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### Column Generation for the Integrated Berth Allocation, Quay Crane Assignment, and Yard Assignment Problem

Kai Wang, a, b Lu Zhen, a Shuaian Wang, b Gilbert Laportec

<sup>a</sup> School of Management, Shanghai University, 200444 Shanghai, China; <sup>b</sup> Department of Logistics and Maritime Studies, Hong Kong Polytechnic University, Kowloon, Hong Kong; Department of Decision Sciences, HEC Montréal, Montréal, Québec H3T 2A7, Canada

Contact: chwangkai@gmail.com, http://orcid.org/0000-0002-3040-0422 (KW); lzhen@shu.edu.cn, 6 http://orcid.org/0000-0001-5209-1109 (LZ); hans.wang@polyu.edu.hk, http://orcid.org/0000-0001-9247-4403 (SW); gilbert.laporte@cirrelt.ca (GL)

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Abstract. This study investigates an integrated optimization problem on the three main types of resources used in container terminals: berths, quay cranes, and yard storage space. It presents a mixed integer linear programming model, which takes account of the decisions of berth allocation, quay crane assignment, and yard storage space unit assignment for incoming vessels. In addition, since the majority of the liner shipping services operate according to a weekly arrival pattern, the periodicity of the plan is also considered in the model and in the proposed algorithm. To solve the model on large-scale instances, a column generation (CG) procedure is developed to provide a lower bound for the integrated problem, in which an exact pseudopolynomial algorithm is designed for the pricing problems. Using this procedure, we propose a CG-based heuristic with different solution strategies and apply dual stabilization techniques to accelerate the algorithm. Based on some realistic instances, we conduct extensive numerical experiments to validate the effectiveness of the proposed model and the efficiency of the algorithm. The results show that the CG-based heuristic can yield a good solution with an approximate 1% optimality gap within a much shorter computation time than that of CPLEX.

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Keywords: column generation • berth allocation • yard management • quay crane assignment

#### 1. Introduction

Today, many ports in the world are propelling the evolution of automated port container terminals (Gharehgozli, Roy, and de Koster 2016). Port operators try to improve their productivity by automating their operations. According to Kannoth (2014), new techniques, equipment, and information systems have been developed and implemented. The current focus lies on developing integrated decision support systems to enhance the scheduling efficiency of equipment. Under the background of automated ports, decision support systems must embed some integrated optimization tools, since the automated systems will replace experienced decision makers in ports for making effective schedules.

Motivated by these industry trends, this paper proposes an integrated optimization methodology for port operations, which combines berth allocation, quay crane (QC) assignment, and yard assignment. Traditionally, port operators have focused on berth allocation decisions since these define the first planning phase. The planned berth locations for vessels are subsequently used as the key input to yard storage, personnel, and equipment deployment planning. When making the berth allocation decision, the QC assignment is usually planned simultaneously, since the number of QCs assigned to the vessels will affect their dwelling time in the port and will thereafter affect the berth allocation for the vessels (Vacca, Salani, and Bierlaire 2013). During a vessel's dwelling time at a port, the number of assigned QCs may change over time (Li, Sheu, and Gao 2015), which inevitably complicates the berth allocation process. In fact, the allocation of berths to vessels is also intertwined with yard assignment, which specifies the allocation of the yard storage space units (subblocks) to vessels (Jiang et al. 2012; Robenek et al. 2014; Jin, Lee, and Hu 2015). The reserved subblocks for a specific vessel will temporarily store export containers and transshipment containers to be loaded on the vessel. This assignment impacts the best berth positions for vessels and hence affects the berth allocation. On the other hand, the berth positions allocated to vessels will impact the assignment of yard space to vessels. Since these decisions interact with each other, an integrated optimization scheme should outperform the traditional sequential optimization processes.

Improving port operations aims at providing highquality handling service for vessels. The vessel arrivals at ports often follow cyclical patterns, especially for some mega transshipment hubs, such as the Port of Shanghai and the Port of Singapore. Based on real data collected by Zhen et al. (2016) for the Port of Shanghai, more than 80% of vessel arrivals followed a weekly pattern. Most shipping liners operate weekly services for shipping routes; that is, they visit each port on a weekly basis (Meng and Wang 2011; Brouer et al. 2013; Li, Qi, and Song 2016). According to Wang, Meng, and Du (2015), liner shipping operators usually plan their shipping services every three to four months, corresponding approximately to seasons. In a season, the arrival pattern and demand pattern of the services normally do not change very much. In response to the fixed patterns of vessels operated by shipping liners, it makes sense to consider periodicity of the berth and yard planning, and to design consistent schedules to be operated for several periods. Henceforth, we extend the integrated planning problem to a periodic one. In this sense, the problem can be viewed as a tactical-level planning problem, like the problems studied by Giallombardo et al. (2010); Vacca, Salani, and Bierlaire (2013); Lalla-Ruiz et al. (2014); and Jin, Lee, and Hu (2015).

This study proposes an integrated model for berth allocation, QC assignment, and yard assignment for container terminals. This model captures the periodicity of the planning problem and is used to design a consistent operational schedule over several periods. To solve the problem for large instances, we have designed a column generation (CG) algorithm, in which an exact pseudopolynomial procedure is applied to solve the pricing problems to optimality. A CG-based heuristic with different solution strategies is developed to derive integer feasible solutions, and dual stabilization techniques are applied to accelerate the algorithm. Numerical experiments are conducted to validate the model and to demonstrate the efficiency of the algorithm. For a set of real-world-like instances, our method can generate good plans within reasonable computation times. We formulate managerial insights based on the results of our experiments.

The remainder of this paper is organized as follows. The related literature is reviewed in Section 2. Section 3 gives a detailed description of the problems. A mixed integer mathematical model is formulated in Section 4. In Section 5, a CG procedure is developed to solve the linear programming (LP) relaxation of a proposed set covering model, while in Section 6, a CG-based heuristic developed to obtain feasible integer solutions is described. Extensive computational experiments are conducted in Section 7, and conclusions are drawn in Section 8.

#### 2. Literature Review

In the areas of container terminal operations and maritime logistics, researchers have devoted significant efforts to various operation management problems. In this section, we briefly introduce some milestone review papers, and we then focus on some highly relevant and state-of-the-art studies.

For a comprehensive overview, we first refer to Vis and de Koster (2003), who provided a classification and summary of the decision problems related to container transshipment. Steenken, Voß, and Stahlbock (2004) described the logistics processes in container terminals and analyzed the related optimization methods. This work was further updated by that of Stahlbock and Voß (2008). Bierwirth and Meisel (2010, 2015) focused on the berth allocation and QC scheduling problems in container terminals and discussed existing scheduling algorithms. Fransoo and Lee (2013) viewed the container transport as a part of the global supply chain and listed relevant research challenges. Meng et al. (2014) concentrated on liner shipping and summarized related routing and scheduling problems.

Our problem includes the berth allocation problem (BAP), which is critical in port management and is normally the first planning phase of making service schedules for shipping liners. The BAP has attracted significant attention among researchers. To our knowledge, Imai, Nagaiwa, and Tat (1997) were the first to solve the BAP for commercial ports. They focused on a static problem, which was extended to the dynamic BAP by Imai, Nishimura, and Papadimitriou (2001). The BAP can be classified into two types, discrete and continuous, depending on whether vessel berthing is performed in a discrete or in a continuous space (Imai et al. 2005; Mauri et al. 2016). Since the BAP is NP-hard both for the discrete and the continuous cases (Bierwirth and Meisel 2010), many researchers have devoted considerable efforts to the design of effective heuristics. For example, Park and Kim (2002) employed a subgradient optimization method, Kim and Moon (2003) proposed a simulated annealing method, Lalla-Ruiz et al. (2014) designed a random key genetic algorithm, and Ribeiro et al. (2016) developed an adaptive large neighborhood search heuristic. Regarding mathematical formulations for the BAP, Cordeau et al. (2005) built a BAP model based on a vehicle routing problem formulation, and Monaco and Sammarra (2007) proposed a compact and stronger formulation. The tactical-level BAP (TBAP) constitutes an important variant of the BAP whose aim is to build berth templates. Moorthy and Teo (2006) studied a berth template planning problem, which maximizes the service level and minimizes the connectivity cost related to the transshipment container groups. Cordeau et al. (2007) studied a tactical service allocation problem arising at the Gioia Tauro transshipment hub in Italy. Giallombardo et al. (2010)



developed a mixed integer quadratic programming formulation for the TBAP, in which they first proposed the concept of QC profiles. Lalla-Ruiz et al. (2014) studied a TBAP that maximizes the values of serving vessels by specific QC profiles and minimizes transshipment costs simultaneously. Zhen (2015) considered the uncertainty of ship operation times at ports in the TBAP. Recently, practical issues have been considered when planning berths, such as the effect of tides, which may influence the water depth of the navigation channels (Du et al. 2015; Lalla-Ruiz et al. 2017). This BAP was defined as the BAP under time-dependent limitations by Xu, Li, and Leung (2012), and the problem was revised and extended by Lalla-Ruiz et al. (2016) from a two-period to a multiperiod planning horizon.

Another stream of BAP studies concerns the integrated planning of the BAP and QC assignment, which takes into account the interaction between two problems. Park and Kim (2003) developed a two-phase heuristic solution procedure for the integrated optimization. Imai et al. (2008) considered the constraint that QCs cannot pass or bypass from one side to the other side of a vessel whose containers are being handled. Meisel and Bierwirth (2009) treated the BAP-QC assignment as a multimode resource-constrained project scheduling problem. Meisel and Bierwirth (2013) proposed a framework for integrating the BAP, QC assignment, and QC scheduling. Vacca, Salani, and Bierlaire (2013) proposed an exact branch-and-price algorithm that can solve instances with up to 20 ships and five berths. Iris et al. (2015) extended existing models by proposing new set partitioning formulations for the integrated BAP and QC assignment. Iris, Pacino, and Ropke (2017) considered the decrease in the marginal productivity of QCs for the integrated planning and developed an adaptive large neighborhood search heuristic. Recently, bunker fuel consumption and emissions were integrated in some BAP studies (Venturini et al. 2017). Du et al. (2011) proposed a mixed integer second-order cone programming model for a BAP by considering the fuel consumption and vessel emissions. Hu, Hu, and Du (2014) further integrated QC allocation into the BAP considering fuel consumption and emissions from vessels, and developed a mixed integer second-order cone programming model. Besides the above studies which are mainly based on mathematical programming, some authors have employed discrete event simulation, for example, Legato and Mazza (2001). A simulation optimization technique was recently applied to optimize the tactical and operational BAP decisions in an integrated way (Legato, Mazza, and Gullì 2014). Randomness in loading and unloading operations and QC assignment were also considered by Legato, Mazza, and Gullì (2014).

Based on the above discussion, researchers have made considerable contributions on the decision problems (e.g., BAP and QC assignment) related to the quay side of container terminals. Some recent work also started to tackle decision problems on the yard side (Won, Zhang, and Kim 2012; Gharehgozli et al. 2014; Gharehgozli et al. 2015). Jin, Lee, and Cao (2016) focused on the daily yard management problem of scheduling yard cranes. Jiang and Jin (2017) integrated yard crane assignment and container allocation among subblocks in the yard. However, very few studies combine the decisions on the quay side and on the yard side. Zhen, Chew, and Lee (2011) integrated the berth template with the yard template planning. Hendriks, Lefeber, and Udding (2013) proposed a heuristic for solving a simultaneous berth allocation and yard planning problem. For bulk ports, Robenek et al. (2014) designed an exact branch-and-price algorithm to solve the integrated berth and yard planning problem. Lee and Jin (2013) studied a feeder vessel management problem that integrates the feeder schedule design, feeder berth allocation, and storage yard assignment for transshipment flows, for which Jin, Lee, and Hu (2015) designed a CG-based solution approach. Among the aforementioned papers, that of Jin, Lee, and Hu (2015) is highly relevant to our study in terms of the problem formulation and algorithm. However, major differences between our study and their work can be easily identified. First, these authors did not include QC assignment in the integrated berth allocation and yard assignment. Second, they did not consider the impact of the vessel's weekly pattern on the integrated planning. Third, they solved the CG pricing problems to optimality by a mathematical model with CPLEX rather than design some ad hoc exact algorithms.

Compared with the related literature, this paper makes the following contributions. First, it extends the traditional berth allocation and QC assignment problem, which is related to the quay-side decision, to the yard-side decision problem (i.e., the yard storage unit assignment problem). In addition, when formulating the integrated model, this study considers the periodicity of the planning process, since most shipping liners operate their shipping services on a weekly pattern. Second, although a few integrated optimization problems in the fields of port operations have been studied, the available solution methods consist of metaheuristics that cannot provide a tight lower bound. This study proposes a CG-based heuristic to solve the model, which provides a strong linear relaxation, and an exact pseudopolynomial algorithm is designed for the pricing problems. Third, we conduct extensive numerical experiments based on some realistic instances, the results of which show that the proposed algorithm exhibits a better performance than



the metaheuristics developed previously. Finally, some managerial implications are derived for the berth allocation and QC assignment based on the results of our experiments.

