CSE 351 - SIGNAL AND SYSTEMS

Assignment - 2

1. Find the inverse z-transform of

a. $\dfrac{8z-19}{(z-2)(z-3)} = X(z)$ $\qquad \dfrac{X(z)}{z} = \dfrac{8z-19}{z(z-2)(z-3)}$

$$\frac{A}{z} + \frac{B}{(z-2)} + \frac{C}{(z-3)} = \frac{8z-19}{z(z-2)(z-3)}$$

$$A(z-2)(z-3) + B\,z(z-3) + C\,z(z-2) = 8z-19$$

$z=0$

$6A = -19 \qquad A = \frac{-19}{6}$

$z=2$

$-2B = -3 \qquad B = \frac{3}{2}$

$z=3$

$3C = 5 \qquad C = \frac{5}{3}$

$$X(z) = -\frac{19}{6} + \frac{\frac{3}{2}z}{z-2} + \frac{\frac{5}{3}z}{z-3}$$

z - Transform Table

$$\boxed{x(n) = \frac{-19}{6}\delta[n] + \frac{3}{2}\,2^n u[n] + \frac{5}{3}\,3^n u[n]}$$

b. $\dfrac{z(2z^2-11z+12)}{(z-1)(z-2)^3} = X(z)$

$$\frac{X(z)}{z} = \frac{2z^2-11z+12}{(z-1)(z-2)^3} \qquad \frac{(2z-3)(z-4)}{(z-1)(z-2)^3} = \frac{X(z)}{z}$$

$$\frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2} + \frac{D}{(z-2)^3} = \frac{(2z-3)(z-4)}{(z-1)(z-2)^3}$$

$$A(z-2)^3 + B(z-1)(z-2)^2 + C(z-1)(z-2) + D(z-1) = (2z-3)(z-4)$$

$z=1$

$-A = 3 \qquad A = 3$

$z=2$

$D = -2 \qquad D = -2$

$z=3$

$A + 2B + 2C + 2D = -3$

$B + C = 2$

$$\boxed{\begin{aligned} A &= -3 \\ B &= 3 \\ C &= -1 \\ D &= -2 \end{aligned}}$$

$z=0$

$-8A - 4B + 2C - D = 12$

$24 - 4B + 2C + 2 = 12$

$-4B + 2C = -14$

$2B + 2C = +4$

$-6B = -18 \qquad B = 3 \qquad C = -1$

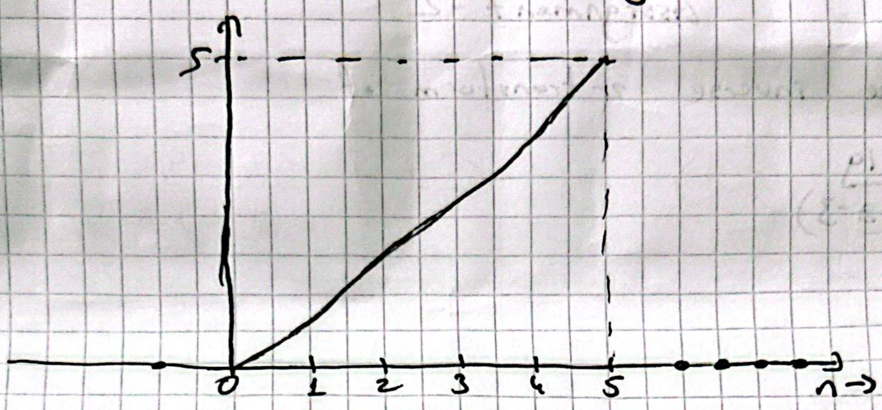
$$X(z) = \frac{-3}{(z-1)} + \frac{3}{(z-2)} - \frac{1}{2}\frac{2}{(z-2)^2} + \frac{-2}{(z-2)^3}$$

$x(n) =$

$$-3[1]^n u[n] + 3\cdot 2^n u[n] - \frac{1}{2} n\,2^n u[n] - 2\,\frac{n\cdot n-1}{2^2\,2}\,2^n u(n)$$

$$\boxed{x(n) = \left[-3[1]^n + 3\cdot 2^n - \frac{1}{2} n\,2^n - \frac{n\cdot n-1}{4}\,2^n\right] u(n)}$$

