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M3

Assignment No: 3 & 4

Q1) Find the correlation coefficient between x & y, given that $n=50$, $\Sigma(x_i-40)=30$; $\Sigma(y_i-20)=70$; $\Sigma(x_i-40)^2=170$; $\Sigma(y_i-20)^2=165$; $\Sigma(x_i-40)(y_i-20)=140$

Soln:-

Consider: $u = x_i - 40$, $v = y_i - 20$

$\bar{u} = \frac{30}{50} = 0.6$ $\quad \bar{v} = \frac{70}{50} = 1.4$

$\Sigma u^2 = 170$ $\quad \Sigma v^2 = 165$ $\quad \Sigma uv = 140$ $\quad \Sigma u = 30$ $\quad \Sigma v = 70$

Coefficient of correlation (r)

$$r = \frac{\Sigma uv - n\bar{u}\bar{v}}{\sqrt{[\Sigma u^2 - n(\bar{u})^2] \times [\Sigma v^2 - n(\bar{v})^2]}}$$

$$= \frac{140 - 50(0.6)(1.4)}{\sqrt{[170-50(0.6)^2] \times [165-50(1.4)^2]}}$$

$$= \frac{140-42}{\sqrt{(152)(67)}} = \frac{98}{100.9158}$$

$\therefore$ $\boxed{\text{coefficient of correlation} = 0.9711066057}$

Q2) The following value of x & y are supposed to follow the law $y = ax^2 + b\log_{10} x$

x	2.85	3.88	4.66	5.69	6.65	7.77	8.67
y	16.7	26.4	35.1	47.5	60.8	77.5	93.4

By using graphical method, find the probable values $a, b = ?$

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Ans The given eqⁿ → $y = ax^2 + b\log_{10} x$

Divide by $\log_{10} x$ on both sides

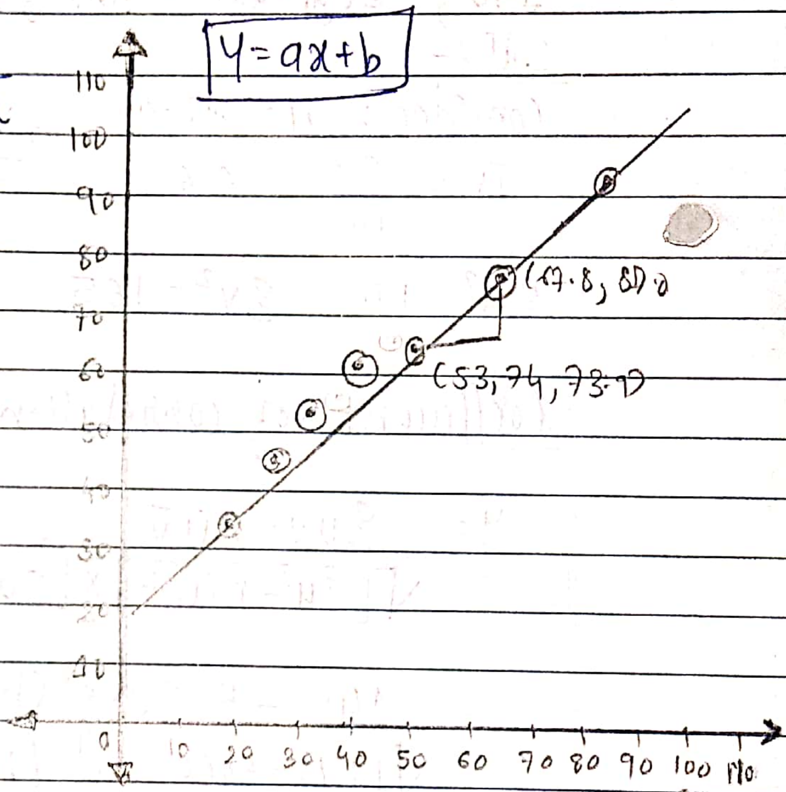
$$\frac{y}{\log_{10} x} = \frac{ax^2}{\log_{10} x} + b$$

$$Y = \frac{y}{\log_{10} x} \qquad X = \frac{x^2}{\log_{10} x}$$

$\boxed{Y = aX + b}$

To convert the table

x	y	Y	X
2.85	16.7	36.8	17.9
3.88	26.4	44.9	25.56
4.66	35.1	52.5	32.49
5.89	47.5	62.9	42.67
6.65	60.8	73.9	63.74
7.77	77.5	89.0	67.8
8.67	93.4	99.5	80.13



$$a = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 = 53.73 \quad x_2 = 67.8 \qquad y_1 = 73.9 \quad y_2 = 87.0$$

$$a = \frac{87.0 - 73.9}{67.8 - 53.73} = 0.9 \approx \therefore \boxed{a = 1}$$

$y = ax + b$ (Substitute the values in eqⁿ):

$73.9 = 1(53.74) + b$

$73.9 - 53.74 = b$

$\therefore \boxed{b = 20.16}$

$\therefore$ a & b values are 1 & 20.16 resp.

