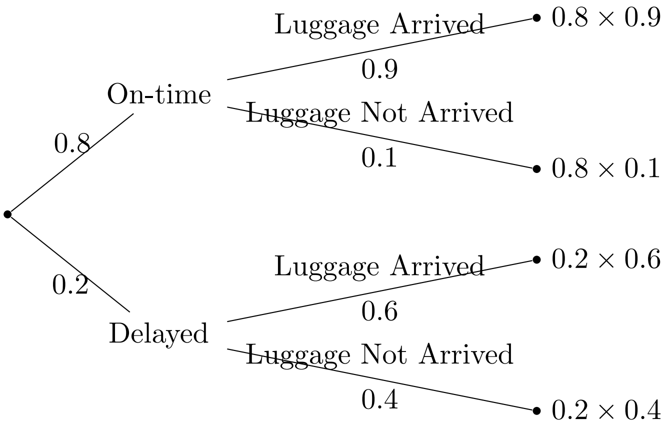
and hence

$$P(\text{dog owner}|\text{no cat}) = \frac{P(\text{dog owner but not cat owner})}{P(\text{no cat})} = \frac{33\%}{100\% - 25\%} \approx \boxed{0.44} = 44\%$$

13. Leah is flying from Boston to Denver with a connection in Chicago. The probability her first flight leaves on time is 0.8. If the flight is on time, the probability that her luggage will make the connecting flight in Chicago and arrive in Denver is 0.9, but if the first flight is delayed, the probability that the luggage will make it is only 0.6.

(13a) (3pts) What is the probability that her luggage arrives in Denver with her? Show your work.

Answer:



$$P(\text{Luggage Arrived}) = 0.8 \times 0.9 + 0.2 \times 0.6 = \boxed{0.84}$$

(13b) (3pts) Suppose you pick her up at the Denver airport, and her luggage is not there. What is the probability that Leah's first flight was delayed? Show your work.

Answer:

$$\begin{aligned} P(\text{First Flight Delayed}|\text{Luggage Not Arrived}) &= \frac{P(\text{First Flight Delayed and Luggage Not Arrived})}{P(\text{Luggage Not Arrived})} \\ &= \frac{0.2 \times 0.4}{0.8 \times 0.1 + 0.2 \times 0.4} = \frac{0.08}{0.16} = 0.5 \end{aligned}$$

(13c) (2pts) Let A be the event that "the first flight leaves on time", and B be the event that "the luggage makes the connection and arrives in Denver." Are the two events A and B independent? Explain briefly.

Answer: Not independent since

$$\begin{aligned} P(B|A) &= P(\text{luggage arrives}|\text{on time}) = 0.9, \quad \text{but} \\ P(B|A^c) &= P(\text{luggage arrives}|\text{delayed}) = 0.6, \end{aligned}$$

which are not equal.

14. You are dealt a hand of 3 cards, one at a time, without replacement. Recall that there are 4 aces and 13 hearts in a deck of 52 poker cards. Three cards are drawn at random WITHOUT replacement from a deck of poker cards.

(14a) (3pts) Find the probability that you get no aces. **Show your calculation**.

Answer: $\frac{48}{52} \times \frac{47}{51} \times \frac{46}{50} \approx 0.7826.$

(14b) (2pts) Find the probability that the third card is an ace given that the first two cards are not aces.

Answer: $\frac{4}{50} = 0.08$

15. The state of Illinois has several state-wide lottery options. One is the *Pick 3* game in which you pick one of the 1000 three-digit numbers between 000 and 999. The lottery selects a three-digit number at random. It takes \$1 to buy a *Pick 3* ticket. You win \$500 if your number is selected and nothing (\$0) otherwise. The distribution of the net profit X from a *Pick 3* ticket is hence:

Net Profit X	$-\$1$	\$499
Probability	0.999	0.001

For all the parts below, please **show your calculation**.

(15a) (2pts) What is the expected value of X?

Answer: $\mathrm{E}(X) = (-\$1) \times 0.999 + \$499 \times 0.001 = -\$0.50$.

(15b) (3pts) What is the standard deviation of X?

Answer:

$$\begin{aligned}
\mathrm{V}(X) &= (-1-(-0.5))^2 \times 0.999 + (499-(-0.5))^2 \times 0.001, \\
&= (0.5)^2 \times 0.999 + (499.5)^2 \times 0.001 \\
&= 0.24975 + 249.50025 = 249.75 \\
\mathrm{SD}(X) &= \sqrt{\mathrm{V}(X)} = \sqrt{249.75} \approx \boxed{\$15.80}.
\end{aligned}$$

(15c) (2pts) *Pick 3* lottery draws a three-digit number twice a day, once during the day and once in the evening. The drawings are independent of each other. So gamblers can make two independent bets a day. If a gambler buys two Pick 3 tickets everyday for a year, one for the midday draw and one for the evening draw, (i.e., he makes $365 \times 2 = 730$ independent bets in total), what is the expected value of the total net profit the gambler can get from the 730 *Pick 3* tickets?

