HW2

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I.Matrix Representation of Multiple Linear Regression

1.Use the following 5 observations and write the simple linear regression model in matrix terms. Then using the least squares calculations in matrix notation, compute estimates for the simple linear regression intercept and slope.

```
y <- c(-0.1, 2.9, 6.2, 7.3, 10.7)
x <- matrix(c(1,1,1,1,1,1,3,5,7,9),nrow=5,ncol=2) ## The design matrix
x; y
```

```
##      [,1] [,2]
## [1,]    1    1
## [2,]    1    3
## [3,]    1    5
## [4,]    1    7
## [5,]    1    9
```

```
## [1] -0.1  2.9  6.2  7.3 10.7
```

$$Y = \begin{bmatrix} -0.1 \\ 2.9 \\ 6.2 \\ 7.3 \\ 10.7 \end{bmatrix}, X = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}, \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \end{bmatrix}$$

Model: $Y = \beta X + \epsilon$
Where $\epsilon_i\ N(0, \sigma^2)$ and $Cov(\epsilon_i, \epsilon_j) = 0$ $\hat{\beta} = (X^T X)^{-1} X^T Y$ So,

$$\hat{\beta} = (\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}^T * \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \\ 1 & 7 \\ 1 & 9 \end{bmatrix})^{-1} * \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}^T * \begin{bmatrix} -0.1 \\ 2.9 \\ 6.2 \\ 7.3 \\ 10.7 \end{bmatrix} = \begin{bmatrix} -1.1 \\ 1.3 \end{bmatrix}$$

```
solve(t(x) %*% x) %*% t(x) %*% y
```

```
##      [,1]
## [1,] -1.1
## [2,]  1.3
```