Task B

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2024-11-05

B.1

(1)

According to the background of B.1, the probability density function $p_\lambda(x)$ of a random variable X is:

$$p_\lambda(x) = \begin{cases} ae^{-\lambda(x-b)} & \text{if } x \geq b, \\ 0 & \text{if } x < b, \end{cases}$$

where: \$ b > 0 \$ is a known constant, $\lambda > 0$ is a parameter of the distribution, \$ a \$ is a constant to be determined in terms of λ and b.

According to the definition of probability density function, $p_\lambda(x)$ must integrate to 1 over its domain, so an equation can be written:

$$\int_{-\infty}^{\infty} p_\lambda(x)\, dx = 1.$$

when $x < b$, $p_\lambda(x) = 0$, so we only need to calculate the integral from $x = b$ to $x = \infty$. To set up the integral, the equation can be written:

$$\int_b^\infty ae^{-\lambda(x-b)}\, dx = 1.$$

a is a constant number, so it can be factored out and just calculate the remaining part:

$$a \int_b^\infty e^{-\lambda(x-b)}\, dx = 1.$$

Let $\mu = x - b$, so domain changes to $\{0, \infty\}$ and the euqation should be transformed:

$$a \int_0^\infty e^{-\lambda\mu}\, d\mu = 1.$$

Solve the integral:

$$\begin{aligned} a \int_0^\infty e^{-\lambda\mu} d\mu &= -\frac{1}{\lambda} \cdot e^{-\lambda\mu}|_0^\infty \\ &= a \cdot [-\frac{1}{\infty} \cdot e^{-\infty\mu} - (-\frac{1}{0} \cdot e^{-0\mu})] \\ &= a \cdot [0 - (-\frac{1}{\lambda})] \\ &= a \cdot \frac{1}{\lambda} = 1. \end{aligned}$$

so it is obvious that

$$a = \lambda$$

The answer of question(1) is $a = \lambda$

(2)