4.7 Exercises

1.

$$P(x) * \left(1 + e^{\beta_0+\beta_1 x}\right) = e^{\beta_0+\beta_1 x}$$

$$\frac{P(x)}{\frac{1}{1 + e^{\beta_0+\beta_1 x}}} = e^{\beta_0+\beta_1 x}$$

$$\frac{P(x)}{1 - \left(\frac{e^{\beta_0+\beta_1 x}}{1 + e^{\beta_0+\beta_1 x}}\right)} = e^{\beta_0+\beta_1 x}$$

$$\frac{P(x)}{1 - P(x)} = e^{\beta_0+\beta_1 x}$$

4.

(a).

Since the set of observations X is uniformly distributed on [0, 1], and we are using a 10% range, that means every X in the distribution are equally probable to be chosen. So, the fraction of available information used is 10% on average.

(b).

From uniformly distribution we can know that the fraction is the intersection of two observations. So the probability = X1 length * X2 length = 0.01, the fraction of available information used is 1% on average.

(c).

Applying the rule above, when p=100, 0.1^p × 100 = 0.1^100 × 100 of the observations are available.

(d).

When p is large there are very few observations which to build test near the given test observation. As the rule above, the fraction of points near a test observation can becomes exponentially smaller when p becomes larger. Because being near in every dimension to a point is a strict condition and this gets less and less likely as the number dimensions increases.

(e).

Backstepping from the rule above, we can know that:

If p=1, length of each side = 0.1^1 = 0.1

If p=2, length of each side = 0.1^ (1/2) = 0.32

If p=1, length of each side = 0.1^ (1/100) = 0.98