#### 3. Problem Background

Before formulating the integrated model for the berth allocation, the QC assignment, and the yard assignment, we provide some background for the problem. In this section, we first introduce the concept of QC profiles in the QC assignment, and we then describe the integrated problem.

#### 3.1. QC-Profile-Based QC Assignment Decision

Most shipping liners plan their services on a seasonal basis, approximately every three to four months. Within the same season, the vessel arrival patterns and shipping demand patterns normally do not change much (Wang, Meng, and Du 2015). Under the fixed patterns, shipping liners will inform the port operators in advance about the feasible and expected turnover time as well as the total container handling workload for their vessel arrivals. From the port perspective, if a berth on the quay side is allocated to an incoming vessel, the port operators will then assign a number of QCs for loading and unloading containers based on the advance information on container handling workload. The number of QCs assigned determines the handling speed of the containers and hence the handling time. More QCs assigned to the vessel means less turnover time for the vessel in the port.

To simplify the QC assignment decision in our problem, we follow the concept of QC profiles proposed by Giallombardo et al. (2010), which has been subsequently applied in several studies (Vacca, Salani, and Bierlaire 2013; Lalla-Ruiz et al. 2013; Expósito-Izquierdo et al. 2016). In a QC profile, the total workload is denoted by the number of QC × time steps, where one QC × time step represents the number of

containers that can be handled by one QC in a time step (e.g., a time step of four hours). For example, if one QC can handle 30 containers in one hour, then one QC  $\times$  time step means a  $4 \times 30 = 120$  container handling capacity. Based on the informed container handling workload, port operators will generate a set of QC profiles for the incoming vessels. Figure 1 shows three possible QC profiles for a vessel with a handling workload of 20 QC  $\times$  time steps (which means  $20 \times 4 \times 30 =$ 2,400 containers to be handled). Two important parameters are defined for each QC profile. One parameter is the handling time by using QC profile p for vessel i, denoted by  $h_{in}$ . In Figure 1, the handling time by using QC profile 1 is six time steps. The other parameter is the number of QCs utilized in the *m*th time step if QC profile p is assigned to vessel i, denoted by  $q_{ipm}$ . For instance, by using QC profile 2, five QCs are utilized in the first time step (i.e.,  $q_{ip1} = 5$ ), four QCs are utilized in the second time step (i.e.,  $q_{ip2} = 4$ ), and so on.

#### 3.2. Integrated Berth Planning and Yard Planning

The integrated planning problem studied in this paper includes three subproblems: the berth allocation problem, the QC assignment problem (i.e., the QC-profile assignment), and the yard assignment problem, which are intertwined with each other in real-world operations. A visualization of the integrated planning problem is shown in Figure 2.

For an incoming vessel, the berth planning determines when and where the vessel moors at the terminal, as well as which QC profile is assigned to the vessel. In Figure 2, vessel 1 is scheduled to arrive at the terminal in time step 1 and moor at berth 1. Meanwhile, the QC profile selected for the vessel is such that it will moor for five time steps. Such a decision is made based on the information provided by the shipping liner. As mentioned earlier, the feasible time interval (denoted by  $[a_i^f, b_i^f]$ ), the expected time interval

Figure 1. An Example of QC Profiles for a Vessel

Total workload for the vessel: 20 QC × time step

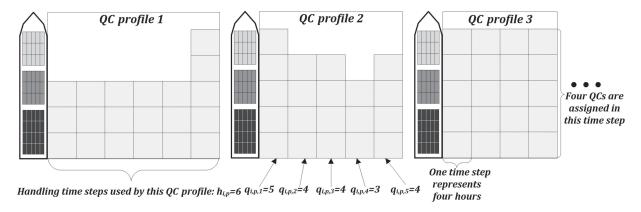
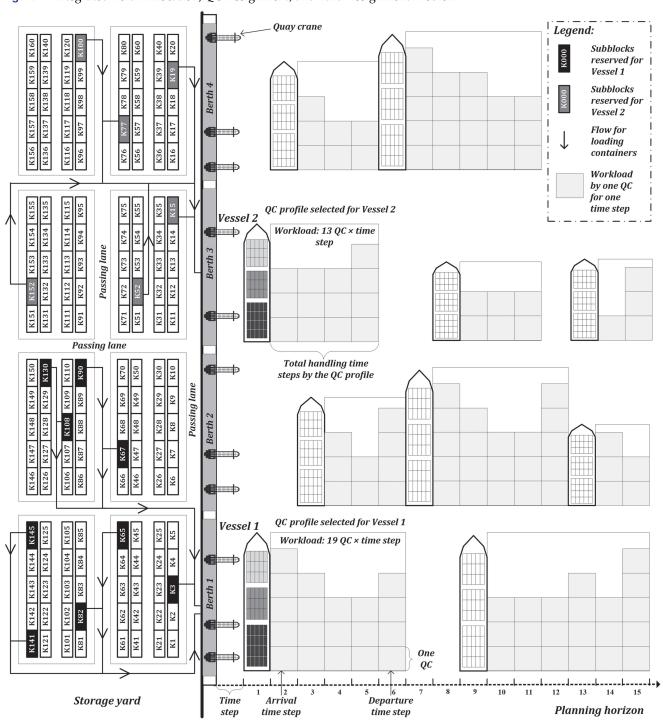




Figure 2. Integrated Berth Allocation, QC Assignment, and Yard Assignment Problem



(denoted by  $[a_i^e, b_i^e]$ ), and the total workload for loading and unloading containers are provided by the shipping liner prior to the vessel arrival. The port operators attempt to construct the berth schedules and assign the QCs in such a way that the vessel can moor at the terminal within the interval  $[a_i^e, b_i^e]$ . If this interval is violated, a penalty cost is charged by the shipping liner. However, the feasible time interval  $[a_i^f, b_i^f]$  provided

by the shipping liner cannot be violated under any circumstances.

In reality, it is extremely difficult for the terminal operators to satisfy all of the shipping liners' requirements on mooring within their expected time intervals. The service quality costs (i.e., the penalty costs) charged by shipping liners are unavoidable, especially when the number of incoming vessels is large with



respect to berth capacity and QC resources. Thus, the objective in berth planning is to minimize the costs incurred when the expected time intervals are violated. Assume that  $\alpha_i$  and  $\beta_i$  are the start time step and the end time step for the handing of vessel *i*, where  $\alpha_i \ge a_i^J$ and  $\beta_i \leq b_i^f$ . If  $\alpha_i < a_i^e$  or  $\beta_i > b_i^e$ , a service quality cost will charged for vessel i, which can be calculated as  $c_i^p[(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+]$  ( $c_i^p$  is the penalty cost coefficient for vessel *i*).

Yard planning is affected by berth planning. Under the consignment strategy in the terminal (Han et al. 2008; Jiang et al. 2012), the yard is utilized for temporary container storage for the shipping liners. Some specific subblocks in the yard are reserved for each vessel. When a vessel arrives, all of the containers stored are loaded from its reserved subblocks to the vessel. In Figure 2, the subblocks K15, K19, K52, K77, K100, and K152 are reserved for vessel 2, which is scheduled to moor at berth 3. When vessel 2 arrives at the terminal, all of the containers stored in the six subblocks K15, K19, K52, K77, K100, and K152 are transported to the berth position along the solid flow lines shown. Here, we assume that each loading route between a subblock and a berth is predetermined as shown by the solid flow lines. We define  $D_{kb}^L$  as the length of the loading route between berth *b* and subblock *k*.

In addition to the loading process, an unloading process also occurs for an incoming vessel. The containers that need to be transshipped to other vessels are unloaded from the incoming vessel and are stored in the subblocks reserved for these vessels. For the unloading process, we assume that if a vessel is allocated to berth b, the route length for unloading a container is the average unloading route length between berth b and all of the subblocks in the yard, denoted by  $D_h^U$ . The reason why we assume the average unloading route length is to avoid complex synchronization for each transshipment vessel pair. The unloaded containers from the incoming vessel can be transshipped

> Start time step for the berth

to any other vessel, and thus can be stored in any subblocks scattered in the yard. Henceforth, it is reasonable to use the average length in the unloading process, even though the assumption may cause some deviations from the optimal assignments that capture the real total length of unloading routes. Essentially, the average unloading route length implies the distance advantage of berth positions to the yard side, which will be discussed in Section 7.

The objective of yard planning is to minimize the total traveled length in the yard for loading and unloading all of the containers from the incoming vessels. Although it focuses on operations cost incurred on the yard side, it is intertwined with the berth planning. Intuitively, the port operators will not assign subblocks K141-K145 to a vessel that moors at berth 4. Henceforth, an integrated model for the berth allocation, QC assignment, and yard assignment is desired.

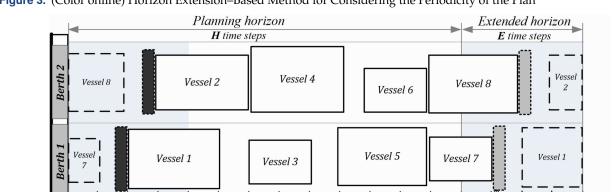
#### 3.3. Cyclical Berth Planning

In a season, most vessels of shipping liners visit ports on a weekly basis with fixed arrival and demand patterns (Zhen et al. 2016; Wang, Meng, and Du 2015), which determines the periodicity of berth planning and yard planning for port operators. In other words, it is economical for port operators to maintain a consistent schedule of berth planning and yard planning over several periods considering the fixed patterns of shipping services. The periodicity does not complicate the yard planning under the consignment strategy, but incurs additional challenges for the berth planning (Moorthy and Teo 2006). The traditional BAP is usually modeled as a constrained two-dimensional bin packing problem (Lim 1998, Kim and Moon 2003) with a time dimension and a space dimension. However, when constructing periodic schedules, the rectangle packing on a plane, as shown in Figure 3, should be extended

H

End time step

H+ETime



 $Gap \leq H$ 

Figure 3. (Color online) Horizon Extension-Based Method for Considering the Periodicity of the Plan



to a packing problem on a cylinder with circumference equal to the length of the planning horizon. To handle periodicity in the planning process, the key idea is to enlarge the original planning horizon from H (e.g., one week) to H + E, where  $E = \max_{i \in V, p \in P_i} \{h_{ip}\}$  ( $P_i$  is the set of QC profiles for vessel i). For each berth, we introduce the first time step (i.e., the start time step  $\varrho_b$ , to be determined) and the last time step (i.e., the end time step  $\varsigma_b$ , to be determined) during which the berth is occupied in the planning horizon. We need to ensure that the berth cannot be occupied by any vessel before the start time step  $\varrho_b$  and after the end time step  $\varsigma_b$ . Meanwhile, to ensure that the berth occupancy can be wrapped around the original planning horizon *H*, the gap between the two time steps (i.e.,  $\zeta_b - \varrho_b$ ) cannot exceed H.

As discussed in Section 3.2, the QC assignment is embedded within the berth planning process. Henceforth, the QC assignment inherits the periodicity from the berth allocation, for which the restrictions of the QC utilization in each time step can be stated as follows: (i) in time step  $t = \{E + 1, ..., H\}$ , the total number of QCs utilized cannot exceed the number of available QCs, and (ii) the sum of the number of QCs utilized in time step t,  $t \in \{1, ..., E\}$  and the number of QCs utilized in its "twin" time step t + H cannot exceed the number of available QCs.

#### 4. Mixed Integer Linear Programming **Formulation**

We now formulate a mixed integer linear programming (MILP) model for the integrated berth allocation, QC assignment, and yard assignment problem. The objective of the model is to minimize the total service quality cost, including the penalty cost caused by the deviation from the vessels' expected service time, and the total operation cost related to the route length of the container transportation flows in the yard.

#### 4.1. Notation

#### Indices:

- i, j Vessels;
- k Subblocks;
- Berths;
- p QC profiles;
- t Time steps.

#### Input Parameters:

- V Set of incoming vessels;
- K Set of available subblocks in the yard;
- Set of berths in the quay;
- Number of time steps in the planning horizon;
- Maximum handling time of all vessels; that is,  $E = \max_{\forall i \in V, p \in P_i} \{h_{ip}\};$  T Set of time steps,  $T = \{1, ..., H + E\};$

- $P_i$  Set of QC profiles for vessel  $i, i \in V$ ;
- Handling time of vessel *i* by using QC profile p with unit of time step,  $i \in V$ ,  $p \in P_i$ ;
- Number of QCs used by QC profile  $p \in P_i$ ,  $i \in V$  at the mth time step,  $m \in \{1, \dots, h_{iv}\}$ ;
  - Maximum number of QCs available at time step  $t, t \in T$ ;
- $[a_i^f, b_i^f]$  Feasible service time steps for vessel  $i, i \in V$ ;  $[a_i^e, b_i^e]$  Expected service time steps for vessel  $i, i \in V$ ;
- Number of subblocks that should be reserved for vessel  $i, i \in V$ , which has incorporated the capacity of each subblock;
  - Number of containers that should be loaded for vessel  $i, i \in V$ ;
  - Number of containers that should be unloaded for vessel  $i, i \in V$ ;
  - Length of loading route from subblock *k* to berth *b* in the yard,  $k \in K$ ,  $b \in B$ ;
  - $D_h^U$ Average length of unloading route from berth b to all of the subblocks in the yard,
  - Coefficient of the penalty cost caused by the deviation from the expected service time of vessel i;
  - Coefficient of the operation cost related to the route length of the container transportation flows in yard;
  - M A sufficiently large positive number.

