# ECE590 HW1

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## Contents

# 1 Assignment 1

Proof Goal:

$$\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$$

Steps:

1. $\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$
   Pick any $P$ and $Q$,

   1.1 $(P \Rightarrow Q) \Rightarrow (\neg P \vee Q)$
   Assume $P \Rightarrow Q$ and

   1.1.1 $\neg P \vee Q$
   By contradiction, assume $\neg(\neg P \vee Q)$

   1.1.2 $\neg\neg P \wedge \neg Q$
   From 1.1.1 by de Morgan's law

   1.1.2.1 $P$
   From 1.1.2 by and elimination and double negation elimination

   1.1.2.2 $Q$
   From 1.1 and 1.1.2.1 by implication

   1.1.2.3 False by case on $\neg\neg P \wedge \neg Q$

   1.2 $(\neg P \vee Q) \Rightarrow (P \Rightarrow Q)$
   Assume $\neg P \vee Q$ and

   1.2.1

# 2 Assignment 2

Proof

$f(n)$ is the statement for

$$\forall n \in \mathbb{N}, \sum_{i=0}^{n} i^3 = \frac{1}{4} n^2 (n+1)^2$$

Base case $f(1)$ :

$$\frac{1}{4} \cdot 1^2 (1+1)^2 = \frac{2^2}{4} = \frac{4}{4} = 1$$

Induction step: Assume $f(n)$ for all $n > 1$
Proof of $f(n+1)$

$$1^3 + 2^3 + \ldots + n^3 = \frac{1}{4} n^2 (n+1)^2$$

$$\begin{aligned} 1^3 + 2^3 + \ldots + n^3 + (n+1)^3 &= \frac{1}{4} n^2 (n+1)^2 + (n+1)^3 \\ &= \frac{(n+1)^2}{4} [n^2 + 4(n+1)] \\ &= \frac{(n+1)^2 (n+2)^2}{4} \end{aligned}$$

So we prove

$$\forall n \in \mathbb{N}, \sum_{i=0}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$$

## 3 Assignment 3

There is something wrong with the induction step. It is not valid when k is 1. When k is 1, consider the set of k+1, which is 2 horses. There is no overlap between the first k horses and last k horses. So the first k horses and last k horses may have different colors. So the k+1 horses may fail to have the same color.

# References