2 Find the z-transform of the signal $x[n]$ depicted below.



$$x(n) = n \cdot (u[n] - u(n-6))$$

$$x(n) = n \cdot u[n] - n\,u(n-6)$$

$$x(n) = n \cdot u[n] - (n-6)\,u[n-6] + 6\,u[n-6]$$

z-Transform

$$X(z) = \frac{z}{(z-1)^2} - \frac{1}{z^6}\,\frac{z}{(z-1)^2} + 6 \cdot \frac{1}{z^6}\,\frac{z}{z-1}$$

$$\boxed{X(z) = \frac{z}{(z-1)^2} - \frac{1}{z^5}\,\frac{1}{(z-1)^2} + 6\,\frac{1}{z^5} \cdot \frac{z}{z-1}}$$

3. Find the response $y[n]$ of an LTID system described by the difference equation

$$y[n+2] + y[n+1] + 0.22\,y[n] = x[n+1] + 0.44\,x[n]$$

for the input $x[n] = (-2)^{-n} u(n)$ with all the initial conditions zero (system in zero state)

$$H(z) = \frac{P(z)}{Q(z)} = \frac{z + 0{,}44}{z^2 + z + 0{,}22}$$

$$x[n] = (-0{,}5)^n \cdot u[n]$$

$$X[z] = \frac{z}{z+0{,}5}$$

$$Y(z) = X(z) \cdot H(z) = \frac{z\,(z+0{,}44)}{(z+0{,}5)\,(z^2+z+0{,}22)}$$

$$\frac{Y(z)}{z} = \frac{(z+0{,}44)}{(z+0{,}5)\,(z^2+z+0{,}22)} \qquad \frac{z+(0{,}44)}{(z+0{,}5)\,(z+0{,}33)\cdot(z+0{,}67)}$$

$$\frac{A}{z+0{,}5} + \frac{B}{z+0{,}33} + \frac{C}{z+0{,}67} = A(z+0{,}33)(z+0{,}67) + B(z+0{,}5)(z+0{,}67) + C(z+0{,}5)(z+0{,}33)$$

Using Calculator

$A = 2{,}08$
$B = 1{,}90$
$C = 3{,}98$

$$\boxed{y(n) = 2{,}08\;0{,}5^n\,u(n) + 1{,}90\,(0{,}33)^n\,u[n] - 3{,}98\,(0{,}67)^n\,u(n)}$$

4.

$$w_0 = \frac{2\pi}{2} = \pi$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi t + b_n \sin n\pi t$$

$$x(t) = \begin{cases} 2At & t<\frac{1}{2} \\ 2A(1-t) & \frac{1}{2}<t<\frac{3}{2} \end{cases}$$

$$a_n = \frac{2}{2}\int_{-1/2}^{3/2} x(t) = \cos n\pi t . dt$$

$$= \int_{-1/2}^{1/2} 2At \cos n\pi t . dt + \int_{1/2}^{3/2} 2A(1-t) \cos n\pi t . dt.$$

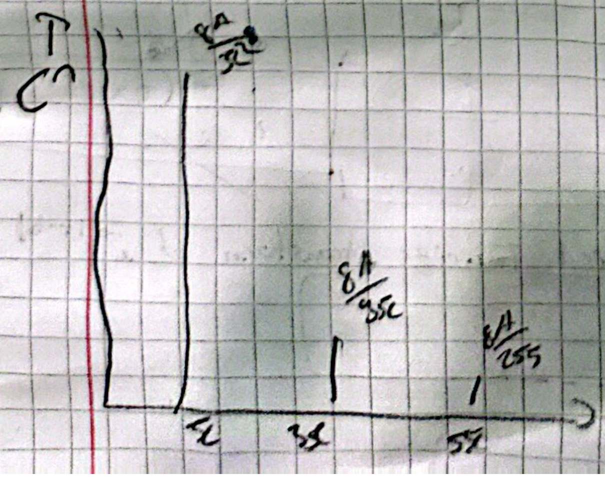
$$b_n = \int_{-1/2}^{1/2} 2At \cdot \sin n\pi t . dt + \int_{1/2}^{3/2} 2A(1-t) \cdot \sin n\pi t . dt)$$