Answer: Let X_i be the payoff from the ith ticket. The net profit he can get from the 730 tickets is $X_1 + X_2 + \cdots + X_{730}$. Observe that the X_i's are independent and the distribution of X_i's are the same as X in the previous part. So, we know $E(X_i) = \$0.5$, and The expected total net profit is hence:

$$E(X_1 + X_2 + \cdots + X_{730}) = E(X_1) + E(X_2) + \cdots + E(X_{730}) = -\$0.5 \times 730 = \boxed{-\$365}.$$

(15d) (3pts) What is the standard deviation of the total net profit the gambler from the 730 *Pick 3* tickets?

Answer: As $V(X_i) = 249.75$ for all i, and the X_i's are independent, the variance total net profit is

$$V(X_1 + X_2 + \cdots + X_{730}) = V(X_1) + V(X_2) + \cdots + V(X_{730}) = 249.75 \times 730 = 182317.5$$

So the SD of his total net profit is $\sqrt{182317.5} \approx 426.9865 \approx \boxed{\$427.0}$.

16. The time it takes a driver to react to the brake lights on a decelerating vehicle is critical in avoiding rear-end collisions. The article "*Fast-Rise Brake Lamp as a Collision-Prevention Device*" (Ergonomics, 1993: 391–95) suggests that reaction time for an in-traffic response to a brake signal from standard brake lights can be modeled with a normal distribution having a mean value 1.25 s and an SD of 0.46 s.

(16a) (3pts) What is the probability that the reaction time is between 1.00 and 1.75 s? If you calculate using R, please show (or write down) the R code you used.

Answer: **0.7309.** Let X be the reaction time $\sim N(\mu = 1.25, \sigma = 0.46)$. So $P(1 < X < 1.75)$ can be found in R using either of the commands below.

```
> pnorm((1.75-1.25)/0.46)-pnorm((1-1.25)/0.46)
[1] 0.5680717
> pnorm(1.75, m=1.25, s=0.46)-pnorm(1, m=1.25, s=0.46)
[1] 0.5680717
```

(16b) (3pts) What is the 99th percentile of the reaction time? That is, find the length of time that is long enough for at least 99% of the drivers to react.
If you calculate using R, please show (or write down) the R code you used.

Answer: The answer is about 2.32 seconds, which can be found in R as follows.

```
> qnorm(0.99, m=1.25, s=0.46)
[1] 2.32012
```

17. The game of roulette involves spinning a wheel with 38 slots: 18 red, 18 black, and 2 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball. One popular bet is that it will stop on a red slot; such a bet has an 18/38 chance of winning. Suppose a gambler bets on red in 6 different spins.

(17a) (3pts) Find the probability the gambler wins the first 3 spins but loses the next 3 spins.

Answer: $\left(\frac{18}{38}\right)^3 \left(\frac{20}{38}\right)^3 \approx 0.0155$

(17b) (3pts) Find the probability the gambler wins exactly 3 of the 6 spins.

Answer: The number of times X the gambler wins the binomial distribution $Bin(n = 6, p = 18/38)$. The chance that $X = 3$ is

$$P(X = 3) = \binom{6}{3}(18/38)^3(20/38)^3 = 20 \cdot (18/38)^3(20/38)^2 \approx \boxed{0.3099}$$

in which $\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$

(17c) (3pts) Find the probability the gambler wins at least 3 of the 6 spins.

Answer:

$$\begin{aligned}
P(X \geq 3) &= P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) \\
&= \binom{6}{3}\left(\frac{18}{38}\right)^3\left(\frac{20}{38}\right)^3 + \binom{6}{4}\left(\frac{18}{38}\right)^4\left(\frac{20}{38}\right)^2 + \binom{6}{5}\left(\frac{18}{38}\right)^5\left(\frac{20}{38}\right)^1 + \binom{6}{6}\left(\frac{18}{38}\right)^6 \\
&= 20\left(\frac{18}{38}\right)^3\left(\frac{20}{38}\right)^3 + 15\left(\frac{18}{38}\right)^4\left(\frac{20}{38}\right)^2 + 6\left(\frac{18}{38}\right)^5\left(\frac{20}{38}\right)^1 + 1\left(\frac{18}{38}\right)^6 \\
&\approx 0.3099 + 0.2092 + 0.0753 + 0.0113 = 0.6057
\end{aligned}$$

For all 3 parts above, please **show your calculation**.

18. (3pts) A box contains 8 red marbles and 3 green ones. Six draws are made at random without replacement. True or false: the probability that the 3 green marbles are drawn equals

$$\frac{6!}{3!\,3!}\left(\frac{8}{11}\right)^3\left(\frac{3}{11}\right)^3$$

Explain briefly.

Answer: False. As the draws are not independent since they are made without placement, the Binomial formula cannot be applied.

Remark. If the draws are made with replacement, then the statement is TRUE.