Stat 22000 Summer 2020 Homework 10 Solutions

Problems to Turn In: due **midnight of Sunday, July 19, on Canvas**.

1. (Similar to Exercise 5.3 on p.257) An random sample is selected from an approximately normal population with an unknown standard deviation. For each the given set of hypotheses, sample size and the T-statistic, find the p-value using the `pt()` function in R.

   (a) $H_A: \mu > 0.5$, $n = 26$, $T = 2.6$

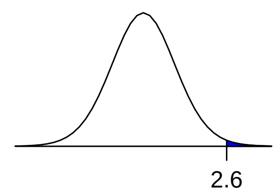
   (b) $H_A: \mu \neq 0.5$, $n = 26$, $T = 2.6$

   (c) $H_A: \mu < 3$, $n = 18$, $T = -2.2$

   (d) $H_A: \mu < 3$, $n = 18$, $T = 2.2$

Answer: *[1pt each. For instructional purpose, the answers below are longer than necessary. Students are NOT required to sketch the t-curves.]*

(a) With $\text{df} = 26 - 1 = 25$, the upper one-sided p-value for $T = 2.6$ is about 0.0077.

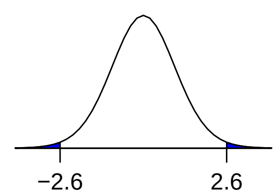



```
> pt(2.6, df=25, lower.tail=F)
[1] 0.007712645
```

Alternatively, as t-curves are symmetric about 0, the area to the right of 2.6 is identical to the area to the left of -2.6. One can also find the p-value as follows.

```
> pt(-2.6, df=25)
[1] 0.007712645
```

(b) With $\text{df} = 26 - 1 = 25$, the two-sided p-value for $T = 2.6$ is about 0.0154.

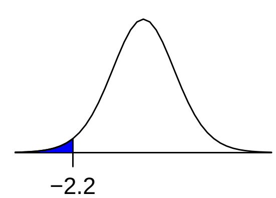



```
> 2*pt(2.6, df=25, lower.tail=F)
[1] 0.01542529
```

Alternatively, as t-curves are symmetric about 0, the area to the right of 2.6 is identical to the area to the left of -2.6. One can also find the p-value as follows.

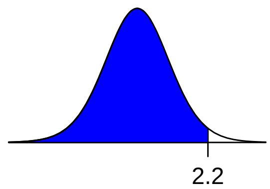
```
> 2*pt(-2.6, df=25)
[1] 0.01542529
```

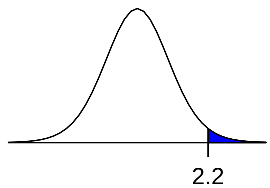
(c) With $\text{df} = 18 - 1 = 17$, the lower one-sided p-value for $T = -2.2$ is about 0.021.




```
> pt(-2.2, df=17)
[1] 0.02096233
```

(d) With df $= 18 - 1 = 17$, the lower one-sided p-value

is the **lower-tail area**  , which is `pt(2.2, df=17)` ≈ 0.979,

NOT the **upper-tail area**  which is `pt(2.2, df=17, lower.tail=F)` ≈ 0.021.

This makes sense since $T = (\bar{(}x) - 3)/\text{SE} = 2.2$ is positive means the sample mean $\bar{x}$ is higher than 3. The null hypotheses H_0: $\mu = 3$ would be more plausible than H_A: $\mu < 3$ since the sample mean $\bar{x}$ is higher than 3. A large P-value would make sense since H_0 shouldn't be rejected.

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2. A study compared different psychological therapies for teenage girls suffering from anorexia, an eating disorder that causes them to become dangerously underweight.

Each girl's weight was measured before and after a period of therapy. The variable of interest was the weight change, defined as weight at the end of the study minus weight at the beginning of the study.

Two therapies were designed to aid weight gain, one of which is the cognitive behavioral therapy. This form of psychotherapy stresses identifying the thinking that causes the undesirable behavior and replacing it with thoughts designed to help improve this behavior. The changes of 29 teenage girls receiving the cognitive behavioral therapy in weight (in lbs) during the study were

```
1.7, 11.7, -1.4, 0.7, 6.1, -0.8, -0.1, 1.1, 2.4, -0.7, -4.0, 12.6, -3.5, 20.9,
1.9, 14.9, -9.3, 3.9, 3.5, 2.1, 0.1, 17.1, 1.4, 15.4, -7.6, -0.3, -0.7,1.6, -3.7
```

The weight change was positive if the girl gained weight (which is desired) and negative if she lost weight. We want to test whether the cognitive behavioral therapy is effective to aid weight gain.

