1. For the given $x(t)$ in Figure 1, plot the following:
   a. $x(t+1)$
   b. $x(-t+1)$
   c. $x\left(\frac{3}{2}t+1\right)$

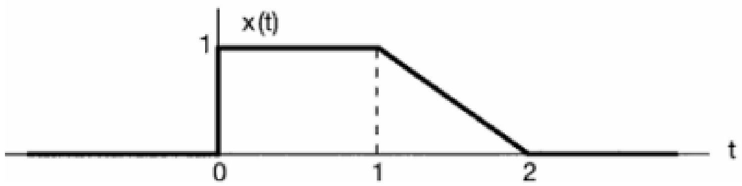
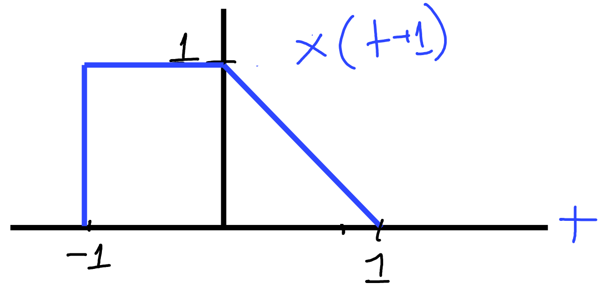
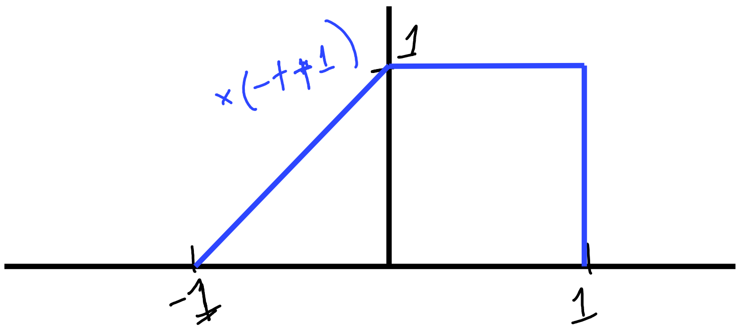
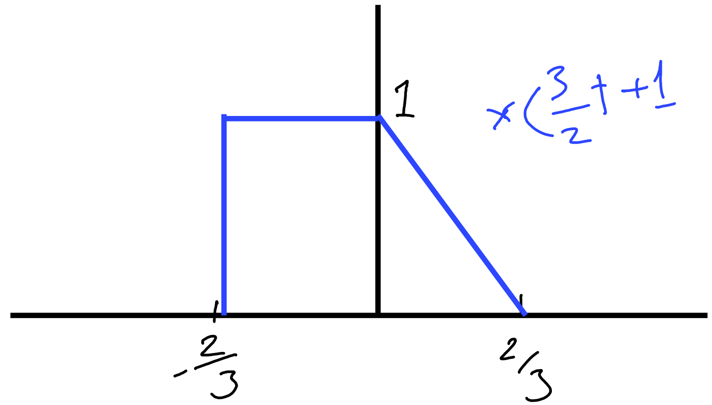


Figure 1: x(t) for Problem 1

a)







2. Is the following (see Figure 2) an energy signal, a power signal, or neither? Please justify. Also calculate the energy/power of the signal (whichever is applicable).

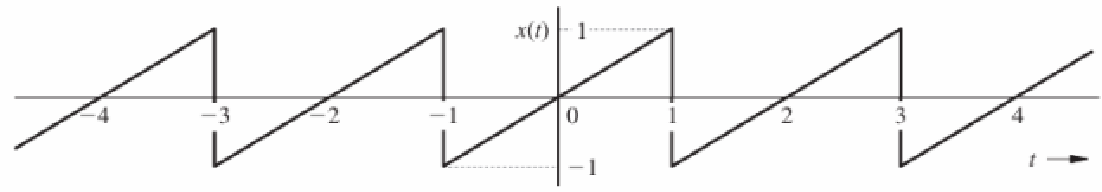


Figure 2: x(t) for Problem 2

The signal x(t) is a power signal, not an energy signal, because it's periodic with period T = 2. Periodic signals extend infinitely in time, making their total energy infinite, while their power remains finite. The power can be calculated by integrating $|x(t)|^2$ over one period and dividing by the period length.RetryClaude can make mistakes. Please double-check responses.