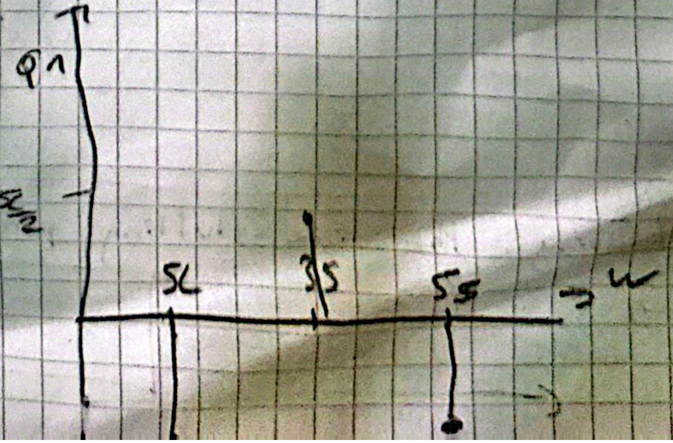
$$b_n = \frac{8A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & n \text{ even} \\ \frac{8A}{n^2\pi^2} & n = 1,5,9 \\ \frac{-8A}{n^2\pi^2} & n = 3,7,11 \end{cases}$$

1

$$x(t) = \frac{8A}{\pi^2}\left(\sin \pi t + \frac{1}{9}\sin 3\pi t + \frac{1}{25}\sin 5\pi t + -\frac{1}{49}\sin 7\pi t + \dots\right)$$

$\sin kt = \cos(kt-90)$ $\qquad$ $-\sin kt = \cos(kt+90)$





5. Determine the fundamental frequency and period of following signals.

a) $x(t) = 2 + 7\cos(\frac{1}{2}t + \theta_1) + 3\cos(\frac{2}{3}t + \theta_2) + 5\cos(\frac{7}{6}t + \theta_3)$

frequencies $= \frac{1}{2}, \frac{2}{3}, \frac{7}{6}$

$\boxed{\frac{2\pi}{T} = w}$

$GCD = \frac{1}{6}$

$\boxed{T = \frac{1}{f}}$

fundamental frequency $= w_0 = \frac{1}{6}$

fundamental period $= T = 12\pi$

b) $x(t) = 2\cos(2t + \theta_1) + 5\sin(\pi t + \theta_2)$

frequencies $= 2, \pi$

the ratio is not rational, thats why it is not periodic.

c) $x(t) = 3\sin(3\sqrt{2}t + \theta) + 7\cos(6\sqrt{2}t + \phi)$

frequencies $= 3\sqrt{2}, 6\sqrt{2}$

$GCD = 3\sqrt{2}$

fundamental frequencies $= w_0 = 3\sqrt{2}$

fundamental period $= T_0 = 2\pi / 3\sqrt{2} = \frac{2\pi\sqrt{2}}{6} = \boxed{\frac{\pi\sqrt{2}}{3}}$

6. Find the Fourier transform of $e^{-a|t|}$

$t<0 \quad t\geq 0$

$$e^{-at} = \begin{cases} e^{at}, & t<0 \\ e^{-at} & t>0 \end{cases}$$

$$x(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$\Downarrow$

$$x(w) = \int_{-\infty}^{0} e^{at} e^{-jwt}\, dt + \int_{0}^{\infty} e^{-at} e^{-jwt}\, dt$$

$x(w) = \int e^{(a-jw)t}\, dt$

$x(w) = \int e^{-(a+jw)t}\, dt$

$$\left. \frac{e^{(a-jw)t}}{a-jw} \right|_{-\infty}^{0} \qquad \left. -\frac{e^{-(a+jw)t}}{a+jw} \right|_{0}^{\infty}$$

$\Downarrow \qquad \Downarrow$

$$\frac{1}{a-jw} + \frac{1}{a+jw} = \boxed{\frac{2a}{a^2+w^2} = x(w)}$$

7. Using the time-shifting property, find Fourier transform of $e^{-a|t-b|}$

$x(w) = \frac{2a}{a^2+w^2}$

$$\boxed{x_{shifted}(w) = e^{-jwb} \cdot \frac{2a}{a^2+w^2}}$$

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$z=2$

$D = -2$ $D = -2$

$z=3$

$A + 2B + 2C + 2D = -3$ (with -3 above A and -4 above $2D$)

$B + C = 2$

$z=0$

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$x(n) =$

$$-3[1]^n u[n] + 3\cdot 2^n u[n] - \frac{1}{2} n \gamma^n u[n] \quad 2\frac{n \cdot n-1}{\gamma^2 2}\gamma^n u(n)$$

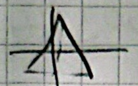
$$\boxed{x(n) = \left[-3[1]^n + 3\cdot 2^n - \frac{1}{2} n 2^n - \frac{n\cdot n-1}{4} 2^n\right]u(n)}$$

8. Consider a signal $x(t) = \text{sinc}^2(5\pi t)$ whose spectrum is $X(\omega) = 0.2\Delta\left(\frac{\omega}{20\pi}\right)$. Plot the frequency spectrum when $f_s = 5, 10, 20$ Hz