(a) Using a calculator or R, find the sample mean and the standard deviation of the weight changes.

Answer: *[1pt]* $\bar{x} = \mathbf{3}$, $s \approx \mathbf{7.32}$, obtained with the R codes below (not required to show)

```
> cognitive = c(1.7,11.7,-1.4,0.7,6.1,-0.8,-0.1,1.1,2.4,-0.7,-4.0,12.6,-3.5,20.9,
                1.9,14.9,-9.3,3.9,3.5,2.1,0.1,17.1,1.4,15.4,-7.6,-0.3,-0.7,1.6,-3.7)
> mean(cognitive)
[1] 3
> sd(cognitive)
[1] 7.320422
```

Or using the `mosaic` library:

```
> library(mosaic)
> favstats(cognitive)
  min   Q1 median  Q3  max mean       sd  n missing
 -9.3 -0.7    1.4 3.9 20.9    3 7.320422 29       0
```

(b) Formulate the null and alternative hypotheses for testing whether teenage girls receiving the cognitive behavioral therapy have a positive mean weight gain.

Answer: [*2pts*] The hypotheses are H_0: $\mu = 0$ (or $\mu \leq 0$) against H_a: $\mu > 0$ where μ is the population mean weight gain of girls receiving the cognitive behavioral therapy. [*1pt off if not explaining what μ is*].

Alternatively, the hypotheses can be stated in words as

- H_0: The mean weight gain of girls receiving the cognitive behavioral therapy is 0 (or less)
- H_a: The mean weight gain of girls receiving the cognitive behavioral therapy is over 0

or

- H_0: Teenage girls receiving the cognitive behavioral therapy have a 0 weight gain on average
- H_a: Teenage girls receiving the cognitive behavioral therapy have a positive weight gain on average

Please note that the effect of the therapy could vary from person to person. It may work for someone and have no effect or even negative effect for another. The hypotheses are only about the "mean" effect of the therapy over the population. The statements of hypotheses should include the word "mean" or "average." It's not quite precise to state the hypotheses as

- H_0: The cognitive behavioral therapy has no effect on girls' weight gain
- H_a: The cognitive behavioral therapy help girls gain weight

since it sounds like the therapy had the same effect for all girls.

Grading:

- *Give 0pts if H_0 and H_a are swapped, i.e., H_0: $\mu > 0$ and H_a: $\mu = 0$ (or $\mu \leq 0$)*
- *Give 0pts for the answer H_0: $\bar{x} = 0$ and H_a: $\bar{x} > 0$. H_0 and H_a should be stated in terms of population means, not sample means.*
- *Take 1pt off if using a 2-sided H_a: $\mu \neq 0$.*
- *Take 1pt off if including the word "significant" in the hypotheses, like*
  - *H_0: The mean weight gain of girls receiving the cognitive behavioral therapy is not significantly over 0*
  - *H_a: The mean weight gain of girls receiving the cognitive behavioral therapy is significantly over 0*

  or

  - *H_0: The cognitive behavioral therapy has no significant effect on girls' weight gain*
  - *H_a: The cognitive behavioral therapy increased girls' weight significantly*

  The word "significant" is only used in the conclusion of a hypothesis test, not in the hypotheses themselves.

(c) Calculate the t-statistic and report the degrees of freedom.

Answer: [*2pts*] t-statistic $= \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}} = \dfrac{3 - 0}{7.32/\sqrt{29}} \approx 2.207$ with df $= 29 - 1 = 28$

(d) Find the p-value and make a conclusion use a significance level of 0.01.

Answer: [*2pts = 1pt for the range of the p-value + 1pt for the conclusion.*]

At df = = $29 - 1 = 28$, the upper one-sided p-value is about 0.01784.

```
> pt(2.207, df=28, lower.tail=F)
[1] 0.01784039
```

Conclusion: As the p-value 0.01784 is over the significance level 0.01, we fail to reject H_0. Girls' mean weight gain is NOT significantly higher than 0 at significance level 0.01.

(e) What are the Type 1 error and Type 2 error respectively for the hypothesis test done in part (b-d)?

Answer: *[2pts]* Type 1 error: Teenage girls receiving the cognitive behavioral therapy have a 0 (or negative) mean weight gain but we conclude that the mean weight gain is significantly higher than 0.

