Portfolio Optimization: A Monte Carlo Study

Zixuan Zhang

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1. Introduction

Portfolio optimization is a core concept in modern finance, aiming to allocate capital among a set of risky assets to achieve the best trade-off between return and risk. Traditional approaches such as Markowitz mean-variance optimization use quadratic programming to identify efficient portfolios. However, when return distributions are uncertain or intractable, Monte Carlo simulation provides a practical alternative for exploring the feasible investment space.

This study investigates portfolio opportunity sets using Monte Carlo simulation under two investment strategies:

Long positions only (no short selling)

Shorts allowed (positions can be negative)

Four assets are selected for analysis: SPY (S&P 500 ETF), GLD (Gold ETF), TLT (Long-Term Treasury ETF), and QQQ (Nasdaq-100 ETF). Their expected returns, volatilities, and correlations are estimated based on historical performance.

2. Methodology

2.1 Asset Assumptions

The selected assets and their assumed annualized return and standard deviation are summarized below:

Asset	Description	Expected Return (μ)	Std Dev (σ)
SPY	S&P 500 ETF	0.10	0.15
GLD	Gold ETF	0.07	0.12
TLT	Long-Term Treasury ETF	0.04	0.08
QQQ	Nasdaq-100 ETF	0.15	0.20

The correlation matrix between the assets is assumed as:

```
cor_matrix = np.array([
    [1.00, 0.20, 0.10, 0.85],
    [0.20, 1.00, 0.30, 0.15],
    [0.10, 0.30, 1.00, 0.05],
    [0.85, 0.15, 0.05, 1.00]
])
```

From this, a 4x4 covariance matrix is computed and used for simulation.

2.2 Monte Carlo Simulation Design

In this study, a Monte Carlo simulation approach is used to explore the opportunity sets of portfolio returns and risks under different investment constraints. I begin by simulating 700 return scenarios for the four selected assets. These returns are generated using a multivariate normal distribution, based on the previously specified mean vector and covariance matrix.

Next, for each investment strategy, shorts allowed and long-only, I generate 700 random portfolio weight vectors. In the shorts-allowed case, weights are permitted to take negative values, as long as they sum to one. In contrast, the long-only strategy requires all weights to lie within the interval [0, 1], also summing to one, reflecting the restriction against short selling.

For each simulated portfolio, I compute two key performance metrics: the expected return and the associated risk. The portfolio return is calculated as the mean of the dot product between the portfolio weight vector and the asset return vector. The formula is below.

$$\mu_p = \mathbb{E}[w^\top r]$$

The portfolio risk is computed as the standard deviation of returns using the formula below, where $\sum \boldsymbol{w}$ is the covariance matrix of asset returns.

$$\sigma_p = \sqrt{w^\top \Sigma w}$$

Finally, visualize the results to compare the distribution and range of portfolio returns and risks under the two strategies. This allows for a direct comparison of the opportunity sets, highlighting the trade-offs between risk and return in each scenario.

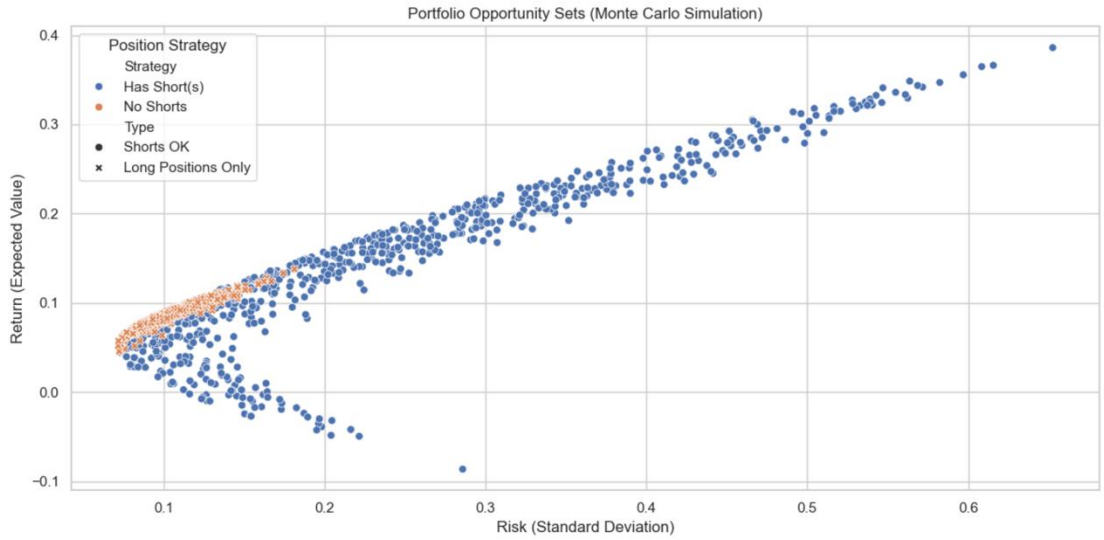
3. Results

The following figure presents the simulation results. Each dot represents one portfolio realization, with:

X-axis: Portfolio risk (standard deviation)

Y-axis: Portfolio expected return

Color/Shape: Investment strategy (shorts allowed vs. long-only)



The simulation results reveal several important patterns in the distribution of portfolio risk and return. Long-only portfolios, represented by orange X markers in the plot, are concentrated in the lower-left region of the graph. This reflects their inherently conservative nature, as they tend to yield relatively low returns while maintaining low levels of risk. In contrast, portfolios that include short positions, shown as blue dots, are distributed across a much broader spectrum. These portfolios reach into areas of both higher expected returns and higher risk, demonstrating the greater flexibility and potential upside that short-selling allows.

The shape of the opportunity set in each strategy suggests the presence of an efficient frontier, visually indicated by the upper-left boundary of the point cloud. This frontier outlines the set of portfolios offering the highest possible return for a given level of risk. Notably, many portfolios with short positions achieve expected returns exceeding 0.20, a level that is unattainable under the constraints of the long-only strategy. This observation underscores the trade-off between risk and return, and highlights how the ability to take short positions can significantly expand the set of viable investment opportunities.

4. Conclusion

Monte Carlo simulation provides an intuitive and flexible method for exploring the feasible set of portfolios under different investment constraints. This study confirms that:

Long-only strategies limit exposure to low-risk, low-return portfolios, suitable for risk-averse investors.

Allowing short positions expands the opportunity set, offering access to higher returns at the cost of increased volatility.

A key takeaway is that investors with moderate to high risk tolerance may benefit from short positions, provided they manage downside risks carefully.