Exercise 1.

the hyperplane equation is $0.5 - 3x_1 + x_2 = 0$.

if they are support vector, the points should be on the margin boundary line.

for $x_1 (\frac{1}{6}, \frac{3}{2}, 1)$. $0.5 - 3\times\frac{1}{6} + \frac{3}{2} = \frac{3}{2} > 1$.

x_1 is not on the hyperplane, either not on margin boundary,

so its is not support vector.

for $x_2 (\frac{1}{3}, -\frac{1}{2}, -1)$, $0.5 - 3\times\frac{1}{3} - \frac{1}{2} = -1$

x_2 is support vector.

for $x_3 (\frac{1}{2}, 2, 1)$ $0.5 - 3\times\frac{1}{2} + 2 = 1$

x_3 is support vector

for $x_4 (\frac{1}{3}, 1, 1)$, $0.5 - 3\times\frac{1}{3} + 1 = 0.5 < 1$

x_4 is not on hyperplane, either not on margin boundary. so it is not support vector

for $x_5 (\frac{1}{4}, \frac{3}{4}, -1)$ $0.5 - 3\times\frac{1}{4} + \frac{3}{4} = 0.5 < 1$.

x_5 is not on hyperplane, either not on margin boundary. so it is not support vector

so, only x_2, x_3 are the support vector of hyperplane $\{x_1, x_2 : 0.5 - 3x_1 + x_2 = 0\}$

Exercise 2

1). using euclidean distance to calculate every points to two centroid $u_1^{(t)}$ and $u_2^{(t)}$.

$$u_1^{(t)} = (0.62, 0.06, 1.63) \qquad u_2^{(t)} = (-0.11, 0.92, 0.02)$$

for points $X_i = (x_{i1}, x_{i2}, x_{i3})$, we can calculate distance:

$$d(x_i, u_1^{(t)}) = \sqrt{(x_{i1} - 0.62)^2 + (x_{i2} - 0.06)^2 + (x_{i3} - 1.63)^2}$$

$$d(x_i, u_2^{(t)}) = \sqrt{(x_{i1} + 0.11)^2 + (x_{i2} - 0.92)^2 + (x_{i3} - 0.02)^2}$$

so. for $X = \begin{bmatrix} 1.76 & -1.30 & 2.39 \\ 2.41 & 0.66 & 0.41 \\ -2.63 & 1.18 & -0.36 \\ 0.82 & 0.64 & 1.66 \\ -0.73 & 0.83 & 0.85 \end{bmatrix}$

so, $d(x_1, u_1^{(t)}) = \sqrt{(1.76-0.62)^2 + (-1.30-0.06)^2 + (2.39-1.63)^2} = 1.93049$

$d(x_1, u_2^{(t)}) = \sqrt{(1.76+0.11)^2 + (-1.30-0.92)^2 + (2.39-0.02)^2} = 3.74729$

$d(x_2, u_1^{(t)}) = \sqrt{(2.41-0.62)^2 + (0.66-0.06)^2 + (0.41-1.63)^2} = 2.24778$

$d(x_2, u_2^{(t)}) = \sqrt{(2.41+0.11)^2 + (0.66-0.92)^2 + (0.41-0.02)^2} = 2.56322$

$d(x_3, u_1^{(t)}) = \sqrt{(-2.63-0.62)^2 + (1.18-0.06)^2 + (-0.36-1.63)^2} = 3.97203$

$d(x_3, u_2^{(t)}) = \sqrt{(-2.63+0.11)^2+(1.18-0.92)^2+(-0.36-0.02)^2} = 2.56172$

$d(x_4, u_1^{(t)}) = \sqrt{(0.82-0.62)^2+(0.64-0.06)^2+(1.66-1.63)^2} = 0.61425$

$d(x_4, u_2^{(t)}) = \sqrt{(0.82+0.11)^2+(0.64-0.92)^2+(1.66-0.02)^2} = 1.80889$

$d(x_5, u_1^{(t)}) = \sqrt{(-0.73-0.62)^2+(0.83-0.06)^2+(0.85-1.63)^2} = 1.73891$

$d(x_5, u_2^{(t)}) = \sqrt{(-0.73+0.11)^2+(0.83-0.92)^2+(0.85-0.02)^2} = 1.03990.$

$d(x_1, u_1^{(t)}) < d(x_1, u_2^{(t)})$, so we give x_1 to cluster 1. $d(x_2, u_1^{(t)}) < d(x_2, u_2^{(t)})$, so we give x_1 to cluster 1.
$d(x_3, u_1^{(t)}) > d(x_3, u_2^{(t)})$, so we give x_1 to cluster 2. $d(x_4, u_1^{(t)}) < d(x_4, u_2^{(t)})$, so we give x_1 to cluster 1
$d(x_5, u_1^{(t)}) > d(x_5, u_2^{(t)})$, so we give x_1 to cluster 2.

and now we know for cluster 1. we have 3 points.

