INCREMENTAL POTENTIAL CONTACT: INTERSECTION- AND INVERSION- FREE LARGE DEFORMATION DYNAMICS

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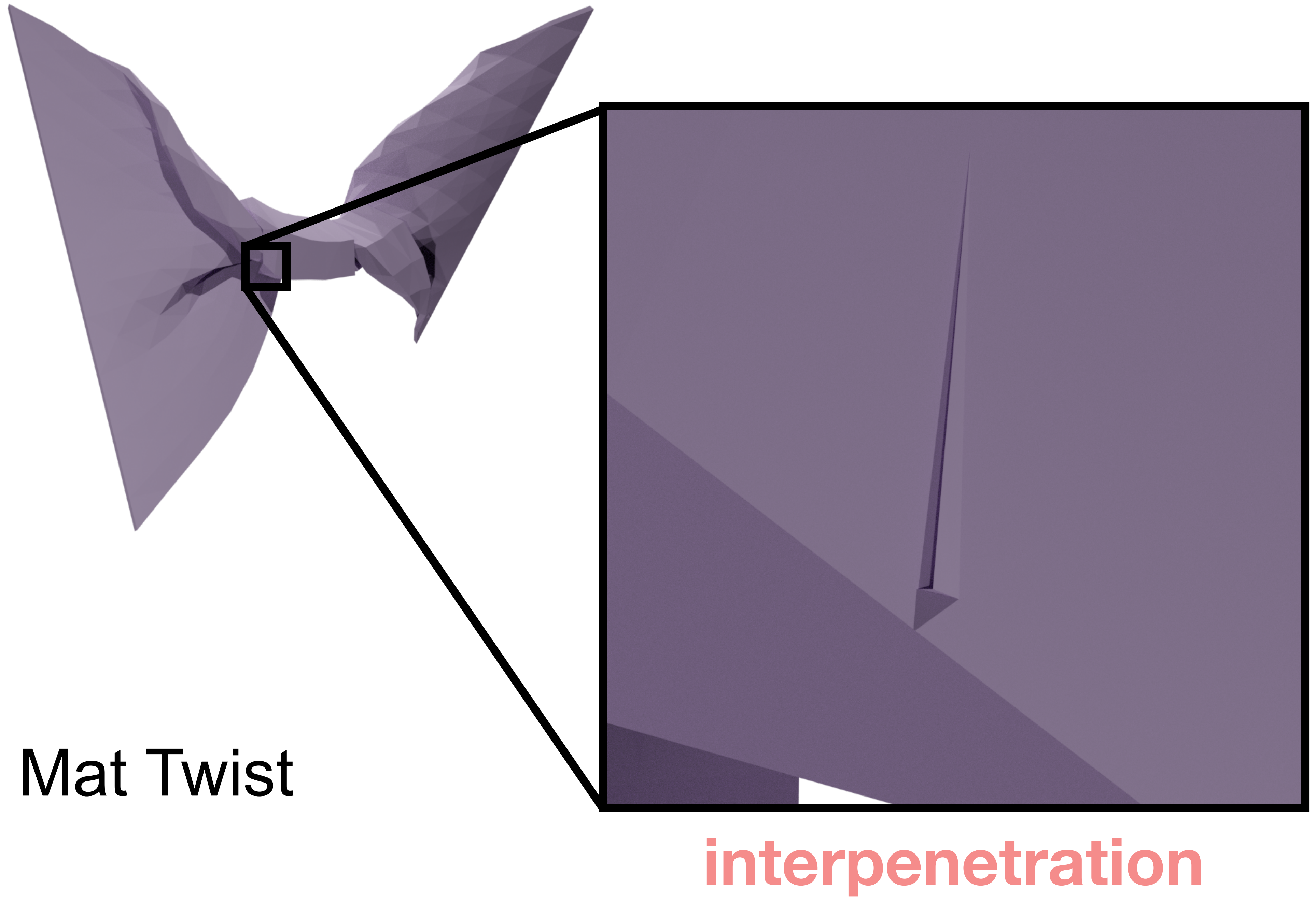
CHALLENGES WITH CONTACT