$X(\omega) = 0.2\Delta\left(\frac{\omega}{20\pi}\right)$ $\quad \left|\frac{\omega}{20\pi}\right| < 1$

$f = \frac{\omega}{2\pi}$

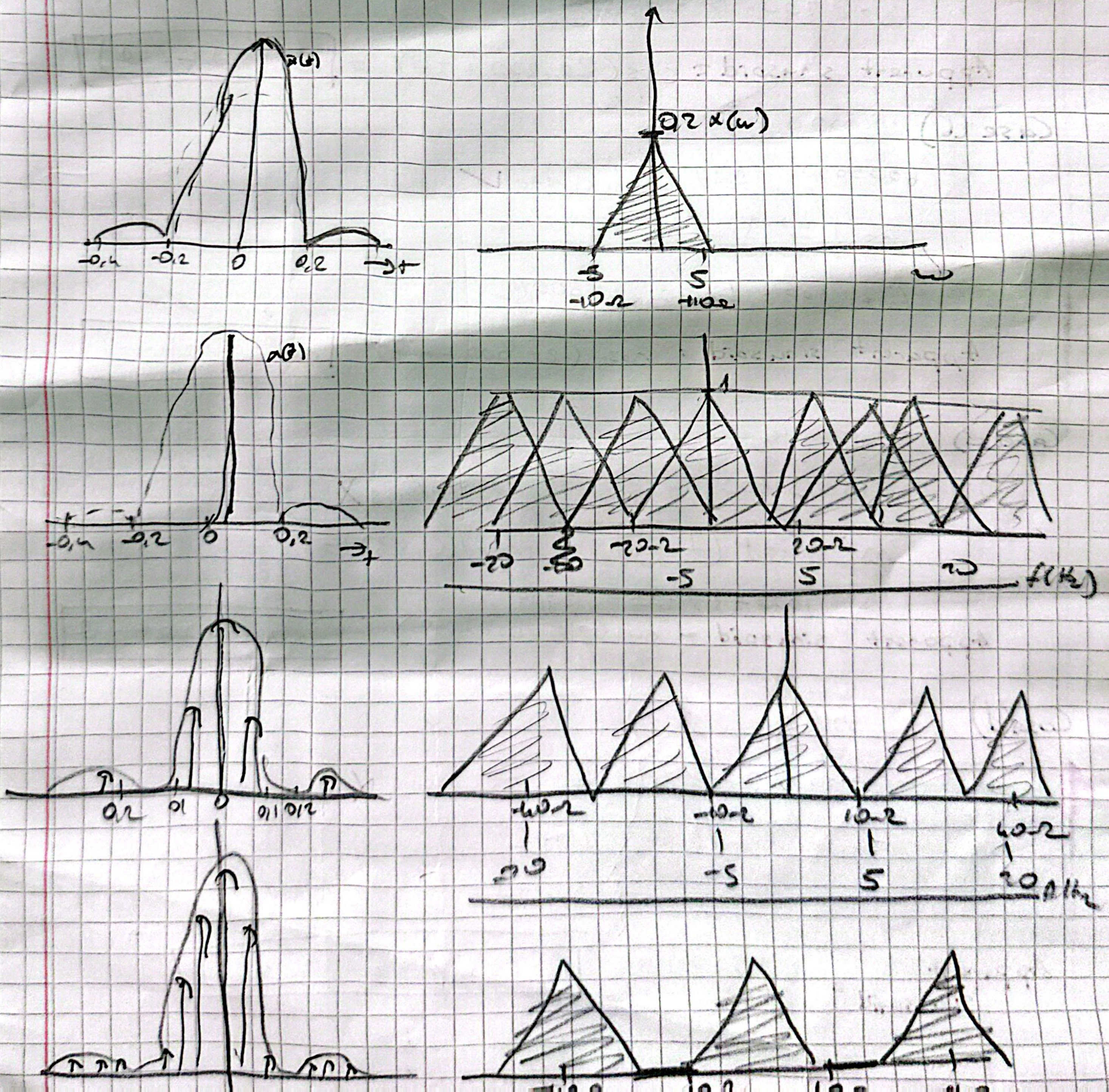
$-20\pi < \omega < 20\pi$



$-10 < f < 10$ Hz

Nyquist rate = $2B = 2 \times 5 = 10$

f_s (Hz)	$T = \frac{1}{f_s}$ (s)	$\frac{1}{T} X(\omega)$	Comments
5	0.2	$\Delta\left(\frac{\omega}{20\pi}\right)$	Under sampling
10	0.1	$2\Delta\left(\frac{\omega}{20\pi}\right)$	Nyquist rate
20	0.05	$4\Delta\left(\frac{\omega}{20\pi}\right)$	Over sampling