#### Decision variables:

- $\omega_{ib} \in \{0,1\}$  Set to one if berth *b* is allocated to vessel *i* and to zero otherwise,  $i \in V$ ,  $b \in B$ ;
- $\delta_{iib} \in \{0, 1\}$ Set to one if both vessel *i* and vessel *j* dwell at berth b, and vessel i dwells at the berth before vessel *j* and to zero otherwise,  $i, j \in V$ ,  $i \neq j$ ,  $b \in B$ ;
- $\varphi_{ik} \in \{0,1\}$ Set to one if subblock *k* is reserved for vessel *i* and to zero otherwise,  $i \in V$ ,  $k \in K$ ;
- $\gamma_{iv} \in \{0,1\}$  Set to one if vessel *i* is served by QC profile p and to zero otherwise,  $i \in V$ ,
- $\mu_{it} \in \{0,1\}$  Set to one if vessel *i* begins handling in the time step t and to zero otherwise,  $i \in V, t \in T$ ;
- $\eta_{ipt} \in \{0,1\}$  Set to one if vessel *i* is served by QC profile p and begins handling by this QC profile in the time step t, and to zero otherwise,  $i \in V$ ,  $p \in P_i$ ,  $t \in T$ ;
  - $\alpha_i \in T$  Integer, the start time step of the handling for vessel i,  $i \in V$ ;
  - $\beta_i \in T$  Integer, the end time step of the handling for vessel  $i, i \in V$ ;
  - $\varrho_b$ ,  $\varsigma_b \in T$  Start and end time steps for berth b,  $b \in B$ ;
    - $\sigma_t \ge 0$  Integer, the number of used QCs at time step  $t, t \in T$ .



#### 4.2. Mathematical Model

Given the input parameters and decision variables, we can formulate the MILP model as follows:

[M1] minimize 
$$\begin{cases} \sum_{i \in V} c_i^p \left[ \left( \frac{a_i^e}{c} - \alpha_i \right)^+ + \left( \beta_i - \frac{b_i^e}{c} \right)^+ \right] \\ + c^o \sum_{i \in V} \sum_{b \in B} \sum_{k \in K} \left[ \omega_{ib} \varphi_{ik} D_{kb}^L (l_i / r_i) \right] \end{cases}$$

$$+c^{o}\sum_{i\in V}\sum_{b\in B}\omega_{ib}D_{b}^{U}u_{i}\bigg\} \tag{1}$$

s.t. 
$$\sum_{i \in V} \varphi_{ik} \le 1$$
,  $k \in K$ , (2)

$$\sum_{k \in K} \varphi_{ik} = r_i, \quad i \in V, \tag{3}$$

$$\sum_{k \in K} \varphi_{ik} = r_i, \quad i \in V,$$

$$\sum_{p \in P_i} \gamma_{ip} = 1, \quad i \in V,$$
(3)

$$\sum_{b \in B} \omega_{ib} = 1, \quad i \in V, \tag{5}$$

$$\sum_{t \in \{1, \dots, H\}} \mu_{it} = 1, \quad i \in V, \tag{6}$$

$$\sum_{t \in T} \mu_{it} t = \alpha_i, \quad i \in V,$$
 (7)

$$\alpha_i + \sum_{p \in P_i} \gamma_{ip} h_{ip} - 1 = \beta_i, \quad i \in V,$$
(8)

$$\alpha_i + \sum_{p \in P_i} \gamma_{ip} h_{ip} \le \alpha_j + (1 - \delta_{ijb}) M,$$

$$i, j \in V, i \ne j, b \in B,$$

$$\delta_{iib} + \delta_{iib} \le \omega_{ib}, \quad i, j \in V, i \ne j, b \in B, \tag{10}$$

(9)

(27)

$$\delta_{ijb} + \delta_{jib} \ge \omega_{ib} + \omega_{jb} - 1, \quad i, j \in V, i \ne j, b \in B, \tag{11}$$

$$\alpha_i \ge a_i^f, \quad i \in V,$$
 (12)

$$\beta_i \leq b_i^f, \quad i \in V, \tag{13}$$

$$\eta_{ipt} \ge \gamma_{ip} + \mu_{it} - 1, \quad i \in V, p \in P_i, t \in T, \tag{14}$$

$$\sigma_{t} = \sum_{i \in V} \sum_{p \in P_{i}} \sum_{m = \max\{1; t - h_{ip} + 1\}}^{t} \eta_{ipm} q_{ip(t - m + 1)}, \quad t \in T, \quad (15)$$

$$\sigma_t \leq Q_t, \quad t \in \{E+1, \dots, H\},\tag{16}$$

$$\sigma_t + \sigma_{t+H} \le Q_t, \quad t \in \{1, \dots, E\},\tag{17}$$

$$\varrho_b \le \alpha_i + (1 - \omega_{ib}) \cdot M, \quad i \in V, b \in B, \tag{18}$$

$$\zeta_b \ge \beta_i + (\omega_{ib} - 1) \cdot M, \quad i \in V, b \in B,$$
(19)

$$\varsigma_b - \varrho_b \le H - 1, \quad b \in B, \tag{20}$$

$$\omega_{ib} \in \{0,1\}, \quad i \in V, b \in B, \tag{21}$$

$$\delta_{iib} \in \{0,1\}, \quad i, j \in V, i \neq j, b \in B,$$
 (22)

$$\varphi_{ik} \in \{0,1\}, \quad i \in V, k \in K,$$
 (23)

$$\gamma_{ip} \in \{0,1\}, \quad i \in V, p \in P_i,$$
 (24)

$$\mu_{it} \in \{0,1\}, \quad i \in V, t \in T,$$
 (25)

$$\eta_{ipt} \in \{0,1\}, \quad i \in V, p \in P_i, t \in T,$$
 (26)

$$\eta_{ipt} \in \{0,1\}, \quad i \in V, p \in \Gamma_i, i \in I, \tag{20}$$

$$\rho_h, \zeta_h \in T, \quad b \in B, \tag{28}$$

where objective (1) minimizes the total cost, including the penalty costs, the operation costs on the loading process, and the operation costs on the unloading process. Note that the third part of the objective function follows from our assumption made in Section 3.2 on the average unloading route length, and the solutions derived under this assumption may be suboptimal for the problem capturing the real unloading route length. However, the average unloading route length exploits the distance advantage of berth positions on the quayside, since central berths that have a short average distance to the yard should be allocated in priority to those vessels with a large amount of unloading containers. This distance advantage will be further investigated in Section 7.

In the above model, constraints (2) guarantee that each subblock is reserved for at most one vessel. Constraints (3) ensure that a given number  $r_i$  of subblocks are reserved to vessel i. Constraints (4) stipulate that only one QC profile is assigned to each vessel. Constraints (5) mean that each vessel can be allocated to only one berth. Constraints (6) state that each vessel starts handling in a certain time step. Constraints (7) connect the two handling start time decision variables (i.e.,  $\mu_{it}$  and  $\alpha_i$ ). Specifically, if vessel *i* begins handling in time step t (i.e.,  $\mu_{it} = 1$ ), the start time step of the handling for vessel i is time step t. Constraints (8) link the start time step and the end time step of the vessels. Constraints (9) ensure that for the same berth, a former dwelling vessel must end its handling activities at the berth before a late dwelling vessel starts its handling activities at the berth. Constraints (10) and (11) guarantee that if two vessels are allocated to the same berth, there must be a time sequence for the two vessels dwelling at the berth. Constraints (12) and (13) enforce the condition that the service time for each vessel must lie within its feasible service time interval. Constraints (14) link two decision variables  $\eta_{ipt}$  and  $\mu_{it}$ that are both related to the start time of handling. Constraints (15) calculate the number of QCs used in each time step. Constraints (16) and (17) guarantee that the number of QCs used in each time step cannot exceed the capacity considering the periodicity of vessel schedules. Constraints (18) and (19) ensure that for each berth,  $\varrho_h$  (or  $\varsigma_h$ ) is no later than (or no earlier than) all of the start (or end) time steps of vessels that occupy berth b. Constraints (20) ensure that the gap between  $\varrho_b$  and  $\varsigma_b$  does not exceed the length of the planning horizon. Constraints (21)-(28) define the domains of decision variables. Note that since our problem is a tactical-level planning problem, model M1 is formulated based on the models for the TBAP (Giallombardo et al. 2010; Expósito-Izquierdo et al. 2016), in which the QC assignment decisions are represented by the selection of QC profiles.

#### 4.3. Linearization for the Model

The first two parts in the objective of the above model are nonlinear, but they can be linearized. To linearize



 $\sigma_t \ge 0$ ,  $t \in T$ ,

the first part, that is,  $\sum_{i \in V} c_i^p [(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+]$ , we define the additional decision variables  $\tau_i^{a+}$ ,  $\tau_i^{a-}$ ,  $\tau_i^{b+}$ , and  $\tau_i^{b-}$ ,  $i \in V$ . By adding the following constraints, the first part in the objective can be reformulated as  $c^p \sum_{i \in V} (\tau_i^{a+} + \tau_i^{b+})$ 

$$a_i^e - \alpha_i = \tau_i^{a+} - \tau_i^{a-}, \quad i \in V;$$
 (29)

$$\beta_i - b_i^e = \tau_i^{b+} - \tau_i^{b-}, \quad i \in V;$$
 (30)

$$\tau_i^{a+}, \ \tau_i^{a-}, \ \tau_i^{b+}, \ \tau_i^{b-} \ge 0, \quad i \in V.$$
 (31)

The second part  $c^o \sum_{i \in V} \sum_{b \in B} \sum_{k \in K} [\omega_{ib} \varphi_{ik} D^L_{kb}(l_i/r_i)]$  can be linearized as follows:

Let  $\theta_{ikb} \in \{0,1\}$  equal one if and only if vessel i dwells at berth b and subblock k is reserved for vessel i,  $i \in V$ ,  $k \in K$ ,  $b \in B$ . Then,

$$\theta_{ikb} \ge \omega_{ib} + \varphi_{ik} - 1, \quad i \in V, k \in K, b \in B;$$
 (32)

$$\theta_{ikh} \in \{0, 1\}, \quad i \in V, k \in K, b \in B.$$
 (33)

Based on these new decision variables and constraints, the integrated model for the berth allocation, QC assignment, and yard assignment problem can be reformulated as an MILP model

[M2] minimize 
$$\left\{ \sum_{i \in V} c_i^p (\tau_i^{a+} + \tau_i^{b+}) + c^o \sum_{i \in V} \sum_{b \in B} \sum_{k \in K} \left[ \theta_{ikb} D_{kb}^L (l_i/r_i) \right] + c^o \sum_{i \in V} \sum_{b \in B} \omega_{ib} D_b^U u_i \right\}$$
(34)

subject to: Constraints (2)–(33).

# 5. Set Covering Model and Column Generation

The mixed integer programming model for the integrated problem becomes hard to solve by some commercial solvers, such as CPLEX, when the size of the problem instances become large. Therefore, in this section, we reformulate the problem as a set covering model by using Dantzig-Wolfe decomposition.

#### 5.1. Set Covering Model

Let  $\mathcal{P}_i$  be the set of all possible assignment plans of vessel  $i, i \in V$  in the given planning horizon. Each assignment plan  $\mathcal{P}_i$  of vessel i represents the allocation of a berth to the vessel in time steps, the reservation of  $r_i$  subblocks in the yard to the vessel, and the number of QCs used by vessel i in each time step. Here, we define  $\mathbb{P} = \bigcup_{i \in V} \mathcal{P}_i$  as the set of all possible assignment plans.

For each assignment plan  $\mathcal{P}_i$  of vessel i, we have the following input parameters:

 $A_{bt}^{\mathcal{P}_i}$  Equals one if berth b is allocated to vessel i in time step t in assignment plan  $\mathcal{P}_i$  and zero otherwise,  $b \in B$ ,  $t \in T$ ;

 $R_k^{\mathcal{P}_i}$  Equals one if subblock k is reserved to vessel i in assignment plan  $\mathcal{P}_i$  and zero otherwise,  $k \in K$ ;

 $U_t^{\mathcal{P}_i}$  Integer, number of QCs used by vessel i in the time step t in assignment plan  $\mathcal{P}_i$ ,  $t \in T$ .