Q3) If $f(x) \begin{cases} 3/4\,(1-x^2) & ; \text{ for } -1 \leq x \leq 1 \\ 0 & ; \text{ otherwise} \end{cases}$

Then, (i) find distribution function, (ii) find the probabilities $P(-1/2 \leq x \leq 1/2)$, (iii) find x such that $P(X \leq x) = 0.98$

Soln:

i) $F(x) = \int_{\infty}^{x} F(x)\,dt$

$= \int_{\infty}^{x} \frac{3}{4}(1-t^2)\,dt$

$= \frac{3}{4}\int_{\infty}^{x}(1-t^2)\,dt$

$= \frac{3}{4}\left[t - \frac{t^3}{3}\right]_{-1}^{x}$

$= \frac{3}{4}\left[\left(x - \frac{x^3}{3}\right) - \left((-1) - \frac{(-1)^3}{3}\right)\right]$

$= \frac{3}{4}\left[\frac{3x+2-x^3}{3}\right]$

$= \frac{3}{4}\left[\frac{3x-x^3+2}{3}\right]$

$= \frac{1}{4}\left[3x-x^3+2\right]$

for $x \geq 1$ $\quad F(x) = 1$

$\therefore F(x) \begin{cases} -1 & ; x \leq -1 \\ (3x-x^3+2)/4 & -1 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$

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ii) $P(-1/2 \leq x \leq 1/2)$

$$P(-1/2 \leq x \leq 1/2) = \frac{3(1/2) - (1/2)^3 + 2}{4} - \frac{3(-1/2) - (-1/2)^3 + 2}{4}$$

$$= \frac{\frac{3}{2} - \frac{1}{8} + 2}{4} - \frac{\frac{-3}{2} - \left(\frac{-1}{8}\right) + 2}{4}$$

$$= \frac{12 - 1 + 16/8}{4} - \frac{-12 + 1 + 16/8}{4}$$

$$= \frac{27/8}{4} - \frac{5/8}{4}$$

$$= \underline{\underline{0.6875}}$$

iii) $P(X \leq x) = 0.98$

$$\frac{3x - x^3 + 2}{4} = 0.98$$

$$3x - x^3 + 2 = 3.92$$

$$3x - x^3 = 1.92$$

$$3x - x^3 - 1.92 = 0$$

$$x_1 = 1.1591$$

$$x_2 = -1.9910$$

$$x_3 = 0.8319$$

Q4) If the probability that a concrete cube fails is 0.001. Determine the probability that, out of 1000 cubes (i) exactly two (ii) more than one cube will fail.

Soln:

$p = 0.01$, $n = 1000$

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$\lambda = nP = 0.001 \times 1000 = 1$

$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-1}1^r}{r!}$

i) exactly two

Here, $r = 2$

$P(2) = \frac{e^{-1}1^2}{2!} = \frac{0.368}{2} = \cancel{0.184}\ 0.184$

ii) more than one cubes will fail

Prob (more than 1) $= P(2) + P(3) + \dots + P(1000)$

$= 1 - [P(0) + P(1)]$

$= 1 - \left[\frac{e^{-1} \times 1^0}{0!} + \frac{e^{-1} 1^1}{1!}\right]$

$= 1 - e^{-1}[1 + 1]$

$= 1 - e^{-1}(2)$

$= 1 - 0.7358$

$= 0.2642$

Q5) If an intelligence test administrated to 1000 student, average score was 42 & standard deviation 24. Find the number of students with score lying between 30 and 54. ?

Soln :

Given, $Z = 0.5$, Area $= 0.1915$

1000 → Students

∴ average mean → 42

standard deviation → 24

→

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Number of student with score lying between 30 and 54.

$z = 0.5$, Area $= 0.1915$

* For 30,

$$z = \frac{\text{Value} - 42}{24} = \frac{30 - 42}{24}$$

$\therefore \boxed{z = -0.5}$

A corresponding to $z = +0.1915$

* For 54,

$$z = \frac{\text{Value} - 42}{24} = \frac{54 - 42}{24}$$

$\therefore \boxed{z = 0.5}$

A corresponding to $z = 0.1915$

$$P(30 < x < 54) = P(-0.5 \leq z \leq 0.5)$$
$$= +0.1915 + 0.1915$$
$$= 0.383$$

$\therefore$ Number of students score lying between.

$$= 1000 \times 0.383$$
$$= 383$$

—x—