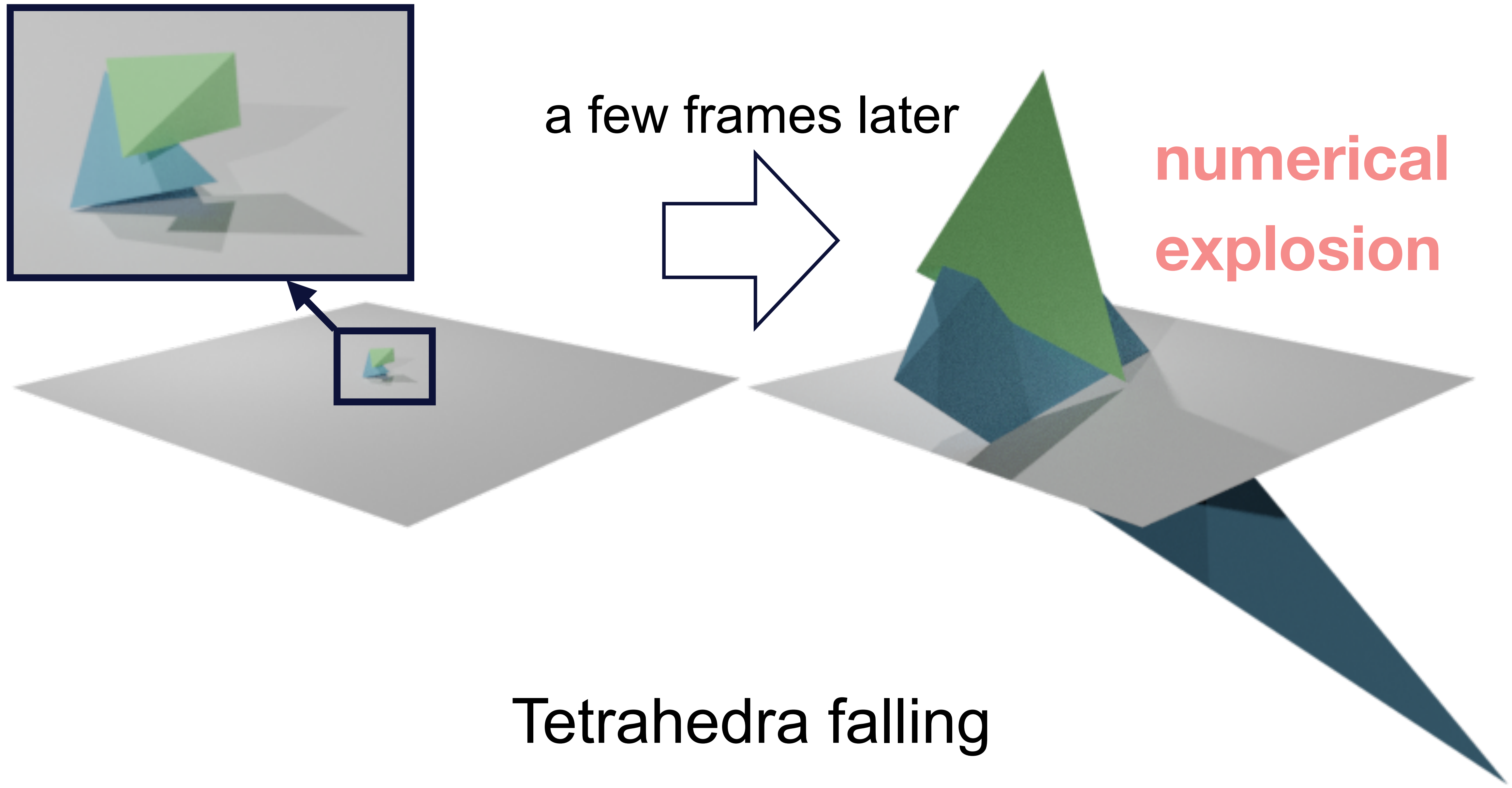
Existing methods

- need small time step sizes for stability and to avoid failures
- need per-example nonphysical parameter/mesh tuning

Common failures

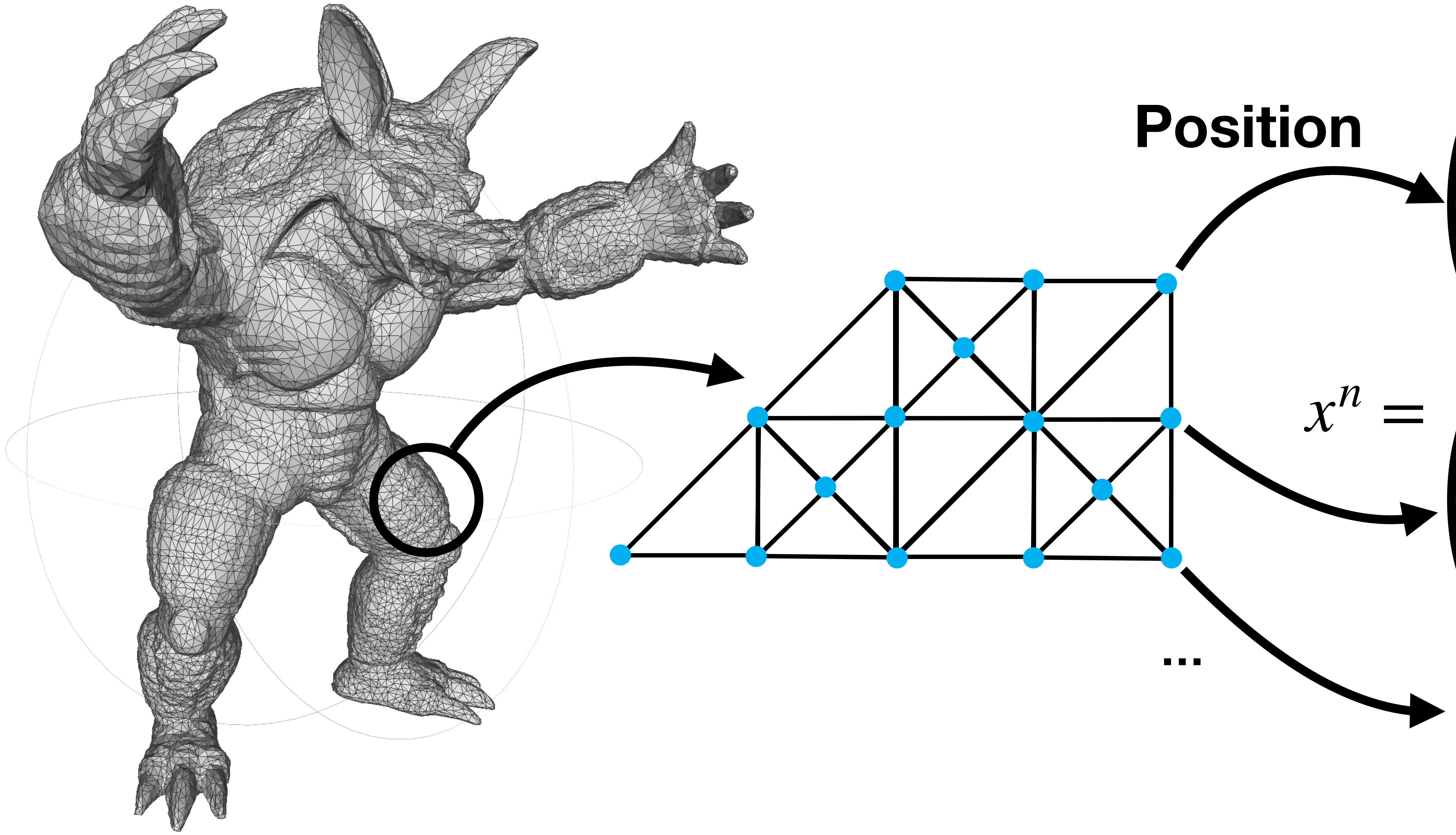


Mat Twist



Tetrahedra falling

SPATIAL DISCRETIZATION



Position

$$x^n = \begin{bmatrix} x^n_{0x} \\ x^n_{0y} \\ x^n_{0z} \\ x^n_{1x} \\ x^n_{1y} \\ x^n_{1z} \\ \vdots \end{bmatrix}$$

Velocity

$$v^n = \begin{bmatrix} v^n_{0x} \\ v^n_{0y} \\ v^n_{0z} \\ v^n_{1x} \\ v^n_{1y} \\ v^n_{1z} \\ \vdots \end{bmatrix}$$

IMPLICIT TIME STEPPING

$$\{x^0, v^0\} \xrightarrow[f]{} \{x^1, v^1\} \xrightarrow[f]{} \{x^2, v^2\} \xrightarrow[f]{} \dots \xrightarrow[f]{} \{x^n, v^n\} \xrightarrow[f]{} \dots$$

Given x^0, v^0**, for time steps** $n = 0,1,2,\ldots$

$$\begin{cases} v^{n+1} = v^n + hM^{-1}f(x^{n+1}) \\ x^{n+1} = x^n + hv^{n+1} \end{cases}$$