Let  $\mathscr{C}_{\mathcal{P}_i}$  be the cost constant of the assignment plan  $\mathcal{P}_i$ , whose calculation will be elaborated in Section 5.2. For each feasible assignment plan  $\mathcal{P}_i \in \mathcal{P}_i$ , we define a binary variable  $\lambda_{\mathcal{P}_i}$ , which equals one if and only if the assignment plan  $\mathcal{P}_i$  is used by vessel i. Based on these parameters, variables, and constants, the set covering model for the problem can be formulated as follows:

[M3] minimize 
$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} \mathcal{C}_{\mathcal{P}_i} \lambda_{\mathcal{P}_i}$$
 (35)

s.t. 
$$\sum_{\mathcal{P}_i \in \mathcal{P}_i} \lambda_{\mathcal{P}_i} = 1, \quad i \in V,$$
 (36)

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} A_{bt}^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \le 1, \quad b \in B, t \in T,$$
(37)

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} R_k^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \le 1, \quad k \in K, \tag{38}$$

$$\sum_{i \in V} \sum_{\mathcal{P}_t \in \mathcal{P}_t} (U_t^{\mathcal{P}_t} + U_{t+H}^{\mathcal{P}_t}) \lambda_{\mathcal{P}_t} \leq Q_t, \quad t \in \{1, \dots, E\}, \quad (39)$$

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} U_t^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \leq Q_t, \quad t \in \{E+1, \dots, H\}, \tag{40}$$

$$t \cdot \sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} A_{bt}^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} + M \left( 1 - \sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} A_{bt}^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \right) - \varrho_b \ge 0,$$

$$b \in B, t \in T, \quad (41)$$

$$t \cdot \sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} A_{bt}^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} + M \left( \sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}_i} A_{bt}^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} - 1 \right) - \zeta_b \le 0,$$

$$\zeta_b - \varrho_b \le H - 1, \quad b \in B, \tag{43}$$

$$\lambda_{\rho_i} \in \{0,1\}, \quad i \in V, \mathcal{P}_i \in \mathcal{P}_i,$$
 (44)

$$\rho_h, \zeta_h \in T, \quad b \in B. \tag{45}$$

In the above formulation, objective (35) minimizes the total cost of serving vessels in the port. Constraints (36) ensure that there is exactly one feasible assignment for each vessel in the solution. Constraints (37) guarantee that each berth is occupied by at most one vessel in each time step. Constraints (38) mean that each subblock can be reserved for at most one vessel. Constraints (39) and (40) state that the QCs used in each time step are within the limited capacity. Constraints (41) and (42) ensure that for each berth,  $\varrho_b$  (or  $\varsigma_b$ ) is no later than (or no earlier than) all of the start (or end) time steps of vessels who occupy berth b. Constraints (43) ensure that the gap between  $\varrho_b$  and  $\varsigma_b$  does



not exceed the length of the planning horizon. Constraints (44) and (45) define the domains of decision variables.

It is worth mentioning that the set covering model is a general and flexible model that can easily accommodate new constraints. For example, one could ignore the periodicity in berth and yard planning and instead solve an integrated berth allocation, quay assignment, and yard assignment problem over a multiperiod planning horizon. Given a planning horizon with N periods (or weeks), the same weekly arrival vessel will visit the port for N times. Here, to be compatible with the model, we denote each visit of each vessel in a period as an incoming vessel arrival  $i \in V$ . Henceforth, the berth allocation and quay assignmentrelated constraints remain the same. However, the yard assignment-related constraints need to be modified through the introduction of new input parameters. For each incoming vessel arrival  $i \in V$ , we introduce a time interval  $[a_i^s, b_i^s]$  for the yard assignment, during which  $r_i$  number of subblocks are assigned to the incoming vessel arrival. We then define a time-indexed parameter for each incoming vessel arrival as

$$g_{it} = \begin{cases} 1 & \text{if } t \in [a_i^s, b_i^s], \\ 0 & \text{otherwise,} \end{cases} \quad i \in V, t \in T.$$
 (46)

Based on the newly defined parameters, we extend a time dimension to constraints (38) for a new set covering model to solve the new research problem, in which constraints (38) are replaced by constraints (47)

$$\sum_{i \in V} g_{it} \sum_{\mathcal{P}_i \in \mathcal{P}_i} R_k^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \le 1, \quad k \in K, \ t \in T. \tag{47}$$

## 5.2. Restricted Master Problem for the Column Generation Procedure

The above formulation contains all of the possible assignment plans for the vessels. Therefore, the size of  $\mathbb{P}$  and the corresponding computational time needed to solve the problem grow exponentially with the instance size. To circumvent this difficulty, we use CG to solve the LP relaxation of the formulation. In the CG procedure, we maintain a restricted master problem (RMP) with a subset of a feasible assignment plan  $\mathbb{P}' = \bigcup_{i \in V} \mathscr{P}'_i \subseteq \mathbb{P}$ . Initially, we derive a  $\mathbb{P}'$  for the RMP by using a heuristic (Section 5.5), which ensures that an initial feasible solution exists in the RMP. The RMP is formulated as follows:

[M4] minimize 
$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}'_i} \mathcal{C}_{\mathcal{P}_i} \lambda_{\mathcal{P}_i}$$
 (48)

subject to: Constraints (36)-(40).

Note that the constraints that ensure the periodicity of the berth allocation (i.e., constraints (42)–(43)) are invalid and removed for the RMP, which is an LP relax-

ation. To guarantee periodicity in feasible integer solutions, a substep is designed in a CG-based heuristic, which will be discussed in Section 6.1. At each iteration of the CG procedure, the dual variables of the RMP are transferred to pricing problems that are used to generate new feasible assignment plans (i.e., columns). These dual variables are defined as follows:

- $\pi_i$  The dual variables for constraints (36),  $i \in V$ ;
- $\varpi_{bt}$  The dual variables for constraints (37),  $b \in B$ ,  $t \in T$ ;
- $\rho_k$  The dual variables for constraints (38),  $k \in K$ ;
- $\phi_t$  The dual variables for constraints (39) and (40),  $t \in T$ :

The dual variables  $\phi_t$  obtained from the RMP are  $\phi_t$ ,  $t \in \{1, \ldots, H\}$ . To ensure periodicity, the planning horizon is enlarged from H to T = H + E. Therefore, the dual variables  $\phi_t$  passing to the pricing problems should be  $\phi_t$ ,  $\forall t \in T$ , where  $\phi_t = \phi_{t-H}$ ,  $\forall t \in \{H+1, \ldots, H+E\}$ . Using these dual variables, the pricing problems will generate feasible assignment plans with the lowest reduced costs (i.e., the objective values of the pricing problems). The CG procedure stops when all minimal reduced costs are positive, which means that no feasible assignment plan can be added to the RMP.

#### 5.3. Pricing Problem

The goal of the pricing problems is to find feasible assignment plans with a negative reduced cost. At each iteration of the CG procedure, there are |V| pricing problems to be solved, each of which corresponds to a vessel (e.g., vessel i), and we will generate one feasible assignment plan  $\mathcal{P}_i^*$  for each vessel. For all of the |V| optimal feasible assignment plans generated by solving the pricing problems, only the feasible assignment plans with a negative reduced cost can be added to the RMP, which means that at each iteration of the CG procedure, there are at most |V| columns to be added into the RMP. The formulation for the pricing problem of each vessel is given next. Note that the index  $i \in V$  is removed from the formulation since the pricing problem for each vessel is solved separately.

Input parameters:

 $\pi$ ,  $\omega_{bt}$ ,  $\rho_k$ ,  $\phi_t$  The dual variables obtained from the RMP;

P Set of QC profiles for the vessel;

 $h_p$  Handling time of the vessel by using QC profile p with unit of time step,  $p \in P$ ;

 $q_{pm}$  Number of QCs used by QC profile  $p \in P$  at the mth time step,  $m \in \{1, ..., h_v\}$ ;

 $[a^f, b^f]$  Feasible service time steps for the vessel;

[ $a^e$ ,  $b^e$ ] Expected service time steps for the vessel;

r Number of subblocks that should be reserved for the vessel;



- l Number of containers that should be loaded for the vessel;
- *u* Number of containers that should be unloaded for the vessel;
- c<sup>p</sup> Coefficient of the penalty cost caused by the deviation from the vessel's expected service time.

#### Decision variables:

- $\varepsilon_{bt} \in \{0,1\}$  Set to one if the vessel dwells at berth b in the time step t and to zero otherwise,  $b \in B$ ,  $t \in T$  (corresponding to  $A_{bt}^{\mathcal{P}_i}$ );
- $\varphi_k \in \{0,1\}$  Set to one if subblock k is reserved for the vessel and to zero otherwise,  $k \in K$  (corresponding to  $R_k^{\mathcal{P}_i}$ );
  - $\zeta_t \ge 0$  Integer, the number of QCs used by the vessel in the time step  $t, t \in T$  (corresponding to  $U_t^{\mathcal{P}_i}$ );
- $v_t \in \{0,1\}$  Set to one if the vessel is served in the time step t and to zero otherwise,  $t \in T$ ;
- $\omega_b \in \{0,1\}$  Set to one if berth b is allocated to the vessel and to zero otherwise,  $b \in B$ ;
- $\gamma_p \in \{0,1\}$  Set to one if the vessel is served by QC profile p and to zero otherwise,  $p \in P$ ;
- $\mu_t \in \{0,1\}$  Set to one if the vessel begins handling in the time step t and to zero otherwise,  $t \in T$ ;
- $\eta_{pt} \in \{0,1\}$  Set to one if the vessel is served by QC profile p and begins handling by this QC profile in the time step t, and to zero otherwise,  $p \in P$ ,  $t \in T$ ;
- $\theta_{kb} \in \{0,1\}$  Set to one if the vessel dwells at berth b and subblock k is reserved for the vessel, and to zero otherwise,  $k \in K$ ,  $b \in B$ ;
  - $\alpha \in T$  Integer, the start time step of the handling for the vessel;
  - $\beta \in T$  Integer, the end time step of the handling for the vessel;
  - $\mathscr{C}_{\mathcal{P}} \geq 0$  The cost for the assignment plan of the vessel.

The variables  $\tau^{a+}$ ,  $\tau^{a-}$ ,  $\tau^{b+}$ , and  $\tau^{b-}$  are additional variables for the linearization. Then, the pricing problem can be formulated as follows:

[M5] minimize  $\left\{ \mathscr{C}_{\mathcal{P}} - \left( \pi + \sum_{b \in B} \sum_{t \in T} \varpi_{bt} \cdot \varepsilon_{bt} + \sum_{k \in K} \rho_k \cdot \varphi_k + \sum_{t \in T} \varphi_t \cdot \zeta_t \right) \right\}$ (49)

$$s.t. \sum_{k=1}^{\infty} \varphi_k = r, \tag{50}$$

$$\sum_{p \in P} \gamma_p = 1,\tag{51}$$

$$\sum_{b \in B} \omega_b = 1,\tag{52}$$

$$\sum_{t \in \{1, \dots, H\}} \mu_t = 1,\tag{53}$$

$$\sum_{t \in T} \mu_t t = \alpha,\tag{54}$$

$$\alpha + \sum_{p \in P} \gamma_p h_p - 1 = \beta, \tag{55}$$

$$\alpha \ge a^f$$
, (56)

$$\beta \le b^f, \tag{57}$$

$$\eta_{pt} \ge \gamma_p + \mu_t - 1, \quad p \in P, t \in T, \tag{58}$$

$$\eta_{vt} \le \gamma_v, \quad p \in P, \ t \in T,$$
(59)

$$\eta_{pt} \le \mu_t, \quad p \in P, \ t \in T,$$
(60)

$$\zeta_t = \sum_{p \in P} \sum_{m = \max\{1; t - h_p + 1\}}^t \eta_{pm} q_{p(t - m + 1)}, \quad t \in T,$$
 (61)

$$t + M(1 - \nu_t) \ge \alpha, \quad t \in T, \tag{62}$$

$$t \le \beta + M(1 - \nu_t), \quad t \in T, \tag{63}$$

$$\sum_{t \in T} \nu_t = \beta - \alpha + 1,\tag{64}$$

$$\varepsilon_{bt} \ge v_t + \omega_b - 1, \quad b \in B, \ t \in T,$$
 (65)

$$\varepsilon_{bt} \le \nu_t, \quad b \in B, \ t \in T,$$
 (66)

$$\varepsilon_{bt} \le \omega_b, \quad b \in B, \ t \in T,$$
 (67)

$$\theta_{kb} \ge \omega_b + \varphi_k - 1, \quad k \in K, b \in B,$$
 (68)

$$a^{e} - \alpha = \tau^{a+} - \tau^{a-}, \tag{69}$$

$$\beta - b^e = \tau^{b+} - \tau^{b-},\tag{70}$$

$$\mathcal{C}_{p} = c^{p}(\tau^{a+} + \tau^{b+}) + c^{o} \sum_{b \in B} \sum_{k \in K} \left[\theta_{kb} D_{kb}^{L}(l/r)\right] + c^{o} \sum_{b \in P} \omega_{b} D_{b}^{U} u, \quad (71)$$

$$\varepsilon_{ht} \in \{0,1\}, \quad b \in B, t \in T, \tag{72}$$

$$\varphi_k \in \{0, 1\}, \quad k \in K,$$
 (73)

$$\omega_b \in \{0,1\}, \quad b \in B, \tag{74}$$

$$\gamma_v \in \{0, 1\}, \quad p \in P,$$
 (75)

$$\mu_t \in \{0,1\}, \quad t \in T,$$
 (76)

$$\eta_{vt} \in \{0, 1\}, \quad p \in P, t \in T,$$
(77)

$$\theta_{kb} \in \{0, 1\}, \quad k \in K, b \in B,$$
 (78)

$$v_t \in \{0, 1\}, \quad t \in T,$$
 (79)  
 $\zeta_t \ge 0, \quad t \in T,$  (80)

$$\alpha, \beta \in T$$
, (81)

$$\tau^{a+}, \tau^{a-}, \tau^{b+}, \tau^{b-}, \mathscr{C}_{\mathcal{P}} \ge 0.$$
 (82)

Once an assignment plan  $\mathcal{P}$  is chosen as the newly added column to the RMP for vessel i, the corresponding cost  $\mathscr{C}_{\mathcal{P}}$  is a cost constant of the newly added assignment plan  $\mathcal{P}_i$  (i.e.,  $\mathscr{C}_{\mathcal{P}_i}$ ), which is included in the objective function of the RMP (i.e., objective (35)). Meanwhile, the decision variables  $\varepsilon_{bt}$ ,  $\varphi_k$ , and  $\zeta_t$  are transferred to the input parameters of the RMP, which are  $A_{bt}^{\mathcal{P}_i}$ ,  $R_k^{\mathcal{P}_i}$ , and  $U_t^{\mathcal{P}_i}$ , respectively.