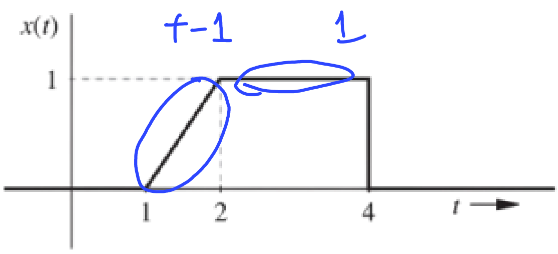
$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)^2| \, .dt \qquad x(t) = x$$

$$P = \frac{1}{2}\int_{-1}^{1} |x^2| \, . dt$$

$$P = \frac{1}{2} \quad \frac{x^3}{3}\Big|^{1} - \frac{x^3}{3}\Big|^{-1}$$

$$P = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{3}\right) \qquad P = \frac{1}{2} \cdot \frac{2}{3} \qquad \boxed{P = \frac{1}{3}}$$

3. Describe the following signal (see Figure 3) mathematically in terms of unit step and ramp functions.



$$x(t) = t-1\,[u(t-1) - u(t-2)] + 1.[u(t-2) - u(t-4)]$$

4. Is the system represented by $y(t) = \frac{d}{dt}x(t)$:

a. Causal or non-causal?

The system represented by y(t) = d/dt x(t) is causal. A system is causal when the output at any time t depends only on the input values up to and including time t, but not on future values. Since differentiation at any point t only requires knowledge of input values infinitesimally close to t (to calculate the derivative), the system doesn't need future values to determine the current output. The derivative operation relies solely on the rate of change at the present moment, making this system causal.

b. Time-varying or invariant?

The system represented by y(t) = d/dt x(t) is time-invariant. A system is time-invariant if delaying the input results in an identically delayed output. When we delay the input by τ to get x(t-τ), the output becomes d/dt[x(t-τ)], which equals dx(t-τ)/dt. This is the same as delaying the original output y(t-τ) = d/dt[x(t-τ)]. Since the relationship between input and output doesn't change with time shifts, the differentiation system is time-invariant

c. Memoryless or with memory?

The system represented by y(t) = d/dt x(t) has memory. This is because calculating a derivative at time t requires knowledge of input values beyond just the precise instant t - it needs information about how the signal behaves in the neighborhood of t (infinitesimally close past and future values). To determine the rate of change, the system must effectively "remember" the recent trajectory of the input signal, making it a system with memory rather than memoryless.

5. Find $y_0(t)$, the zero-input response of an LTIC system described by the repeated root system $(D^2 + 2D + 1)y(t) = \cancel{(D+2)x(t)}$ with initial conditions $y_0(0) = 1$ and $\dot{y}_0(0) = 2$.

↳ zero input

Characteristic Equation

$r^2 + 2r + 1 = 0$

$(r+1)^2 = \quad r_{1,2} = -1$

$c_1 e^{-rt} + c_2 \cdot e^{-rt} \cdot t$

$c_1 e^{-t} + c_2 \cdot t \cdot e^{-t} = y(t)$

$-c_1 \cdot e^{-t} + c_2 \cdot e^{-t} - c_2 \cdot t \cdot e^{-t} = y'(t)$

$y(0) = 1 = c_1 e^0 + c_2 \cdot 0 \cdot e^0 = \boxed{c_1 = 1}$

$y'(0) = 2 = -c_1 \cdot e^{-t} + c_2 e^{-t} - c_2 t \cdot e^{-t} = 2$

$= -1 e^0 + c_2^0 - \cancel{c_2 \cdot 0 \cdot e^{-t}} = 2$

$-1 + c_2 = 2$

$c_2 = 3$

$$\boxed{\begin{aligned} y_0 &= e^{-t} + 3t \cdot e^{-t} \\ y_0 &= (1+3t)e^{-t} \end{aligned}}$$