Velocity update

Position update

$$x^{n+1} - h^2M^{-1}f(x^{n+1}) = x^n + hv^n$$

$$\Rightarrow \min_{x^{n+1}} \frac{1}{2}||x^{n+1} - \tilde{x}^n||_M^2 + h^2\Psi(x^{n+1})$$

where

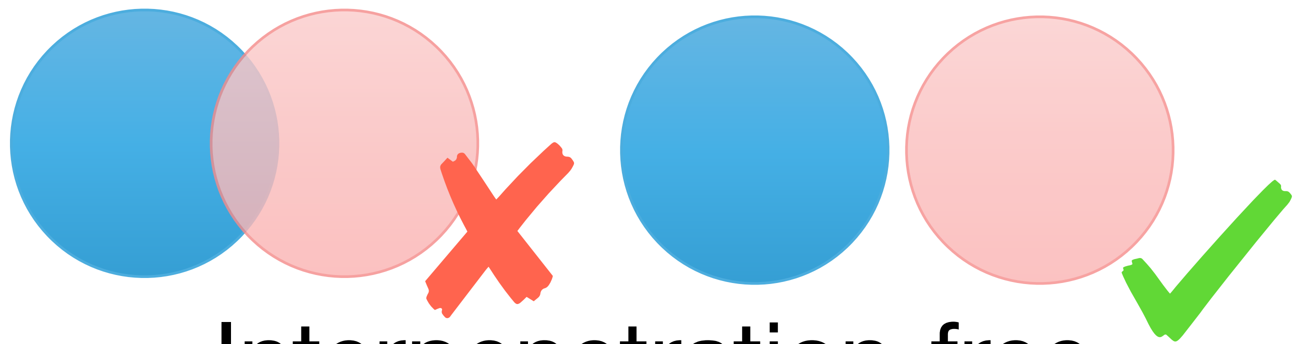
$$\Psi(x) = -\int f(x)dx$$

$$\tilde{x}^n = x^n + hv^n$$

$$\Psi = \sum_e V_e(\frac{\mu}{2}(\sum_i S_{ii}^2 - d) - \mu \ln(\det A_e) + \frac{\lambda}{2}(\ln(\det A_e))^2)$$

$A = USV$ is the transformation matrix of an element

OPTIMIZATION TIME INTEGRATION



For each time step t

Predictive position

Mass matrix

Time step Size

Interpenetration-free

$$\underbrace{x^{t+1}}_{\text{New node positions}} = \textbf{argmin}_x \underbrace{E(x)}_{\text{Incremental potential}} = \underbrace{\frac{1}{2}||x - \tilde{x}||_M^2}_{\text{Inertia term}} + \underbrace{h^2\Psi(x)}_{\text{Elasticity Potential}} \qquad s.t. \quad g(x) \geq 0$$

New node positions

Incremental potential

[Ortiz and Stainier 1999]

Inertia term

Elasticity Potential

Nonlinear, nonconvex! Hard!

Without contact:

Solve with line search methods with 2nd-order information [Liu et al. 2017, Li et al. 2019]

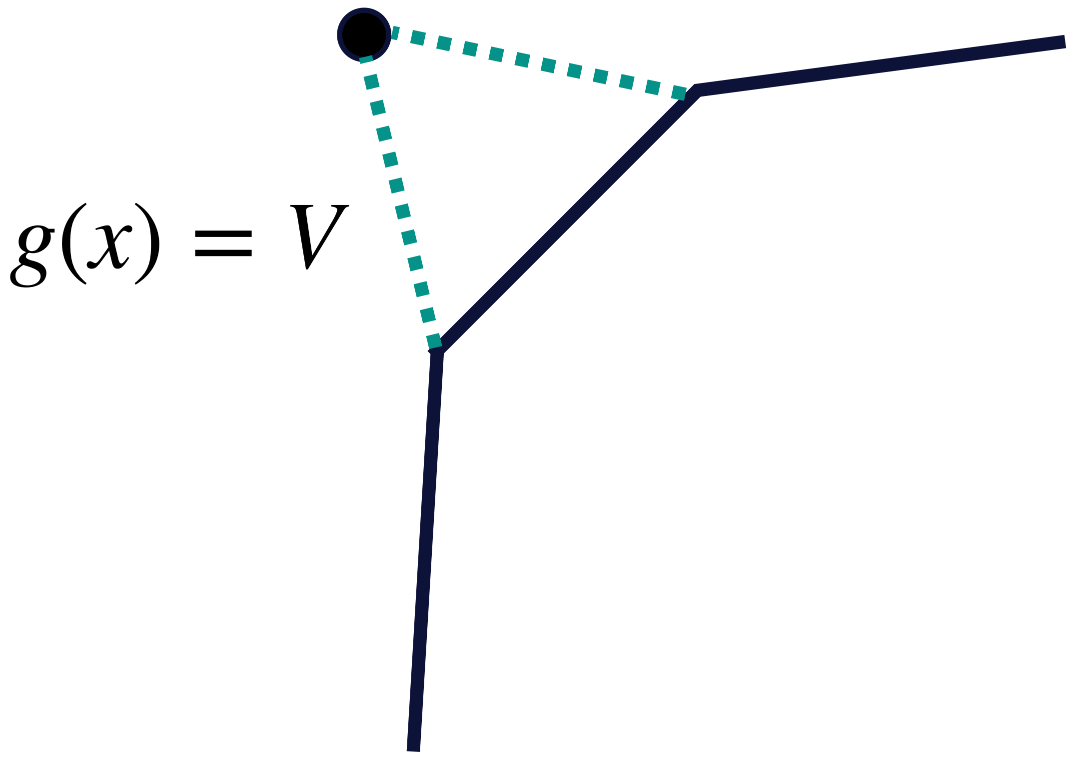
Challenges with contact:

1. How to **define** $g(x)$
2. How to **solve** the constrained problem

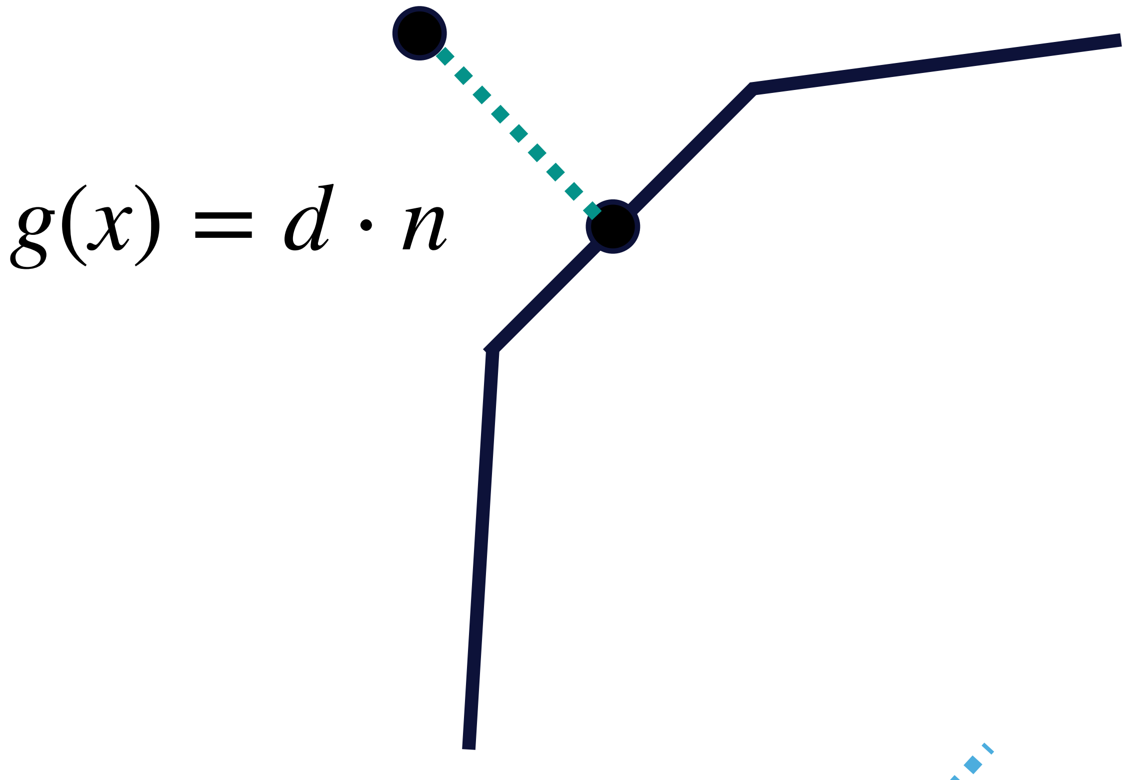
RELATED WORK

Constraint definition:

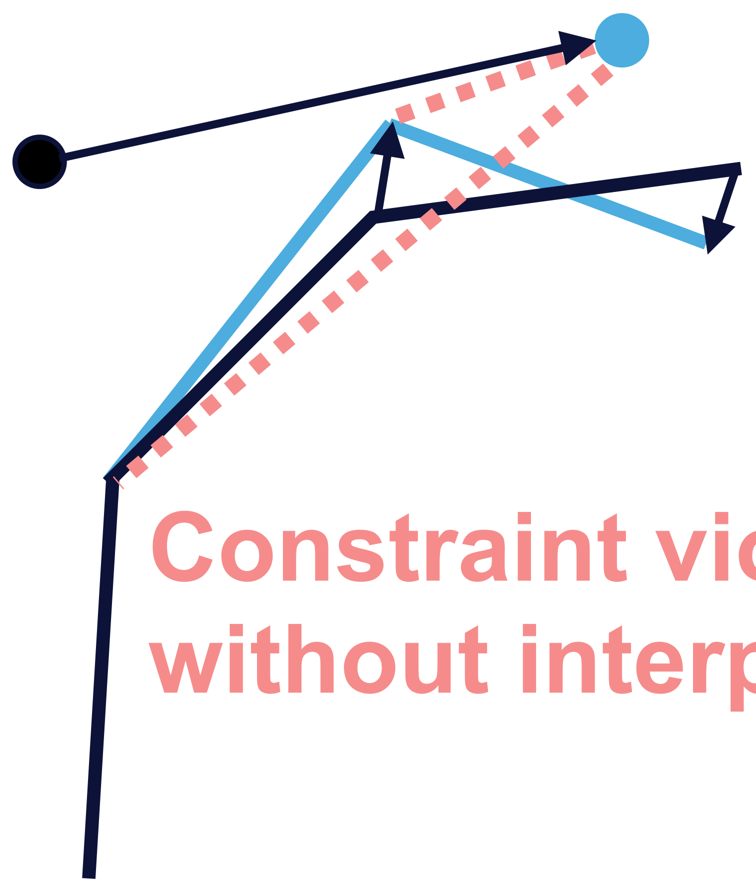
Volume constraints [Kane 1999, Müller 2015, etc]:



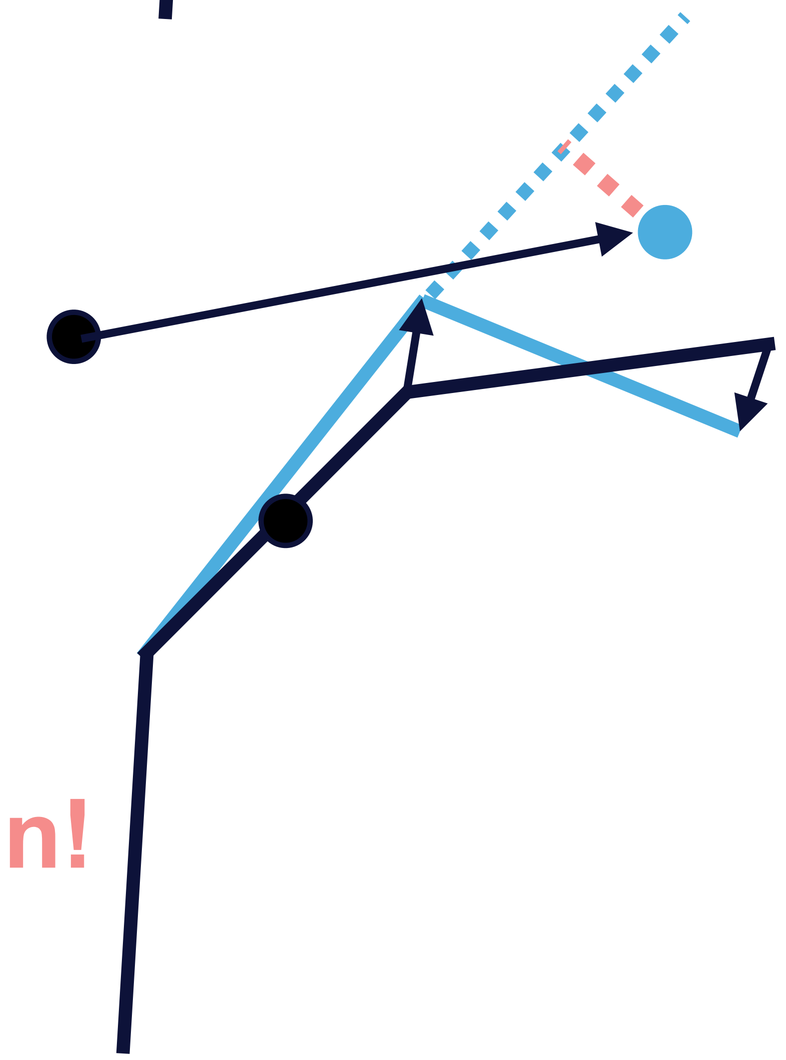
Gap constraints [Harmon 2008, Verschoor 2019, Otaduy 2009, etc]:



Large displacement:



Constraint violated without interpenetration!