In the above formulation, objective (49) minimizes the reduced cost of the optimal assignment plan. Constraints (50) states that *r* subblocks should be reserved for the vessel. Constraint (51) guarantees that only one QC profile is selected for the vessel. Constraint (52)



ensures that exactly one berth is allocated to the vessel. Constraint (53) states that the vessel starts handling in a certain time step. Constraint (54) connects the two handling start-related decision variables (i.e.,  $\pi_i$  and  $\alpha$ ). Constraint (55) links the start time step and the end time step of the vessel. Constraints (56) and (57) force the service time for the vessel to be within the feasible service time span. Constraints (58)-(60) link two decision variables  $\eta_{pt}$  and  $\mu_t$  that are both related to the start time of handling. Constraint (61) calculate the number of QCs used by the vessel in each time step. Constraints (66)–(68) connect the three service-related decision variables (i.e.,  $\alpha$ ,  $\beta$ , and  $\nu_t$ ). Constraints (65)–(67) link two decision variables,  $\varepsilon_{ht}$ and  $\omega_b$ , that are both related to berth allocation. Constraints (68)–(70) are additional constraints for the linearization. Constraint (71) calculates the cost for the assignment plan of the vessel. Constraints (72)–(82) define the domains of decision variables.

#### 5.4. Solving the Pricing Problem

In this section, we propose an efficient algorithm for the pricing problem, which can compute optimal solutions for the problem in pseudopolynomial time. The basic idea of this method is as follows: for a given vessel, we list all of the possible time steps at which the vessel starts to be served (i.e., t:  $\mu_t = 1$ ) and all of the possible numbers of time steps during which the vessel dwells at the port (i.e.,  $\beta - \alpha + 1$ ). Here, for simplicity, we define the time step at which the vessel starts to be served as  $\chi$ , and the number of time steps that the vessel dwells at the port as  $\psi$ . Based on the input parameters, the handling time of the vessel using QC profile p (i.e.,  $h_n$ ) is used to measure the efficiency of the QC profiles. However, to improve the berth availability in the optimal solution,  $h_{\nu}$  can also be used to narrow down the range of  $\psi$  since the vessel will be served immediately on arrival and can depart immediately after the service is finished, which means that  $\psi \in [\min(h_n), \max(h_n)]$ . Regarding  $\chi$ , we use another input parameter to reduce its possible range, which is  $[a^f, b^f]$  (i.e., the feasible service time steps for the vessel). Given a value of  $\psi$  (i.e., the dwelling time for the vessel is given), we can further conclude that  $\chi \in$  $[a^f, b^f - \psi + 1].$ 

We denote the combination of a given starting time step (i.e.,  $\chi$ ) and of a dwelling time (i.e.,  $\psi$ ) as a scenario of the vessel. Here, note that  $\chi$  can also be deemed as the arrival time step of the vessel, and  $\chi$  +  $\psi$  – 1 as its departure time step. The cardinality of the scenarios remains unchanged even if the size of the problem instance increases, because it is related to the service level of the port and to the flexibility of the vessel. Given a scenario,  $\chi$  and  $\psi$  can be determined, which brings the following changes for the pricing problem: (i) the penalty cost (i.e.,  $c^p(\tau^{a+} + \tau^{b+})$ ) caused by the deviation can be written as  $[(a^e - \chi)^+ +$   $(\chi + \psi - 1 - b^e)^+$ ], which helps avoid complex linearizations in the pricing problem; (ii) the selection of QC profile p is isolated from the pricing problem, which can be implemented by solving M6. This model can be solved very easily by an exact polynomial algorithm (called Subalgorithm 1). The pseudocode for this algorithm is given in the online appendix; (iii) what is left for the pricing problem is to allocate a berth and a certain number of subblocks to the vessel. The berth allocation and the subblock assignment still interact with each other even in the scenario. However, we can formulate a simple model for the berth allocation and the subblock assignment, given as M7. The exact polynomial algorithm (called Subalgorithm 2) for this model is also elaborated in the online appendix

[M6] maximize 
$$\left\{ \sum_{t \in T} \phi_t \cdot \zeta_t \right\}$$
 (83)

s.t. 
$$\sum_{p \in P} \gamma_p = 1,$$
 (84)  
$$\sum_{p \in P} \gamma_p \cdot h_p = \psi,$$
 (85)

$$\sum_{p \in P} \gamma_p \cdot h_p = \psi, \tag{85}$$

$$\zeta_t = \sum_{p \in P} \gamma_p \cdot q_{p(t-\chi+1)}, \quad t \in [\chi, \chi + \psi - 1], \tag{86}$$

$$\zeta_t = 0, \quad t \in T \setminus [\chi, \chi + \psi - 1],$$
 (87)

$$\gamma_p \in \{0, 1\}, \quad p \in P.$$
 (88)

In the above model, objective (83) aims to optimize the QC-related reduced cost of the scenario. Constraint (84) ensures that exactly one QC profile is selected. Constraint (85) guarantees that the selected QC profile must serve the vessel for exactly  $\psi$  time steps. Constraints (86) and (87) calculate the number of QCs used by the vessel in each time step. Constraints (88) define the domains of decision variables

[M7] minimize 
$$\left\{ c^{o} \sum_{b \in B} \sum_{k \in K} \left[ \theta_{kb} D_{kb}^{L} (l/r) \right] + c^{o} \sum_{b \in B} \omega_{b} D_{b}^{U} u - \sum_{b \in B} \sum_{t \in T} \omega_{bt} \cdot \varepsilon_{bt} - \sum_{k \in K} \rho_{k} \cdot \varphi_{k} \right\}$$
 (89)

s.t. 
$$\sum_{k \in K} \varphi_k = r,$$
 (90)  
$$\sum_{k \in R} \omega_b = 1,$$
 (91)

$$\sum_{b \in B} \omega_b = 1, \tag{91}$$

$$\theta_{kb} \ge \omega_b + \varphi_k - 1, \quad k \in K, b \in B,$$
 (92)

$$\varepsilon_{ht} = \omega_h, \quad t \in [\chi, \chi + \psi - 1], b \in B,$$
 (93)

$$\varepsilon_{bt} = 0, \quad t \in T \setminus [\chi, \chi + \psi - 1], \ b \in B,$$
 (94)

$$\varepsilon_{bt} \in \{0,1\}, \quad b \in B, \ t \in T, \tag{95}$$

$$\varphi_k \in \{0, 1\}, \quad k \in K,$$
 (96)

$$\omega_b \in \{0, 1\}, \quad b \in B, \tag{97}$$

$$\theta_{kb} \in \{0, 1\}, \quad k \in K, b \in B.$$
 (98)

In the above formulation, objective (89) minimizes the berth- and subblock-related reduced cost of the



scenario. Constraint (90) states that r subblocks should be reserved for the vessel. Constraint (91) ensures that exactly one berth is allocated to the vessel. Constraints (92) link the two decision variables  $\omega_b$  and  $\varphi_k$ , which are related to the berth allocation and the subblock assignment, respectively. Constraints (93) and (94) aim to derive the allocation of berths to the vessel in each time step. Constraints (95)–(98) define the domains of the decision variables. Based on the above analysis, the detailed procedure of this exact algorithm for solving the pricing problem is elaborated in Algorithm 1.

5.4.1. QC Assignment Decision in the Pricing Problem. Model M6 makes the QC assignment decision for the vessel by choosing a feasible QC profile from the pregenerated set P of QC profiles. Initially, the major motivation for using the concept of QC profiles was to release the optimization pressure of the MILP model. Here, because of the hierarchical structure of the pricing algorithm and the compatibility of the RMP, we can abandon the concept of QC profiles and fully make the QC assignment decision by introducing a new model with an ad hoc algorithm. The reason why we can make the QC assignment decision in the CG (in other words, generate QC profiles in the solution approach) is as follows: Passing a QC assignment decision for the vessel to the RMP needs to input the value of  $\zeta_t$ , which is an integer value equal to the number of QCs used by the vessel on each time step t. In the pricing algorithm, as we fix the handling time window  $[\chi, \chi + \psi - 1]$ for a vessel as a scenario in the outer circulation, we can assign QCs to each time step during the handling time window to maximize  $\sum_{t \in T} \phi_t \cdot \zeta_t$ , subject to some underlying constraints.

Before introducing the new model and the algorithm for making the QC assignment decision without using QC profiles, we first reveal some underlying inputs and constraints related to the QC assignment, which are actually suggested in Section 7.1. We define  $z_i^{\text{max}}$ and  $z_i^{\min}$  as the maximum and minimum numbers of QCs that can be assigned to vessel i, respectively, in each handling time step considering the size of the vessel and possible interference between working QCs (Giallombardo et al. 2010; Bierwirth and Meisel 2010); we define  $wl_i$  as the total handling workload for vessel i with the unit of QC  $\times$  time step, given that the handling speed of one QC is v containers per time step. Then,  $wl_i$  can be computed as  $wl_i = \lceil (l_i + u_i)/v \rceil$ . Given a vessel i in the pricing problem, we can replace the model M6 with the following model to make a general QC assignment decision:

[M6'] maximize 
$$\left\{ \sum_{t \in T} \phi_t \cdot \zeta_t \right\}$$
 (99)

$$s.t. \sum_{t \in T} \zeta_t = w l_i, \tag{100}$$

$$z_i^{\min} \le \zeta_t \le z_i^{\max}, \quad t \in [\chi, \chi + \psi - 1], \tag{101}$$

$$\zeta_t = 0, \quad t \in T \setminus [\chi, \chi + \psi - 1].$$
 (102)

The above model can be solved exactly. We have designed Subalgorithm 3 which is described in the online appendix. Based on the model **M6**′ and the exact algorithm, we can derive the following property.

**Proposition 1.** Given a handling time window  $[\chi, \chi + \psi - 1]$ , in the optimal QC assignment, there is at most one time step in which the number of assigned QCs in the time step is neither  $z_i^{\min}$  nor  $z_i^{\max}$ , that is, is in the range  $[z_i^{\min} + 1, z_i^{\max} - 1]$ . In all other time steps, the number of the assigned QCs is either  $z_i^{\min}$  or  $z_i^{\max}$ .

**Proof.** The exact algorithm Subalgorithm 3 reveals the proof procedure. Given the total workload  $wl_i$ ,  $z_i^{\min}$  QCs (or workload in one time step) should first be assigned in each handling time step to guarantee the feasibility of the QC assignment. The remaining workload should be allocated to the time step that has the highest dual value  $\phi_t$  until the assigned QCs in the step reaches the upper limit  $z_i^{\max}$ . In the end, there must be a time step that carries all of the remaining workload. The total QCs assigned in this time step can be any value in the range  $[z_i^{\min}, z_i^{\max}]$ . In all other time steps, the number of the assigned QCs is either  $z_i^{\min}$  or  $z_i^{\max}$ .

