
ECE590 HW1

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1 Assignment 1

Proof Goal:

$$\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$$

Steps:

1. $\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$

Pick any P and Q ,

1.1 $(P \Rightarrow Q) \Rightarrow (\neg P \vee Q)$

Assume $P \Rightarrow Q$ and

1.1.1 $\neg P \vee Q$

By contradiction, assume $\neg(\neg P \vee Q)$

1.1.2 $\neg\neg P \wedge \neg Q$

From 1.1.1 by de Morgan's law

1.1.2.1 P

From 1.1.2 by and elimination and double negation elimination

1.1.2.2 Q

From 1.1 and 1.1.2.1 by implication

1.1.2.3 False by case on $\neg\neg P \wedge \neg Q$

1.2 $(\neg P \vee Q) \Rightarrow (P \Rightarrow Q)$

Assume $\neg P \vee Q$ and

1.2.1 $P \Rightarrow Q$

By contradiction, assume $\neg P \Rightarrow Q$

1.2.1.1 P

From 1.2.1

1.2.1.2 $\neg Q$

From 1.2.1

1.2.1.3 False by case on $(\neg P \vee Q)$

1.3 $\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$

From 1.1 to 1.2

2 Assignment 2

Proof

$f(n)$ is the statement for

$$\forall n \in \mathbb{N}, \sum_{i=0}^n i^3 = \frac{1}{4}n^2(n+1)^2$$

Base case $f(1)$:

$$\frac{1}{4} \cdot 1^2(1+1)^2 = \frac{2^2}{4} = \frac{4}{4} = 1$$

Induction step: Assume $f(n)$ for all $n > 1$

Proof of $f(n+1)$

$$1^3 + 2^3 + \dots + n^3 = \frac{1}{4}n^2(n+1)^2$$

$$\begin{aligned} 1^3 + 2^3 + \dots + n^3 + (n+1)^3 &= \frac{1}{4}n^2(n+1)^2 + (n+1)^3 \\ &= \frac{(n+1)^2}{4}[n^2 + 4(n+1)] \\ &= \frac{(n+1)^2(n+2)^2}{4} \end{aligned}$$

So we prove

$$\forall n \in \mathbb{N}, \sum_{i=0}^n i^3 = \frac{1}{4}n^2(n+1)^2$$

3 Assignment 3

There is something wrong with the induction step. It is not valid when k is 1. When k is 1, consider the set of $k+1$, which is 2 horses. There is no overlap between the first k horses and last k horses. So the first k horses and last k horses may have different colors. So the $k+1$ horses may fail to have the same color.

References