it's $X_1 = \begin{bmatrix} 1.76 & -1.30 & 2.39 \\ 2.41 & 0.66 & 0.41 \\ 0.82 & 0.64 & 1.66 \end{bmatrix}$

So the new centroid $u_1^{(t+1)} = \left(\frac{\sum x_{i1}}{n_1}, \frac{\sum x_{i2}}{n_1}, \frac{\sum x_{i3}}{n_1}\right) = (1.663, 0, 1.487)$

and for cluster 2, we have 2 points.

it's $X_2 = \begin{bmatrix} -2.63 & 1.18 & -0.36 \\ -0.73 & 0.83 & 0.85 \end{bmatrix}$

so the new centroid $u_2^{(t+1)} = \left(\frac{\sum x_{i1}}{n_2}, \frac{\sum x_{i2}}{n_2}, \frac{\sum x_{i3}}{n_3}\right) = (-1.68, 1.005, 0.245)$

2. for the cluster sum of squares moving from iteration t to iteration $t+1$:

for iteration t:

points in cluster 1: x_1, x_4, x_5 $\quad u_1^{(t)} = (0.62, 0.06, 1.63)$
points in cluster 2: x_2, x_3, $\quad u_2^{(t)} = (-0.11, 0.92, 0.02)$.

So for cluster 1:

$$WCSS_1 = (x_{11}-u_{11}^{(t)})^2 + (x_{12}-u_{12}^{(t)})^2 + (x_{13}-u_{13}^{(t)})^2 + (x_{51}-u_{11}^{(t)})^2 + (x_{52}-u_{12}^{(t)})^2 + (x_{53}-u_{13}^{(t)})^2 + (x_{41}-u_{11}^{(t)})^2 + (x_{42}-u_{12}^{(t)})^2 + (x_{43}-u_{13}^{(t)})^2$$
$$= (1.76-0.62)^2+(-1.3-0.06)^2+(2.39-1.63)^2+(-0.73-0.62)^2+(0.83-0.06)^2+(0.85-1.63)^2+(0.82-0.62)^2+(0.64-0.06)^2+(1.66-1.63)^2$$
$$= 7.1279$$

for cluster 2:

$$WCSS_2 = (x_{21}-u_{21}^{(t)})^2 + (x_{22}-u_{22}^{(t)})^2 + (x_{23}-u_{23}^{(t)})^2 + (x_{31}-u_{21}^{(t)})^2 + (x_{32}-u_{22}^{(t)})^2 + (x_{33}-u_{23}^{(t)})^2$$
$$= (2.41+0.11)^2+(0.66-0.92)^2+(0.41-0.02)^2+(-2.63+0.11)^2+(1.18-0.92)^2+(-0.36-0.02)^2$$
$$= 13.1325$$

for iteration $t+1$:

points in cluster 1: x_1, x_2, x_4 $\quad u_1^{(t+1)} = (1.663, 0, 1.487)$.

points in cluster 2: x_3, x_5. $\quad u_2^{(t+1)} = (-1.68, 1.005, 0.245)$

for cluster 1:

$$WCSS_1 = (x_{11} - u_{11}^{(t+1)})^2 + (x_{12} - u_{12}^{(t+1)})^2 + (x_{13} - u_{13}^{(t+1)})^2 + (x_{21} - u_{11}^{(t+1)})^2 + (x_{22} - u_{12}^{(t+1)})^2 + (x_{23} - u_{13}^{(t+1)})^2 + (x_{41} - u_{11}^{(t+1)})^2 + (x_{42} - u_{12}^{(t+1)})^2 + (x_{43} - u_{13}^{(t+1)})^2$$

$$= (1.76 - 1.663)^2 + (-1.3 - 0)^2 + (2.39 - 1.487)^2 + (2.41 - 1.663)^2 + (0.66 - 0)^2 + (0.41 - 1.487)^2 + (0.82 - 1.663)^2 + (0.64 - 0)^2 + (1.66 - 1.487)^2$$

$$= 5.818534$$

for cluster 2:

$$WCSS_2 = (x_{31} - u_{21}^{(t+1)})^2 + (x_{32} - u_{22}^{(t+1)})^2 + (x_{33} - u_{23}^{(t+1)})^2 + (x_{51} - u_{21}^{(t+1)})^2 + (x_{52} - u_{21}^{(t+1)})^2 + (x_{53} - u_{23}^{(t+1)})^2$$

$$= (-2.63 + 1.68)^2 + (1.18 - 1.005)^2 + (-0.36 - 0.245)^2 + (-0.73 + 1.68)^2 + (0.83 - 1.005)^2 + (0.85 - 0.245)^2$$

$$= 2.5983.$$

WCSS of cluster 1 decreased from 7.1279 to 5.818534, which indicates that the points in cluster 1 are more tightly clustered around the new centroid.

WCSS of cluster 2 decreased significantly from 13.1325 to 2.5983, which indicates points in cluster 2 are clustered more closely after updating the centroid.