Essentially, the property suggests a managerial insight for the decision makers in ports if they use the QC profiles to make the QC assignment decision. When generating QC profiles for a given vessel, the procedure may not be effective if more than two time steps in the handling time window have a number of assigned QCs that is neither  $z_i^{\min}$  nor  $z_i^{\max}$ . A suggested method for generating QC profiles is to assign  $z_i^{\min}$  or  $z_i^{\max}$  QCs to all time steps first, and then select one time step with  $z_i^{\min}$  assigned QCs to take charge of all of the remaining workload. For example, consider the QC profiles presented in Figure 1. Supposing  $z_i^{\min} = 3$  and  $z_i^{\max} = 5$ , QC profile 1 is a qualified QC profile, and QC profile 2 is not a suggested one, since three handling time steps have neither  $z_i^{\min}$  nor  $z_i^{\max}$  assigned QCs.

#### 5.5. Heuristic for the Initial Set of Feasible Assignment Plans

To apply the CG procedure, we need to generate an initial set of feasible assignment plans for the RMP, so that the RMP can yield at least one feasible solution. Here, we propose a heuristic to derive an initial feasible solution. Since solving the integrated problem of berth, QC, and yard arrangement is still intractable, even heuristically, we divide the integrated problem into two stages. The berth allocation and the QC assignment are solved in the first stage, and the yard assignment is solved in the second stage given that the berth-related variables are determined.



#### Algorithm 1 (Exact algorithm for the pricing problem)

of the dwelling time  $\psi^*$ ).

```
1: Input: A given vessel
 2: Output: An optimal assignment plan and its minimal reduced cost
 3: for all the \psi, \psi \in [\min(h_p), \max(h_p)] do
        for all the \chi, \chi \in [a^f, b^f - \psi + 1] do
            Define V_{\chi,\psi} as the minimal reduced cost if the vessel starts to be served in time step \chi and its dwelling time at
 5:
            Initialize V_{\chi,\psi} = c^p [(a^e - \chi)^+ + (\chi + \psi - 1 - b^e)^+]
 6:
 7:
            Solve model M6 with the objective value denoted by Z_1^*, by Subalgorithm 1
 8:
            Solve model M7 with the objective value denoted by Z_2^*, by Subalgorithm 2
 9:
            Set V_{\chi,\psi} = V_{\chi,\psi} - Z_1^* + Z_2^*, which is the minimal reduced cost of the scenario
10:
        end for
11: end for
12: Solve \min(V_{\chi,\psi} \mid \psi \in [\min(h_p), \max(h_p)], \chi \in [a^f, b^f - \psi + 1],) - \pi which is the minimal reduced cost of the pricing
          problem of the vessel, and the new optimal assignment plan for the vessel can be extracted from the values of the
          decision variables (i.e., \epsilon_{kt}^*, \phi_k^*, and \zeta_t^*) in the optimal scenario (the combination of the starting time step \chi^* and
```

When solving the first-stage problem (i.e., the berth allocation and the QC assignment), we apply a sequential method (Zhen, Chew, and Lee 2011), which consists of solving the berthing schedule for the vessels one at a time. To implement this method, a sequence of vessels must be generated at the beginning. Here, we generate this sequence in decreasing order of the  $c_i^p$  value, which reflects the priority of vessels in the sense of penalty. A berth-QC planning model denoted by M8 is then solved for each vessel. After solving the model **M8** for a vessel, the remaining time-berth space and the number of available QCs in each time step are updated before solving the next vessel. The formulation of M8 and the procedure for the first stage are given in the online appendix. In the second stage (i.e., the yard assignment), given the berth position of the vessels (i.e.,  $\omega_{ih}$ ), we can derive the decisions for the yard assignment by solving another model denoted by M9, which is formulated as follows:

[M9] minimize 
$$c^o \sum_{i \in V} \sum_{b \in B} \sum_{k \in K} \left[ \theta_{ikb} D_{kb}^L(l_i/r_i) \right]$$
 (103)

subject to: Constraints (2), (3), (23), (32), and (33).

#### 5.6. Overall CG Procedure

The overall scheme of the CG procedure is described as follows:

Step 0. Run the initial heuristic in Section 5.5 to obtain a set of feasible assignment plans. Input the initial set to the RMP formulated in Section 5.2.

Step 1. Solve the RMP by an LP solver (e.g., CPLEX) and obtain a dual vector  $(\pi_i, \omega_{bt}, \rho_k, \phi_t, \iota_{bt}, \kappa_{bt})$ .

Step 2. Pass the dual vector to the pricing problem defined in Section 5.3 and use Algorithm 1 to find the optimal assignment plan for each vessel  $i \in V$ .

Step 3. Add the optimal assignment plans with negative reduced cost to the RMP. If all assignment plans have a nonnegative reduced cost, stop the CG procedure; otherwise, go to Step 1.

#### 6. A Column Generation-Based Heuristic

The proposed CG procedure only solves the linear relaxation of the set covering model and does not guarantee that feasible integer solutions will be found. Therefore, we propose a CG-based heuristic to compute near-optimal integer solutions using different assignment plan selection strategies. The assignment plans are chosen from the subset of feasible assignment plans maintained in the RMP.

#### 6.1. Framework of the CG-Based Heuristic

Here, we describe the framework of our proposed CG-based heuristic. The outer procedure is the selection heuristic used to obtain an integer solution. The strategies for the selection procedure will be detailed in Section 6.2. The inner procedure is the CG procedure proposed in Section 5. Before elaborating on the framework of the algorithm, we define three port resources limited in the RMP, and we initially set their values. These are  $Berth\_time_{bt} = 1$ , for all  $b \in B$ ,  $t \in T$ (i.e., berth resource over time),  $Subblock_k = 1$ , for all  $k \in K$  (i.e., subblock resource), and  $QCs_t = Q_t$ , for all  $t \in \{1, \dots, H\}$  (i.e., QC resource over time). These three resources correspond to the right-hand sides of constraints (37)–(40) in the RMP, respectively, and are set as input parameters for the right-hand sides of the constraints in the algorithm. The detailed framework of our algorithm is as follows:

Step 0. Initialize a vessel waiting list, which includes all of the vessels that have not been designated with an assignment plan  $\mathcal{P}_i$ . Initialize the set  $\Omega$  for the final solution plans as empty. Pass the initial three port resources (i.e.,  $Berth\_time_{bt} = 1$ ,  $Subblock_k = 1$  and  $QCs_t = Q_t$ ) to the right-hand sides of the constraints in the RMP.

Step 1. Invoke the CG procedure. When the CG procedure ends, an LP solution is obtained by solving the RMP. Update a column pool with assignment plans



whose corresponding decision variables  $\lambda_{\mathcal{P}_i}$  are not equal to zero.

Step 2. Test whether the assignment plans in the column pool satisfy constraints (37)–(40) with the current port resources. If not, delete these assignment plans.

Step 3. Select one assignment plan  $\mathcal{P}_i$  from the column pool based on the strategies proposed in Section 6.2 and pass it to the set  $\Omega$ . Remove the corresponding vessel i from the vessel waiting list.

Step 4. Update the three port resources based on the selected assignment plan. For example, if the selected assignment plan  $\mathcal{P}_i$  occupies berth b' in time steps t' and t+1', then set  $Berth\_time_{b't'}=0$  and  $Berth\_time_{b't+1'}=0$ .

Substep 4.1. Assume the selected assignment plan is for vessel i, its arrival time step is  $\bar{\alpha}$  (i.e., the handling start time), and its departure time step is  $\bar{\beta}$  (i.e., the handling end time). To guarantee periodicity, we further update the berth resource (i.e.,  $Berth\_time_{bt}$ ) as follows: If  $\bar{\beta} - (H-1) \ge 1$ , we set  $Berth\_time_{b\tau} = 0$ ,  $\forall \tau \in [1, \bar{\beta} - (H-1)]$ . If  $\bar{\alpha} + (H-1) \le H + E$ , we set  $Berth\_time_{b\tau} = 0$ ,  $\forall \tau \in [\bar{\alpha} + (H-1), H + E]$ .

After the update, pass the current three port resources to the right-hand sides of the constraints in the RMP.

Step 5. Repeat Steps 1–4 until the vessel waiting list is empty. At the end of the algorithm, an integer solution for the problem can be derived from the set  $\Omega$ .

Note that in Section 5.2, the berth allocation periodicity cannot be considered in the RMP since the problem is an LP relaxation. Here, we insert Substep 4.1 to guarantee periodicity in the final solution. Periodicity enforces the condition that the time gap between the start time step  $\varrho_b$  for berth b and the end time step  $\varsigma_b$ for berth b is less than H-1 (i.e., constraint (20)), which essentially implies that the time gaps between all of the arrival time steps of the vessels allocated to berth b and all of the departure time steps of the vessels allocated to berth b are less than H-1. The principle behind Substep 4.1 is that if a vessel has been allocated to berth b in the solution set  $\Omega$ , we must ensure that no other assignment plan can be selected if the assignment plan allocates its corresponding vessel to berth b and the gaps between its dwelling time steps and  $\bar{\alpha}$  or  $\beta$  are greater than H-1. Thereafter, periodicity in the final solution can be ensured.

#### 6.2. Strategies to Select the Assignment Plan

After Step 2 of the heuristic algorithm, the column pool with feasible assignment plans is obtained. We propose four heuristic strategies to select an assignment plan from the pool.

Strategy 1. Select from the column pool the assignment plan corresponding to the largest fractional value of the decision variables  $\lambda_{\mathcal{P}_i}$ . If there are two assignment plans with the same fractional value, select the

one with the lower plan cost. The principle behind this strategy is that the assignment plan with the highest fractional value is more likely to be part of an optimal solution.

Strategy 2. Select from the column pool the assignment plan corresponding to the lowest plan cost (i.e.,  $\mathcal{C}_{\mathcal{P}}$ ). If there are two assignment plans with the same plan cost, select the one with the higher fractional value of the decision variable. The principle behind this strategy is to select the assignment plan that contributes the least to the total cost under current port resources.

Strategy 3. Select from the column pool the assignment plan corresponding to the lowest reduced cost with the current values of the dual variables. The reduced cost can be calculated as

$$\mathcal{C}_{\mathcal{P}} - \left(\pi_i + \sum_{b \in B} \sum_{t \in T} \omega_{bt} \cdot A_{bt}^{\mathcal{P}_i} + \sum_{k \in K} \rho_k \cdot R_k^{\mathcal{P}_i} + \sum_{t \in T} \phi_t \cdot U_t^{\mathcal{P}_i}\right).$$

If there are two assignment plans with the same reduced cost, select the one with the lower cost. The principle of this strategy is to find the assignment plan that has the lowest sum of the contribution cost to the total cost and to the usage cost of port resources.

Strategy 4. To implement this strategy, we initially rank all berths  $b \in B$  from lowest to highest, based on their average distance to all of the subblocks in the yard (i.e., the input parameter  $D_b^U$ ). Under this strategy, we first pick all of the assignment plans from the column pool that allocate its vessel to the lowest berth. If no assignment plan exists, we further check the assignment plans with the next lowest berth until the assignment plans are picked. If there is more than one assignment plan picked with the lowest berth, select the one with the lowest reduced cost. The principle of this strategy is to maximize the utilization of the berths that are close to the subblocks in the yard. Thus the transportation cost in the yard can be reduced.

#### 6.3. Accelerating the CG Procedure by Dual Stabilization

In the proposed algorithm, CG is the core procedure to derive an LP solution. However, CG is known to suffer from instability, which causes slow convergence. The instability of CG is due to the following. Suppose that we can build the master problem with all possible columns and the dual problem for the master problem (DMP). At each iteration of the CG procedure, an RMP is solved with a subset belonging to the full set of all possible columns, which means that some columns are missing from the RMP compared with the master problem. A column in the master problem denotes a constraint in the DMP, which suggests that the dual problem of the RMP lacks some constraints in the DMP. Thus, the optimal dual solution  $\Pi = (\pi_i, \omega_{bt}, \rho_k, \phi_t, \iota_{bt}, \kappa_{bt})$  obtained by the RMP could



be feasible for the DMP, and thereafter optimal, or could be infeasible and superoptimal for the DMP.

To overcome such a problem in the CG procedure and to improve the efficiency of our algorithm, we have designed an ad hoc dual stabilization method, which is inspired from Addis, Carello, and Ceselli (2012). This method aims to pass a dual vector  $\Pi$  =  $(\tilde{\pi}_i, \tilde{\omega}_{bt}, \tilde{\rho}_k, \phi_t, \tilde{\iota}_{bt}, \tilde{\kappa}_{bt})$  to the pricing problem, which is close to the optimal dual vector of the DMP. To obtain a near-optimal dual vector (i.e.,  $\tilde{\Pi}$ ), we maintain a stability center  $\Pi = (\bar{\pi}_i, \bar{\omega}_{bt}, \bar{\rho}_k, \phi_t, \bar{\iota}_{bt}, \bar{\kappa}_{bt})$ , which represents our incumbent best guess for the optimal dual vector. Initially, we set  $\bar{\Pi}$  with zeros in all components of the vector, which is a feasible solution for the DMP. At each iteration of the CG procedure, we obtain a dual vector by solving an RMP (i.e., computing  $\Pi$ ) and pass a modified dual vector (i.e.,  $\Pi$ ) to the pricing problems by the updated equation

$$\tilde{\Pi} = (\tilde{\pi}_i, \tilde{\omega}_{bt}, \tilde{\rho}_k, \tilde{\phi}_t, \tilde{\iota}_{bt}, \tilde{\kappa}_{bt}) = a \cdot \Pi + (1 - a) \cdot \bar{\Pi}, \quad (104)$$

where  $a \in [0,1]$ . Initially, we set a = 0.5. Given a specific a, the CG procedure is executed with all negative reduced cost columns added to the RMP. When no columns can be added with the current setting of a, this means that  $\tilde{\Pi}$  satisfies all of the constraints in the dual problem and is a feasible dual solution. Thus, we update the  $\bar{\Pi} = \tilde{\Pi}$  for the incumbent best guess, and we then increase a by 0.05 for a new iteration of the above process. The CG procedure terminates when a = 1 and no negative reduced cost columns can be found.

