ECE590 HW1

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Contents

1 Assignment 1

Proof Goal:

$$\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$$

Steps:

1. $\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$
   Pick any P and Q,

   1.1 $(P \Rightarrow Q) \Rightarrow (\neg P \vee Q)$
       Assume $P \Rightarrow Q$ and

       1.1.1 $\neg P \vee Q$
             By contradiction, assume $\neg(\neg P \vee Q)$

       1.1.2 $\neg\neg P \wedge \neg Q$
             From 1.1.1 by de Morgan's law

          1.1.2.1 P
                  From 1.1.2 by and elimination and double negation elimination

          1.1.2.2 Q
                  From 1.1 and 1.1.2.1 by implication

          1.1.2.3 False by case on $\neg\neg P \wedge \neg Q$

   1.2 $(\neg P \vee Q) \Rightarrow (P \Rightarrow Q)$
       Assume $\neg P \vee Q$ and

       1.2.1 $\neg P \vee P$

       1.2.2 $\neg Q \vee Q$

       1.2.3

       case P

          1.2.3.1 Q
                  From $\neg P \vee Q$

          1.2.3.2 $P \Rightarrow Q$
                  From P, Q

       case $\neg P$

          case $\neg Q$

          1.2.3.3 $P \Rightarrow Q$

          case Q

          1.2.3.4 $P \Rightarrow Q$

       1.2.4 $(\neg P \vee Q) \Rightarrow (P \Rightarrow Q)$ From 1.2.1 to 1.2.3

   1.3 $\forall P, Q \in \mathbb{N}, (P \Rightarrow Q) \iff (\neg P \vee Q)$
       From 1.1 to 1.2

2 Assignment 2

Proof

$f(n)$ is the statement for

$$\forall n \in \mathbb{N}, \sum_{i=0}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$$

Base case $f(1)$:

$$\frac{1}{4} \cdot 1^2(1+1)^2 = \frac{2^2}{4} = \frac{4}{4} = 1$$

Induction step: Assume $f(n)$ for all $n > 1$
Proof of $f(n+1)$

$$1^3 + 2^3 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2$$

$$\begin{aligned}
1^3 + 2^3 + \ldots + n^3 + (n+1)^3 &= \frac{1}{4}n^2(n+1)^2 + (n+1)^3 \\
&= \frac{(n+1)^2}{4}[n^2 + 4(n+1)] \\
&= \frac{(n+1)^2(n+2)^2}{4}
\end{aligned}$$

So we prove

$$\forall n \in \mathbb{N}, \sum_{i=0}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$$

3 Assignment 3

There is something wrong with the induction step. It is not valid when k is 1. When k is 1, consider the set of k+1, which is 2 horses. There is no overlap between the first k horses and last k horses. So the first k horses and last k horses may have different colors. So the k+1 horses may fail to have the same color.

References