TABLE 2.1 Select Convolution Integrals

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^N u(t)$	$e^{\lambda t}u(t)$	$\frac{N!e^{\lambda t}}{\lambda^{N+1}}u(t) - \sum_{k=0}^{N}\frac{N!t^{N-k}}{\lambda^{k+1}(N-k)!}u(t)$
8	$t^M u(t)$	$t^N u(t)$	$\frac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$

6. Using **convolution tables**, determine the output of the system represented by $h(t) = (3e^{-3t} + 2e^{-t})u(t)$ for an input $x(t) = 5e^{-2t}$.

$y(t) = h(t) \cdot x(t)$

$y(t) = (3e^{-3t} + 2e^{-t})u(t) \cdot 5e^{-2t}$

$y(t) = 5e^{-2t} \cdot 3e^{-3t} \cdot u(t) + 5e^{-2t} \cdot 2e^{-t} \cdot u(t)$

$\lambda_1 = -2 \quad \lambda_2 = -3$

$15 \cdot \frac{e^{-2t} - e^{-3t}}{-2-(-3)}$

$\lambda_1 = -2 \quad \lambda_2 = -1$

$10 \cdot \frac{e^{-2t} - e^{-t}}{1-2}$

$y(t) = (15 \cdot (e^{-2t} - e^{-3t}) - 10(e^{-2t} - e^{-t})) \cdot u(t)$

$y(t) = [15(e^{-2t} - e^{-3t}) - 10(e^{-2t} - e^{-t})] \cdot u(t).$

7. During semester n, $x[n]$ students enroll in a course requiring a certain textbook while the publisher sells $y[n]$ new copies of the same book. On average, one-third of students with books in salable condition resell the texts at the end of the semester, and the book life is three semesters. Write the equation relating $y[n]$, the new books sold by the publisher, to $x[n]$, the number of students enrolled in the n-th semester, assuming that every student buys a book.

$$y[n] = x[n] - \frac{1}{3}\left(x[n-1] + \boxed{\frac{2}{3}}\, x[n-2] + \boxed{\frac{4}{9}}\, x[n-3]\right)$$

This loose $\frac{1}{3}$ salable books on it current semester

$1-\frac{1}{3} = \boxed{\frac{2}{3}}$ remain

This loose $\frac{1}{3}$ salable books on current semester

$\frac{2}{3} \cdot \frac{1}{3}$ loose on next semester

$\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$ $\quad 1-\frac{5}{9} = \boxed{\frac{4}{9}}$ remain

8. Solve the following iteratively (3 iterations)

$$y[n+2] - y[n+1] + 0.24y[n] = x[n+2] - 2x[n+1]$$

when $y[-1] = 2, y[-2] = 1$ and $x[n] = nu[n]$.