Type 2 error: Teenage girls receiving the cognitive behavioral therapy have a positive mean weight gain but we fail to reject the null of 0 mean weight gain.

[Take 1pt off if the errors are not stated in context of the study, i.e., simply saying a Type 1 error is rejecting H_0 when it is true and a Type 1 error is failing to reject H_0 when the H_a is true.]

(f) Repeat part (b)(c)(d) but for testing whether the population mean weight gain is not 0, rather than higher than 0.

Answer: *[3pts = 1pt for the hypotheses + 1pt for the z-statistic + 1pt for the p-value]* The H_0: $\mu = 0$ is unchanged, but the H_a becomes $\mu \neq 0$ where μ is as defined in part (b). The t-statistic $= 2.207$ remains unchanged. The two-sided p-value is twice of the one-sided P-value, about $2 \times 0.01784 = 0.03568$.

(g) Find the 95% t-confidence interval for μ, where μ is the population mean weight gain of girls receiving the cognitive behavioral therapy. Based on the constructed interval, explain why it suggests that the true mean change in weight is positive, but possibly quite small.

Answer: *[4pts = 3pts for the CI + 1pt for the comment]* With df = 28, the critical value for 95% CI is $t^* = 2.0484$ can be found with either of the following R commands

```
> qt(0.05/2, df=28, lower.tail=F)
[1] 2.048407
> qt(1-0.05/2, df=28)
[1] 2.048407
```

The 95% CI is

$$\bar{x} \pm t^* s/\sqrt{n} = \underbrace{3}_{1pt} \pm \underbrace{2.0484}_{1pt} \times \underbrace{7.32/\sqrt{29}}_{1pt} \approx 3 \pm 2.79 = (0.21, 5.79).$$

Comment: *[1pt]* The 95% CI contains positive values only, suggesting the mean weight gain is positive. However, the CI suggests the mean weight gain can possibly be as small as 0.21 lbs, which is quite small.

(h) Verify your computations in part (c)(d)(f)(g) with the R commend `t.test`.

```
cognitive = c(1.7,11.7,-1.4,0.7,6.1,-0.8,-0.1,1.1,2.4,-0.7,-4.0,12.6,-3.5,20.9,
              1.9,14.9,-9.3,3.9,3.5,2.1,0.1,17.1,1.4,15.4,-7.6,-0.3,-0.7,1.6,-3.7)
t.test(cognitive, alternative = "greater")
t.test(cognitive, alternative = "two.sided")
```

Answer: [*0pt*] The R output for the upper one-sided t test is as follows.

```
        One Sample t-test

data:  cognitive
t = 2.2069, df = 28, p-value = 0.01784
alternative hypothesis: true mean is greater than 0
95 percent confidence interval:
 0.6875368       Inf
sample estimates:
mean of x
        3
```

We see the t-statistic 2.2069, df = 28 and the one-sided p-value 0.01784 lies between 0.01 and 0.025. Both agree with our computations in (c) and (d). Note that the 95% CI given (0.6875368, Inf) = $(0.6875368, \infty)$ is one sided (which we didn't cover) as we asked R to conduct a one-sided test.
The R output for the two-sided t test is as follows.

```
        One Sample t-test

data:  cognitive
t = 2.2069, df = 28, p-value = 0.03569
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.2154606 5.7845394
sample estimates:
mean of x
        3
```

We see the t-statistic 2.2069, df = 28 are unchanged, the two-sided P-value 0.03569 lies between 0.02 and 0.05, and the 95% CI $(0.2154606, 5.7845394)$. All agree with our computations in (f) and (g). Observe that the two-sided p-value 0.03569 is twice the one-sided p-value 0.01784.

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3. Determine if the following statements are true or false, and explain your reasoning. If false, state how it could be corrected.

   (a) Decreasing the significance level (α) will increase the probability of making a Type 1 Error.

   (b) If a given value is within a 90% confidence interval for a parameter, it will also be within a 95% confidence interval.

   (c) Suppose the null hypothesis is $\mu = 5$ and we fail to reject H_0. Under this scenario, the true population mean is 5.

Answer: [*Give 0pt if the reason is wrong*]

(a) [*2pts*] False. The significance level is the probability of the Type 1 Error. So increasing the significance level will increase the chance of making a Type 1 error, not decrease it.

(b) [*1pt*] True.

(c) [*2pts*] False. Failure to reject H_0 only means there wasn't sufficient evidence to reject it, not that it has been confirmed.