#### 7. Computational Experiments

We have conducted extensive numerical experiments to validate the effectiveness of the proposed model and the efficiency of the CG-based heuristic. The experiments were run on a PC equipped with a 3.3 GHz Intel Core i5 CPU and 16 GB RAM. All of the algorithms were programmed in C# (VS2012), and the RMP was solved by CPLEX 12.5. The time limit for all test instances was three hours (10,800 seconds).

#### 7.1. Generation of the Test Instances

The planning horizon considered is one week. Each day is divided into six time steps of four hours each. In total, there are 42 time steps for the planning horizon (i.e., H = 42). In the computational experiments, we randomly generated test instances with seven different scales. The parameter settings for the seven instance groups (ISGs) are listed in Table 1.

All of the incoming vessels are classified into three classes, that is, feeder vessels, medium vessels, and jumbo vessels. Table 2 illustrates the QC-profile generation for the three vessel classes. The available QC profiles for each vessel are randomly generated based on the table. We can calculate the average handling time for all of the vessels as (3 + 4 + 5)/3 = 4,

**Table 1.** Scale of Instance Groups in Experiments

Group ID	No. of vessels $( V )$	No. of berths $( B )$	No. of QCs (Q)	No. of subblocks $( K )$	No. of time steps ( <i>H</i> )
ISG1	15	2	5	80	42
ISG2	20	3	7	120	42
ISG3	30	4	11	160	42
ISG4	35	5	12	200	42
ISG5	45	6	16	240	42
ISG6	50	7	18	280	42
ISG7	60	8	21	320	42

and the average workload for all of the vessels as (3.5 + 10.0 + 17.5)/3 = 10.3.

Given the QC-profile generation table, it can be concluded that each vessel will occupy a berth for four time steps, on average, and use QC resources for 10.3 QC × time steps, on average. Thereafter, for all of the instance groups, the berth utilization rate and the QC utilization rate, when all incoming vessels are served, can be calculated as shown in Table 3. As can be seen, the berth utilization rate and QC utilization rate for all instance groups are in the 63%–74% range, which is realistic.

The coefficient  $c_i^p$  for the penalty cost for each vessel is randomly generated in the ranges of [2, 6], [6, 10], and [10,14] for the three vessel classes, respectively (Meisel and Bierwirth 2009). The coefficient for the operation cost in the yard is set as  $c^{\circ} = 5 \times 10^{-6}$  (Zhen, Chew, and Lee 2011). The workload of each vessel is generated based on Table 1 with the unit of QC  $\times$  time step, where a QC can handle about 30 containers per hour. Thus, the total number of handled containers for each vessel can be calculated by multiplying its workload, four hours, and 30 containers. For each vessel, we assume a random  $\epsilon \in [40\%, 60\%]$  proportion of loading containers and a  $1-\epsilon$  proportion of unloading containers among all handled containers, providing the input data  $l_i$  and  $u_i$ . The number of reserved subblocks for each vessel is generated in the sets of  $\{2,3\}$ ,  $\{4,5,6,7\}$ , and {8,9,10} for the three vessel classes, respectively.

#### 7.2. Efficiency of Two Column Generators

We initially conducted some experiments to compare the efficiency of two methods to solve the pricing problems. The first way is to use CPLEX to solve the pricing model M5 directly. The second way is to use the proposed exact algorithm to solve the pricing model. Both methods are called column generators. To compare the efficiency of the two methods, the RMP was solved to optimality during the CG procedure. Using two column generators, the optimal result of LP relaxation for the problem and the computational time were recorded in Table 4 by group of instances, where each group contains five instances with the same problem scale.



Table 2. QC-Profile Generation for Different Vessel Classes

Vessel		QC-profile specifications							
Class	F. F. Class Proportion u		Range of handing time (time step)	Average handling time (time step)	Range of workload (QC × time step)	Average workload (QC × time step)			
Feeder	1/3	1 to 3	2 to 4	3	2 to 5	3.5			
Medium	1/3	2 to 4	3 to 5	4	6 to 14	10.0			
Jumbo	1/3	3 to 5	4 to 6	5	15 to 20	17.5			

**Table 3.** Berth and QC Utilization Rates of the Instances in the Experiments

		Berth utilization		QC utilization				
Group ID	Vessel usage $( V  \times 4)$	Port resource $( B  \times H)$	Utilization rate (%)	Vessel usage $( V  \times 10.3)$	Port resource $(Q \times H)$	Utilization rate (%)		
ISG1	60	84	71.4	154.5	210	73.6		
ISG2	80	126	63.5	206.0	294	70.1		
ISG3	120	168	71.4	309.0	462	66.9		
ISG4	140	210	66.7	360.5	504	71.5		
ISG5	180	252	71.4	463.5	672	69.0		
ISG6	200	294	68.0	515.0	756	68.1		
ISG7	240	336	71.4	618.0	882	70.1		

Table 4. Comparison of the Efficiency of Two Ways to Solve the Pricing Problems

Insta	nce	Solving PF	by CPLEX	Solving PP b	Solving PP by Algorithm 1			
Group	ID	LP optimum	CPU time (s)	LP optimum	CPU time (s)	Time ratio		
ISG1	4-1	43.45	159	43.45	10	0.06		
	4-2	35.62	118	35.62	8	0.07		
	4-3	32.65	133	32.65	9	0.07		
	4-4	44.75	161	44.75	8	0.05		
	4-5	31.43	150	31.43	11	0.07		
ISG2	4-6	47.17	351	47.17	22	0.06		
	4-7	44.64	247	44.64	13	0.05		
	4-8	47.70	283	47.70	17	0.06		
	4-9	45.05	308	45.05	23	0.07		
	4-10	54.03	236	54.03	14	0.06		
ISG3	4-11	79.70	656	79.70	54	0.08		
	4-12	78.99	402	78.99	42	0.10		
	4-13	77.53	566	77.53	36	0.06		
	4-14	84.66	594	84.66	51	0.09		
	4-15	77.38	551	77.38	40	0.07		
Average			328		24	0.07		

 $\it Note.$  "Time ratio" is the computational time of solving the pricing problem (PP) by Algorithm 1 divided by the computational time of solving PP by CPLEX.

As can be seen from Table 4, both column generators obtain the same optimal objective values for the LP relaxation over all instances, which means that Algorithm 1 can solve the pricing problems to optimality. However, the efficiencies of the two column generators are significantly different. According to the "time ratio" in Table 4, Algorithm 1 needs only 7% of the CPU time of CPLEX, which demonstrates that the proposed exact algorithm is highly efficient to solve the pricing problems. The reason is that solving the pricing problems by CPLEX needs to invoke the procedure to build

a model in the MILP solver, which is time consuming. However, solving the pricing problems by Algorithm 1 requires only a simple circulation procedure in programming without invoking any MIP solver, which leads to a higher efficiency.

## 7.3. Comparison of the Four Proposed Selection Strategies

In Section 6.2, we proposed four assignment plan selection strategies for the CG-based heuristic. Here, we conduct extensive numerical experiments to test the



Table 5. Comparison of Four Assignment Plan Selection Strategies

Instance CPLEX		PLEX	Strategy 1				Strategy 2			Strategy 3			Strategy 4		
Group	ID	Obj.	Seconds	Obj.	Gap (%)	Seconds	Obj.	Gap (%)	Seconds	Obj.	Gap (%)	Seconds	Obj.	Gap (%)	Seconds
ISG1	5-1	59.18	74	62.40	5.44	31	61.68	4.22	24	59.62	0.74	32	59.97	1.33	34
	5-2	54.53	56	59.41	8.95	23	57.96	6.30	26	55.03	0.92	29	55.21	1.26	20
	5-3	57.64	135	60.40	4.78	34	60.62	5.17	40	58.44	1.39	37	58.40	1.31	44
	5-4	54.72	31	58.23	6.42	19	57.38	4.86	23	55.13	0.75	31	55.27	1.01	27
	5-5	46.81	34	50.26	7.38	21	49.50	5.73	24	46.91	0.21	21	47.01	0.44	30
ISG2	5-6	62.97	1,256	65.63	4.23	97	65.85	4.57	89	63.48	0.81	142	64.01	1.66	112
	5-7	66.34	1,328	68.12	2.68	164	68.01	2.50	148	67.36	1.53	173	67.32	1.48	132
	5-8	60.65	2,250	62.21	2.58	186	62.40	2.89	202	61.10	0.75	189	61.53	1.45	230
	5-9	64.29	1,928	68.86	7.10	210	67.99	5.76	231	65.45	1.81	253	65.53	1.93	231
	5-10	67.46	3,515	69.74	3.38	231	69.44	2.93	240	68.32	1.28	221	67.98	0.78	195
ISG3	5-11	103.22	9,775	107.09	3.75	532	106.74	3.41	472	104.27	1.01	528	104.01	0.76	547
	5-12	_	_	104.33	_	643	105.00	_	534	101.82	_	493	101.54	_	476
	5-13	_	_	99.48	_	542	99.30	_	478	97.05	_	503	97.23	_	673
	5-14	_	_	103.25	_	674	102.99	_	525	101.12	_	596	100.87	_	553
	5-15	_	_	96.29	_	540	96.02	_	609	93.78	_	525	93.70	_	601
Average	e (%)				5.15			4.39			1.02			1.22	

Notes. "CPLEX" shows the solution method that solves the problem directly by CPLEX, which provides the optimal solution. "Strategy 1," "Strategy 2," "Strategy 3," and "Strategy 4" show the solution methods for the proposed CG-based heuristic by using the four proposed assignment plan selection strategies, respectively. "Obj." is the objective value of the solution obtained by the corresponding solution method. "Gap" is the optimality gap between the optimal solution obtained by CPLEX and the solution obtained by the CG-based heuristic by using the corresponding strategy. "Seconds" is the number of CPU seconds needed for the solution method to obtain the solution. A dash means the computational time for the instance is more than 10,800 seconds.

efficiency and the effectiveness of the algorithm by using the four strategies. To test whether our proposed algorithm can identify near-optimal solutions within reasonable computational times, we also use CPLEX to solve model **M2** optimally. Small-scale instance groups ISG1, ISG2, and ISG3 were used in this experiment.

Table 5 illustrates the comparisons between CPLEX and the proposed algorithm using different strategies. As can be seen, CPLEX can solve the problem only for some small-scale instances; that is, Instance 5-1 to Instance 5-11. The majority of instances in ISG3 cannot be solved to optimality by CPLEX within three hours, which means that the optimal solution is achievable only for the instances in ISG1 and ISG2. However, all instances in the table can be solved efficiently by the proposed algorithm under different strategies. The choice of a strategy has nearly no effect on the computational time of the proposed algorithm, but has a significant effect on the quality of the solution obtained by the algorithm. Strategies 3 and 4 outperform Strategies 1 and 2 since using the former two strategies leads to average small optimality gaps of 1.02% and 1.22% compared with 5.15% and 4.39%. This demonstrates that using a tailored strategy in the CG-based heuristic can yield near-optimal solutions.

#### 7.4. Effectiveness of the Proposed CG-Based Heuristic Algorithm

To validate the effectiveness of the proposed model and of the CG-based heuristic, we further conducted experiments to compare our algorithm by applying the two strategies with the first come, first served (FCFS) rule and the squeaky wheel optimization (SWO) metaheuristic on large-scale instance groups (i.e., ISG4, ISG5, ISG6, and ISG7), which are commonly used in berth and yard allocation problems (Lim and Xu 2006; Meisel and Bierwirth 2009; Zhen, Chew, and Lee 2011). The implementations of FCFS and SWO for the problem in this paper are similar to those of Zhen, Chew, and Lee (2011).