9. A continuous-time sinusoid $\cos(2\pi ft + \theta)$ is sampled at a rate $f_s = 1200\,Hz$. Determine the apparent aliased sinusoid of the resulting samples if the input signal frequency f is (a) 200 Hz (b) 600 Hz (c) 1000 Hz (d) 2400 Hz

Fundamental Band

$-\frac{f_s}{2} < \quad < \frac{f_s}{2}$ $\quad [-600, 600]\,Hz$

$f_a = f - m.f_s$

$m = \text{round}\left(\frac{f}{f_s}\right)$

Case (a) $f = 200\,Hz$

$-600 < 200 < 600$ ✓ No alising

$m = \left(\frac{200}{1200}\right) = \text{round}(0{,}167) = 0$

$f_a = 200 - 0 = 200\,Hz$

Apparent sinusoid $= \cos(2\pi.200.t + \theta) = \boxed{\cos(400\pi t + \theta)}$

Case (b) $f = 600\,Hz$

$600 = 600$ folding frequenc ✓

$m = \left[\frac{600}{1200}\right] = 0{,}5 = 1$

$f_a = 600 - 1 \times 1200 = -600\,Hz$

Apparent sinusoid $= \cos(2\pi\, 600\, t - \theta) = \boxed{\cos(1200\pi t - \theta)}$

Case (c) $f = 1000\,Hz$

$1000 > 600$ Yes alising occurs ✗

$m = \text{round}\left(\frac{1000}{1200}\right) = \text{round}(0{,}833) = 1$

$f_a = 1000 - 1 \times 1200 = -200\,H$

Apparent sinusoid $= \cos(2\pi\, 200 t - \theta) = \boxed{\cos(400\pi t - \theta}$

Case (d) $f = 2400\,Hz$

$2400 > 600$ Yes alising occurs. ✗

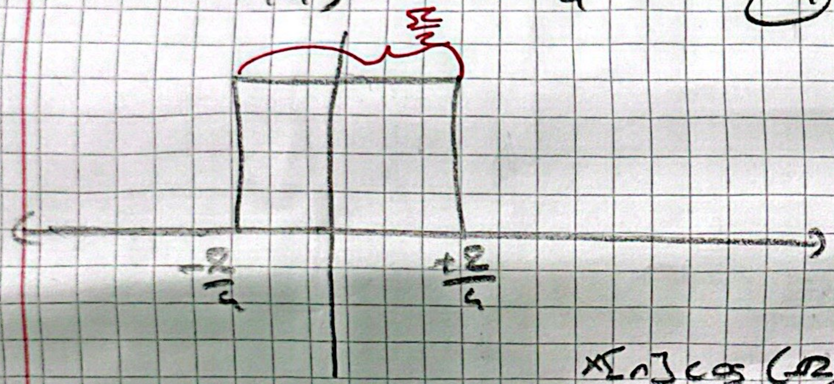
$m = \text{round}\left(\frac{2400}{1200}\right) = 2$

$f_a = 2400 - 2 \times 1200 = 0\,Hz$

Apparent sinusoid $= \cos(0 + \theta) = \boxed{\cos(\theta) \text{ Constant for all } t}$

10) A signal $x[n]$ $\text{sinc}(\pi n/4)$ modulates a carrier $\cos \Omega_c n$. Find and sketch the spectrum of the modulated signal $x[n] \cos \Omega_c n$ for $\Omega_c = \frac{\pi}{2}$.

$\text{sinc}(\pi n/4) = \quad k = \frac{\pi}{4} \Rightarrow \overset{\times 2}{\frac{2\pi}{4}} = \frac{\pi}{2}$



$x[n]\cos(\Omega_c n)\ \frac{1}{2}\left[X(e^{j(\Omega+\Omega_c)}) + X(e^{j(\Omega-\Omega_c)})\right]$

Left shift $-\frac{\pi}{4}-\frac{\pi}{2}, \frac{\pi}{4}-\frac{\pi}{2} = \left[-\frac{3\pi}{4}, -\frac{\pi}{4}\right]$

Right shift $\left[\frac{\pi}{4}+\frac{\pi}{2}, \frac{\pi}{4}+\frac{\pi}{2}\right] = \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$