SQP solve [Kane 1999, Kaufman 2008, Otaduy 2009, Verschoor 2019, etc]:

For iteration i in a time step:

$$x^{i+1} = \text{argmin}_x \frac{1}{2} x^T \nabla^2 E(x^i) x + x^T \nabla E(x^i)$$

$$s.t. \quad \forall k \in C(x^0), \quad g_k(x^i) + x^T \nabla g_k(x^i) \geq \epsilon$$

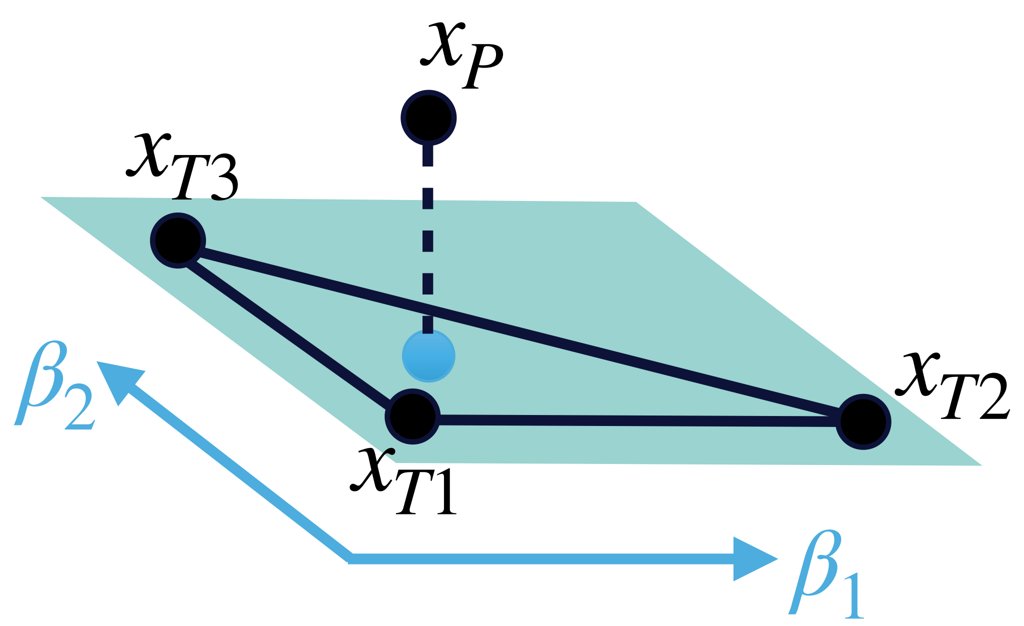
Heuristically updating C, require small h

Houdini FEM

SOFA [Faure et al. 2012]

IPC METHOD

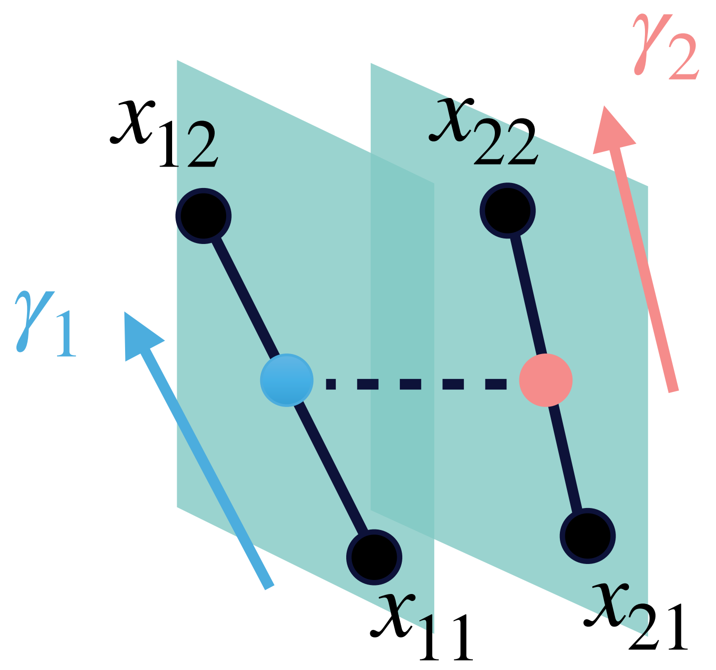
CONSISTENT UNSIGNED DISTANCE



Always compute the precise distance

k-th point (x_P) - triangle ($x_{T1}x_{T2}x_{T3}$) pair:

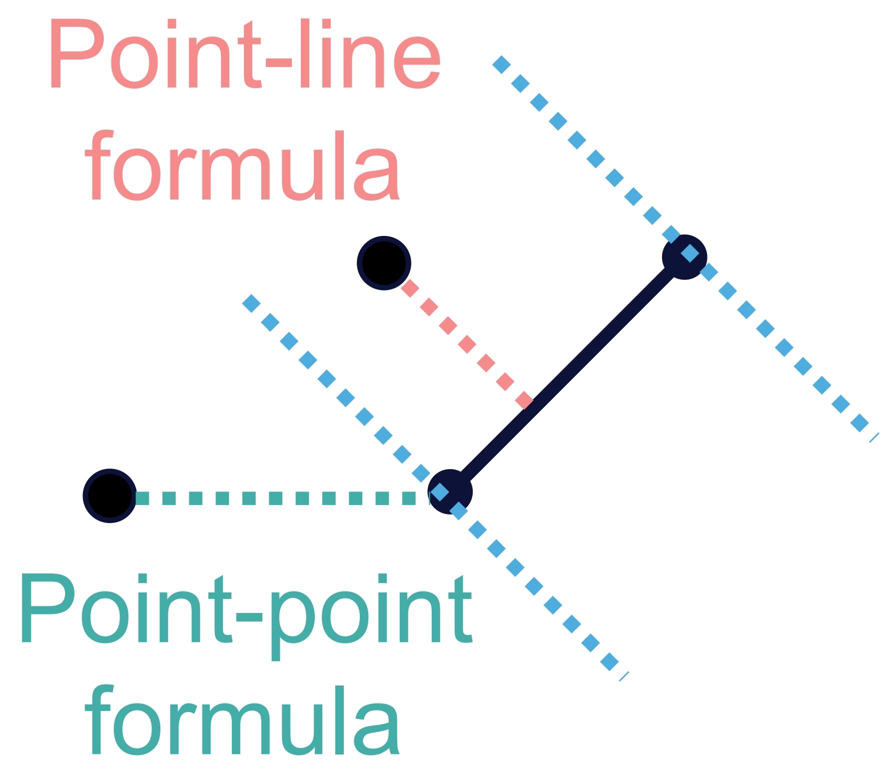
$$D_k^{PT}(x) = \min_{\beta_1,\beta_2} ||x_P - (x_{T1} + \beta_1(x_{T2} - x_{T1}) + \beta_2(x_{T3} - x_{T1}))|| \quad s.t. \quad \beta_1, \beta_2 \geq 0 \quad \beta_1 + \beta_2 \leq 1$$



k-th edge ($x_{11}x_{12}$) - edge ($x_{21}x_{22}$) pair:

$$D_k^{EE}(x) = \min_{\gamma_1,\gamma_2} ||x_{11} + \gamma_1(x_{12} - x_{11}) - (x_{21} + \gamma_2(x_{22} - x_{21}))|| \quad s.t. \quad 0 \leq \gamma_1, \gamma_2 \leq 0$$

— both are piecewise smooth analytical functions



$g(x) = D(x) \geq 0$ always holds… Use $g(x) = D(x) > 0$:

IPC Constraint definition: (with intersection-free $x^{0,0}$)
$\forall \alpha \in [0,1]$, time step t, iteration i, contact primitive pair k:

$$D_k^{PT}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0, \quad D_k^{EE}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0$$

CCD LINE SEARCH AND BARRIER FORMULATION

IPC Constraint definition: (with intersection-free $x^{0,0}$)