Table 6 provides comparisons between the proposed CG-based heuristics, the FCFS rule, and the SWObased metaheuristic. From Table 6, we can see that the CG-based heuristics and the SWO-based metaheuristic significantly outperform the commonly used FCFS decision rule. The SWO based metaheuristic can improve the objective by 9.45%, on average. However, the proposed CG-based heuristics under Strategy 3 and Strategy 4 improve it by 10.40% and 10.31%, respectively. The results demonstrate that the CG-based heuristic outperforms the SWO-based metaheuristic for the integrated problem with respect to both the computation time and the solution quality. The SWO-based metaheuristic cannot converge within three hours for each of the instances in ISG7. This shows that the proposed heuristic algorithm is more efficient than the SWO-based metaheuristic.

#### 7.5. Cost Analysis of the Integrated Optimization

One major motivation of this study is to integrate berth and yard planning. The berth planning problem embeds berth allocation and QC assignment, which is



Table 6. Comparison Between the FCFS Rule and SWO Metaheuristic for Large-Scale Instances

Insta	nce	FCFS		SWO			Strategy 3	3		Strategy 4	1
Group	ID	Obj.	Obj.	Gap (%)	Seconds	Obj.	Gap (%)	Seconds	Obj.	Gap (%)	Seconds
ISG4	6-1	130.91	118.80	9.25	1,386	116.53	10.98	1,119	116.86	10.73	1,073
	6-2	135.62	123.83	8.69	1,517	122.83	9.43	1,046	122.50	9.68	1,228
	6-3	137.35	123.94	9.76	1,414	122.85	10.55	996	124.09	9.65	1,137
	6-4	134.42	120.64	10.25	1,276	119.15	11.36	876	118.89	11.55	1,045
	6-5	129.29	118.72	8.17	1,257	117.33	9.25	1,058	117.46	9.15	997
ISG5	6-6	194.56	175.32	9.89	3,782	174.43	10.35	2,750	175.48	9.81	3,012
	6-7	191.10	174.70	8.58	3,532	172.98	9.48	2,672	173.32	9.31	2,977
	6-8	186.67	168.34	9.82	3,398	162.20	13.11	2,828	162.51	12.94	2,764
	6-9	184.82	165.97	10.20	4,078	163.56	11.50	2,499	163.33	11.63	2,542
	6-10	188.98	171.21	9.40	3,123	168.46	10.86	2,375	167.93	11.14	2,212
ISG6	6-11	232.34	213.21	8.23	6,732	210.20	9.53	5,534	212.06	8.73	5,768
	6-12	237.37	213.47	10.07	7,071	213.81	9.92	5,774	212.52	10.47	5,423
	6-13	231.53	208.88	9.78	7,290	207.50	10.38	4,632	206.99	10.60	4,212
	6-14	233.97	211.74	9.50	6,786	207.51	11.31	5,654	207.93	11.13	6,043
	6-15	225.58	202.77	10.11	7,343	201.02	10.88	5,850	201.28	10.77	5,723
ISG7	6-16	292.30	_	_	_	262.61	10.16	9,190	262.25	10.28	9,289
	6-17	302.87	_	_	_	273.71	9.63	8,928	274.12	9.49	9,813
	6-18	294.21	_	_	_	264.37	10.14	9,561	265.04	9.92	8,972
	6-19	300.90	_	_	_	272.25	9.52	9,821	270.78	10.01	9,312
	6-20	302.12	_	_	_	273.24	9.56	8,722	274.33	9.20	8,834
Average	e (%)			9.45			10.40			10.31	

*Notes.* "SWO" shows the solution method for the SWO metaheuristic. "Strategy 3" and "Strategy 4" show the solution methods for the proposed CG-based heuristic by using the two proposed assignment plan selection strategies, respectively. "Obj." is the objective value of the solution obtained by the corresponding solution method. "Gap" lists the objective gap between the solution obtained by the FCFS rule and the solution obtained by the CG-based heuristic by using the corresponding strategy. "Seconds" is the number of CPU seconds needed for the solution method to obtain the solution. A dash means the computational time for the instance is more than 10,800 seconds (i.e., three hours).

an elementary optimization integration on the quay side. This integration aims to minimize the penalty cost incurred by the deviation of the expected service time for vessels, which is captured by the first part of objective (1) (i.e.,  $\sum_{i \in V} c_i^p [(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+]$ ). The yard planning problem focuses on the container loading and unloading processes on the yard side, which leads to the second part of objective (1) for loading travel cost (i.e.,  $c^o \sum_{i \in V} \sum_{b \in B} \sum_{k \in K} [\omega_{ib} \varphi_{ik} D_{kb}^L(l_i/r_i)]$ ) and the third part of objective (1) for unloading travel cost (i.e.,  $c^o \sum_{i \in V} \sum_{b \in B} \omega_{ib} D_b^U u_i$ ). Here, we call the three costs the penalty cost, the loading cost, and the unloading cost. In this section, we use different combinations of the three costs as the optimization objective, and each combination is denoted as an optimization integration level. Numerical experiments are conducted on different optimization integration levels to investigate their effects on the three types of costs, as well as on the

Four optimization integration levels are involved in this investigation. (i) The first one (referred to as the baseline) is the proposed integrated model, in which we minimize the sum of the three costs (the total cost) in the objective. (ii) The second one (denoted as BAP+QC) minimizes the penalty cost first, which is the traditional integration of the berth allocation and QC

assignment. Given the decisions made in BAP+QC, we further optimize the yard planning that offers the loading and unloading cost. (iii) The third one (denoted as BAP+QC+L) tackles the BAP+QC and the loading process simultaneously, which optimizes the sum of the penalty cost and of the loading cost. Based on the decisions made in BAP+QC+L, the unloading cost is then derived. (iv) The fourth one (denoted as BAP+QC+U) deals with the BAP+QC and the unloading process simultaneously, which minimizes the sum of the penalty cost and the unloading cost. Given the decisions made in BAP+QC+U, we further optimize the loading process which gives the loading cost.

In the experiments, under each of the three instance groups ISG2, ISG3, and ISG4, we first generate 10 randomly generated instances. We then use the four optimization integrations to solve the 10 instances in each instance group, during which the average values of the three types of costs and of the total cost are recorded for comparison. By setting the costs derived by the proposed integrated model (the baseline) as the benchmark, we calculate the relative proportion of the costs derived by the other three optimization integrations (BAP+QC, BAP+QC+L, and BAP+QC+U) to measure the changes in the costs obtained by using different integration levels. Figure 4 illustrates the results of



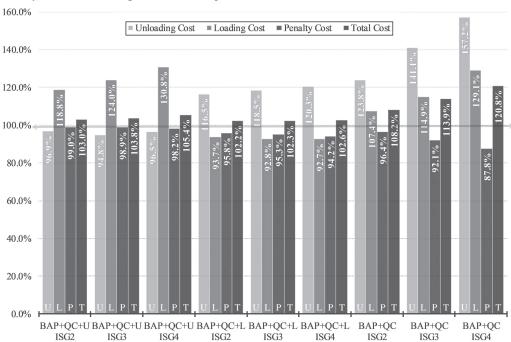


Figure 4. Cost Analysis in Different Optimization Integration Levels

these experiments. In the figure, each bar group shows the average total cost, penalty cost, loading cost, and unloading cost derived by an optimization integration (e.g., BAP+QC+U) in each instance group (e.g., ISG2). In each bar, the capital letter at the bottom indicates the cost type (e.g., "T" for total cost) and the percentage at the top represents the relative proportion of the cost compared with the cost derived from the baseline. A percentage larger than 100% (respectively, smaller than 100%) corresponds to the cost increase (respectively, cost decrease) with respect to the baseline.

We first analyze the integration of BAP+QC. This traditional integration focuses on the decisions on the quay side to minimize the penalty cost by attempting to satisfy the expected service time of all vessels and ignores the loading and unloading process on the yard side. This omission leads to a very large growth in the loading and unloading costs. Under the instance group ISG4, the loading and unloading costs increase by 29.1% and 57.2% compared with the baseline. By prioritizing the penalty cost, BAP+QC reduces it moderately, but this does not compensate for the loss in the loading and unloading process. The total cost increases more significantly as the problem size grows (from 8.2% to 20.8%). Such results support our justification in Section 1: The berth planning and the yard planning are intertwined with each other and the separate optimizations may increase the total cost significantly. The reasons behind this cost increase are revealed in the following analysis and discussion.

Second, the integration of BAP+QC+L minimizes the sum of the penalty cost and of the loading cost.

Without considering the unloading process, this integration cuts down the penalty cost and loading cost slightly. However, this drives the unloading cost much higher, which induces an increase in the total cost. One reason behind the phenomenon is that without considering the unloading process, some vessels with a large number of unloading containers may be allocated to "remote berths" that have longer average berthsubblock distance, such as berth 1 and berth 4 in Figure 2. Therefore, serving these vessels at "remote berths" will incur high unloading costs. For example, a vessel dwelling at berth 1 will incur a higher unloading cost compared with the same vessel dwelling at berth 2. Here, it is worth mentioning that the integration of BAP+QC+L yields a solution close to that of the baseline, since the total cost increase is less than 2.6% under the three instance groups.

Finally, the integration of BAP+QC+U focuses on the berth allocation and unloading process without involving the loading process, which causes a slight decrease in the penalty and unloading costs. In essence, this integration considers the position advantage of berths in the traditional BAP+QC, since the unloading cost is accounted for. Under this case, some "central berths" that are close to the yard, such as berth 2 and berth 3 in Figure 2, will be allocated in priority to the vessels with a large number of unloading containers to minimize unloading cost. However, this priority leads to a significant growth in the loading cost, as shown in Figure 4: the increases are 18.8%, 27.0%, and 39.1% for the three instance groups. The reason is that the vessels with a large number of unloading containers



also have a large number of loading containers, since they may belong to jumbo vessels. When several jumbo vessels are allocated to central berths, these are less likely to be assigned with the subblocks that are close to central berths, since jumbo vessels require a larger number of reserved subblocks compared with medium vessels and feeder vessels. When jumbo vessels are assigned with the subblocks that are far away from their berth positions, a significant increase in loading cost is unavoidable.

The above data analysis helps formulate an important managerial insight for port operators. The container unloading process will take longer if some jumbo vessels are allocated to berths that are far away from the yard side. The central berths should indeed be allocated in priority to these jumbo vessels. However, the port operators should also avoid situations in which too many jumbo vessels dwell at the central berths. It is more cost-effective to insert some smaller-size vessels in berth positions between jumbo vessels.

#### 8. Conclusions

We have modeled and solved a periodical integrated berth allocation, QC assignment, and yard assignment problem in container terminals. To tackle the optimization, we first proposed a compact mixed integer linear programming model and then developed a CG-based solution approach. By conducting extensive numerical experiments, our results show that the CG-based solution approach can yield near-optimal solutions within a much shorter computation time than a direct application of CPLEX. Based on the results, some managerial implications can be recommended for port operators. It is critical to integrate the decisions on the quay side and on the yard side. Without involving the yard planning, the berth planning cannot make appropriate allocations that consider the advantages of berth positions. As a result, the operations cost will increase significantly in the loading and unloading process on the yard side when vessels arrive. For those port operators who currently do not have an integrated optimization tool for berth and yard planning, we make the following suggestion for berth allocation. It is economical to allocate central berths in priority to large-size vessels that normally have a large number of unloading containers, but allocating too many large-size vessels to the central berths is not recommended, since this may negatively impact the loading process. The port operators may insert some small-size vessels among the largesize vessels in central berths. As for the QC assignment, if port operators apply the concept of QC profiles to make decisions, the following approach is recommended for generating a set of QC profiles. Given an incoming vessel with a total container handling workload and a handling time window, the port operators may first assign either the maximum number of QCs or the minimum number of QCs at each handling time step. The sum of all assigned handling workload at time steps must be less than the total container handling workload. Then, the remaining unassigned handling workload can be assigned to any time step that has a minimum number of assigned QCs.

We have designed an optimization tool for the integrated berth and yard planning in port operations. Four meaningful extensions of this work can be envisaged. First, the current model is oriented toward the decision environment of the discrete berth allocation. Continuous berth allocation is another way of tackling the berth allocation, which requires a further study to adjust the optimization tool. Second, this study assumed the average unloading route length. In real situations, the unloading routes of one vessel are affected by all reserved subblocks of other vessels receiving transshipment containers from that vessel. This consideration complicates the column generation. Indeed, when generating the assignment plan for one vessel in its pricing problem, the best berth position of this vessel depends on the subblock assignments of other vessels. Further study may be needed to incorporate this feature. Third, the integrated planning studied in this paper is a tactical-level planning problem for port operators and was solved in a deterministic context. However, the information on container handling workload and actual arrival time of vessels is often uncertain in practice. It may therefore be interesting to study the problem as a two-stage stochastic programming problem in which the berth allocation and yard assignment are determined in the first stage and the QC assignment is optimized in the second stage in response to uncertainty. Finally, some practical operations concerns in the quay side and yard side are to be embedded into the optimization tool to enhance its practicability. For example, on the quay side, the effect of tides considered in some studies can be incorporated into the berth allocation process. On the yard side, during the loading and unloading process, the traffic flow of trucks inside the yard can also be considered to avoid traffic congestion.

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