$n = -2$

$y[0] - \underbrace{y[-1]}_{2} + 0.24\, y[-2] = x[0] - 2x[-1]$

$2 \qquad + 0.24 \qquad = \cancel{0 \cdot u(n)} + 2u[n]$

$y[0] - 2 + 0,24 = 2u[n]$

$\boxed{y[0] = 3,76}$

$n = -1$

$y[1] - \underline{y[0]} + 0,24\, \underbrace{y[-1]}_{2} = x[+1] - 2x[0]$

$= 1 \cdot u[n] \quad \cancel{-2 \cdot 0 \cdot u[n]}$

$y[1] - 3,76 + 0,48$

$\boxed{y[1] = 2,28}$

$n = 0$

$y[2] - y[1] + 0,24\, y[0] = x[2] - 2x[1]$

$y[2] - 2,28 + 0,90 = \cancel{2u[n] - 2u[n]} \to 0$

$y[2] = 2,28 - 0,90$

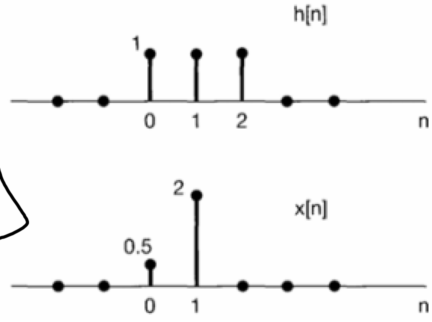
$\boxed{y[2] = 1,38}$

9. For the given $h[n]$ and $x[n]$ (see Figure 4), determine the output signal $y[n]$.

$x[n] \Rightarrow y[n]$

$Y[n] \Rightarrow h[n]$

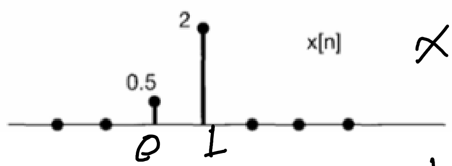
$\delta[n-m] \Rightarrow h[n-m]$



$x[m]\delta[n-m]$

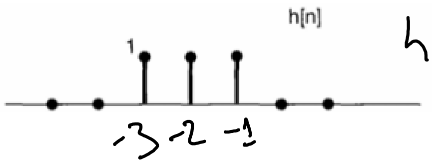
Figure 4: h[n] and x[n] for Problem 9

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m] =$$

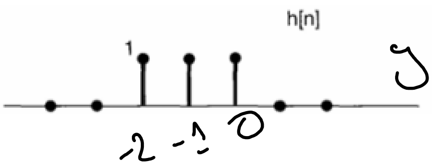


$x[k]$

$h[n-k] = h[-1-k]$

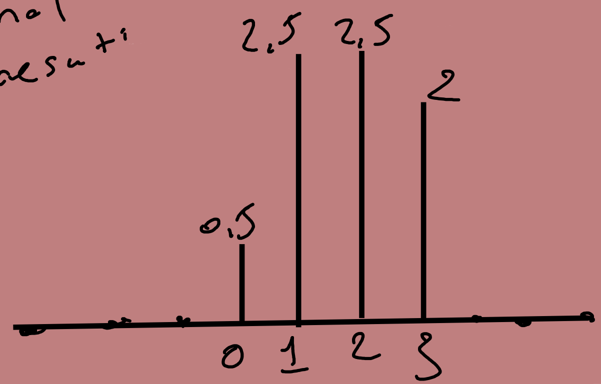


$y[n]=0$

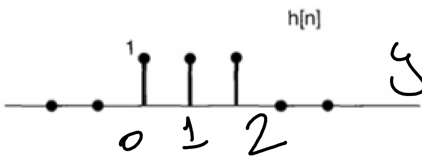


$y[n]=0,5$

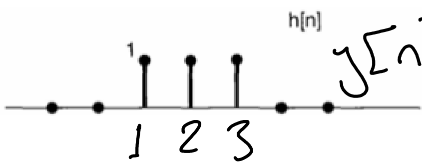
Final result



$y[n]=2,5$



$y[n]=2,5$



$y[n]=2$

10. Find the unilateral Laplace transform of

$$\frac{8s+10}{(s+1)(s+2)^3}$$

$$\frac{8s+10}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

$s = -1$ $\quad 8s+10 = A$ $\quad A = 2$

$D = -1D = -6$ $\quad D = 6$

$s = -2$

$s = 0$ $\quad \cancel{8A}^{16} + 4B + 2C + \cancel{D}^{6} = 10$

$4B + 2C = -12$

$s = 1$ $\quad \cancel{27A}^{54} + 18B + 6C + \cancel{2D}^{12} = 18$

$18B + 6C = -48$

$-3/\ \ 4B + 2C = -12$

$+6B = -12$

$B = -2$

$C = -2$

$$\frac{8s+10}{(s+1)(s+2)^3} = \frac{2}{s+1} + \frac{2}{s+2} - \frac{2}{(s+2)^2} + \frac{6}{(s+2)^3}$$

TABLE 4.1 Select (Unilateral) Laplace Transform Pairs

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$

$$x(t) = 2e^{-t} - 2e^{-2t} - 2te^{-2t} + 3t^2 \cdot e^{-2t}$$