$\forall \alpha \in [0,1]$, time step t, iteration i, contact primitive pair k:

$$D_k^{PT}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0, \quad D_k^{EE}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0$$

IPC model problem becomes:

CCD in each iteration:

$\alpha_{CCD} \leftarrow CCD(x^{t,i}, x^{t,i+1})$

$x^{t,i+1} \leftarrow \alpha x^{t,i} + (1-\alpha)x^{t,i+1}$ with $\alpha \in (0, \alpha_{CCD})$

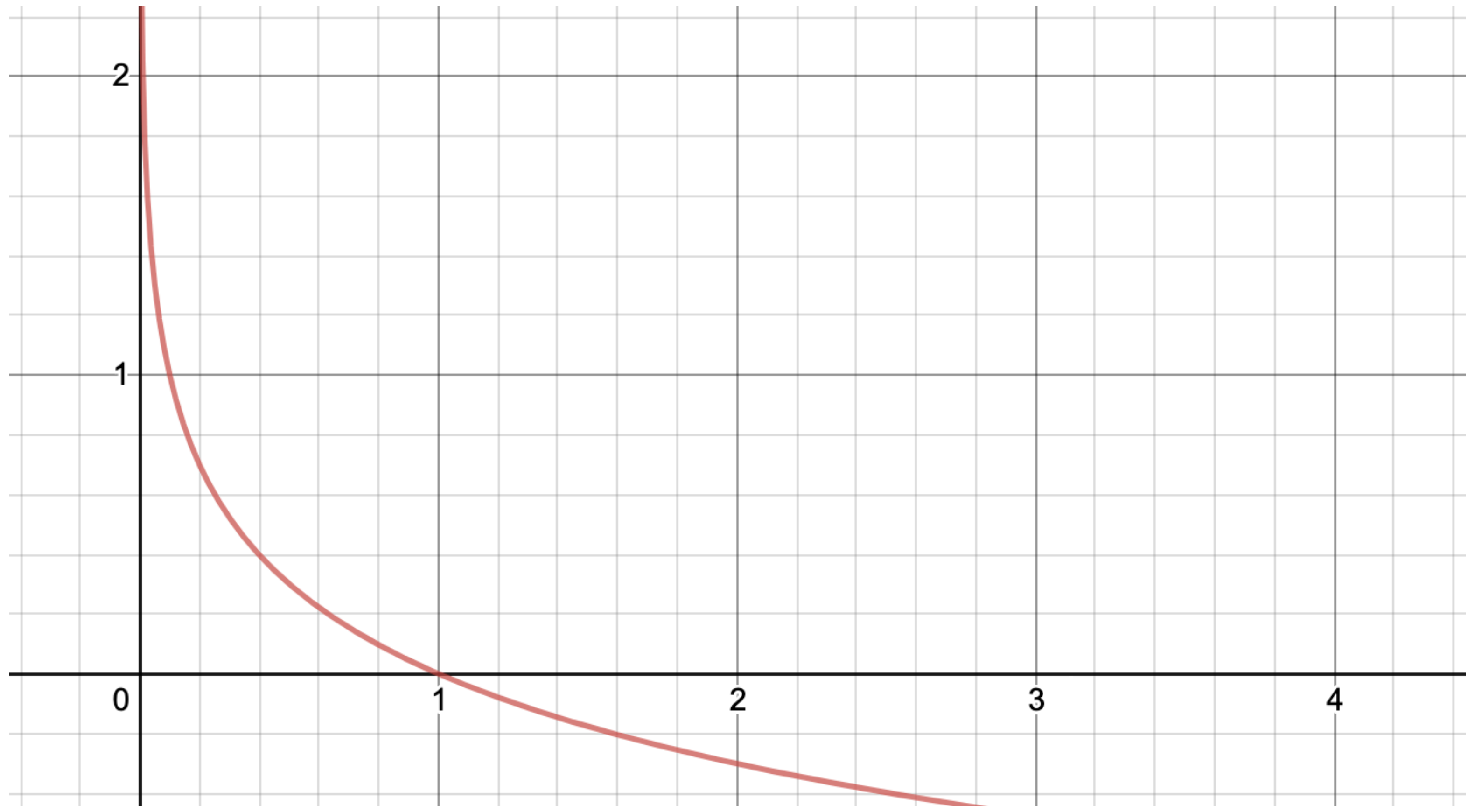
$x^{t+1} = \text{argmin}_x E(x) \quad s.t. \quad \forall k, \quad D_k(x) > 0$

Barrier method

$$x^{t+1} = \text{argmin}_x \left(E(x) + \kappa \sum_k b(D_k(x)) \right)$$

unconstrained!

Barrier function example: $b(D) = -\log(D)$



Newton iteration **quadratically** approximate $E(x) + \kappa \sum_k b(g_k(x))$ but not forming

$$\min_x x^T \nabla^2 E(x^i) x + 2x^T \nabla E(x^i)$$

$$s.t. \quad \forall k, \quad g_k(x^i) + x^T \nabla g_k(x^i) \geq 0$$

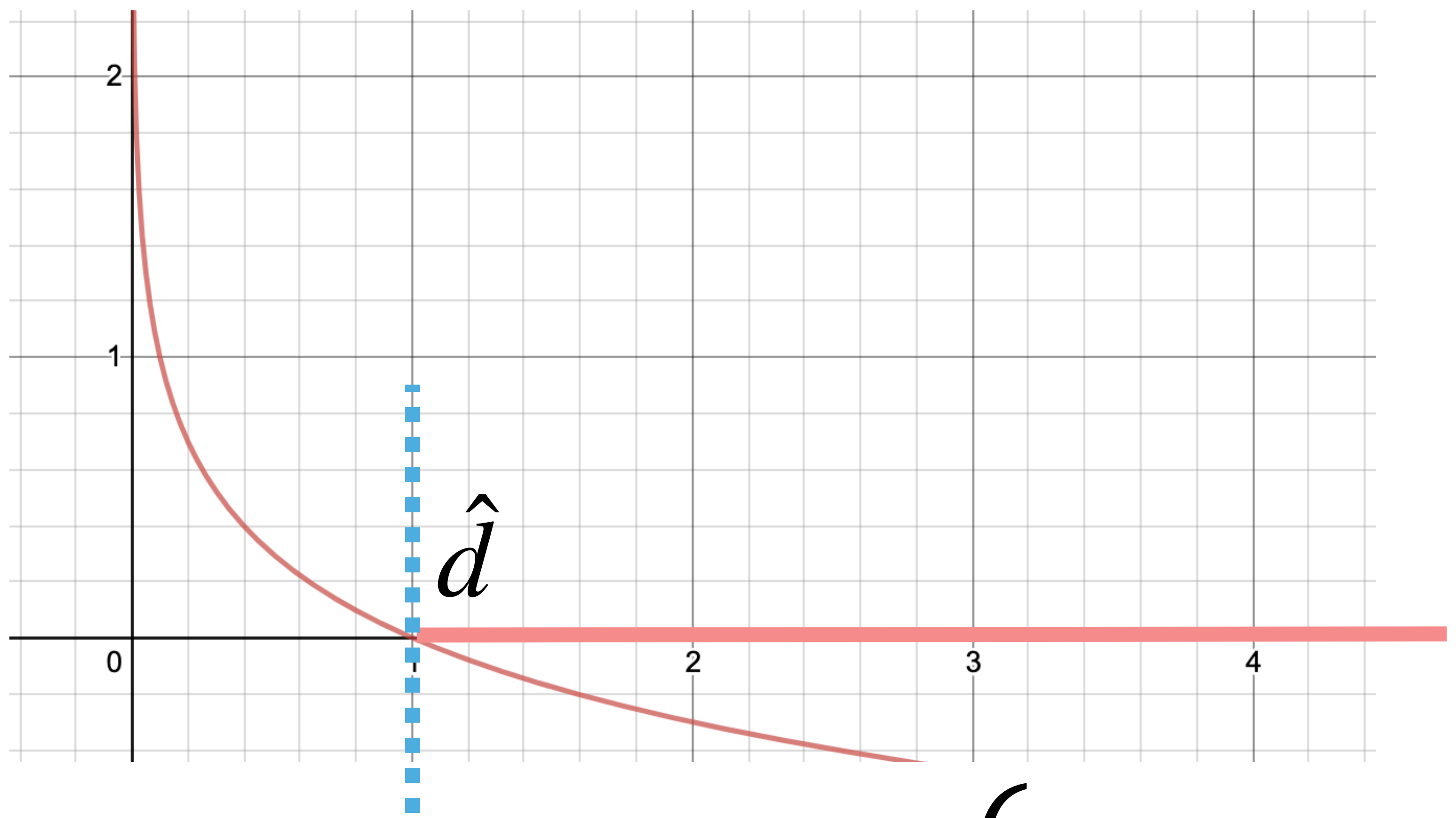
Hence no infeasible subproblems!

SMOOTHLY CLAMPED BARRIER FOR SCALABILITY

$$x^{t+1} = \text{argmin}_x \left(E(x) + \kappa \sum_k b(D_k(x)) \right)$$

quadratically increases with # surface primitives

Traditional barrier function: $b(D) = -\log(D)$

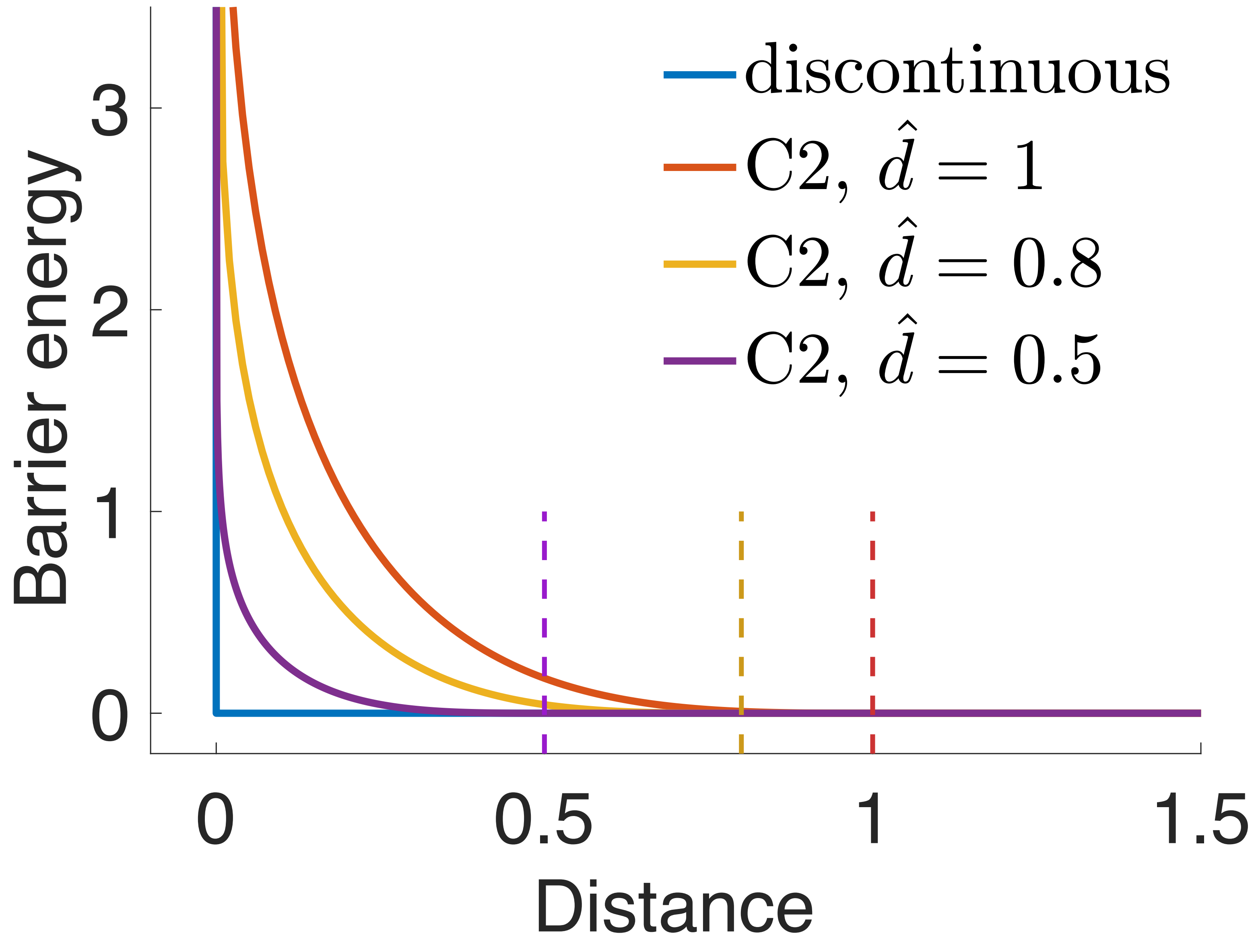


C0 clamping $b(D, \hat{d}) = \begin{cases} -\log(D/\hat{d}) & \text{if } D < \hat{d} \\ 0 & \text{if } D \geq \hat{d} \end{cases}$

harms convergence!

IPC's C2 clamping:

$$b(D, \hat{d}) = \begin{cases} -(D - \hat{d})^2 \log(D/\hat{d}) & \text{if } D < \hat{d} \\ 0 & \text{if } D \geq \hat{d} \end{cases}$$



IPC PSEUDO-CODE

Initialization: $x \leftarrow x^t,\ x_{prev} \leftarrow x,\ E_{prev} \leftarrow E(x)$

Newton loop (no contact):

Do

$H \leftarrow$ SPDProject($\nabla_x^2 E(x)$)

$p \leftarrow -H^{-1} \nabla_x E(x)$

$\alpha \leftarrow 1$

Do

$x \leftarrow x_{prev} + \alpha p$

$\alpha \leftarrow \alpha/2$

While $E(x) > E_{prev}$

$E_{prev} \leftarrow E(x),\ x_{prev} \leftarrow x$

While $\|p\|_\infty / h > \epsilon_d$

with IPC

$\hat{C} \leftarrow$ ComputeConstraintSet$(x, \hat{d})$, $E_{prev} \leftarrow B_t(x, \hat{d}, \hat{C})$

Newton loop (IPC): $B_t(x, \hat{d}, \hat{C}) = E(x) + \kappa \sum_{k \in \hat{C}} b(D_k(x), \hat{d})$

Do

$H \leftarrow$ SPDProject($\nabla_x^2 B_t(x, \hat{d}, \hat{C})$)

$p \leftarrow -H^{-1} \nabla_x B_t(x, \hat{d}, \hat{C})$

$\alpha \leftarrow \min(1, CCD(x, x+p))$

Do

$x \leftarrow x_{prev} + \alpha p$

$\hat{C} \leftarrow$ ComputeConstraintSet$(x, \hat{d})$

$\alpha \leftarrow \alpha/2$

While $B_t(x, \hat{d}, \hat{C}) > E_{prev}$

$E_{prev} \leftarrow B_t(x, \hat{d}, \hat{C}),\ x_{prev} \leftarrow x$

Update κ, boundary conditions, etc

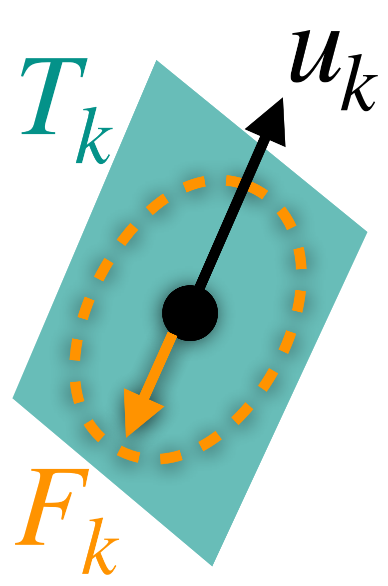
While $\|p\|_\infty / h > \epsilon_d$

FRICTION

Maximum Dissipation Principle (MDP) [Moreau 1973]

With $u_k = T_k(x)^T(x - x^t)$, the local relative sliding displacement at contact k:

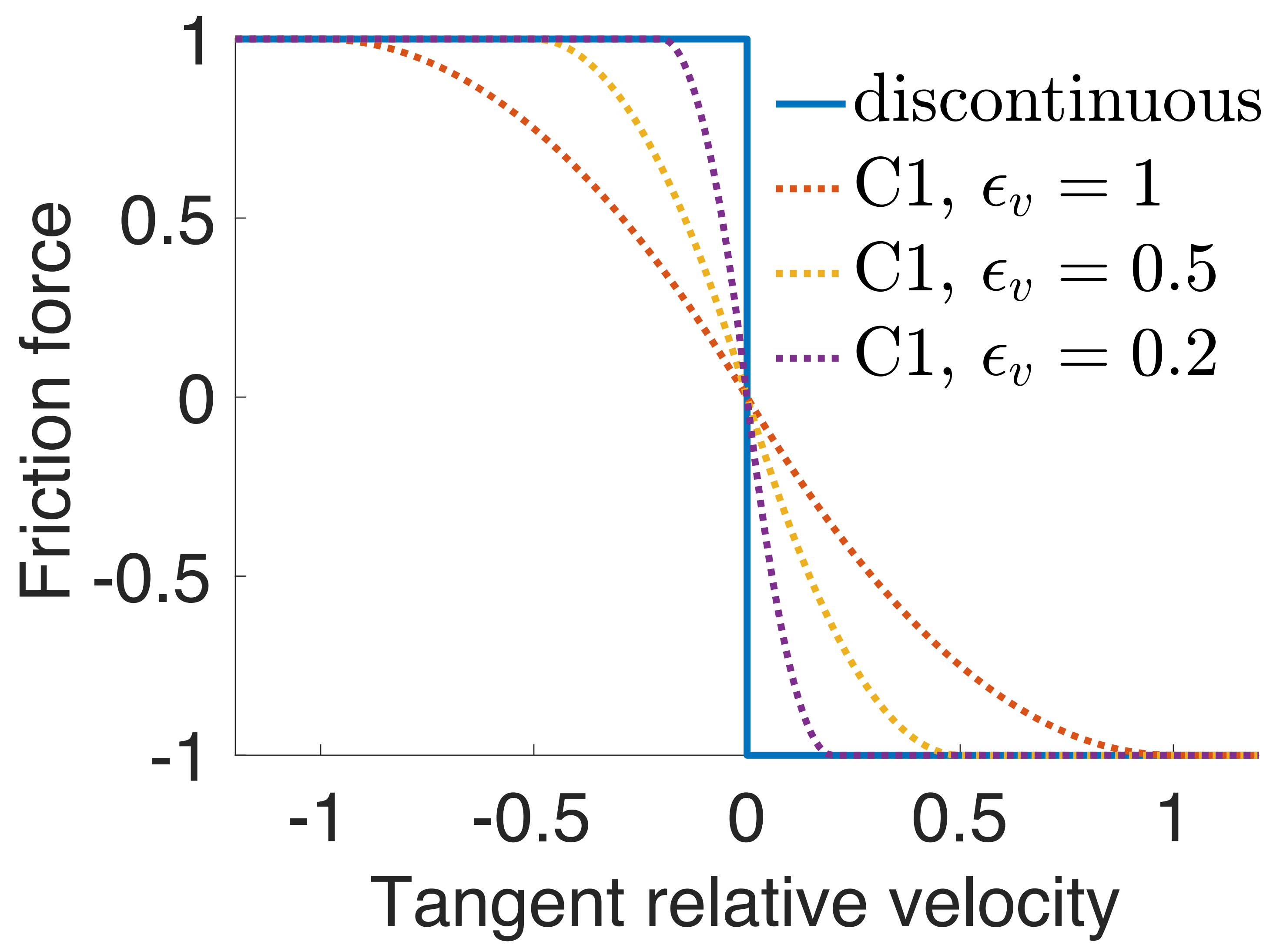
Friction force $F_k = \begin{cases} -\mu\lambda_k T_k u_k / \|u_k\| & \|u_k\| > 0 \\ -\mu\lambda_k T_k f & \|u_k\| = 0 \end{cases}$



λ_k is the normal force magnitude

f can take any vector with $\|f\| \leq 1$

IPC Friction smoothing:



Challenges:

1. When $\|u_k\| = 0$, F_k can have non-smooth jumps
2. Displacement alone is not enough to F_k
3. No well-defined $P_k(x)$ where $-\partial P_k(x)/\partial x = F_k$

IPC lags λ_k and T_k to last time step or last nonlinear solve to approximately define variational friction

$F_k(x) = -\mu\lambda_k(x^j)T_k(x^j)f(x)$